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B→A transitions in the light-cone QCD sum rules with the chiral current*

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Abstract: In this article, we calculate the form-factors of the transitions $B \to a_1(1260)$, $b_1(1235)$ in the leading-order approximation using the light-cone QCD sum rules. In calculations, we choose the chiral current to interpolate the B-meson, which has the outstanding advantage that the twist-3 light-cone distribution amplitudes of the axial-vector mesons make no contributions, and the resulting sum rules for the form-factors suffer from far fewer uncertainties. Then we study the semi-leptonic decays $B \to a_1(1260)l\bar{\nu}_l$, $b_1(1235)l\bar{\nu}_l$ ($l = e$, μ, τ), and make predictions for the differential decay widths and decay widths, which can be compared with the experimental data in the coming future.

Key words: B-meson decay, axial-vector mesons, light-cone QCD sum rules, chiral current

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1 Introduction

The semi-leptonic B-decays are excellent subjects for exploring the CKM matrix elements and CP violations. We can use both the exclusive and inclusive $b \to u$ transitions to study the CKM matrix element V_{ub}. Although the inclusive decays are relatively easier in theoretical studies, the experimental measurements are very difficult. Furthermore, the perturbative QCD calculations in the region near the end-point of the lepton spectrum are less reliable as many resonances appear [1]. We can resort to the exclusive processes, which are easy to measure experimentally, to overcome the difficulty, and study the hadronic matrix elements with some nonperturbative methods, such as the light-cone QCD sum rules and lattice QCD.

The relevant exclusive semi-leptonic decays in determining the CKM matrix element V_{ub} are $B \to \pi l\bar{\nu}_l$, $\rho l\bar{\nu}_l$, $Al\bar{\nu}_l$, where A denotes the axial-vector mesons. The semi-leptonic decays $B \to \pi l\bar{\nu}_l$, $\rho l\bar{\nu}_l$, which were firstly observed by the CLEO collaboration [2], have been extensively studied theoretically. The semi-leptonic decays $B \to Al\bar{\nu}_l$ are expected to be observed at the LHCb, where the $b\bar{b}$ pairs will be copiously produced with the cross section about 500 μb [3]. The $B \to a_1(1260)$ form-factors have been studied with the constituent quark meson (CQM) model [1], the covariant light-front (CLF) approach [4], the improved Isgur-Scora-Grinstein-Wise (ISGW2) model [5], the QCD sum rules (QCDSR) [6], the light-cone QCD sum rules (LCSR) [7, 8] and the perturbative QCD (pQCD) [9], and the values differ from each other remarkably. It is interesting to restudy the semi-leptonic decays $B \to a_1(1260)l\bar{\nu}_l$, $b_1(1235)l\bar{\nu}_l$ with the chiral current using the LCSR [10–14].

In the light-cone QCD sum rules [15], we carry out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$, while the nonperturbative hadronic matrix elements are

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parameterized by the light-cone distribution amplitudes (LCDAs) of increasing twist instead of the vacuum condensates. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side, for example, the form-factors. The twist-2 and twist-3 LCDAs usually enter the sum rules and play an important role in the LCSR for the form-factors. A better understanding of those LCDAs is critical to make the calculations more reliable. In Refs. [16, 17], K. C. Yang proposes model LCDAs for the axial-vector mesons, which are expanded in terms of the Gegenbauer polynomials, and estimates the coefficients of the LCDAs with the QCD sum rules. If we choose the chiral currents, the twist-3 LCDAs make no contributions to the form-factors, the uncertainties originating from the LCDAs can be reduced remarkably [10–14]. In this article, we extend our previous works to study the semi-leptonic decays $B \to a_1(1260)l\bar{\nu}_l$, $b_1(1235)l\bar{\nu}_l$.

The paper is organized as follows: In Section 2, we study the $B \to a_1(1260)$, $b_1(1235)$ form-factors with the chiral current using the LCSR; in Section 3, we present the numerical results of the form-factors, the differential decay widths and decay widths of the $B \to a_1(1260)l\bar{\nu}_l$, $b_1(1235)l\bar{\nu}_l$; Section 4 is reserved for summary and discussion.

2 The light-cone sum rules with the chiral current

We extend our previous work [10–14] to study the $B \to A$ form-factors with the chiral current in the framework of the LCSR. The chiral current warrants that the LCDAs of the same (opposite) chirality remain (disappear).

In the standard model, the semi-leptonic decays $B \to Al\bar{\nu}_l$ take place through the following effective Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} V_{\rm ub}\bar{u}\gamma_\mu(1-\gamma_5)b\bar{l}\gamma^\mu(1-\gamma_5)\nu_l\,,$$

where $V_{\rm ub}$ is the CKM matrix element and $G_{\rm F}$ is the Fermi constant. In calculations, we are confronted with the hadronic matrix elements $\langle A(P,\epsilon^*)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}(P+q)\rangle$ and $\langle A(P,\epsilon^*)|\bar{q}\gamma_\mu b|\bar{B}(P+q)\rangle$, which can be parameterized in terms of the form-factors $A(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $A_3(q^2)$ and $A_0(q^2)$ [4],

$$\begin{aligned}&\langle A(P,\epsilon^*)|\bar{q}\gamma_\mu\gamma_5 b|\bar{B}(P+q)\rangle\\ &= -\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}q^\rho P^\sigma\frac{2{\rm i}A(q^2)}{m_{\rm B}-m_{\rm A}},\end{aligned} \quad (1)$$

$$\begin{aligned}&\langle A(P,\epsilon^*)|\bar{q}\gamma_\mu b|\bar{B}(P+q)\rangle\\ &= -\epsilon^*_\mu(m_{\rm B}-m_{\rm A})A_1(q^2)+\epsilon^*\cdot qP_\mu\frac{2A_2(q^2)}{m_{\rm B}-m_{\rm A}}\\ &\quad+\epsilon^*\cdot qq_\mu\left[\frac{A_2(q^2)}{m_{\rm B}-m_{\rm A}}+2m_{\rm A}\frac{A_3(q^2)-A_0(q^2)}{q^2}\right],\end{aligned} \quad (2)$$

where $A_3(q^2) = \frac{m_{\rm B}-m_{\rm A}}{2m_{\rm A}}A_1(q^2)-\frac{m_{\rm B}+m_{\rm A}}{2m_{\rm A}}A_2(q^2)$, $A_3(0)=A_0(0)$, $\epsilon^{0123}=1$, and the ϵ^*_ν is the polarization vector of the axial-vector meson. The hadronic matrix element $\langle A(P,\epsilon^*)|\bar{q}\gamma_\mu b|\bar{B}(P+q)\rangle$ can be redefined as

$$\begin{aligned}&\langle A(P,\epsilon^*)|\bar{q}\gamma_\mu b|\bar{B}(P+q)\rangle\\ &= -\epsilon^*_\mu(m_{\rm B}-m_{\rm A})A_1(q^2)+\epsilon^*\cdot qP_\mu\frac{2A_+(q^2)}{m_{\rm B}-m_{\rm A}}\\ &\quad+\epsilon^*\cdot qq_\mu\frac{A_+(q^2)+A_-(q^2)}{m_{\rm B}-m_{\rm A}},\end{aligned} \quad (3)$$

where

$$A_2(q^2) = A_+(q^2)\,, \quad (4)$$

$$A_3(q^2) = \frac{m_{\rm B}-m_{\rm A}}{2m_{\rm A}}A_1(q^2)-\frac{m_{\rm B}+m_{\rm A}}{2m_{\rm A}}A_+(q^2), \quad (5)$$

$$\begin{aligned}A_0(q^2) &= \frac{m_{\rm B}-m_{\rm A}}{2m_{\rm A}}A_1(q^2)-\frac{m_{\rm B}+m_{\rm A}}{2m_{\rm A}}A_+(q^2)\\ &\quad-\frac{q^2}{2m_{\rm A}(m_{\rm B}-m_{\rm A})}A_-(q^2).\end{aligned} \quad (6)$$

In the following, we write the correlation function with a chiral current,

$$\begin{aligned}\Pi_\mu(P,q) &= {\rm i}\int{\rm d}^4x{\rm e}^{{\rm i}qx}\langle A(P,\perp)|T\{\bar{q}_1(x)\gamma_\mu(1-\gamma_5)b(x),\\ &\quad\bar{b}(0){\rm i}(1+\gamma_5)q_2(0)\}|0\rangle\,,\end{aligned} \quad (7)$$

where $P^2=m^2_{\rm A}$. We study the relevant form-factors with the transversely polarized axial-vector mesons [8], and obtain simple relations among the form-factors as the corresponding ones in the $B \to V$ transitions.

According to the quark-hadron duality [18] and unitarity, we can insert a complete set of intermediate states with the same quantum numbers as the current operator $\bar{b}(0){\rm i}(1-\gamma_5)q_1(0)$ in the correlation function to obtain the hadronic representation. After isolating the ground state contribution from the pole term of the pseudoscalar B meson, we obtain the result,

$$\Pi_\mu(P,q) = \frac{\langle A(P,\perp)|\bar{q}_1\gamma_\mu(1-\gamma_5)b|\bar{B}(P+q)\rangle\langle\bar{B}(P+q)|\bar{b}i\gamma_5 q_2|0\rangle}{m_{\rm B}^2-(P+q)^2}$$
$$+\sum_{\rm h}\frac{\langle A(p,\perp)|\bar{q}_1\gamma_\mu(1-\gamma_5)b|\bar{B}^{\rm h}(P+q)\rangle\langle\bar{B}^{\rm h}(P+q)|\bar{b}{\rm i}(1+\gamma_5)q_2|0\rangle}{m_{\rm B^h}^2-(P+q)^2}. \quad (8)$$

It should be stressed that there are contributions from the scalar B-meson, the pseudoscalar B-meson, and their resonances [19], we can attribute the (ground state) scalar B-meson into the higher resonances and continuum states $|B^{\rm h}\rangle$. Taking into account the definition of the B-meson decay constant $\langle\bar{B}|\bar{b}i\gamma_5 q_2|0\rangle = \frac{f_{\rm B}m_{\rm B}^2}{m_{\rm q_2}+m_{\rm b}}$, we can obtain the hadronic representation,

$$\Pi_\mu(P,q) = \left[-(m_{\rm B}-m_{\rm A})A_1\epsilon^*_{\perp\mu}+\left(\frac{A_2(q^2)}{m_{\rm B}-m_{\rm A}}+2m_{\rm A}\frac{A_3(q^2)-A_0(q^2)}{q^2}\right)\epsilon^*_\perp\cdot q q_\mu\right.$$
$$\left.+\frac{2A_2(q^2)}{m_{\rm B}-m_{\rm A}}\epsilon^*_\perp\cdot qP_\mu+\frac{2iA(q^2)}{m_{\rm B}-m_{\rm A}}\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}_\perp q^\rho P^\sigma\right]\frac{1}{m_{\rm B}^2-(P+q)^2}\frac{m_{\rm B}^2 f_{\rm B}}{m_{\rm q_2}+m_{\rm b}}+\frac{1}{\pi}\int_{s_0}^\infty {\rm d}s\frac{\rho^{\rm h}_\mu(s)}{s-(P+q)^2}. \quad (9)$$

The spectral density $\rho^{\rm h}_\mu(s)$ can be approximated as

$$\rho^{\rm h}_\mu(s) = \rho^{\rm QCD}_\mu(s)\theta(s-s_0), \quad (10)$$

by invoking the quark-hadron duality ansatz, where the $\rho^{\rm QCD}_\mu(s)$ is the perturbative QCD spectral density. Here the threshold s_0 is near the squared mass of the lowest scalar B-meson.

Now, we briefly outline the operator product expansion for the correlation function in perturbative QCD. The calculations are performed at the large space-like momentum region $(P+q)^2 \ll m_{\rm b}^2$ and $0\leqslant q^2<(m_{\rm b}-m_{\rm A})^2-2(m_{\rm b}-m_{\rm A})\Lambda_{\rm QCD}$ [20], or more specific, $0\leqslant q^2<12\ {\rm GeV}^2$ for the axial-vector mesons $\rm a_1(1260)$ and $\rm b_1(1235)$. We contract the b-quark fields in the correlation function, substitute it with the free b-quark propagator, and obtain the result,

$$\Pi_\mu(P,q) = {\rm i}\int\frac{{\rm d}^4k{\rm d}^4x}{(2\pi)^4}\frac{{\rm e}^{{\rm i}(q-k)x}}{m_{\rm b}^2-k^2}{\rm Tr}\left\{[\gamma_\mu(1-\gamma_5)(\not{k}+m_{\rm b})(1+\gamma_5)]_{\delta\alpha}\langle A(P,\perp)|\bar{q}_{1\delta}(x)q_{2\alpha}(0)|0\rangle\right\}. \quad (11)$$

The light-cone distribution amplitudes of the axial-vector mesons are defined by [8]

$$\langle A(P,\lambda)|\bar{q}_{1\,\delta}(x)\,q_{2\,\alpha}(0)|0\rangle = -\frac{\rm i}{4}\int_0^1{\rm d}u\,{\rm e}^{{\rm i}u\,Px}\left\{f_{\rm A}m_{\rm A}\left[\not{P}\gamma_5\frac{\epsilon^*_{(\lambda)}x}{Px}\left(\Phi_\parallel(u)+\frac{m_{\rm A}^2x^2}{16}\boldsymbol{A}^2_\parallel(u)\right)\right.\right.$$
$$+\left(\not{\epsilon}^*-\not{P}\frac{\epsilon^*_{(\lambda)}z}{Px}\right)\gamma_5 g^{(\rm a)}_\perp(u)-\not{x}\gamma_5\frac{\epsilon^*_{(\lambda)}x}{2(Px)^2}m_{\rm A}^2\bar{g}_3(u)+\epsilon_{\mu\nu\rho\sigma}\,\epsilon^{*}_{(\lambda)}{}^{\mu}P^\rho x^\sigma\gamma^\mu\,\left.\frac{g^{(v)}_\perp(u)}{4}\right]$$
$$+f_{\rm A}^\perp\left[\frac{1}{2}\left(\not{P}\,\not{\epsilon}^*_{(\lambda)}-\not{\epsilon}^*_{(\lambda)}\,\not{P}\right)\gamma_5\left(\Phi_\perp(u)+\frac{m_{\rm A}^2x^2}{16}\boldsymbol{A}^2_\perp(u)\right)\right.$$
$$-\frac{1}{2}\left(\not{P}\,\not{x}-\not{x}\,\not{P}\right)\gamma_5\frac{\epsilon^*_{(\lambda)}x}{(Px)^2}m_{\rm A}^2\bar{h}^{(\rm t)}_\parallel(u)-\frac{1}{4}\left(\not{\epsilon}^*_{(\lambda)}\,\not{x}-\not{x}\,\not{\epsilon}^*_{(\lambda)}\right)\gamma_5\frac{m_{\rm A}^2}{Px}\bar{h}_3(u)$$
$$\left.\left.+{\rm i}(\epsilon^*_{(\lambda)}x)m_{\rm A}^2\gamma_5\frac{h^{(\rm p)}_\parallel(u)}{2}\right]\right\}_{\alpha\delta}, \quad (12)$$

where u is the fraction of the light-cone momentum of the axial-vector meson carried by the quark, and $\bar{u}=1-u$. After carrying out the integrals of x and k, we obtain the following result,

$$\Pi_\mu(P,q) = {\rm i}\int{\rm d}u{\rm Tr}\left\{[\gamma_\mu(1-\gamma_5)(\not{k}+m_{\rm b})(1+\gamma_5)]_{\delta\alpha}M^{\rm A}_{\perp\alpha\delta}\right\}\left.\frac{1}{m_{\rm b}^2-k^2}\right|_{k=q+uP}, \quad (13)$$

where the transverse projectors, which project the transverse components of the axial-vector meson, are given by [8],

$$M_{\perp}^{A} = \mathrm{i}\frac{f_{\mathrm{A}}^{\perp}}{4}E\Bigg\{ \not\epsilon_{\perp}^{*(\lambda)} \not n_{-}\gamma_5\,\Phi_{\perp}(u) - \frac{f_{\mathrm{A}}}{f_{\mathrm{A}}^{\perp}}\frac{m_{\mathrm{A}}}{E} \times\Bigg[\not\epsilon_{\perp}^{*(\lambda)}\gamma_5\, g_{\perp}^{(\mathrm{a})}(u) - E\int_0^u \mathrm{d}v\,(\Phi_{\parallel}(v) - g_{\perp}^{(a)}(v))\,\not n_{-}\gamma_5\,\epsilon_{\perp\mu}^{*(\lambda)}\frac{\partial}{\partial k_{\perp\mu}} + \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\,\gamma^{\mu}\epsilon_{\perp}^{*(\lambda)\nu} n_{-}^{\rho} \times\left(n_{+}^{\sigma}\,\frac{g_{\perp}^{(v)\prime}(u)}{8} - E\,\frac{g_{\perp}^{(v)}(u)}{4}\,\frac{\partial}{\partial k_{\perp\sigma}}\right)\Bigg]\Bigg|_{k=uP} + \mathcal{O}\left(\frac{m_{\mathrm{A}}^2}{E^2}\right)\Bigg\}, \tag{14}$$

here we have taken $P^{\mu} = En_{-}^{\mu} + m_{\mathrm{A}}^2 n_{+}^{\mu}/4E \approx En_{-}^{\mu}$ and the exactly longitudinal and transverse polarization vectors of the axial-vector meson, independent of the coordinate variable x, are defined as

$$\epsilon_{\perp}^{*(L)\mu} = \frac{E}{m_{\mathrm{A}}}\left[\left(1-\frac{m_{\mathrm{A}}^2}{4E^2}\right)n_{-}^{\mu} - \frac{m_{\mathrm{A}}^2}{4E^2}n_{+}^{\mu}\right], \tag{15}$$

$$\epsilon_{\perp}^{*(\lambda)\mu} = \epsilon^{*(\lambda)\mu} - \frac{\epsilon^{*(\lambda)}n_{+}}{2}n_{-}^{\mu} - \frac{\epsilon^{*(\lambda)}n_{-}}{2}n_{+}^{\mu},\ (\lambda=\pm). \tag{16}$$

We carry out the trace in Eq. (13), and observe that only the leading-twist LCDAs $\Phi_{\perp}(u)$ contribute,

$$\Pi_{\mu}(P,q) = f_{\mathrm{A}}^{\perp}\int_0^1 \mathrm{d}u\frac{\Phi_{\perp}(u)}{m_{\mathrm{b}}^2-(q+uP)^2}[2P\cdot(q+uP)\epsilon_{\perp\mu}^{*} - 2(\epsilon_{\perp}^{*}\cdot q)P_{\mu} - 2\mathrm{i}\epsilon_{\mu\nu\rho\sigma}\epsilon_{\perp}^{*\nu}q^{\rho}P^{\sigma}].$$

With reference to the LCDAs and decay constants of the axial-vector mesons, a few words should be given. In the flavor $SU(3)$ symmetry limit, due to G-parity the twist-2 LCDA $\Phi_{\perp}(u)$ obeys the normalization

$$\int_0^1 \mathrm{d}u\Phi_{\perp}(u) = 0 \tag{17}$$

for the ${}^3P_1$ meson and

$$\int_0^1 \mathrm{d}u\Phi_{\perp}(u) = 1 \tag{18}$$

for the ${}^1P_1$ meson. Based on the conformal symmetry of the QCD Lagrangian, $\Phi_{\perp}(u,\mu)$ can be expanded in terms of a series of Gegenbauer polynomials $C_{\mathrm{m}}^{3/2}(\xi)$ with increasing conformal spin [16, 17],

$$\Phi_{\perp}(u,\mu) = 6u\bar{u}\Big[a_0^{\perp}(\mu) + a_1^{\perp}(\mu)\,C_1^{3/2}(\xi) + a_2^{\perp}(\mu)\,C_2^{3/2}(\xi) + \cdots\Big], \tag{19}$$

where $\xi = 2u-1$, the values of the coefficients $a_{\mathrm{m}}^{\perp}(\mu)$ at the energy scale $\mu = 1$ GeV are $a_0^{\perp} = a_2^{\perp} = 0$, $a_1^{\perp} = -1.04\pm0.34$ for the $\mathrm{a}_1(1260)$ and $a_0^{\perp} = 1$, $a_1^{\perp} = 0$, $a_2^{\perp} = 0.03\pm0.19$ for the $\mathrm{b}_1(1235)$, respectively. We plot the LCDAs $\Phi_{\perp}(u,\mu)$ of the axial-vector mesons $\mathrm{a}_1(1260)$ and $\mathrm{b}_1(1235)$ at the energy scale $\mu = 1.0$ GeV in Fig. 1. The G-parity conserving decay constants of the axial-vector mesons are defined by [8]

$$\langle 1^3P_1(P,\lambda)|\bar{q}_1\gamma_{\mu}\gamma_5 q_2|0\rangle = \mathrm{i}f_{{}^3P_1}m_{{}^3P_1}\epsilon_{\mu}^{*(\lambda)}, \tag{20}$$

$$\langle 1^1P_1(P,\lambda)|\bar{q}_1\sigma_{\mu\nu}\gamma_5 q_2|0\rangle = f_{{}^1P_1}^{\perp}(\epsilon_{\mu}^{*(\lambda)}P_{\nu} - \epsilon_{\nu}^{*(\lambda)}P_{\mu}), \tag{21}$$

where the decay constant $f_{{}^3P_1}$ ($f_{{}^1P_1}^{\perp}$) is scale independent (dependent). The G-parity violating decay constants are defined by $f_{{}^3P_1}^{\perp} = f_{{}^3P_1}$ and $f_{{}^1P_1} = f_{{}^1P_1}^{\perp}$ at the energy scale $\mu = 1$ GeV.

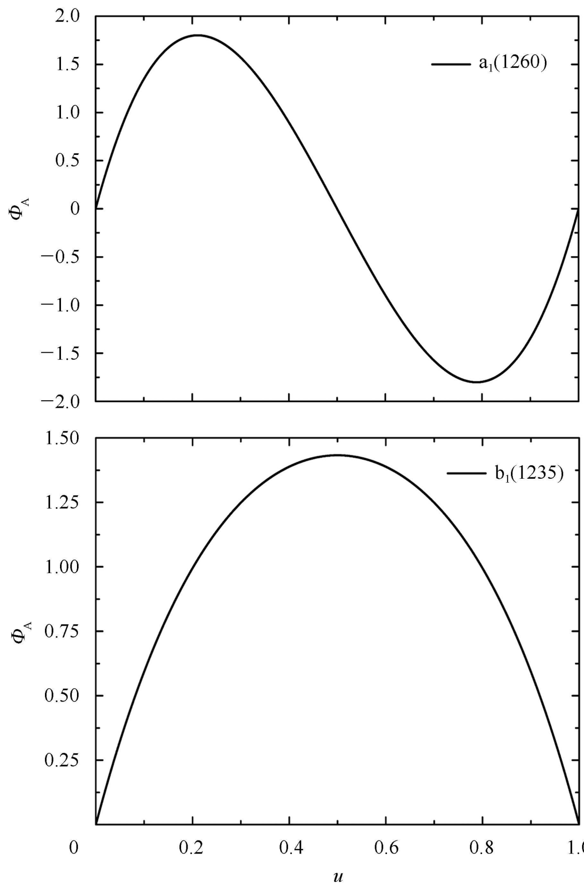

Fig. 1. The twist-2 LCDAs $\Phi_{\perp}(u,\mu)$ of the axial-vector mesons $\mathrm{a}_1(1260)$ and $\mathrm{b}_1(1235)$ at the energy scale $\mu = 1.0$ GeV.

After matching with the hadronic representation and performing the Borel transformation with respect to the variable $(P+q)^2$, we obtain the sum rules for the form-factors:

$$A(q^2) = -\frac{m_{q_2}+m_b}{m_B^2 f_B}(m_B-m_A)f_A^{\perp}\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}e^{FF}, \tag{22}$$

$$A_1(q^2) = -\frac{m_{q_2}+m_b}{m_B^2 f_B}\frac{f_A^{\perp}}{m_B-m_A}\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}\frac{m_b^2-q^2+u^2P^2}{u}e^{FF}, \tag{23}$$

$$A_2(q^2) = -\frac{m_{q_2}+m_b}{m_B^2 f_B}(m_B-m_A)f_A^{\perp}\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}e^{FF}, \tag{24}$$

$$\begin{aligned}A_3(q^2) = &-\frac{m_{q_2}+m_b}{m_B^2 f_B}\frac{f_A^{\perp}}{2m_A}\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}\frac{m_b^2-q^2+u^2P^2}{u}e^{FF}\\ &+\frac{m_{q_2}+m_b}{m_B^2 f_B}\frac{f_A^{\perp}}{2m_A}(m_B^2-m_A^2)\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}e^{FF},\end{aligned} \tag{25}$$

$$\begin{aligned}A_0(q^2) = &-\frac{m_{q_2}+m_b}{m_B^2 f_B}\frac{f_A^{\perp}}{2m_A}\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}\frac{m_b^2-q^2+u^2P^2}{u}e^{FF}\\ &+\frac{m_{q_2}+m_b}{m_B^2 f_B}\frac{f_A^{\perp}}{2m_A}(m_B^2-m_A^2)\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}e^{FF}+\frac{m_{q_2}+m_b}{m_B^2 f_B}\frac{q^2 f_A^{\perp}}{2m_A}\int_{\Delta}^{1}du\frac{\Phi_{\perp}(u)}{u}e^{FF},\end{aligned} \tag{26}$$

where

$$\Delta = \frac{1}{2m_A^2}\left[\sqrt{(s_0-m_A^2+Q^2)^2+4(m_b^2+Q^2)m_A^2}-(s_0-m_A^2+Q^2)\right],$$

$$FF = -\frac{1}{uM^2}[m_b^2+u(1-u)m_A^2+(1-u)Q^2]+\frac{m_B^2}{M^2},$$

M^2 is the Borel parameter and $Q^2=-q^2$. The form factors $A_+(q^2)$ and $A_-(q^2)$ can be obtained from the relations (4), (5) and (6).

It is surprising that the expressions of the form-factors are very simple, and only the leading twist LCDA $\Phi_{\perp}(u,\mu)$ appears in the final sum rules. The form-factors A_+ and A_- have the following simple relations,

$$A_-(q^2) = -A_+(q^2), \tag{27}$$

$$A(q^2) = A_+(q^2). \tag{28}$$

Similar relations can be obtained for the $B\to V$ form-factors if we use the chiral current in the LCSR [21]. The simple relations obtained for the $B\to S$, V, P, A form-factors in Refs. [14, 21, 22] and the present work, up to the hard-exchange corrections, are consistent with the predictions of the soft collinear effective theory [23].

3 Numerical results and discussions

The input parameters for the semi-leptonic decays $B\to a_1(1260)l\bar{\nu}_l$, $b_1(1235)l\bar{\nu}_l$ are taken as [16, 17, 24–26]:

$$\begin{aligned}G_F &= 1.166\times10^{-2}\text{GeV}^{-2},\\ |V_{ub}| &= 3.96^{+0.09}_{-0.09}\times10^{-3},\\ m_u(1\text{ GeV}) &= 2.8\text{ MeV},\\ m_d(1\text{ GeV}) &= 6.8\text{ MeV},\\ m_b &= (4.8\pm0.1)\text{ GeV},\\ m_{e,\mu} &= 0\text{ MeV},\\ m_\tau &= 1776.82\text{ MeV},\\ m_{a_1(1260)} &= 1.23\pm0.06\text{ GeV},\\ m_{b_1(1235)} &= 1.21\pm0.07\text{ GeV},\\ f_{a_1(1260)}^{\perp} &= 0.238\pm0.010\text{ GeV},\\ f_{b_1(1235)}^{\perp} &= 0.180\pm0.008\text{ GeV},\\ m_B &= 5.279\text{ GeV},\\ f_B &= (0.19\pm0.02)\text{ GeV}.\end{aligned} \tag{29}$$

We take into account the binding energy difference between the scalar and pseudoscalar B mesons from the QCD sum rules in the heavy quark effective theory [27], and choose the suitable threshold parameter s_0 to avoid contamination from the scalar B-meson [19], and obtain the value $s_0 = (32\pm1)$ GeV^2, which is smaller than the ones used in the conventional QCD sum rules to reproduce the experimental values of the pseudoscalar B-meson. On the other hand, we can estimate the mass of the scalar B_0 meson with the scalar current $\bar{\mathrm{b}}(x)\mathrm{q}(x)$ using the conventional two-point QCD sum rules, and obtain the value $M_{\mathrm{B}_0} = 5.74$ GeV. The strong decays $\mathrm{B}_0 \to \mathrm{B}\pi$ are Okubo-Zweig-Iizuka allowed; we can take the value $g_{\mathrm{B}_0\mathrm{B}\pi} = 21$ GeV or $g_{\mathrm{B}_0\mathrm{B}\pi} \approx g_{\mathrm{B}_{s0}\mathrm{BK}} = 20$ GeV from the light-cone QCD sum rules [28, 29], and obtain the decay width $\Gamma_{\mathrm{B}_0\to\mathrm{B}\pi} \approx 0.2$ GeV, which is supposed to saturate the width of the scalar B_0 meson. $\left(M_{\mathrm{B}_0} - \frac{\Gamma_{\mathrm{B}_0}}{2}\right)^2 \approx 32$ GeV^2, the contaminations from the scalar B_0 meson and thereafter the $\mathrm{B}\pi$ continuum states are very small, and can be neglected. Also, it is possible to determine the threshold parameters in other approaches, among which the scenario suggested in Ref. [30] is more effective. The Borel parameter M^2 shared by all the QCD sum rules in the pseudoscalar channel is $M^2 = (10-15)$ GeV^2. In this interval, the higher resonances and continuum states contribute less than 20% and the uncertainties originating from the Borel parameter M^2 are about (0.7–1.5)%.

The values of the form-factors $\mathrm{B} \to \mathrm{a}_1(1260)$, $\mathrm{b}_1(1235)$ at zero momentum transfer are rather stable with variations of the Borel parameter M^2. In Fig. 2, we present numerical results for the $A_1(0)$ with the central values of the input parameters as an example.

The LCDAs of the axial-vector mesons $^3P_1$ and $^1P_1$ have been evaluated using the QCD sum rules [16, 17]. Owing to the G-parity, the chiral-even two-particle LCDAs of the $^3P_1$ ($^1P_1$) mesons are symmetric (antisymmetric) under the exchange of the quark and antiquark momentum fractions in the flavor $SU(3)$ symmetry limit. For the chiral-odd LCDAs, the situation is opposite. We show the numerical values of the LCDAs $\Phi_\perp(u,\mu)$ of the axial-vector mesons $\mathrm{a}_1(1260)$ and $\mathrm{b}_1(1235)$ at the energy scale $\mu = 1.0$ GeV explicitly in Fig. 1. The integral interval in the sum rules is about 0.7–1, and the decay constants of the $\mathrm{a}_1(1260)$ and $\mathrm{b}_1(1235)$ mesons have the same sign, therefore the form-factors A_1, A_2, A_0, A for the $\mathrm{B} \to \mathrm{a}_1(1260)$, $\mathrm{b}_1(1235)$ transitions have opposite signs, see Table 1. The uncertainties of the LCDAs $\Phi_\perp(u)$ and b-quark mass m_b both result in errors for the form-factors, which are shown as the first and second errors respectively in Table 1.

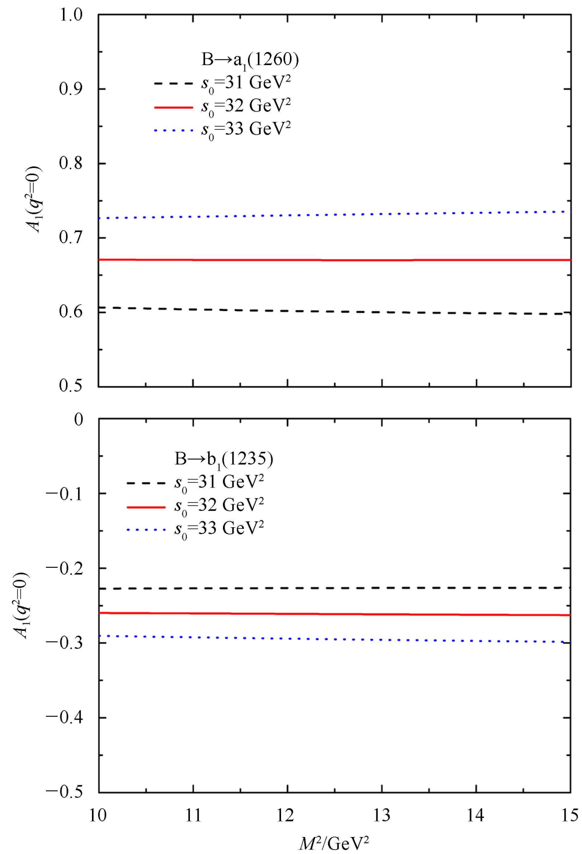

Fig. 2. The form-factor $A_1(0)$ with variation of the Borel parameter M^2 at the energy scale $\mu = 1.0$ GeV. The threshold parameter $s_0 = 31, 32, 33$ GeV^2.

Here we will take a short digression to discuss the LCDAs of the axial-vector mesons. The distribution amplitudes of an energetic light hadron moving nearly on the light-cone can be described by a set of LCDAs, which are governed by the special collinear subgroup $SL(2,\mathbb{R})$ of the conformal group and characterized by the conformal spin j [16, 17]. The large widths of the axial-vector mesons reflect the fact that their lives are short and they are not stable (for example, the width of the $\mathrm{a}_1(1260)$ is mainly determined by the Okubo-Zweig-Iizuka allowed strong decay $\mathrm{a}_1(1260) \to \rho(770)\pi$, as both the phase space and the coupling constant are large), and do not mean that they cannot move nearly on the light-cone and the momentum fractions carried by the u and d quarks are changed. If we take into account the widths of the axial-vector mesons in estimating the coefficients of the LCDAs using the QCD sum rules, the net effects of the widths can lead to a numerical factor, which can be absorbed in the decay constants.

Table 1. The $B \to a_1(1260)$, $b_1(1235)$ form-factors at zero momentum transfer, where the first and second errors originate from the uncertainties of the LCDA $\Phi_\perp(u)$ and the b-quark mass m_b, respectively. In calculations, we have taken the values $M^2 = 12$ GeV^2 and $s_0 = 32$ GeV^2.

$B \to A$	$A_1(0)$	$A_2(0)$	$A_0(0)$	$A(0)$
$B \to a_1(1260)$	$0.67\pm0.11\pm0.06$	$0.38\pm0.07\pm0.03$	$0.09\pm0.02\pm0.01$	$0.38\pm0.07\pm0.03$
$B \to b_1(1235)$	$-0.26\pm0.05\pm0.03$	$-0.15\pm0.03\pm0.02$	$-0.04\pm0.01\pm0.01$	$-0.15\pm0.03\pm0.02$

Table 2. The $B \to a_1(1260)$ form-factors $A_1(0), A_2(0), A_0(0)$ and $A(0)$ from different theoretical approaches.

	CQM [1]	CLF [4]	ISGW2 [5]	QCDSR [6]	LCSR [7]	LCSR [8]	pQCD [9]	this work
$A_1(0)$	2.10	0.59	0.87	0.68	0.67	0.60	0.43	0.67
$A_2(0)$	0.21	0.11	−0.03	0.33	0.31	0.26	0.13	0.38
$A_0(0)$	1.20	0.13	1.01	0.23	0.29	0.30	0.34	0.09
$A(0)$	0.06	0.16	0.13	0.42	0.41	0.30	0.26	0.38

We present the central values of the $B \to a_1(1260)$ form-factors $A_1(0)$, $A_2(0)$, $A_0(0)$, $A(0)$ in Table 2 compared with the predictions from the CQM model [1], CLF approach [4], ISGW2 model [5], QCDSR [6], LCSR [7, 8], and pQCD [9]. From the table, we can see that the present predictions are consistent with the ones from QCDSR [6] and LCSR [7, 8] except for the $A_0(0)$, and differ from the values from other theoretical approaches remarkably. It has been pointed out by K.C.Yang in Ref.[8] that the higher twist effects might be negligible, while we exclude all contributions from the higher twist LCDAs by using the chiral current in the correlation function. In addition, the form-factors A_1, A_2, A_0, A are not independent; they are related with the formulae like (27) and (28).

In Fig. 3, we plot the q^2 dependence of the form-factors $A_1(q^2)$, $A_2(q^2)$, $A_0(q^2)$, $A(q^2)$ for the transitions $B \to a_1(1260)$, $b_1(1235)$ in the region $0 \leqslant q^2 < 12$ GeV^2, which is similar to the accessible region $0 \leqslant q^2 < 10$ GeV^2 in the QCD sum rules [6], beyond that values the nonperturbative contributions become large and the operator product expansion breaks down. The pole models are merely suitable for describing those form-factors with momentum transfers q^2 near the squared pole masses $m^2_{\rm pole}$. In the present $B \to A$ case, the $m^2_{\rm pole}$ are far away from their kinematical regions, we do not extrapolate the form-factors from small q^2 to large ones with the pole models.

Now, we study the differential decay widths of the $B \to A$ semi-leptonic decays, which can be written as [6, 9]

$$\frac{{\rm d}\Gamma_{\rm L}(\bar{\rm B}\to {\rm A}l\bar{\nu}_l)}{{\rm d}q^2} = \left(\frac{q^2-m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda(m_{\rm B}^2,m_{\rm A}^2,q^2)}G_{\rm F}^2V_{\rm ub}^2}{384m_{\rm B}^3\pi^3}\times\frac{1}{q^2}\Bigg\{3m_l^2\lambda(m_{\rm B}^2,m_{\rm A}^2,q^2)V_0^2(q^2) + (m_l^2+2q^2)\left|\frac{1}{2m_{\rm A}}\left[(m_{\rm B}^2-m_{\rm A}^2-q^2)(m_{\rm B}-m_{\rm A})V_1(q^2)-\frac{\lambda(m_{\rm B}^2,m_{\rm A}^2,q^2)}{m_{\rm B}-m_{\rm A}}V_2(q^2)\right]\right|^2\Bigg\}, \quad (30)$$

$$\frac{{\rm d}\Gamma_\pm(\bar{B}\to {\rm A}l\bar{\nu}_l)}{{\rm d}q^2} = \left(\frac{q^2-m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda(m_{\rm B}^2,m_{\rm A}^2,q^2)}G_{\rm F}^2V_{\rm ub}^2}{384m_{\rm B}^3\pi^3} \times\left\{(m_l^2+2q^2)\lambda(m_{\rm B}^2,m_{\rm A}^2,q^2)\left|\frac{A(q^2)}{m_{\rm B}-m_{\rm A}}\mp\frac{(m_{\rm B}-m_{\rm A})V_1(q^2)}{\sqrt{\lambda(m_{\rm B}^2,m_{\rm A}^2,q^2)}}\right|^2\right\}, \quad (31)$$

where $\lambda(m_{\rm B}^2,\ m_{\rm A}^2,\ q^2) = (m_{\rm B}^2+m_{\rm A}^2-q^2)^2-4m_{\rm B}^2m_{\rm A}^2$, and L, $+$, $-$ denote the helicities of the axial-vector mesons.

We plot the differential decays widths of the $B \to a_1(1260)l\bar{\nu}_l$, $b_1(1235)l\bar{\nu}_l$ in the effective regions $m_l^2 \leqslant q^2 \leqslant (m_{\rm B}-m_{\rm A})^2$ in Figs. 4–5, where we take $m_e = m_\mu = 0$. We can integrate the differential decay widths over the variable q^2, and obtain the decay widths, which satisfy the relation $\Gamma_- > \Gamma_{\rm L} \gg \Gamma_+$, and are consistent with the results of Ref. [8].

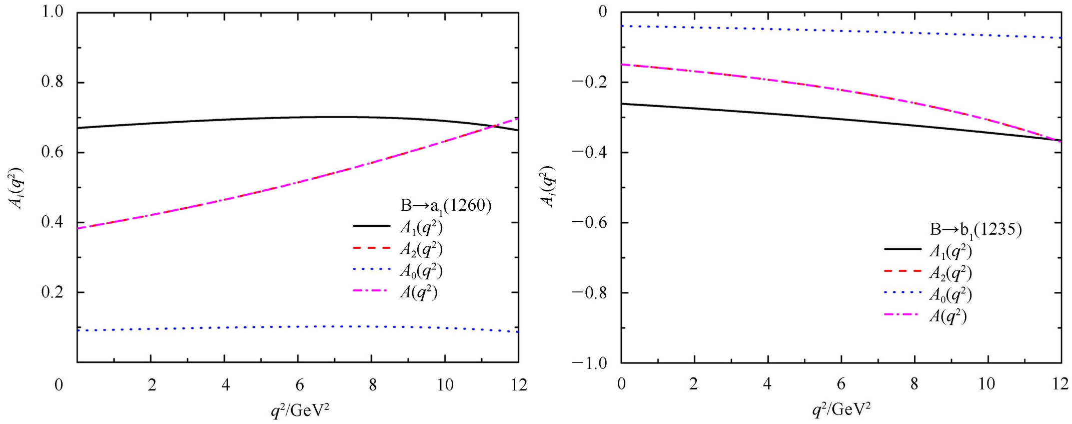

Fig. 3. The $\mathrm{B}\to\mathrm{a}_1(1260)$, $\mathrm{b}_1(1235)$ form-factors $A_1(q^2)$, $A_2(q^2)$ and $A_0(q^2)$ with the momentum transfer q^2, where we have taken the values $M^2=12\ \mathrm{GeV}^2$, $s_0=32\ \mathrm{GeV}^2$ and $A_2=A$.

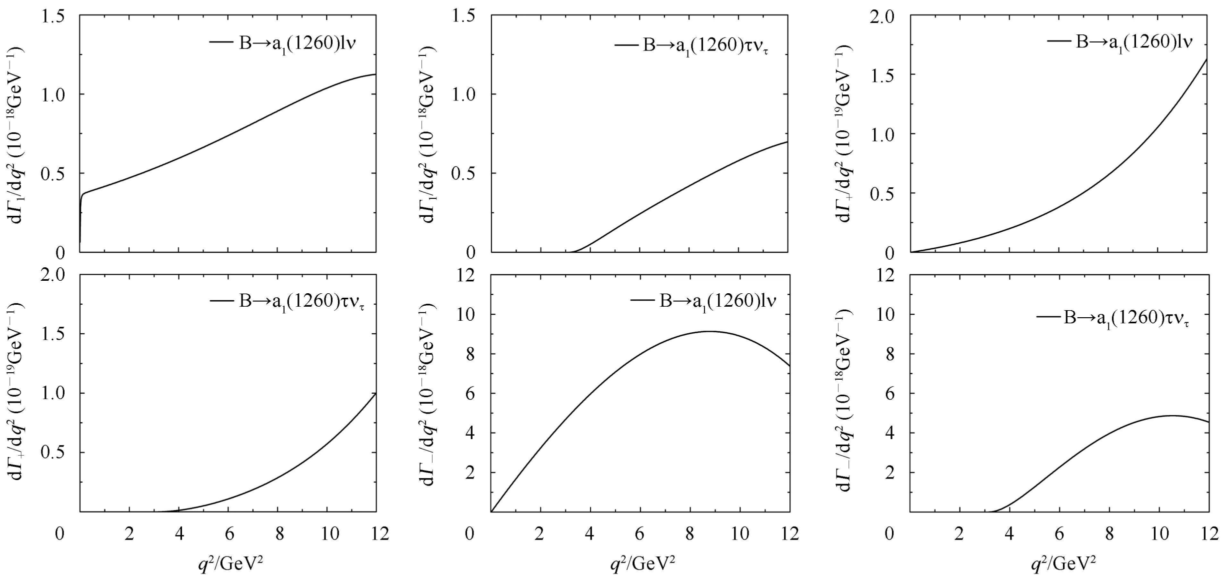

Fig. 4. Differential decay widths of the $\mathrm{B}\to\mathrm{a}_1(1260)\mathrm{l}\bar{\nu}_\mathrm{l}$ as functions of q^2. Here $\mathrm{l}=\mathrm{e},\ \mu$ in the left diagram.

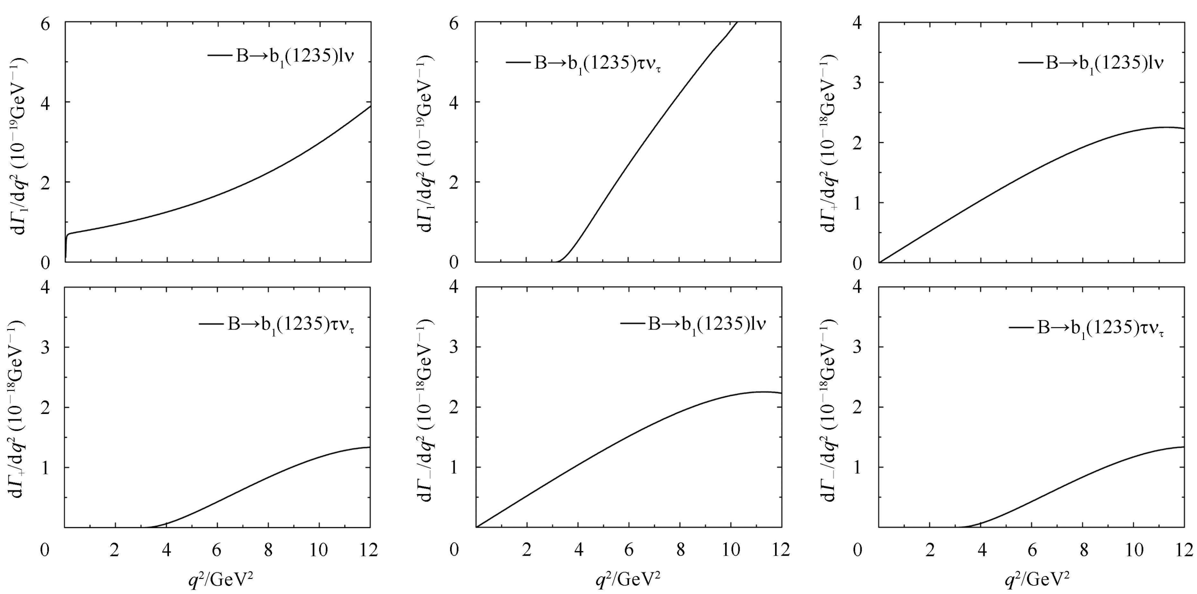

Fig. 5. Differential decay widths of the $\mathrm{B}\to\mathrm{b}_1(1235)\mathrm{l}\bar{\nu}_\mathrm{l}$ as functions of q^2. Here $\mathrm{l}=\mathrm{e},\ \mu$ in the left diagram.

4 Summary and discussion

In this article, we calculate the $B \to a_1(1260)$, $b_1(1235)$ form-factors in the accessible region $0 \leqslant q^2 < 12$ GeV^2 with the light-cone QCD sum rules at the leading order approximation, then study the differential decay widths and decay widths of the semi-leptonic decays $B \to a_1(1260) l\bar{\nu}_l$, $b_1(1235) l\bar{\nu}_l$.

(1) In this paper, we choose the chiral current to interpolate the B-meson, and observe that only the leading-twist LCDAs of the axial-vector mesons contribute to the form-factors after taking account of the transversely polarization of the axial-vector mesons. We avoid contributions from the twist-3 LCDAs, which have the most uncertainty in the form-factors, by using the chiral current. The uncertainties originating from the LCDAs are reduced remarkably.

(2) Owing to the G-parity of the axial-vector mesons $^3P_1$ and $^1P_1$, the form-factors of the $B \to a_1(1260)$, $b_1(1235)$ transitions have opposite signs. There exist relations among the $B \to A$ transition form-factors which are in accordance with the prediction of the soft collinear effective theory [23].

(3) The present predictions of the differential decay widths and decay widths of the semi-leptonic decays $B \to a_1(1260) l\bar{\nu}_l$, $b_1(1235) l\bar{\nu}_l$ can be compared with the experimental data at the KEK-B and LHCb in the future. If the perturbative $\mathcal{O}(\alpha_s)$ corrections are taken into account, the predictions may be improved, however, the improvements are not expected to be large considering the corresponding calculations of the $B \to V$ form-factors.

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