Electromagnetic transition properties of $\Delta \rightarrow N\gamma$ in a hypercentral scheme

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Abstract: The electromagnetic transition properties of the decuplet to octet baryon ($\Delta \rightarrow N\gamma$) is studied within the framework of a hypercentral quark model. The confinement potential is assumed as a hypercentral coloumb plus linear potential. The transition magnetic moment and transition amplitude f_{M_1} for the $\Delta \rightarrow N\gamma$ are in agreement with other theoretical predictions. The present result of the radiative decay width is found to be in excellent agreement with the experimental values reported by the particle data group over other theoretical model predictions.

Key words: light flavour baryons, transition magnetic moment, radiative decay width

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1 Introduction

The Δ (1232) and N (939) are the two lowest decuplet and octet baryon states in the low flavour (u, d) sector. Their descriptions and properties thus play a very important role in the understanding of strong interaction. There are only two decay channels for a Δ baryon, the first one is $\Delta \rightarrow N\pi$ and the second one is $\Delta \rightarrow N\gamma$. The first decay channel is more dominant (almost 100%) while the branching ratio for the second one is less than 1% [1]. Due to this very small branching ratio the electromagnetic transition of $\Delta \rightarrow N\gamma$ has been the subject of intense study, starting in the early nineties and continuing until very recently [2–8]. However, the high precision measurements of the N→ Δ transition by means of electromagnetic probes became possible with the advent of the new generation of electron beam facilities such as BATES, LEGS, MAMI, ELSA and at the Jefferson Lab. Several experimental programs devoted to the study of the electromagnetic properties of the Δ have been reported in the past few years [1, 9, 10]. These experimental efforts provide new incentives for the theoretical study of these observables.

The electromagnetic transition of decuplet to octet baryons is very important in order to understand the internal quark structure and their dynamics. The decuplet to octet electromagnetic transitions allowed by spin-parity selection rules are the magnetic dipole (M_1), electric quadrapole (E_2) and Coulomb quadrupole (C_2) moments. The transition can also give essential information about the shape of the baryon. When the shape of the baryon is spherically symmetric, then the E_2 and C_2 amplitudes must vanish. However, the experiments show a non zero, though very small, contribution from E_2 and C_2 over the dominant M_1 transition. In the rest frame of Δ the $\Delta \leftrightarrow N\gamma$ process is predominantly an M_1 transition involving the spin and isospin flip of a single quark. The quadrupole amplitudes are only about 1/40 of the dominant magnetic dipole amplitude. Though there exist many model predictions on the radiative decay width of the $\Delta \rightarrow N\gamma$ transition based on lattice calculations, light cone QCD, chiral quark model etc [2, 6, 11–14], their predictions vary widely with the experimental values [1]. In this article we study the N-Δ system through a phenomenological hypercentral quark model. The confinement of

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the three quark system is described through a hypercentral coloumb plus linear potential. It is expected that the quark confinement effect plays a decisive role in the transition properties of the baryons. So, we define an effective mass to the confined quarks within the baryon for the parameter-free predictions of the transition properties of $\Delta \rightarrow N\gamma$.

The article is organized as follows. In Section 2 the hypercentral scheme and a brief introduction of hypercentral coloumb plus linear potential employed for the present study are described. Section 3 describes the computational details of transition magnetic moments and the transition amplitude of $\Delta \rightarrow N\gamma$ incorporated with and without the effective mass of the bound quarks. In Section 4 we present the calculation of the radiative decay width (Γ_{M_1}) of $\Delta \rightarrow N\gamma$ channel. In Section 5, we discuss our results in comparison with other theoretical predictions and experimental results.

2 Hypercentral scheme for baryons

The most general Jacobi co-ordinates to describe a three-body system of unequal masses can be written as [15]

$$\vec{\rho} = \sqrt{\frac{m_2 m_3}{m(m_2+m_3)}}(\vec{r}_2-\vec{r}_3), \tag{1}$$

$$\vec{\lambda} = \sqrt{\frac{m_1(m_2+m_3)}{mM}}\left(\vec{r}_1-\frac{m_2\vec{r}_2+m_3\vec{r}_3}{m_2+m_3}\right), \tag{2}$$

$$\vec{R} = \frac{1}{M}(m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3), \tag{3}$$

where m_1, m_2, m_3 in our case are the constituent quark mass parameters, $M = m_1+m_2+m_3$ is the center of mass of the system and

$$m = \frac{1}{M}(m_1m_2+m_2m_3+m_1m_3) \tag{4}$$

is equivalent to the reduced mass of the system.
The total kinetic energy operator can now be expressed as

$$T = \frac{P_{\rho}^2}{2m}+\frac{P_{\lambda}^2}{2m}+\frac{P_{R}^2}{2M}. \tag{5}$$

Introducing the hyper spherical coordinates given by the angles

$$\Omega_{\rho} = (\theta_{\rho},\phi_{\rho})\,;\ \Omega_{\lambda} = (\theta_{\lambda},\phi_{\lambda}) \tag{6}$$

together with the hyper radius, x and hyper angle ξ respectively as [16]

$$x = \sqrt{\rho^2+\lambda^2}\,;\ \xi = \arctan\left(\frac{\rho}{\lambda}\right), \tag{7}$$

and assuming the translational invariance, the Hamiltonian in the hyper central model (hcm) can be written as

$$H = \frac{P_x^2}{2m}+V(x). \tag{8}$$

By expressing the interaction potential of the three-body bound system in terms of the hypercentral coordinate, x enables us to incorporate not only the two-body interaction but also the three-body effects. Such three-body effects are desirable in the study of hadrons since the non-abelian nature of QCD leads to gluon-gluon couplings which produce three-body forces. In the six dimensional hyperspherical coordinates, the kinetic energy operator $\frac{P_x^2}{2m}$ of the three-body system can be expressed as

$$\frac{P_x^2}{2m} = \frac{-1}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{5}{x}\frac{\partial}{\partial x}-\frac{L^2(\Omega_{\rho},\Omega_{\lambda},\xi)}{x^2}\right), \tag{9}$$

where $L^2(\Omega_{\rho},\Omega_{\lambda},\xi)$ is the quadratic Casimir operator of the six dimensional rotational group $O(6)$ and its eigen functions are the hyperspherical harmonics, $Y_{[\gamma]l_{\rho}l_{\lambda}}(\Omega_{\rho},\Omega_{\lambda},\xi)$ satisfying the eigenvalue relation

$$L^2 Y_{[\gamma]l_{\rho}l_{\lambda}}(\Omega_{\rho},\Omega_{\lambda},\xi) = \gamma(\gamma+4)Y_{[\gamma]l_{\rho}l_{\lambda}}(\Omega_{\rho},\Omega_{\lambda},\xi). \tag{10}$$

Here, γ is the grand angular quantum number and it takes values 0,1,2....

As the interaction potential is hypercentral such that the potential depends only on the hyper radius x, the hyper radial Schrodinger equation which corresponds to the Hamiltonian given by Eq. (8) can be written as

$$\left[\frac{d^2}{dx^2}+\frac{5}{x}\frac{d}{dx}-\frac{\gamma(\gamma+4)}{x^2}\right]\phi_{\gamma}(x) = -2m[E-V(x)]\,\phi_{\gamma}(x). \tag{11}$$

Following our earlier study on heavy flavour baryons [17], for the present study we consider the hyper central potential $V(x)$ as the hyper color coulomb plus linear potential form given by

$$V(x) = -\frac{2}{3}\frac{\alpha_s}{x}+\beta x. \tag{12}$$

Here $\frac{2}{3}$ is the color factor for the baryon, β corresponds to the string tension of the confining term and α_s is the strong running coupling constant. To account for the spin dependent part of the three-body interaction, we add a separate spin dependent potential given by [17, 18]

$$V_{\rm spin}(x) = -\frac{1}{4}\alpha_s\frac{e^{\frac{-x}{x_0}}}{xx_0^2}\sum_{i<j}\frac{\vec{\sigma}_i\cdot\vec{\sigma}_j}{6m_im_j}\vec{\lambda}_i\cdot\vec{\lambda}_j\,, \tag{13}$$

to the Hamiltonian. Here, x_0 is the hyperfine parameter of the model.

The six dimensional radial Schrodinger equation described by Eq. (11) has been obtained in the variational scheme with the hyper coloumb trial radial wave function given by [19]

$$\psi_{\omega\gamma}=\left[\frac{(\omega-\gamma)!(2g)^6}{(2\omega+5)(\omega+\gamma+4)!}\right]^{\frac{1}{2}}(2gx)^{\gamma}\mathrm{e}^{-gx}L_{\omega-\gamma}^{2\gamma+4}(2gx). \tag{14}$$

The wave function parameter g and hence the energy eigenvalue are obtained by applying virial theorem.

The baryon mass is then obtained as

$$M_{\rm B}=\sum_i m_i+\langle H\rangle. \tag{15}$$

The model parameters listed in Table 1 are fixed using the experimental center of weight (spin-average) mass and hyper fine splitting of the N-Δ ground state. To account for the quark confinement effect, we define an effective mass to the bound quarks as [17]

$$m_i^{\rm eff}=m_i\left(1+\frac{\langle H\rangle}{\sum\limits_i m_i}\right), \tag{16}$$

such that mass of the baryon is given by $M_{\rm B}=\sum\limits_{i=1}^{3}m_i^{\rm eff}$. Accordingly, within the baryon the mass of the quarks may get modified due to its binding interactions with the other two quarks.

Table 1. The hypercentral quark model parameters and the experimental masses of N and Δ.

$m_{\rm u}$/MeV	$m_{\rm d}$/MeV	$\alpha_{\rm s}$	β/GeV^2	x_0/MeV^{-1}	N/MeV	Δ/MeV
338	338	1.0	4.4	−0.00496	939	1232

3 Transition magnetic moment and transition amplitude

The transition magnetic moment corresponding to $\Delta\to N\gamma$ can be computed in terms of the orbital and spin-flavour wave function of the constituent quarks as [20, 21]

$$\mu_{\Delta\to N\gamma}=\left|\left\langle \Delta_{\rm orb}\left|j_0\left(\frac{qx}{2}\right)\right|N_{\rm orb}\right\rangle\right|^2\times\sum_i\langle\Delta_{\rm sf}\,|\,\mu_i\sigma_{iz}\,|\,N_{\rm sf}\rangle. \tag{17}$$

Here, the first term corresponds to the contribution from the orbital part of the transition, while the second term is related to the spin-flavour contribution to the transition magnetic moment. Here, $j_0\left(\frac{qx}{2}\right)$ is the spherical Bessel function, $\mu_i=\frac{\mathrm{e}_i}{2m_i^*}$ and q is the photon energy.

For the transition $\Delta^+\to\mathrm{p}$, the contribution from the spin-flavor wave function is given by

$$(\mu_{\Delta^+\to{\rm p}\gamma})_{\rm sf}=\sum_i\langle\,\Delta^+\,|\,\mu_i\sigma_{iz}\,|\,p\,\rangle. \tag{18}$$

The spin-flavour wave function is given by [21]

$$\left|p,s_z=\frac{1}{2}\right\rangle=\frac{1}{\sqrt{18}}[2u^\uparrow d^\downarrow u^\uparrow+2u^\uparrow u^\uparrow d^\downarrow+2d^\downarrow u^\uparrow u^\uparrow -u^\uparrow u^\downarrow d^\uparrow-u^\uparrow d^\uparrow u^\downarrow-u^\downarrow d^\uparrow u^\uparrow -d^\uparrow u^\downarrow u^\uparrow-d^\uparrow u^\uparrow u^\downarrow-u^\downarrow u^\uparrow d^\uparrow] \tag{19}$$

and for Δ^+ state there are two possibilities to write down the spin-flavour wave function,

$$1)\ \left|\Delta^+,\ S_z=\frac{1}{2}\right\rangle\ \text{and}\ 2)\ \left|\Delta^+,\ S_z=\frac{3}{2}\right\rangle.$$

The spin-flavour wave function of these two states can be written as

$$\left|\Delta^+,\ S_z=\frac{1}{2}\right\rangle=\frac{1}{3}[u^\uparrow u^\uparrow d^\downarrow+u^\uparrow d^\uparrow u^\downarrow+d^\uparrow u^\uparrow u^\downarrow +u^\uparrow u^\downarrow d^\uparrow+u^\uparrow d^\downarrow u^\uparrow+d^\uparrow u^\downarrow u^\uparrow +u^\downarrow u^\uparrow d^\uparrow+u^\downarrow d^\uparrow u^\uparrow+d^\downarrow u^\uparrow u^\uparrow], \tag{20}$$

$$\left|\Delta^+,\ S_z=\frac{3}{2}\right\rangle=\frac{1}{\sqrt{3}}(u^\uparrow u^\uparrow d^\uparrow+u^\uparrow d^\uparrow u^\uparrow +d^\uparrow u^\uparrow u^\uparrow). \tag{21}$$

All there spin-flavour wave functions of p and $\Delta^+\left(S_z=\frac{1}{2},\ \frac{3}{2}\right)$ are obviously orthogonal to each other. However, the transition matrix element as given by Eq. (17) can provide non zero contribution coming only from $\left\langle\Delta^+,\ S_z=\frac{1}{2}\right|\mu_i\sigma_{iz}\left|p,\ S_z=\frac{1}{2}\right\rangle$ with all other combinations leading to zero. The resulting transition magnetic moment is obtained as

$$\langle\Delta_{\rm sf}^+|\mu_i\sigma_{iz}|p_{\rm sf}\rangle = \frac{1}{3\sqrt{18}}[2(\mu_{\rm u}-\mu_{\rm d}+\mu_{\rm u})\langle u^\uparrow d^\downarrow u^\uparrow|u^\uparrow d^\downarrow u^\uparrow\rangle +2(\mu_{\rm u}+\mu_{\rm u}-\mu_{\rm d})\langle u^\uparrow u^\uparrow d^\downarrow|u^\uparrow u^\uparrow d^\downarrow\rangle +2(-\mu_{\rm d}+\mu_{\rm u}+\mu_{\rm u})\langle d^\downarrow u^\uparrow u^\uparrow|d^\downarrow u^\uparrow u^\uparrow\rangle -(\mu_{\rm u}-\mu_{\rm u}+\mu_{\rm d})\langle u^\uparrow u^\downarrow d^\uparrow|u^\uparrow u^\downarrow d^\uparrow\rangle$$

$$-(\mu_u+\mu_d-\mu_u)\langle u^{\uparrow}d^{\uparrow}u^{\downarrow}|u^{\uparrow}d^{\uparrow}u^{\downarrow}\rangle$$
$$-(-\mu_u+\mu_d-\mu_u)\langle u^{\downarrow}d^{\uparrow}u^{\uparrow}|u^{\downarrow}d^{\uparrow}u^{\uparrow}\rangle$$
$$-(\mu_d-\mu_u+\mu_u)\langle d^{\uparrow}u^{\downarrow}u^{\uparrow}|d^{\uparrow}u^{\downarrow}u^{\uparrow}\rangle$$
$$-(\mu_d+\mu_u-\mu_u)\langle d^{\uparrow}u^{\uparrow}u^{\downarrow}|d^{\uparrow}u^{\uparrow}u^{\downarrow}\rangle$$
$$-(-\mu_u+\mu_u+\mu_d)\langle u^{\downarrow}u^{\uparrow}d^{\uparrow}|u^{\downarrow}u^{\uparrow}d^{\uparrow}\rangle], \quad (22)$$

$$(\mu_{\Delta^+\to p\gamma})_{sf} = \frac{1}{3\sqrt{18}}[6(2\mu_u-\mu_d)-6\mu_d] = \frac{2\sqrt{2}}{3}[\mu_u-\mu_d]. \quad (23)$$

Similarly we can find out the transition magnetic moment of

$$(\mu_{\Delta^0\to n\gamma})_{sf} = \frac{2\sqrt{2}}{3}[\mu_d-\mu_u]. \quad (24)$$

As we are not differentiating the different charge states of Delta and the nucleon states (p, n), we express the transition magnetic moment of $\Delta^+\to N\gamma$ in terms of its magnitude only. Finally Eq. (17) can be reduced into

$$|\mu_{\Delta\to N\gamma}| = \left|\langle\Delta_{orb}|j_0\left(\frac{qx}{2}\right)|N_{orb}\rangle\right|^2 \frac{2\sqrt{2}}{3}[\mu_u-\mu_d]. \quad (25)$$

In Eq. (17) e_i and σ_{iz} represents the charge and the spin projection of the quark constituting the baryonic state. While, m_i^* is equivalent to the effective mass of the bound quarks of the N-Δ system. It is defined in terms of the respective mass of the bound quarks constituting the N and Δ states as [20]

$$m_i^{eff} = \sqrt{m_{i(\Delta)}^{eff} m_{i(N)}^{eff}}. \quad (26)$$

Using the spin-flavour wave function of the N (octet) and Δ (decuplet) states [21], the transition magnetic moment is computed with (WEM) and without (WOM) considering the quark confinement effect through the effective mass of the bound quarks. The transition amplitude in terms of the transition magnetic moment can now be computed from the relation

$$f_{M_1}^2 = \frac{\pi\alpha}{M_N^2}|q|\ |\mu_{\Delta\to N\gamma}|^2, \quad (27)$$

where, α is the electromagnetic fine structure constant, M_N is the nucleon mass, $\mu_{\Delta\to N}$ is the radiative transition magnetic moments, q is the photon energy and in the non-relativistic case, photon energy is given by $M_\Delta - M_N$. The present values of transition magnetic moment and transition ammplitude are listed along with other model predictions and with the experimental value in Table 2.

4 Radiative decay

The radiative decays of baryons provide a much better understanding of the underlying structure of baryons and the dependence on the constituent quark mass. Though the nonrelativistic model of Isgur and Karl successfully predicted the electromagnetic properties of the low-lying octet baryons, it fails to provide a good description of the radiative decay of the decuplet baryons [22]. Thus, the successful prediction of the electromagnetic properties of octet baryons as well as the decuplet baryons become detrimental to any phenomenological attempts. The radiative decay width of the baryon is related to the transition magnetic moment as [23]

$$\Gamma_{M_1} = q^3\frac{2}{2J+1}\frac{\alpha}{M_N^2}|\mu_{\Delta\to N}|^2, \quad (28)$$

where, J is the total angular momentum quantum number of the decaying baryon. Thus, in terms of the transition amplitude given by Eq. (27), we write the decay width corresponding to $\Delta\to N\gamma$ as

$$\Gamma_{M_1} = \frac{q^2}{2\pi}f_{M_1}^2. \quad (29)$$

The computed values of radiative decay width and the branching ratio $\frac{\Gamma_{M_1}}{\Gamma}$ are listed in Table 3. For the branching ratio we have used the total decay width of Δ reported by PDG (2010) [1].

Table 2. Transition magnetic moments ($\mu_{\Delta\to N}$) in μ_N and transition amplitude f_{M1} in $GeV^{-\frac{1}{2}}$.

decay mode	transition magnetic moments (μ_N)					transition amplitude		
	expression	WEM	WOM	others	Expt. [9]	WEM	WOM	others
$\Delta\to N\gamma$	$\frac{2\sqrt{2}}{3}(\mu_u-\mu_d)$	2.6199	2.4699	2.46 [2]	3.23±0.1	0.2299	0.2199	0.23[2]
				2.76 [3]				
				2.50 [6]				
				2.48 [20]				
				2.47 [11]				

WEM - With effective mass, WOM - Without effective mass.

Table 3. Radiative Decay Widths (Γ_{M_1} in MeV) and Branching Ratio.

decay mode	radiative decay width(Γ_{M_1}) in MeV				branching ratio $\left(\frac{\Gamma_{M_1}}{\Gamma}\right)$ in %			
	WEM	WOM	others	Expt.	symbol	WEM	WOM	Expt. [1]
$\Delta \to N\gamma$	0.7139	0.6389	0.430 [2]	0.61-0.71 [1]	$\frac{\Gamma_{M_1}}{\Gamma(\Delta)}$	0.6049	0.5409	0.52-0.60
			0.900 [6]					
			0.334 [11]					
			0.36 [12]					
			0.343 [13]					
			0.67-0.79 [14]					

[2]-Lattice, [6]-LCQCD, [11]-χQM, [12]-NRQM, [13]-Algebraic model, [14]-HBχPT.

5 Results and discussion

The properties of a N, Δ system are studied within the frame work of a non-relativistic hypercentral quark model. After fixing the model parameters using the ground state masses of N and Δ states, the electromagnetic transition properties are computed without any additional parameters. The transition magnetic moment and the transition amplitude for $\Delta \to N\gamma$ obtained in the present study are in good agreement with the lattice result [2] as well as being in accordance with other model predictions. However, all the theoretical predictions are found to be lower by about 15 to 24% compared with the experimental value of $\mu_{\Delta^0 \to n} = 3.23$ [9]. The transition magnetic moments predicted in our model by considering the confinement effect on the bound quark mass (WEM) and without considering the confinement effect (WOM) are in good agreement with other theoretical predictions. The transition amplitude predicted with the effective mass of the bound quarks is in good agreement with that of the lattice predictions [2]. The disagreement with the theoretical result and experimental value reported by [9] for the transition magnetic moment of $\Delta \to N\gamma$ demands more intense experimental measurements.

In the case of radiative decay width, our results with and without the bound state effect on the quark mass are in good agreement with the average range of experimental values (0.61–0.71 MeV) reported by PDG (2010), while other theoretical values except that by HBχPT [14] are widely off by 35 to 50%. Thus we conclude here that the hypercentral model with the color coloumb plus the linear form of the three quarks interactions within baryon is one of the successful schemes that describes the electromagnetic properties of the decuplet (Δ) - octet (N) transition. We look forward to extending the scheme for all the octet and decuplet baryons in the (u, d, s) sector.

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