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Azimuthal distributions of radial momentum and velocity in relativistic heavy ion collisions*

LI Lin(里霖)[1;1)] LI Na(李娜)[2] WU Yuan-Fang(吴元芳)[1,3,4]

[1] Institute of Particle Physics, Central China Normal University, Wuhan 430079, China
[2] Huazhong University of Science and Technology, Wuhan 430074, China
[3] Brookhaven National Laboratory, Upton, NY 11973, USA
[4] Key Laboratory of Quark & Lepton Physics (Huazhong Normal University), Ministry of Education, Wuhan 430079, China

Abstract: Azimuthal distributions of radial (transverse) momentum, mean radial momentum, and mean radial velocity of final-state particles are suggested for relativistic heavy ion collisions. Using the AMPT transport model with string melting, the distributions of Au+Au collisions at 200 GeV are presented and studied. It is demonstrated that the distribution of total radial momentum is more sensitive to the anisotropic expansion, as the anisotropies of final-state particles and their associated transverse momentums are both counted in the measurement. The mean radial velocity distribution is compared with the radial flow velocity. The thermal motion contributes an isotropic constant to the mean radial velocity.

Key words: azimuthal distribution, radial momentum, radial velocity

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1 Introduction

One of the main goals of current relativistic heavy ion collisions is to understand the properties of quark-gluon plasma (QGP) [1]. It is well known that one important character of this formed matter is an anisotropic collective flow. In non-central collisions, the overlap area of two incident nuclei is an almond shape in the transverse coordinate plane [2]. This initial geometric asymmetry leads to a larger density gradient along the short axis. It in turn pushes the formed system to expand anisotropically, i.e., large collective flow velocity in the short side direction, which is perpendicular to the anisotropy in coordinate space. Therefore, the measurement of the anisotropic distribution of final-state particles should provide valuable information about the system evolution [2, 3].

Conventionally, the azimuthal distribution of the multiplicity of final-state particles is presented. Its anisotropy is quantified by the coefficients of the Fourier expansion of the distribution [4]

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1+\sum_{n=1}^{\infty} 2v_n(N)\cos(n\phi), \tag{1}$$

where ϕ is the azimuthal angle between the transverse momentum of the particle and the reaction plane. The Fourier coefficients are evaluated by,

$$v_n(N) = \langle\cos(n\phi)\rangle, \tag{2}$$

where $\langle\cdots\rangle$ is an average over all particles in all events, and $v_n(N)$ refers to the anisotropy coefficient of the azimuthal multiplicity distribution. The second harmonic coefficient $v_2(N)$ is the so-called elliptic flow parameter. It presents the anisotropy of the colliding system and has the biggest ellipticity at high energy heavy ion collisions [5, 6]. In addition, the azimuthal asymmetry distribution of energy loss and its Fourier expansion coefficients are also studied [7].

However, the multiplicity distribution only counts the number of particles emitted at a certain azimuthal angle. The expansion of the system results in not only

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1) E-mail: lilin@iopp.ccnu.edu.cn

the anisotropy of multiplicity distribution but also their associate radial (transverse) momentum. The total radial momenta at a given azimuthal angle is the combination of them. Therefore, the azimuthal distribution of radial momentum should be a more sensitive measure of the anisotropic expansion, which has not been directly explored before.

In addition to the radial momentum, the radial flow velocity is another interesting and important quantity. It directly relates to the equation of state [8] and shear viscous interactions. For an ideal flow, the radial flow velocity is isotropic. While, if there are shear interactions, the radial flow velocity will be different from layer to layer. In hydrodynamics, the shear viscous interactions are supposed to be proportional to the gradient of flow velocity [9]. The proportional constant is defined as shear viscosity. The gradient of radial flow velocity along the azimuthal direction is directly related to the shear viscous interactions.

Theoretically, the radial flow velocity is a parameter in model calculations. It is usually obtained by fitting the spectrum of transverse momentum [10]. Recently, it is further suggested to extract the radial flow velocity from photon and dilepton spectra [11].

Experimentally, only the radial velocity of final-state particles ($\vec{v}$) is measurable. It should be a combination of the flow velocities ($\vec{v}_{\rm flow}$) and the random thermal motion ($\vec{v}_{\rm th}$) [12]. How to extract the random thermal motion from the radial velocity of final-state particles and get the radial flow velocity is not clear. This is why the radial velocity of final-state particles has not been explored for a long time. It is interesting to see how the radial velocity of a final-state particle relates to the radial flow velocity. Therefore, we further suggest the measurement of the azimuthal distribution of the mean radial velocity of final-state particles.

In the second section of this article, we will give the definitions of the suggested azimuthal distributions of radial momentum and velocity of final-state particles, and the corresponding anisotropic parameters. In the third section, using the samples generated using AMPT with the string melting model, we show the azimuthal distributions of radial momentum and the centrality dependence of its anisotropic parameters. The results are compared with those of the corresponding azimuthal multiplicity distribution. In the fourth section, the azimuthal distributions of mean radial velocity at different centralities are presented, and compared with those given by the anisotropic blast-wave model [13–15]. Finally, a summary and the conclusion are presented.

2 Azimuthal distributions of radial momentum and velocity

As indicated, the initial anisotropy in coordinate space in non-central collisions makes the formed system expand in a perpendicular almond shape in momentum space. The final state particles move outward anisotropically. Both the particle density and the associated momentum behaves anisotropically during the expansion. The distribution of total transverse momentum at the different azimuthal directions should be a good measurement for both of these two effects. The total transverse momentum in the mth azimuthal bin can be defined as

$$\langle P_{\rm t}(\phi_m)\rangle = \frac{1}{N_{\rm event}}\sum_{j=1}^{N_{\rm event}}\left(\sum_{i=1}^{N_m} p_{{\rm t},i}(\phi_m)\right), \qquad (3)$$

where $p_{{\rm t},i}$ is the transverse momentum of the ith particle, N_m is the total number of particles, and $\langle\cdots\rangle$ denotes the average over all events.

In order to see the contributions of radial momentum in particular, the mean radial momentum in the mth azimuthal bin can be defined accordingly as,

$$\langle\langle p_{\rm t}(\phi_m)\rangle\rangle = \frac{1}{N_{\rm event}}\sum_{j=1}^{N_{\rm event}}\left(\frac{1}{N_m}\sum_{i=1}^{N_m} p_{{\rm t},i}(\phi_m)\right). \qquad (4)$$

Here, the averages $\langle\langle\cdots\rangle\rangle$ are over all particles in the mth angle bin and all events. It records only the contributions from the transverse momentum of final particles, the multiplicity effect is canceled by the average over all particles.

The anisotropic parameters of all those azimuthal distributions can be directly obtained from their Fourier expansions, respectively,

$$\frac{{\rm d}\langle P_{\rm t}\rangle}{{\rm d}\phi} \propto\ 1+\sum_{n=1}^{\infty} 2v_n(\langle P_{\rm t}\rangle)\cos(n\phi), \qquad (5)$$

and

$$\frac{{\rm d}\langle\langle p_{\rm t}\rangle\rangle}{{\rm d}\phi} \propto\ 1+\sum_{n=1}^{\infty} 2v_n(\langle\langle p_{\rm t}\rangle\rangle)\cos(n\phi). \qquad (6)$$

$\frac{{\rm d}\langle P_{\rm t}\rangle}{{\rm d}\phi}$ and $\frac{{\rm d}\langle\langle p_{\rm t}\rangle\rangle}{{\rm d}\phi}$ are the azimuthal distribution functions of total radial momentum and mean radial momentum. $v_n(\langle P_{\rm t}\rangle)$ and $v_n(\langle\langle p_{\rm t}\rangle\rangle)$ are their anisotropic parameters, respectively.

Considering the relativistic effect, the transverse (radial) velocity of the ith particle can be written as,

$$v_{\mathrm{t},i}=\frac{p_{\mathrm{t},i}}{m_{\mathrm{t}}}=\frac{p_{\mathrm{t},i}}{\sqrt{m_{0,i}^2+p_{\mathrm{t},i}^2}}, \tag{7}$$

where $p_{\mathrm{t},i}$ and $m_{\mathrm{t},i}$ are the transverse momentum and mass of the ith particle, respectively. $m_{0,i}$ is the mass of the ith particle in the rest frame. The radial velocity fluctuates from particle to particle. In a given azimuthal direction, the mean radial velocity can be considered as a good approximation. Analogously, the azimuthal distribution of mean radial velocity can be defined as

$$\langle\langle v_{\mathrm{t}}(\phi_m)\rangle\rangle=\frac{1}{N_{\mathrm{event}}}\sum_{j=1}^{N_{\mathrm{event}}}\left(\frac{1}{N_m}\sum_{i=1}^{N_m}v_{\mathrm{t},i}(\phi_m)\right). \tag{8}$$

Here, the average is over all the particles in the mth bin and events.

The behavior of those suggested observables should provide more information about anisotropic expansion. In the following, as a demonstration, we use the generated AMPT sample with string melting [16, 17]. A partonic phase is implemented in the model and the elliptic flow data from RHIC are well reproduced by it [18]. For Au+Au at $\sqrt{s_{\mathrm{NN}}}=200$ GeV, about 1.6 million minimum bias events are generated.

3 Azimuthal distributions of radial momentum in the AMPT model

The azimuthal distributions of radial momentum, mean radial momentum, and multiplicity are presented in Fig. 1(a), (b) and (c), respectively. Error is statistical only and smaller than the size of the points. The particles within rapidity range $y\in[-5,5]$ are counted. These cases are kept in all the following figures.

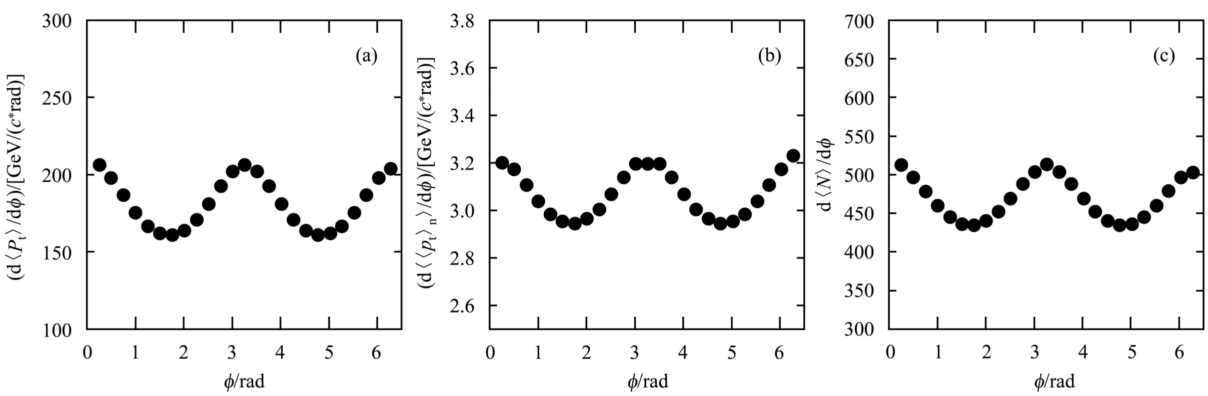

Fig. 1. The azimuthal distributions of (a) radial momentum, (b) mean radial momentum, and (c) multiplicity, for the sample of Au+Au collisions at $\sqrt{s_{\mathrm{NN}}}=200$ GeV generated by using AMPT with string melting.

We can see from Fig. 1 that all the observables as a function of the azimuthal angle show an anisotropic shape, $\cos(2\phi)$. It is the same as multiplicity distribution, the biggest anisotropy of mean radial momentum distribution appears in the in-plane direction, as shown in Fig. 1(b). It indicates that not only the particle density, but also the associated p_{t} are larger in the in-plane direction. It is interesting to see if the data at RHIC show the same character as the model.

In order to compare the anisotropy effects of these three distributions qualitatively, the centrality dependence of the corresponding anisotropic parameter, v_2, is presented in Fig. 2. The anisotropy parameter v_2 from different measurements shows similar centrality dependencies. At each centrality, the anisotropy parameter of the multiplicity distribution, $v_2(N)$, is larger than that of the mean radial momentum distribution $v_2(\langle\langle p_{\mathrm{t}}\rangle\rangle)$. The anisotropy parameter of the radial momentum, $v_2(\langle P_{\mathrm{t}}\rangle)$, is the largest one among the three variables. It confirms the anisotropy of the radial momentum distribution including the contributions from a number of particles and their associated transverse momentum. Therefore, the azimuthal distribution of radial momentum gives a full count of anisotropic expansion.

As we know, the anisotropy parameters v_2 also depend on p_{t}, and it increases with p_{t} when $p_{\mathrm{t}}<2$ GeV/c [19]. The p_{t} dependence of the anisotropy parameters of radial momentum and multiplicity distributions are presented in Fig. 3. The anisotropy parameter increases with p_{t} when $p_{\mathrm{t}}<2$ GeV/c, the same as the data shown. We can also see that the v_2

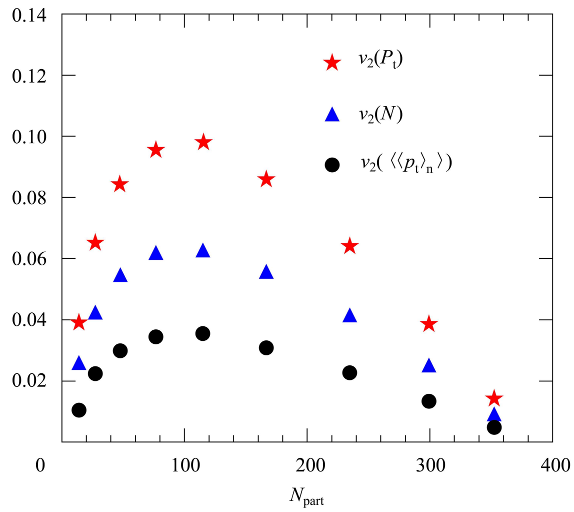

Fig. 2. (color online)The centrality dependence of elliptic flow parameters deduced from the azimuthal distributions of radial momentum (solid red stars), mean radial momentum (solid black cycles), and multiplicity (solid blue triangles) for the sample of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV generated by using AMPT with string melting.

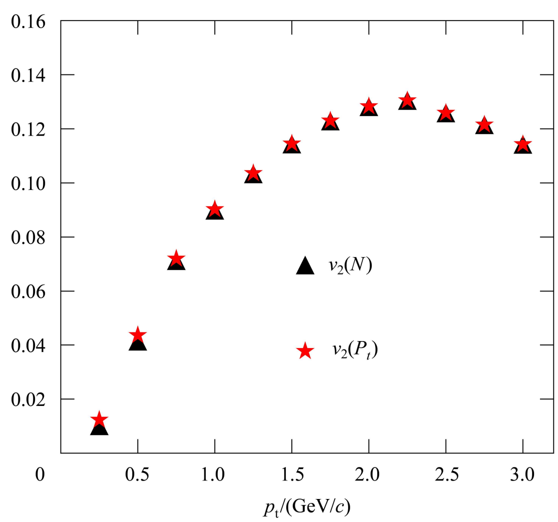

Fig. 3. (color online) p_t dependence of the anisot-ropic parameter of azimuthal distributions of radial momentum (red solid stars), and multiplicity (black triangles) for the sample of Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV generated by using AMPT with string melting.

slightly decreases with p_t when $p_t > 2$ GeV/c, and it may be contributed by the hard components [20]. At a fixed p_t bin, the anisotropy of the radial momentum is almost the same as that of multiplicity. This is because the p_t of all particles in a small given p_t bin are almost the same. The anisotropy of radial momentum is dominated by that of multiplicity.

4 Azimuthal distributions of radial velocity in the AMPT model

The azimuthal distribution of mean radial velocity is presented in Fig. 4(a). It is a period function and can be well fitted by

$$\langle\langle V_t\rangle\rangle = V_0 + V_a\cos(2\phi). \tag{9}$$

It is the same mode as the flow velocity,

$$\beta = \beta_0 + \beta_a\cos(2\phi), \tag{10}$$

which is usually assumed in the blast-wave model in counting the anisotropic expansion [13, 15].

In order to see the contribution of random thermal motion, the mean radial velocity of final-state particles in three typical centralities are presented in Fig. 4(b). In mid-central (30%–40%) and peripheral (60%–70%) collisions, the mean radial velocities are anisotropic, while it becomes an approximately isotropic constant in central (0–5%) collisions. This suggests that the interactions between azimuthal layers are negligible in central collisions, which is consistent with the expectations of viscous hydrodynamics [21, 22]. It also shows that the thermal motion only contributes an isotropic constant to the mean radial velocity.

As we know, for a system with a fixed temperature, the lighter particle has higher thermal velocity. In order to test if the V_0 is mainly caused by thermal motion, the mean radial velocities of three different particles and their corresponding fitting parameters are presented in Fig. 5. Indeed, the lightest pion has the highest V_0, while the heaviest proton has the lowest one.

To see the anisotropy effect alone, we can calculate the gradient of mean radial velocity along the azimuthal direction. In this case, the constant part of the mean radial velocity is canceled. Fig. 4(c) shows the corresponding gradients of Fig. 4(b). In central collisions, it is approximately zero. The amplitudes in mid-central collisions are larger than those in peripheral collisions. These results show that there is almost no gradient of mean radial velocity in central collisions and becomes largest in mid-central collisions.

Conventionally, the parameters of flow velocity Eq. (10), β_0 and β_a, are obtained by fitting the spectra of the produced particles. Here, we choose the spectra of pion, proton and kaon from the AMPT string melting model and get, $\beta = 0.35 + 0.04\cos(2\phi)$.

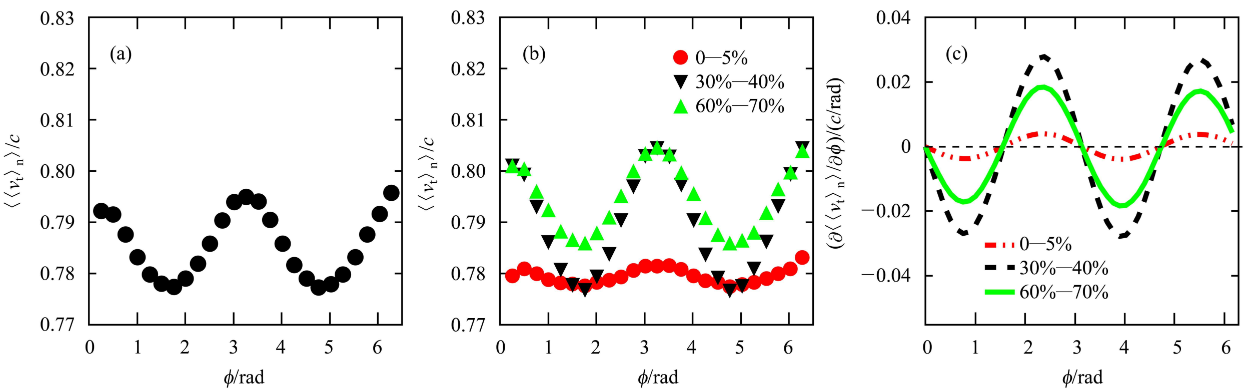

Fig. 4. (color online) The azimuthal distributions of mean radial velocities of minimum bias sample (a) and the samples of three different centralities (b), and the azimuthal gradients of (b) in (c).

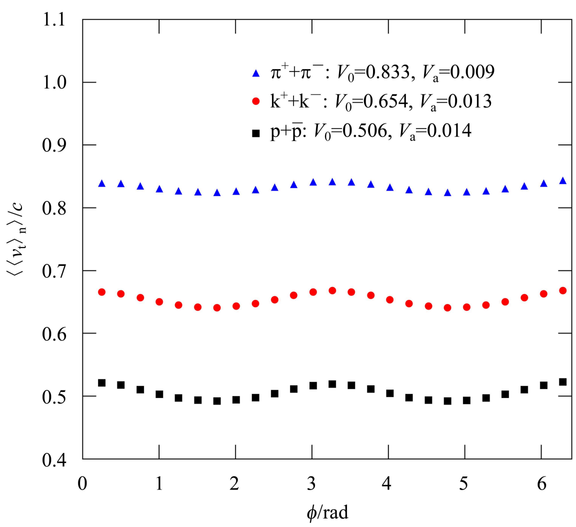

Fig. 5. (color online)The azimuthal distributions of mean radial velocity of charged pion, charged kaon and (anti)proton. The lines are fitted by Eq. (8), and the corresponding fitted parameters are listed. The errors of the parameters are less than 1% relative values.

Due to thermal motion, the β_0 is not directly comparable with V_0. However, $V_{\rm a}\sim 0.01$ from the corresponding mean radial velocity may be a good approximation of flow velocity estimated by the blast-wave model, where $\beta_{\rm a}\sim 0.04$.

Certainly, the flow velocities obtained from directly measured radial velocity and from the spectrum fitting based on the blast-wave model should be better as compared with the experimental data sample, where the spectrum is precisely presented. The comparison of these two methods will lead to a better understanding of the flow velocity.

5 Summary and conclusion

In the paper, we suggest the studies for azimuthal distributions of radial momentum, mean radial momentum, and mean radial velocity in relativistic heavy heavy ion collisions.

Using the sample of Au+Au collisions at $\sqrt{s_{\rm NN}}=$ 200 GeV produced by a multiphase transport model (AMPT), we find that the azimuthal distribution of radial transverse momentum indeed counts the anisotropy of final-state particles and their associated transverse momenta. Thus it presents a full description of anisotropic expansion at various centralities. The azimuthal distribution of radial momentum shows the same anisotropy as that of the multiplicity distribution in a small $p_{\rm t}$ bin only.

The azimuthal distribution of mean radial velocity is shown to be the same mode as the flow velocity that is usually assumed in the generalized blast-wave model. Its centrality dependency indicates that thermal motion only contributes an isotropic constant to mean radial velocity. Its particle mass dependency further shows that the mass ordering of isotropic mean radial velocity is the same as thermal motion. The anisotropic mean radial velocity is approximated to flow velocity, which is obtained from fitting the spectrum of corresponding particles based on the blast-wave model.

Therefore, it is interesting to measure the azimuthal distributions of radial momentum, mean radial momentum, and mean radial velocity in current relativistic heavy ion collisions.

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