Mass effect in polarization investigation at BEPC/BES and the B-factory *

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Abstract: We consider the annihilation process of an electron-positron pair into a pair of heavier fermions when the initial electron and positron beams are polarized. By calculating the polarization of the final-state particles, we discuss in detail the effect due to the produced particle masses in the τ-charm energy region at BEPC/BES, and also compare the effect with that at the B-factory. Such a study is useful for the design of possible polarization investigation at the BEPC/BES facility and the B-factory.

Key words: BEPC/BES, B-factory, polarization, helicity

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1 Introduction

Quantum chromodynamics (QCD) has been considered the fundamental theory of strong interactions, and the theoretical predictions so far are all compatible with the phenomenological observations. However, in many cases the accuracy of theoretical calculations and experimental measurements is still rather low for the purpose of seeking for new physics beyond the Standard Model (SM).

For testing the reliability of the SM and searching for new physics, it is still desirable to compare more precise predictions derived from the theory with experimental measurements of higher accuracy. In addition to high energy experimental projects with high precision, lower energy facilities with high luminosity can also address such an issue, and the upgraded BEPCⅡ/BESⅢ facility is important for high precision studies.

The BEPCⅡ facility is a double-ring e^+e^- collider running in the τ-charm energy region ($E_{\rm cm}$ = 2.0–4.6 GeV), and it can reach a design luminosity of 1×10^{33} cm^{-2}·s^{-1} at a center-of-mass energy of 3.78 GeV. With this luminosity, the BESⅢ detector can collect, for example, 10 billion J/ψ events in one year of running [1–4]. The comprehensive studies of e^+e^- annihilation in the τ-charm threshold region can provide us with a novel and unique chance to study hadronization and nonperturbative dynamics. This is very valuable to the investigations of, for example, the structure of hadrons and the spectrum of hadronic states. The upgraded BEPCⅡ/BESⅡ can play an important role in not only testing the SM, but also in searching for new physics beyond the SM [5]. There is also an optional consideration to include the polarization of at least the incident electrons for the future upgrade of the BEPCⅡ/BESⅢ facility [6], and it is thus necessary to consider some more detailed features of the polarized e^+e^- processes.

According to QCD, the process $e^+e^-\to q\bar{q}$ (a quark-antiquark pair) is the simplest e^+e^- process that ends in hadrons. This process is extraordinarily useful in determining the properties of elementary particles and investigating the hadronic structure. In this paper we compute the polarized cross section for $e^+e^-\to\mu^+\mu^-$, to the lowest order. It is the simplest of all QED processes, but also fundamental to the understanding of all reactions in the e^+e^- collision. It is easy to extend the results for muon production to production of other leptons and quarks. Thus our study is useful for further study on possible polarization investigations at e^+e^- colliders.

2 Polarized cross section of muon-antimuon pair production

Using the Feynman rules, we can write down the amplitude for the polarized $e^+e^-\to\mu^+\mu^-$ process:

$$\bar{v}^{s'}(p')(-ie\gamma^{\mu})u^{s}(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^{r}(k)(-ie\gamma^{\nu})v^{r'}(k'),$$

where the superscripts s, s′, r, and r′ denote the polarized states of the incoming electron and positron and the outgoing muon and anti-muon pair.

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The squared amplitude of this process is

$$|M(e^-_{\rm s}(p)e^+_{\rm s'}(p')\to\mu^-_{\rm r}(k)\mu^+_{\rm r'}(k'))|^2$$

$$=\frac{e^4}{q^4}(\bar{v}^{\rm s'}(p')\gamma^\mu u^{\rm s}(p)\bar{u}^{\rm s}(p)\gamma_\nu v^{\rm s'}(p'))$$

$$\times(\bar{u}^{\rm r}(k)\gamma_\mu v^{\rm r'}(k')\bar{v}^{\rm r'}(k')\gamma_\nu u^{\rm r}(k)).$$

We compute the polarized $e^+e^-\to\mu^+\mu^-$ cross section by using the trace technology with the addition of helicity projection operators to project out the desired left- or right-handed spinors. For a massless particle, chirality and helicity are equivalent and Lorentz-invarient. Thus $(1+\gamma_5)/2$ becomes a right-(left-)handed helicity projection operator for a massless particle (antiparticle), and $(1-\gamma_5)/2$ becomes a left-(right-)handed helicity projection operator for a massless particle (antiparticle). γ_5 anticommutes with γ_μ, and thus in the electron-positron annihilation the helicities of e^- and e^+ must be opposite to each other in order to annihilate when the electron mass is ignored. On the other hand, for a particle with a nonvanishing mass, chirality is not well defined, but states of such a particle can still be identified by their helicities in a specific reference system. In our calculation, we take the masses of both the incident electrons and the final-state particles into account.

Let us first calculate the muon half of the squared amplitude

$$\bar{u}^{\rm r}(k)\gamma_\mu v^{\rm r'}(k')\bar{v}^{\rm r'}(k')\gamma_\nu u^{\rm r}(k).$$

The polarization projection operator for a massive particle should be

$$\hat{P}(\pm)=\frac{1\pm\gamma_5\not{n}}{2},$$

where n^μ is a normalized spacelike vector, $n_\mu n^\mu=-1$, and is orthogonal to the particle momentum, $n_\mu k^\mu=0$. $\hat{P}(+)$ and $\hat{P}(-)$ project out the states with polarization in direction $\boldsymbol{n}$ and $-\boldsymbol{n}$, respectively.

We consider the helicities of the final-state particles, so we choose

$$n=\left(\frac{|\boldsymbol{k}|}{m},\frac{E}{m}\frac{\boldsymbol{k}}{|\boldsymbol{k}|}\right),\ n'=\left(\frac{|\boldsymbol{k}'|}{m},\frac{E}{m}\frac{\boldsymbol{k}'}{|\boldsymbol{k}'|}\right).$$

Then $\hat{P}(h)=\dfrac{1+h\gamma_5\not{n}}{2}$ projects out the h-helicity ($h=\pm1$) component from the final-state muon, and similarly, $\hat{P}'(h')=\dfrac{1+h'\gamma_5\not{n}'}{2}$ projects out the h'-helicity ($h'=\pm1$) component from the final-state anti-muon.

The final-state muon half of $|M|^2$, for an h-helicity muon and an h'-helicity anti-muon, is then

$$\bar{u}^{\rm r}(k)\gamma_\mu v^{\rm r'}(k')\bar{v}^{\rm r'}(k')\gamma_\nu u^{\rm r}(k)$$

$$=\sum_{\rm spins}\bar{u}(k)\gamma_\mu\hat{P}'(h')v(k')\bar{v}(k')\gamma_\nu\hat{P}(h)u(k)$$

$$=\mathrm{tr}\left[(\not{k}+m)\gamma_\mu\frac{1+h'\gamma_5\not{n}'}{2}(\not{k}'-m)\gamma_\nu\frac{1+h\gamma_5\not{n}}{2}\right].$$

Similarly, the electron half of $|M|^2$ for a λ-helicity electron and a λ'-helicity positron is then

$$\bar{v}^{\rm s'}(p')\gamma^\mu u^{\rm s}(p)\bar{u}^{\rm s}(p')\gamma^\nu v^{\rm s'}(p')$$

$$=\sum_{\rm spins}\bar{v}(p')\gamma^\mu\frac{1+\lambda\gamma_5\not{s}}{2}u(p)\bar{u}(p')\gamma^\nu\frac{1+\lambda'\gamma_5\not{s}'}{2}v(p')$$

$$=\mathrm{tr}\left[(\not{p}'-m_{\rm e})\gamma^\mu\frac{1+\lambda\gamma_5\not{s}}{2}(\not{p}+m_{\rm e})\gamma^\nu\frac{1+\lambda'\gamma_5\not{s}'}{2}\right],$$

where

$$s=\left(\frac{|\boldsymbol{p}|}{m_{\rm e}},\frac{E}{m_{\rm e}}\frac{\boldsymbol{p}}{|\boldsymbol{p}|}\right),\ s'=\left(\frac{|\boldsymbol{p}'|}{m_{\rm e}},\frac{E}{m_{\rm e}}\frac{\boldsymbol{p}'}{|\boldsymbol{p}'|}\right).$$

Calculating the trace explicitly and dotting the electron half into the final-state muon half, we find that the squared matrix element for the polarized process $e^+e^-\to\mu^+\mu^-$ in the center-of-mass frame is

$$|M(e^-_\lambda e^+_{\lambda'}\to\mu^-_h\mu^+_{h'})|^2$$

$$=\frac{e^4}{4}\Big[(1-hh'-\lambda\lambda'+hh'\lambda\lambda')(1+\cos^2\theta)$$

$$+2(h\lambda+h'\lambda'-h\lambda'-h'\lambda)\cos\theta$$

$$+\frac{m^2}{E^2}(1+hh'-\lambda\lambda'-hh'\lambda\lambda')(1-\cos^2\theta)$$

$$+\frac{m_{\rm e}^2}{E^2}(1-hh'+\lambda\lambda'-hh'\lambda\lambda')(1-\cos^2\theta)$$

$$+\frac{m^2m_{\rm e}^2}{E^4}(1+hh'+\lambda\lambda'+hh'\lambda\lambda')\cos^2\theta\Big],$$

where θ is the angle between the directions of the final-state muon and the incident electron.

The formula of differential cross section in the center-of-mass frame is

$$\frac{{\rm d}\sigma}{{\rm d}\Omega}=\frac{1}{2E_{\rm A}2E_{\rm B}|v_{\rm A}-v_{\rm B}|}\frac{|\boldsymbol{p}_1|}{(2\pi)^24E_{\rm cm}}|M(p_{\rm A}p_{\rm B}\to p_1p_2)|^2.$$

For our problem, $E_{\rm A}=E_{\rm B}=\dfrac{E_{\rm cm}}{2}=E$, $|\boldsymbol{p}_1|=\sqrt{E^2-m^2}$, so we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\mathrm{e}^-_\lambda\mathrm{e}^+_{\lambda'}\to\mu^-_h\mu^+_{h'})=\frac{1}{4E^2|v_\mathrm{A}-v_\mathrm{B}|}\frac{|\boldsymbol{k}_1|}{(2\pi)^2 8E}|M(\mathrm{e}^-_\lambda\mathrm{e}^+_{\lambda'}\to\mu^-_h\mu^+_{h'})|^2$$

$$=\frac{\alpha^2}{32E^2|v_\mathrm{A}-v_\mathrm{B}|}\sqrt{1-\frac{m^2}{E^2}}[(1-hh'-\lambda\lambda'+hh'\lambda\lambda')(1+\cos^2\theta)+2(h\lambda+h'\lambda'-h\lambda'-h'\lambda)\cos\theta$$

$$+\frac{m^2}{E^2}(1+hh'-\lambda\lambda'-hh'\lambda\lambda')(1-\cos^2\theta)+\frac{m_\mathrm{e}^2}{E^2}(1+\lambda\lambda'-hh'-hh'\lambda\lambda')(1-\cos^2\theta)$$

$$+\frac{m^2m_\mathrm{e}^2}{E^4}(1+hh'+\lambda\lambda'+hh'\lambda\lambda')\cos^2\theta].$$

3 Longitudinal polarizations of particles produced at BEPC/BES

By definition, the longitudinal polarizations of the incident electrons and positrons [7] are

$$\omega_1 = \frac{N_{1+}-N_{1-}}{N_1}, \tag{1}$$

$$\omega_2 = -\frac{N_{2+}-N_{2-}}{N_2}, \tag{2}$$

where $N_1=N_{1+}+N_{1-}$ and $N_2=N_{2+}+N_{2-}$. N_{1+} and N_{1-} are the numbers of electrons with positive-helicity and negative-helicity, respectively. N_{2+} and N_{2-} are the numbers of positrons with positive-helicity and negative-helicity, respectively.

Let $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\lambda\lambda')$ denote the polarized differential cross section for a λ-helicity electron and a λ'-helicity positron. Thus the total number of events in the unit solid angle is proportional to

$$N_{1+}N_{2-}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(+-)+N_{1-}N_{2+}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(-+)$$

$$+N_{1+}N_{2+}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(++)+N_{1-}N_{2-}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(--). \tag{3}$$

When both the electron and positron beams are polarized, substituting Eq. (1) and Eq. (2) into Eq. (3) we obtain the differential cross section as follows

$$\frac{1+\omega_1\omega_2}{4}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(+-)+\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(-+)\right]$$

$$+\frac{\omega_1+\omega_2}{4}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(+-)-\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(-+)\right]$$

$$+\frac{1-\omega_1\omega_2}{4}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(++)+\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(--)\right]$$

$$+\frac{\omega_1-\omega_2}{4}\left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(++)-\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(--)\right].$$

Let (h,h') denote the differential cross section with the helicities of the final-state muon and anti-muon being h and h', respectively. Thus the degree of longitudinal polarization of the final-state muon is

$$P=\frac{(h=1,h'=1)+(h=1,h'=-1)-(h=-1,h'=1)-(h=-1,h'=-1)}{(h=1,h'=1)+(h=1,h'=-1)+(h=-1,h'=1)+(h=-1,h'=-1)}=\frac{2(\omega_1+\omega_2)\cos\theta}{D}, \tag{4}$$

where

$$D = (1+\omega_1\omega_2)(1+\cos^2\theta)+\frac{m^2}{E^2}(1+\omega_1\omega_2)(1-\cos^2\theta)+\frac{m_\mathrm{e}^2}{E^2}(1-\omega_1\omega_2)(1-\cos^2\theta)+\frac{m_\mathrm{e}^2}{E^2}\frac{m^2}{E^2}(1-\omega_1\omega_2)\cos^2\theta.$$

Just substituting the corresponding final-state fermion mass for m, Eq. (4) is also valid for other $\mathrm{e^+e^-}\to\mathrm{f\bar{f}}$ processes, where $\mathrm{f\bar{f}}$ represents fermion pairs including $\tau^+\tau^-$ lepton pair and quark-antiquark pairs of which the production thresholds are below the center-of-mass energy.

When the masses of both the final-state fermion and the incident electron are ignored, Eq. (4) becomes

$$P_{m,m_\mathrm{e}\to 0}=\frac{2(\omega_1+\omega_2)\cos\theta}{(1+\omega_1\omega_2)(1+\cos^2\theta)}. \tag{5}$$

At BEPC/BES, where E=2.087 GeV, we consider the case that only the electron beam is polarized while the positron beam is unpolarized. Substituting $\omega_1=80\%$ and $\omega_2=0$ into Eq. (4) and Eq. (5), and also substituting the masses [8] of the incident electron and the final-state fermions into Eq. (4), we can plot the curves of final-state fermion longitudinal polarizations P and $P_{m,m_e\to 0}$ versus $\cos\theta$ as shown in Fig. 1 and Fig. 2.

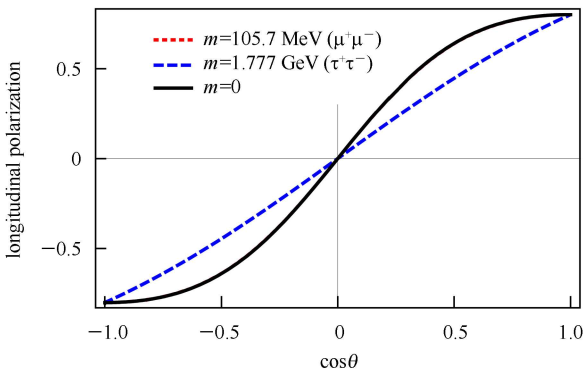

Fig. 1. The longitudinal polarizations of the final-state leptons produced at BEPC/BES (E=2.087 GeV). In the figure the dotted curve is in overlap with the solid curve.

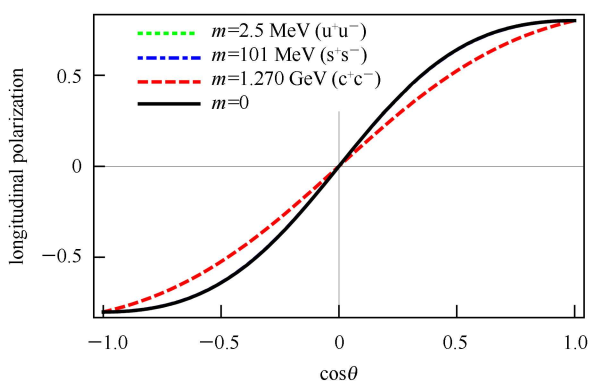

Fig. 2. The longitudinal polarizations of the final-state quarks produced at BEPC/BES (E=2.087 GeV). In the figure the dotted and dot-dashed curves are in overlap with the solid curve.

It can be seen from the figures that the longitudinal polarization curves P and $P_{m,m_e\to 0}$ are almost overlapped when the final-state particles are $\mu^+\mu^-$ or $u\bar{u}$. This indicates that ignoring the masses of the initial and final state particles has no significant effect on the final-state fermion longitudinal polarization in this case; when the final-state particles are $\tau^+\tau^-$ or $c\bar{c}$, the curve $P_{m,m_e\to 0}$ clearly separates from the curve P, and this indicates that ignoring the masses of the initial and final state particles has a significant effect on the final-state fermion longitudinal polarization in this case.

When $\cos\theta$=0.5, the deviation caused by ignoring the masses of the initial and final state particles for various fermion pair productions, i. e. $\Delta P_{m,m_e\to 0}=P_{m,m_e\to 0}-P$, is shown in Table 1.

Table 1. The deviation caused by ignoring the masses of the initial and final state particles for various fermion pair productions (E=2.087 GeV, $\cos\theta$=0.5).

	m/MeV	$\Delta P_{m,m_e\to 0}$
$\mu^+\mu^-$	105.7	9.83509×10^{-4}
$\tau^+\tau^-$	1777	1.94004×10^{-1}
$u\bar{u}$	2.5	5.74039×10^{-7}
$d\bar{d}$	5.0	2.22709×10^{-6}
$s\bar{s}$	101	8.98112×10^{-4}
$c\bar{c}$	1270	1.16347×10^{-1}

When taking the final-state particle masses into account while ignoring the incident electron mass, Eq. (4) becomes

$$P_{m_e\to 0}=\frac{2(\omega_1+\omega_2)\cos\theta}{(1+\omega_1\omega_2)(1+\cos^2\theta)+\dfrac{m^2}{E^2}(1+\omega_1\omega_2)(1-\cos^2\theta)}. \quad (6)$$

When $\cos\theta=0.5$, the deviation caused by this approximation, i. e. $\Delta P_{m_e\to 0}=P_{m_e\to 0}-P$, is shown in Table 2.

Table 2. The deviation caused by ignoring the incident electron mass for various fermion pair productions (E=2.087 GeV, $\cos\theta$=0.5).

	m/MeV	$\Delta P_{m_e\to 0}$
$\mu^+\mu^-$	105.7	2.29701×10^{-8}
$\tau^+\tau^-$	1777	1.38814×10^{-8}
$u\bar{u}$	2.5	2.30212×10^{-8}
$d\bar{d}$	5.0	2.30211×10^{-8}
$s\bar{s}$	101	2.29746×10^{-8}
$c\bar{c}$	1270	1.73142×10^{-8}

4 Longitudinal polarizations of particles produced at the B-factory

Then we consider the case at the B-factory. As discussed above, substituting the B-factory energy E = 6.0 GeV for BEPC/BES energy E=2.087 GeV, and including the case of $b\bar{b}$ production, we can plot the curves of the final-state fermion longitudinal polarization P and $P_{m,m_e\to 0}$ versus $\cos\theta$ as shown in Fig. 3 and Fig. 4.

At the B-factory, as shown in the figures, only the mass of b quark has significant effect on the final-state fermion longitudinal polarization, while the masses of τ lepton and c quark are negligible.

The deviation caused by ignoring the masses of the initial and final state particles for various fermion pair productions at the B-factory, i.e., $\Delta P_{m,m_e\to 0}=P_{m,m_e\to 0}-P$, is shown in Table 3.

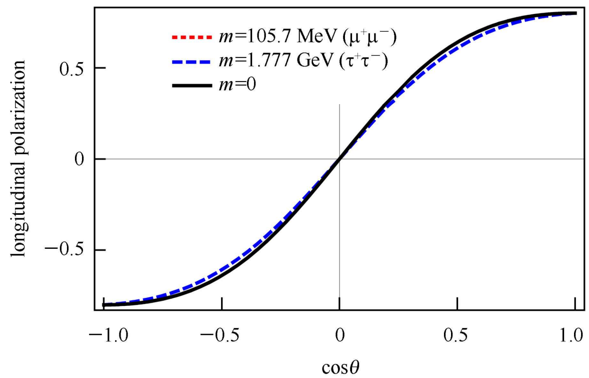

Fig. 3. The longitudinal polarizations of the final-state leptons produced at the B-factory (E=6.0 GeV). In the figure the dotted curve is in overlap with the solid curve.

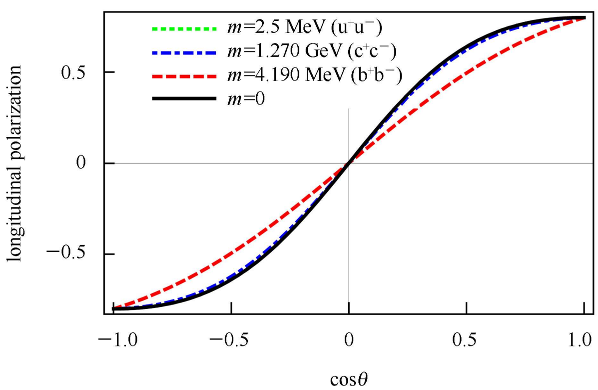

Fig. 4. The longitudinal polarizations of the final-state quarks produced at the B-factory (E=6.0 GeV). In the figure the dotted and dot-dashed curves are in overlap with the solid curve.

Table 3. The deviation caused by ignoring the masses of the initial and final state particles for various fermion pair productions (E=6.0 GeV, $\cos\theta$=0.5).

	m/MeV	$\Delta P_{m,m_e\to 0}$
$\mu^+\mu^-$	105.7	1.19154×10^{-4}
$\tau^+\tau^-$	1777	3.19984×10^{-2}
$u\bar{u}$	2.5	6.94519×10^{-8}
$d\bar{d}$	5.0	2.69452×10^{-7}
$s\bar{s}$	101	1.08795×10^{-4}
$c\bar{c}$	1270	1.67539×10^{-2}
$b\bar{b}$	4190	1.44875×10^{-1}

5 Conclusion

In conclusion, we study the mass effect for the polarized electron-positron annihilation processes at the BEPC/BES facility and the B-factory. From our study, we find that the masses of the final state fermion pairs may have some effects in the higher precision measurements of these facilities with high luminosity. Thus our study is useful for the design of possible polarization investigation at these electron-positron colliders, and also for the physical programs involving the production of heavy flavored hadrons. The super-tau-charm factory has advantages for the study of $\tau^+\tau^-$ and $c\bar{c}$ polarization effects compared with the B-factory, especially, observables for CP violation in τ decays can be well defined with polarized electron/positron beams at the super-tau-charm factory. More studies are still needed for making clear the influences due to the mass effect in various processes for the future super-tau-charm factory.

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