Electric and magnetic screenings of gluons in a model with a dimension-2 gluon condensate*

XU Fu-Kun(徐甫坤)[1] HUANG Mei(黄梅)[1,2]

[1] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
[2] Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, China

Abstract: Electric and magnetic screenings of thermal gluons are studied using the background expansion method in a gluodynamic model with a gauge invariant dimension-2 gluon condensate at zero momentum. At low temperature, the electric and magnetic gluons are degenerate. With the increase of temperature, it is found that the electric and magnetic gluons start to split at a certain temperature T_0. The electric screening mass changes rapidly with temperature when $T>T_0$, and the Polyakov loop expectation value rises sharply around T_0 from zero in the vacuum to a value around 0.8 at a high temperature. This suggests that the color electric deconfinement phase transition is driven by electric gluons. It is also observed that the magnetic screening mass remains almost the same as its vacuum value, which manifests that the magnetic gluons remain confined. Both the screening masses and the Polyakov loop results are qualitatively in agreement with the Lattice calculations.

Key words: dimension-2 gluon condensate, color eletric deconfinement, color magnetic confinement

PACS: 12.38.Aw, 14.70.Dj, 21.65.Qr **DOI:** 10.1088/1674-1137/37/1/014103

1 Introduction

A QCD vacuum is characterized by spontaneous chiral symmetry breaking and color confinement. It is expected that chiral symmetry can be restored and color degrees of freedom can be freed at high temperature and/or density.

The spontaneous breaking of chiral symmetry is well understood by the dimension-3 quark condensate $\langle \bar{q}q \rangle$ [1] in the vacuum, which is the order parameter in the chiral limit when the current quark mass is zero $m=0$, and the chiral restoration is characterized by the vanishing of the quark condensate.

The mechanism of confinement still remains a challenge. The confinement is normally taken as the color singlet nature of the spectrum. However, the nature of the color singlet spectrum is not unique for the QCD, but also holds for gauge-Higgs theories in which the gauge group is spontaneously broken. From the specific features of QCD dynamics, the Regge trajectories of hadrons indicate the string-picture of the hadrons, and the confinement can be described by the string picture of the hadrons or the linear potential between two quarks at large distances, i.e. $V_{\bar{\mathrm{Q}}\mathrm{Q}}(R)=\sigma R$ with σ as the string tension. There has been a great effort made to understand the emergence of the string-like object, e.g. the Abrikosov flux tubes [2], the dual superconductor scenario induced by monopole condensation [3], and the center vortices [4]. In the limits of an infinite heavy current quark mass, the flux tube never breaks, and it corresponds to the scenario of "permanent confinement". From the symmetry point of view, when the current quark mass goes to infinity $m\rightarrow\infty$, QCD becomes the pure gauge $SU(3)$ theory, which is centrally symmetric in the vacuum. The non-vanishing string tension corresponds to the area law for the Wilson loop, vanishing Polyakov lines, perimeter-law for the 't Hooft loops or the area-law falloff for the vortex free energy [5]. The deconfinement phase transition referring to the "permanent confinement" is characterized by the breaking of central symmetry, and the order parameter usually used is the Polyakov loop expectation value $\langle L \rangle$ [6].

In the framework of QCD effective models, there is still no dynamical model which can describe the breaking and restoration of the chiral symmetry, and the confinement deconfinement phase transition simultaneously. The main difficulty in creating an effective QCD model to include a confinement mechanism lies in that it is hard to calculate the Polyakov loop analytically. Currently, popular models used to investigate the chiral and deconfinement phase transitions are the Polyakov Nambu-Jona-Lasinio model (PNJL) and Polyakov linear sigma

Received 28 April 2012
* Supported by NSFC (11275213), DFG and NSFC (CRC110), CAS (KJCX2-EW-N01), K. C. Wong Education Foundation, CAS Program "Outstanding Young Scientists Abroad Brought in and Youth Innovation Promotion Association of CAS

model (PLSM) [7]. However, in this kind of models, the Polyakov loop potential part is not dynamically induced. A dynamical model for describing confinement as well as the deconfinement phase transition is still missing.

There has also been a great effort to understand confinement and deconfinement from low-energy Gluodynamics. Various vacuum condensates provide important information to understand the non-perturbative dynamics of QCD. For example, the gauge invariant dimension-4 gluon condensate $\langle g^2G^2\rangle$ has been widely investigated in both QCD sum rules and lattice calculations [8–10], and the non-vanishing value of the condensate does not signal the breaking of any symmetry directly, but rather the non-perturbative dynamics of strongly interacting gluon fields. In the last decade, there has been a growing interest in dimension-2 gluon condensates $\langle g^2A^2(x)\rangle$ in the $SU(N_c)$ gauge theory [11–22], with the local dimension-2 operator

$$A^2(x)=\sum_{a=1}^{N_c^2-1}\sum_{\mu=1}^{4}A_\mu^a(x)A_\mu^a(x). \tag{1}$$

The dimension-2 gluon condensate breaks the properties of gauge invariance, and it has been investigated in various gauges. For example, the dimension-2 operator A^2 gets a special meaning in the Landau gauge [16, 20], in which the condensate is at an extreme and plays as a saddle point on its gauge orbit, and a BRST-invariant mixed gluon-ghost condensate has been introduced in [17]. Though it is not gauge invariant, the growing interest in the dimension-2 gluon condensate lies in that it is related to the production of the dynamical gluon mass, and the possible connection between the minimal value of the $\langle A^2\rangle_{\min}$ and the topological defects (e.g. the magnetic monopoles [16]). Furthermore, the dimension-2 gluon condensate has a closer relationship with confinement, the dimension-2 gluon condensate yields the UV corrections Λ^2/Q^2 in the QCD running coupling constant $\alpha_s(Q^2)$, which leads to the linear potential $\sigma_s R$ at short distances with $\sigma_s\simeq g_R^2\langle A_\mu^2\rangle$.

By extracting the lattice data of the Polyakov loop, the work in Ref. [23] shows that to create the correct behavior of the Polyakov loop around T_c, the dimension-2 gluon condensate is essential. It is also shown in holographic QCD models [24, 25], that the dimension-2 gluon condensate plays an essential role in realizing the linear heavy quark potential as well as the deconfinement phase transition. It is of great interest to investigate the behavior of the dimension-2 gluon condensate at a finite temperature and its role in the deconfinement phase transition in a gluodynamic model.

Under the zero temperature case the space-time space is symmetric under the $O(4)$ rotation, i.e. all Lorentz components of the gauge field A_μ contribute equally to the vacuum. In the finite temperature, it is more appropriate to divide the gauge boson into time-like (electric) and space-like (magnetic) components [26, 27]. This can be viewed as the different components of the overall variable, because the rotational symmetry is broken down to (approximate) $O(3)$ spatial symmetry as the time direction deduces to a finite volume with $\beta=1/T$. In fact, as we will show, the electric and magnetic components are quite nontrivial and behave quite differently at a finite temperature.

On the other hand, the color screening effect is one of the main features of the quark-gluon plasma (QGP) and has been widely investigated in lattice and effective theories [28–39]. Significant evidence shows that gluon confinement is not affected by a small (physical) number of light quarks [33, 38] and the nonperturbative features of QCD are most probably generated in the gauge sector. It is therefore reasonable to study the behavior of screening of gluons at a finite temperature. The Lattice result shows that the QCD coupling constant strength near the critical temperature T_c is still of the order of one [34], and the perturbation theory cannot be applied in this region. Especially in the regime right above the critical temperature, the nonperturbative effects are supposed to be important.

Therefore, in this work we extend the pure gluodynamic model with gauge invariant dimension-2 gluon condensate at zero momentum in the vacuum [40], and estimate the electric as well as magnetic screening masses of gluons at a finite temperature. We also investigate the contribution of the dimension-2 gluon condensate to the deconfinement phase transition.

This paper is structured as follows. In Section 2 we introduce the pure gluodynamic model with dimension-2 gluon condensate in the vacuum, which was developed by Celenza and Shakin [40]. Then in Section 3, we extend the gluodynamic model to finite temperature and define the electric and magnetic screening masses from the gluon self-energy tensor. We give the numerical results of the electric and magnetic screening masses as well as the Polyakov loop expectation value in Section 4 and give the summary in Section 5.

2 The gluodynamic model with a dimension-2 gluon condensate

In this section, we follow Ref. [40] to introduce the Celenza-Shakin model which gives an effective action for a pure gluon system with a dimension-2 gluon condensate. As an overall notation the paper is in the framework of Euclidean space.

The pure gluon part of QCD Lagrangian is described by

$$\mathscr{L}_G=-\frac{1}{4}G_{\mu\nu}^aG_{\mu\nu}^a, \tag{2}$$

with

$$G^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu. \tag{3}$$

Motivated by the Nambu–Jona-Lasinio model with quark-antiquark condensate in the vacuum, which is similar to the BCS pairing condensation in the superconductor, Celenza-Shakin proposed the “pairing” of two gluon condensates in the vacuum in Ref. [40]. The Bose-Einstein condensation of two “pairing” gluons at zero momentum is gauge invariant as demonstrated in Ref. [40]. The key observation is that the condensation is determined by its value at zero momentum $\langle g^2A^2(k=0)\rangle$, which also means that the condensate is coordinate independent and the coherence length is macroscopically large. Here we clearly demonstrate why the dimension-2 gluon condensate is gauge invariant. The local gauge symmetry is defined as

$$\begin{aligned}A_\mu(x)\to A'_\mu(x) \;=\; & U(x)A_\mu(x)U^{-1}(x)\\ &+\frac{\mathrm{i}}{g}(\partial_\mu U(x))U^{-1}(x).\end{aligned} \tag{4}$$

Accordingly, the dimension-2 gluon opertator in momentum space is changed as

$$\langle g^2A^2(k)\rangle\to\langle g^2A'^{\,2}(k)\rangle=\langle g^2A^2(k)\rangle+2\langle g^2k\cdot A\rangle+\langle g^2k\cdot k\rangle. \tag{5}$$

For a condensate, only the $k=0$ mode should be considered, and we arrive at

$$\langle g^2A'^{\,2}(k=0)\rangle=\langle g^2A^2(k=0)\rangle, \tag{6}$$

which simply means that the dimension-2 gluon condensate is gauge invariant.

The gluon field can be decomposed into a condensate field $\mathbb{A}^a_\mu$ and a fluctuating field $\mathscr{A}^a_\mu$ [40, 41] as,

$$A^a_\mu(x):=\mathbb{A}^a_\mu+\mathscr{A}^a_\mu(x), \tag{7}$$

where $\mathbb{A}^a_\mu$ is macroscopically occupied and independent of x, which carries a zero vacuum expectation value, i.e. $\langle \mathrm{vac}|\mathbb{A}^a_\mu|\mathrm{vac}\rangle=0$. In the system of our work, the gluons condense into each of eight color states and various spatial directions, and both the gauge invariance and Lorentz invariance are preserved, which is different from the Savvidy vacuum [42] with an external chromomagnetic field in the background. However, it is worth mentioning that the Nielsen-Olesen instability [43] in the Savvidy vacuum can be resolved by the dimension-2 gluon condensate as demonstrated in Refs. [44, 45].

The Fourier transformation of Eq. (7) has the form of

$$A^a_\mu(k):=\mathbb{A}^a_\mu(k=0)+\mathscr{A}^a_\mu(k)\equiv\mathbb{A}^a_\mu+\mathscr{A}^a_\mu(k), \tag{8}$$

where the background $\mathbb{A}^a_\mu$ carries only zero momentum mode, and for simplicity we assume it to be a constant.

By using the expansion Eq.(8), the gluon part of the QCD Lagrangian becomes

$$\begin{aligned}\mathscr{L}_G \;=\; -\frac{1}{4}\Big[&\mathscr{G}_{\mu\nu}\mathscr{G}_{\mu\nu}+2gf^{abc}\mathscr{G}^a_{\mu\nu}(\mathbb{A}^b_\mu\mathscr{A}^c_\nu+\mathscr{A}^b_\mu\mathbb{A}^c_\nu+\mathbb{A}^b_\mu\mathbb{A}^c_\nu)\\ &+g^2f^{eab}f^{ecd}(\mathbb{A}^a_\mu\mathscr{A}^b_\nu+\mathscr{A}^a_\mu\mathbb{A}^b_\nu)(\mathbb{A}^c_\mu\mathscr{A}^d_\nu+\mathscr{A}^c_\mu\mathbb{A}^d_\nu)\\ &+2g^2f^{eab}f^{ecd}\mathbb{A}^a_\mu\mathbb{A}^b_\nu(\mathbb{A}^c_\mu\mathscr{A}^d_\nu+\mathscr{A}^c_\mu\mathbb{A}^d_\nu)\\ &+g^2f^{eab}f^{ecd}\mathbb{A}^a_\mu\mathbb{A}^b_\nu\mathbb{A}^c_\mu\mathbb{A}^d_\nu\Big].\end{aligned} \tag{9}$$

As a further assumption one can treat $\mathbb{A}^a_\mu$ as a classical variable:

$$\mathbb{A}^a_\mu:=\phi_0\hat{\eta}^a_\mu, \tag{10}$$

where ϕ_0 is a constant and $\hat{\eta}^a_\mu$ is a vacuum vector. The vector $\hat{\eta}^a_\mu$ has the following properties:

$$\hat{\eta}\equiv\frac{\eta}{|\eta|},\quad \eta^a_\mu\equiv(\eta^a_4,\,\vec{\eta}^a),\quad (\hat{\eta}^a_\mu)^2=1,\quad \eta^2=\eta^a_\mu\eta^a_\mu=32.$$

The averaging procedure for an operator $O[\hat{\eta}]$ may be written as

$$\langle O[\hat{\eta}]\rangle_{\hat{\eta}}=\frac{\int\prod_{a'}\hat{\eta}_{a'}\delta(\hat{\eta}\cdot\hat{\eta}-1)O[\hat{\eta}]}{\int\prod_{a'}\hat{\eta}_{a'}\delta(\hat{\eta}\cdot\hat{\eta}-1)}.$$

Now that the field η^a_μ plays as the vacuum degree of freedom, then one can consider the expectation value of this averaging as the vacuum expectation i.e.

$$\langle \mathrm{vac}|O[A^a_\mu]|\mathrm{vac}\rangle\equiv\langle \mathrm{vac}|O[\mathbb{A}^a_\mu]|\mathrm{vac}\rangle\equiv\langle O[\hat{\eta}^a_\mu]\rangle_{\hat{\eta}}.$$

After taking the expecting value in terms of η^a_μ, one gets

$$\langle\mathbb{A}^a_\mu\mathbb{A}^b_\nu\rangle_{\hat{\eta}}=\frac{\delta^{ab}}{8}\frac{\delta_{\mu\nu}}{4}\phi_0^2,\qquad \langle\mathbb{A}^a_\mu\mathbb{A}^a_\mu\rangle_{\hat{\eta}}=\phi_0^2. \tag{11}$$

Actually it is the nonzero expectation value of the double combination $\mathbb{A}^2$ that plays as an order parameter representing the existence of condensate but not the gauge field A^a_μ as one spontaneously has the constraint of

$$\langle O[(\mathbb{A}^a_\mu)^{\mathrm{odd}}]\rangle_{\hat{\eta}}=0. \tag{12}$$

Then the Lagrangian after this background expansion becomes

$$\langle\mathscr{L}\rangle_{\hat{\eta}}=-\frac{1}{4}\langle GG\rangle_{\hat{\eta}}=-\frac{1}{4}\left[\mathscr{G}\mathscr{G}+2m_{\mathrm{g}}^2\mathscr{A}^2+4b\phi_0^4\right], \tag{13}$$

with

$$m_{\mathrm{g}}^2=\frac{9}{32}g^2\phi_0^2,\quad b=\frac{9}{136}g^2. \tag{14}$$

The gluon gets mass because of the existence of nonperturbative dimension-2 gluon condensate. We note that the dimension-four gluon condensate $\langle g^2G^2\rangle_{\hat{\eta}}$ is proportional to the dimension-2 gluon condensate $\langle g^2A^2\rangle^2_{\hat{\eta}}$.

3 Electric and magnetic screening at finite temperature

We now use the Lagrangian in Eq. (13) as the effective model of a pure gluon system. At a finite temperature, the temporal and spatial direction of the gluon field is generally different, i.e.

$$\mathscr{A}:=(\mathscr{A}_4,\vec{\mathscr{A}}), \tag{15}$$

and the Lagrangian can be written as

$$\langle\mathscr{L}\rangle_{\hat{\eta}}=-\frac{1}{4}\langle GG\rangle_{\hat{\eta}}=-\frac{1}{4}\left[\mathscr{G}\mathscr{G}+2(m_{\rm E}^2\mathscr{A}_4^2+m_{\rm M}^2\vec{\mathscr{A}}^2)+4b\phi_0^4\right], \tag{16}$$

In the zero temperature limit, one has $m_{\rm E}^2=m_{\rm M}^2\equiv m_{\rm g}^2$.

By adding the gauge-fixing term in Lagrangian i.e. $\mathscr{L}_{\rm fix}=-\frac{1}{2\xi}(\partial_\mu\mathscr{A}_\mu)^2$, one can solve the gluon propagator of the fluctuating field $\mathscr{A}_\mu^a$ from the equation of

$$\left[K^2\delta_{\mu\nu}-(1-1/\xi)K_\mu K_\nu+m_{\rm E}^2\delta_{44}+m_{\rm M}^2\delta_{ij}^{\mu\nu}\right]\cdot D_{\nu\sigma}(K)=\delta_{\mu\sigma}. \tag{17}$$

The gluon propagator has the form of

$$D_{\mu\nu}(K)=\frac{P_{\mu\nu}^T}{K^2+m_{\rm M}^2}+\frac{K^2P_{\mu\nu}^L+\xi\left(m_{\rm M}^2\delta_{44}+K_\mu K_\nu+m_{\rm E}^2k_\mu k_\nu/k^2\right)}{K^2(K^2+m_{\rm E}^2)-K_4^2(m_{\rm E}^2-m_{\rm M}^2)+\xi(k^2m_{\rm M}^2+K_4^2m_{\rm E}^2+m_{\rm M}^2m_{\rm E}^2)}. \tag{18}$$

In the limit of $\xi\to\infty$, i.e. in the unitary gauge, the gluon propagator takes the form of

$$D_{\mu\nu}(K)=\frac{1}{K^2+m_{\rm M}^2}\left(\delta_{ij}-\frac{k_\mu k_\nu}{k^2}\right)+\frac{1}{k^2m_{\rm M}^2+K_4^2m_{\rm E}^2+m_{\rm E}^2m_{\rm M}^2}\left(\delta_{44}m_{\rm M}^2+K_\mu K_\nu+m_{\rm E}^2\frac{k_\mu k_\nu}{k^2}\right). \tag{19}$$

In the zero temperature limit ($m_{\rm E}^2=m_{\rm M}^2=m_{\rm g}^2$) it becomes a simple form

$$D_{\mu\nu}(K)=\frac{1}{K^2+m_{\rm g}^2}\left(\delta_{\mu\nu}-\frac{K_\mu K_\nu}{m_{\rm g}^2}\right).$$

The screening masses are defined as the gluon self-energy tensor $\Pi_{\mu\nu}^{ab}(p_4,p)$ at the static limit ($p_4=0,\ p\to 0$) [46, 47], and the electric and magnetic screening masses take the following expressions:

$$m_{\rm E}^2\delta_{44}\delta^{ab}=-\Pi_{44}^{ab}(0,p\to 0),\quad m_{\rm M}^2\delta_{ij}\delta^{ab}=-\Pi_{ij}^{ab}(0,p\to 0). \tag{20}$$

Here the gluon self-energy tensor is with a full propagator so that it contains both the perturbative and the nonperturbative contributions of the interaction of the gauge field. As pointed out by some authors, the above definition does not yield a gauge invariant definition of the screening masses in a strict sense.

On the other hand, we suggest a nonperturbative iterative relation of the gluon mass similar to the Dyson-Schwinger method [48], i.e. the value of the screening mass especially at a finite temperature is decided by the gluon self-energy as shown in Fig. 1.

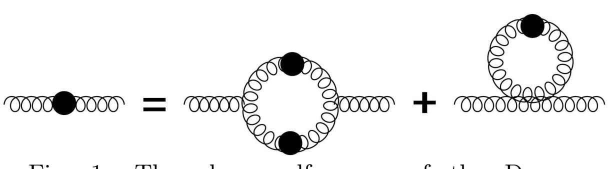

Fig. 1. The gluon self-energy of the Dyson-Schwinger-equation like method.

The direct calculation by using propagator Eq. (19) gives

$$\begin{aligned}&\Pi_{G,44}^{ab}(P=0)\\ &=-g^2N_{\rm c}\delta_{44}\delta^{ab}T\sum_n\int\frac{{\rm d}^3k}{(2\pi)^3}\left(2\frac{-\omega_n^2+k^2+m_{\rm M}^2}{(K^2+m_M^2)^2}\right.\\ &\qquad\left.+m_{\rm E}^2\frac{k^2m_{\rm M}^2-\omega_n^2m_{\rm E}^2+m_{\rm E}^2m_{\rm M}^2}{(k^2m_{\rm M}^2+\omega_n^2m_{\rm E}^2+m_{\rm E}^2m_{\rm M}^2)^2}\right),\end{aligned} \tag{21}$$

$$\begin{aligned}&\Pi_{G,ij}^{ab}(P=0)\\ &=-g^2N_{\rm c}\delta_{ij}\delta^{ab}T\sum_n\int\frac{{\rm d}^3k}{(2\pi)^3}\left(2\frac{\omega_n^2+k^2/3+m_{\rm M}^2}{(K^2+m_{\rm M}^2)^2}\right.\\ &\qquad\left.+m_{\rm M}^2\frac{k^2m_{\rm M}^2/3+\omega_n^2m_{\rm E}^2+m_{\rm E}^2m_{\rm M}^2}{(k^2m_{\rm M}^2+\omega_n^2m_{\rm E}^2+m_{\rm E}^2m_{\rm M}^2)^2}\right).\end{aligned} \tag{22}$$

Then one immediately gets the results with parameters given.

4 Results and discussions

We firstly investigate the thermal behavior of electric and magnetic screening masses by using the definition Eq. (20) and the electric and magnetic gluon self-energy in Eqs. (21) and (22).

In our model, there are two input parameters, i.e. the dimension-four gluon condensate g^2G^2 or the nonperturbative coupling constant g, and the momentum cutoff parameter Λ at zero temperature. For simplicity we assume that the coupling constant g and cutoff parameter Λ remain constants even at a finite temperature. The value of dimension-four gluon condensate at zero temperature is derived both in QCD sum-rules (lower range of the interval) [8, 49] and in lattice (higher range of the interval) [50, 51]. (The dimension-four gluon condensate in the gluon plasma has been extracted in Ref. [53].) Different authors give different results but an acceptable candidate is $\langle g^2G^2\rangle=(0.009\pm0.006)\times4\pi^2$ GeV^4 [52]. We take the value of dimension-four gluon condensate as $\langle g^2G^2\rangle=0.009\times4\pi^2$ GeV^4, which corresponds to the dimension-2 gluon condensate $\langle g^2A^2\rangle=1.16$ GeV^2 and the gluon mass $m_{\rm g}=571$ MeV.

For the calculation of the momentum integral, we employ a soft-cutoff function (for example see [54]), which takes the the form of

$$f(K)=\mathrm{e}^{-\Lambda^2K^2}\equiv\mathrm{e}^{-\Lambda^2(\omega_n^2+k^2)}. \tag{23}$$

In the following numerical calculation, we choose $\Lambda=0.3$ [GeV^{-1}].

4.1 The electric and magnetic screening masses

The electric and magnetic screening masses as functions of the temperature are shown in Fig. 2. The solid line and the dashed-dotted line are for the electric and magnetic parts, respectively.

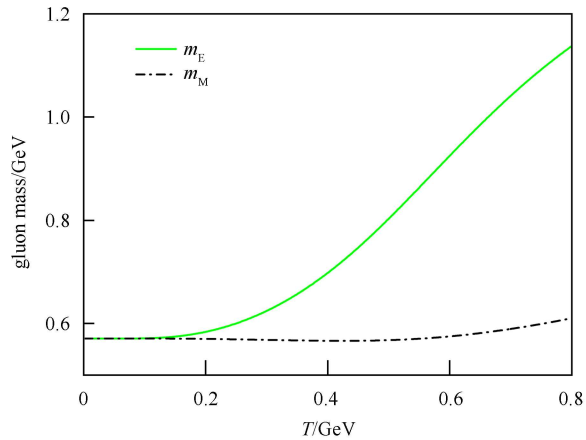

Fig. 2. The electric and magnetic screening masses as functions of temperature.

It is found that both electric and magnetic screening masses are degenerate and remain unchanged at a low temperature, and the electric and magnetic components start to split at the temperature $T_0=150$ MeV. In the temperature region $T>T_0$, the electric screening mass rises rapidly with the increase of temperature, however, the magnetic screening mass of the gluons remains almost the same as its vacuum value.

In order to compare with the lattice data in Ref. [36], we divide the screening masses by the temperature. We also assume the critical temperature $T_c=T_0=150$ MeV, where m_E and m_M start to split. (The exact value of T_c is not important here, and will not affect the qualitative property of the ratio of the screening mass over the temperature.) Fig. 3 shows the ratios of m_E/T and m_M/T as functions of T/T_c and compare between them and with the lattice data in Ref. [36]. The solid line and the dashed-dotted line are for the electric and magnetic parts, respectively.

It is found that the ratio of the electric screening mass over temperature m_E/T is around $\sim$1.8 in the region of $2<T/T_c<5$, which is qualitatively in agreement with the lattice result $m_E/T\sim2.3$. The ratio of the magnetic screening mass over temperature m_M/T is around 1 in the region of $2<T/T_c<5$, which is almost the same as the lattice result $m_M/T\sim1$. It is worth mentioning that $m_E/T>m_M/T$ in the temperature region of $T/T_c<3$ cannot be explained by using the perturbative scaling $m_E\sim gT$ and $m_M\sim g^2T$, because of the coupling constant $g(T)>1$ in this region.

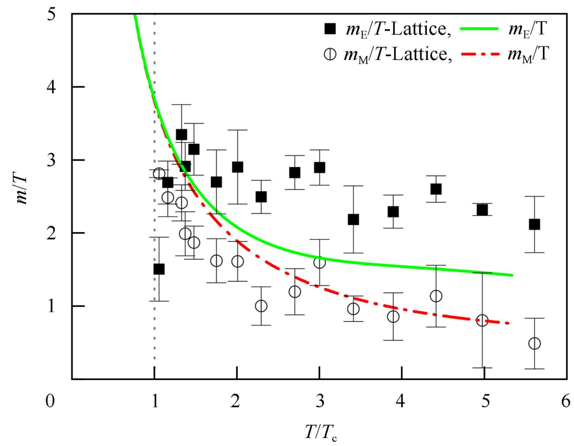

Fig. 3. The ratios of the screening masses m_E/T and m_M/T as functions of T/T_c. The lattice data are taken from Ref. [36].

4.2 Gauge dependence investigation

We have demonstrated clearly in Section 2 that the dimension-2 gluon condensate in zero momentum is gauge invariance. In this part, we will check the gauge dependence of the screening mass.

We have shown the screening masses by using the gluon propagator Eq. (18) in the unitary gauge, i.e. $\xi\to\infty$. By fixing the model parameters $\langle g^2G^2\rangle=0.009\times4\pi^2$ GeV^4 and Λ=0.3 [GeV^{-1}], in Fig. 4 we show the screening masses in different gauges. The solid lines are for the the Landau gauge ξ=0, the dash-dotted lines are for the Feynman gauge $\xi=1$, and the dotted lines

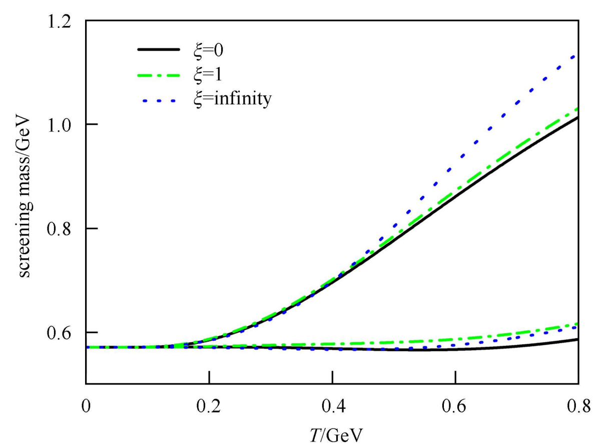

Fig. 4. The gauge dependence of the screening mass as a function of temperature.

are for the unitary gauge $\xi\to\infty$. It is found that below $T=500$ MeV, the screening masses are independent of different gauges. The gauge dependence starts to show up when $T>500$ MeV, the electric screening mass is more sensitive to the gauge fixing than the magnetic screening mass. In the temperature region that we are interested in, both the electric and magnetic screening masses are not sensitive to the gauge fixing.

4.3 The Polyakov loop expectation value

The deconfinement phase transition is characterized by the Polyakov-loop expectation value. The Polyakov-loop is defined as

$$L(x)=\mathscr{P}\exp[\mathrm{i}g\int_0^\beta \mathrm{d}\tau A_4(\boldsymbol{x},\tau)]. \tag{24}$$

In order to investigate the relationship between the dimension-2 gluon condensate and the deconfinement phase transition, it is necessary to calculate the Polyakov-loop expectation value. By using perturbative expansion [55], it has been observed in Ref. [23] that the Polyakov loop expectation value is associated with the electric dimension-2 gluon condensate by the following relationship:

$$\langle L\rangle=\exp\left[-\frac{g^2\langle A_4^2\rangle}{4N_{\mathrm{c}}T^2}\right]. \tag{25}$$

In our model, the electric dimension-2 gluon condensate has a simple relationship with the electric screening mass square, i.e. $\langle A_4^2\rangle=m_{\mathrm{E}}^2$.

We show the Polyakov loop expectation value as a function of T/T_{c} in Fig. 5, and compare the results with the lattice data in Ref. [56]. It is found that the Polyakov loop expectation value is zero in the vacuum and low temperature region, but it starts to rise at around $0.5T_{\mathrm{c}}$, then rises sharply to a value of 0.8 at a high temperature.

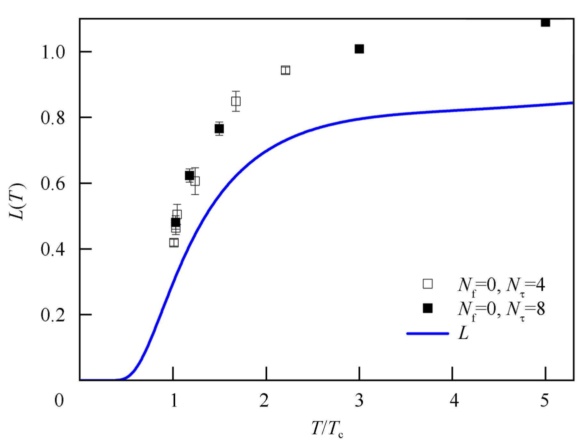

Fig. 5. The Polyakov loop expectation value as a function of T/T_{c} comparing with the lattice result in Ref. [56].

We have taken $T_{\mathrm{c}}=T_0=150$ MeV, where the electric and magnetic gluons start to split. It is worth mentioning that the susceptibility of the Polyakov loop expectation value indeed gives the critical temperature at around $T_{\mathrm{c}}=T_0$. Our simple model indicates that the color electric deconfinement phase transition is driven by the electric gluons, and although the nonperturbative dimension-2 gluon condensate plays an important role, it still gives at least an 80% contribution to the Polyakov loop expectation value even in the temperature region $T>3T_{\mathrm{c}}$.

5 Conclusions

We have investigated the electric and magnetic screenings of thermal gluons in a gluodynamic model with a dimension-2 gluon condensate in zero momentum, which spontaneously generates the effect of dynamical gluon mass in the vacuum.

It is found that the electric and magnetic gluons are degenerate at low temperature. With an increase in temperature, the electric and magnetic gluons start to split at a certain temperature around $T_0=150$ MeV. The electric screening mass changes rapidly with temperature at $T>T_0$, and the Polyakov loop expectation value rises sharply around T_0 from zero in the vacuum to a value around 0.8 at high temperature. This suggests that the color electric deconfinement phase transition is driven by electric gluons. It is also observed that the magnetic screening mass remains almost the same as its vacuum value, which manifests that the magnetic gluons remain confined. Both the screening masses and the Polyakov loop results are qualitatively in agreement with the Lattice calculations.

The Polyakov loop expectation value in this work is calculated by using the perturbation expansion, a more convenient way to derive the Polyakov loop expectation value is by using AdS/CFT method in the 5D holographic model, e.g. in Ref. [25] with a dimension-2 dilaton field background. It is worth mentioning that the dimension-2 dilaton field corresponds to a dimension-2 gluon condensate operator, and in Ref. [25], the Polyakov loop expectation value at finite temperature agrees well with the lattice data [56].

The model we used in this paper is quite simple, but it captures some important features of gluon dynamics in the vacuum as well as at a finite temperature. We can conclude that the dimension-2 gluon condensate plays an essential role both in confinement as well as in deconfinement phase transition.

The authors wish to thank H. Chen, T. Hatsuda, T. Mukherjee, N. Su, and Q.S. Yan for valuable discussions.

References

1 Nambu Y. Phys. Rev., 1960, **117**: 648–663

2 Nambu Y. In Symmetries and Quark Models. Ed. Chand R, Gordon and Breach. 1970; Nambu Y. Phys. Rev. D, 1974, **10**: 4262; Submitted to the 15th International Conference on High Energy Physics. Kiev, 1970; Nielsen H B, Olesen P. Nucl. Phys. B, 1973, **61**: 45; Susskind L. Nuovo Cim. A, 1970, **69**: 457

3 't Hooft. Nucl. Phys. B, 1981, **190**: 455; Mandelstam. Phys. Rep. C, 1976, **23**: 245

4 't Hooft G. Nucl. Phys. B, 1978, **138**: 1

5 Greensite J. Eur. Phys. J. ST, 2007, **140**: 1

6 Polyakov A M. Phys. Lett. B, 1978, **72**: 477

7 Fukushima K. Phys. Lett. B, 2004, **591**: 277; Ratti C, Thaler M A, Weise W. Phys. Rev. D, 2006, **73**: 014019; Sasaki C, Friman B, Redlich K. Phys. Rev. D, 2007, **75**: 074013; Schaefer B J, Pawlowski J M, Wambach J. Phys. Rev. D, 2007, **76**: 074023; MAO H, JIN J, HUANG M. J. Phys. G, 2010, **37**: 035001

8 Shifman M A, Vainshtein A I, Zakharov V I. Nucl. Phys. B, 1979, **147**: 385–447; 1979, **147**: 448–518

9 Boyd G, Engels J, Karsch F, Laermann E, Legeland C, Lutgemeier M, Petersson B. Nucl. Phys. B, 1996, **469**: 419–444. [hep-lat/9602007]

10 Schafer T, Shuryak E V. Rev. Mod. Phys., 1998, **70**: 323–426 [hep-ph/9610451]

11 Lavelle M J, Schaden M. Phys. Lett. B, 1988, **208**: 297

12 Lavelle M, Oleszczuk M. Mod. Phys. Lett. A, 1992, **7**: 3617–3630

13 Gubarev F V, Stodolsky L, Zakharov V I. Phys. Rev. Lett., 2001, **86**: 2220–2222. [hep-ph/0010057]

14 Verschelde H, Knecht K, van Acoleyen K, Vanderkelen M. Phys. Lett. B, 2001, **516**: 307–313. [hep-th/0105018]

15 Chetyrkin K G, Narison S, Zakharov V I. Nucl. Phys. B, 1999, **550**: 353–374. [hep-ph/9811275]

16 Gubarev F V, Zakharov V I. Phys. Lett. B, 2001, **501**: 28–36. [hep-ph/0010096]

17 Kondo K I. Phys. Lett. B, 2001, **514**: 335. [arXiv:hep-th/0105299]

18 Slavnov A A. Theor. Math. Phys., 2005, **143**: 489; Teor. Mat. Fiz., 2005, **143**: 3. [arXiv:hep-th/0407194]

19 Blossier B, Boucaud P, Brinet M, de Soto F, LIU Z, Morenas V, Pene O, Petrov K et al. Phys. Rev. D, 2011, **83**: 074506; Blossier B, Boucaud P, Brinet M, de Soto F, DU X, Gravina M, LIU Z, Morenas V et al. arXiv:1111.3023 [hep-lat]

20 Boucaud P, Le Yaouanc A, Leroy J P, Micheli J, Pene O, Rodriguez-Quintero J. Phys. Rev. D, 2001, **63**: 114003. [arXiv:hep-ph/0101302]

21 Dudal D, Verschelde H, Gracey J A, Lemes V E R, Sarandy M S, Sobreiro R F, Sorella S P. JHEP, 2004, **0401**: 044. [hep-th/0311194]

22 Dudal D, Verschelde H, Browne R E, Gracey J A. Phys. Lett. B, 2003, **562**: 87–96. [hep-th/0302128]

23 Megias E, Ruiz Arriola E, Salcedo L L. JHEP, 2006, **0601**: 073. [arXiv:hep-ph/0505215]

24 Andreev O, Zakharov V I. Phys. Rev. D, 2006, **74**: 025023. [arXiv:hep-ph/0604204]

25 HE S, HUANG M, YAN Q S. Phys. Rev. D, 2011, **83**: 045034; LI D, HE S, HUANG M, YAN Q S. JHEP, 2011, **1109**: 041. [arXiv:1103.5389 [hep-th]]

26 Chernodub M N, Ilgenfritz E M. Phys. Rev. D, 2008, **78**: 034036. [arXiv:0805.3714 [hep-lat]]

27 Vercauteren D, Verschelde H. Phys. Rev. D, 2010, **82**: 085026. [arXiv:1007.2789 [hep-th]]

28 GAO M. Phys. Rev. D, 1990, **41**: 626

29 Heller U M, Karsch F, Rank J. Phys. Rev. D, 1998, **57**: 1438–1448. [hep-lat/9710033]

30 Kaczmarek O, Karsch F, Laermann E, Lutgemeier M. Phys. Rev. D, 2000, **62**: 034021. [hep-lat/9908010]

31 Kraemmer U, Rebhan A. Rept. Prog. Phys., 2004, **67**: 351. [hep-ph/0310337]

32 Chakraborty P, Mustafa M G, Thoma M H. arXiv:1109.1971 [hep-ph]

33 Bowman P O, Heller U M, Leinweber D B, Parappilly M B, Williams A G. Phys. Rev. D, 2004, **70**: 034509. [hep-lat/0402032]

34 Kaczmarek O, Karsch F, Zantow F, Petreczky P. Phys. Rev. D, 2004, **70**: 074505. [hep-lat/0406036]; Kaczmarek O, Zantow F. Phys. Rev. D, 2005, **71**: 114510. [hep-lat/0503017]

35 Peshier A. [hep-ph/0601119]

36 Nakamura A, Saito T, Sakai S. Phys. Rev. D, 2004, **69**: 014506. [hep-lat/0311024]

37 Maezawa Y et al. (WHOT-QCD collaboration). Phys. Rev. D, 2010, **81**: 091501. [arXiv:1003.1361 [hep-lat]]

38 Fischer C S, Alkofer R. Phys. Rev. D, 2003, **67**: 094020. [arXiv:hep-ph/0301094]

39 Fischer C S, Maas A, Muller J A. Eur. Phys. J. C, 2010, **68**: 165. [arXiv:1003.1960 [hep-ph]]; Cucchieri A, Maas A, Mendes T. Phys. Rev. D, 2007, **75**: 076003. [arXiv:hep-lat/0702022]

40 Celenza L S, Shakin C M. Phys. Rev. D, 1986, **34**: 1591

41 LI X D, Shakin C M. Phys. Rev. D, 2005, **71**: 074007. [arXiv:hep-ph/0410404]

42 Savvidy G K. Phys. Lett. B, 1977, **71**: 133

43 Nielsen N K, Olesen P. Nucl. Phys. B, 1978, **144**: 376

44 Kondo K I. Phys. Lett. B, 2004, **600**: 287. [arXiv:hep-th/0404252]

45 Vercauteren D, Verschelde H. Phys. Lett. B, 2008, **660**: 432. [arXiv:0712.0570 [hep-th]]

46 Kapusta J I. Finite Temperature Field Theory. Cambridge: Cambridge University Press, 1989

47 Le Bellac M. Thermal Field Theory. Cambridge: Cambridge University Press, 1996

48 Roberts C D, Schmidt S M. Prog. Part. Nucl. Phys., 2000, **45**: S1-S103. [nucl-th/0005064]; Alkofer R, von Smekal L. Phys. Rept., 2001, **353**: 281. [arXiv:hep-ph/0007355]; Maris P, Roberts C D. Int. J. Mod. Phys. E, 2003, **12**: 297. [arXiv:nucl-th/0301049]

49 Geshkenbein B V. Sov. J. Nucl. Phys., 1990, **51**: 719–725; Yndurain F J. Phys. Rept., 1999, **320**: 287. [arXiv:hep-ph/9903457]; Ioffe B L, Zyablyuk K N. Eur. Phys. J. C, 2003, **27**: 229–241. [hep-ph/0207183]; Zyablyuk K. JHEP, 2003, **0301**: 081. [arXiv:hep-ph/0210103]; Samsonov A. [arXiv:hep-ph/0407199]

50 Boyd G, Miller D E. arXiv:hep-ph/9608482

51 Giacomo A Di, Rossi G C. Phys. Lett. B, 1981, **100**: 481; Giacomo A Di, Paffuti G. Phys. Lett. B, 1982, **108**: 327

52 Ioffe B L. Phys. Atom. Nucl., 2003, **66**: 30–43. [hep-ph/0207191]

53 Hietanen A, Kajantie K, Laine M, Rummukainen K, Schroder Y. JHEP, 2005, **0501**: 013. [arXiv:hep-lat/0412008]

54 Schmidt S M, Blaschke D, Kalinovsky Y L. Phys. Rev. C, 1994, **50**: 435–446; Gocke C, Blaschke D, Khalatyan A, Grigorian H. [hep-ph/0104183]; Grigorian H. Phys. Part. Nucl. Lett., 2007, **4**: 223–231. [hep-ph/0602238]; Gomez Dumm D, Scoccola N N. Phys. Rev. D, 2002, **65**: 074021. [arXiv:hep-ph/0107251]

55 Gava E, Jengo R. Phys. Lett. B, 1981, **105**: 285

56 Kaczmarek O, Karsch F, Petreczky P, Zantow F. Phys. Lett. B, 2002, **543**: 41–47. [hep-lat/0207002]