Heavy mesons in the Nambu–Jona-Lasinio model

GUO Xiao-Yu(郭霄雨) CHEN Xiao-Lin(陈晓林) DENG Wei-Zhen(邓卫真)[1)]

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing, 100871, China

Abstract: We propose an extended Nambu–Jona-Lasinio model to include heavy mesons with heavy quark symmetry. The quark current–current interaction is generalized to include the heavy quark currents. In order to comply with the heavy quark spin symmetry at the heavy quark limit, the dependence of the quark mass on the interaction strength is introduced. The light and heavy pseudo-scalar and vector mesons, their masses and the weak decay constants are calculated in the unified frame.

Key words: NJL model, heavy meson, heavy quark limit

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1 Introduction

In recent years, some exotic hadron states have been observed experimentally, and many of them cannot be explained easily as conventional quarkonia. A possible interpretation of this is the molecular state hypothesis [1–3]. Many of the studies on exotic states are based on heavy quark effective theory (HQET), and recently the chiral quark model has been used to solve the molecular state [4, 5]. Currently, the greatest difficulty in identifying a molecular state is the uncertainty surrounding the interaction strengths and form factor parameters.

In principle, these parameters can be calculated from QCD at the quark level. However, in the low-energy region, where the QCD perturbation method fails, we have to rely on effective theories. Among these, the Nambu–Jona–Lasinio (NJL) model [6, 7] is widely used to investigate many low-energy hadron problems related to QCD symmetries in a simple way [8–10].

By means of the Dyson-Schwinger equation (DSE), the dynamic quark mass is generated from spontaneous chiral symmetry breaking. After solving the Bethe-Salpeter equation (BSE), pseudo-scalar mesons are obtained as Goldstone bosons [6, 7]. Other mesons, such as vector mesons and axial-vector mesons, are included by introducing more chiral invariant interactions [11–13]. The model is also extended to comprise the strange flavor [14, 15]. In addition, a bosonization technique has been developed [16] and many studies have been performed using this approach [17–20].

Because of QCD color coulomb interaction, a heavy quark spin symmetry is reached in the heavy quark limit where the dependency of hadronic matrix elements on the orientation of the heavy quark spin vanishes [21]. From the heavy quark symmetry, HQET formalism was developed (for a review, see Refs. [22, 23]).

Some efforts have been made to study the heavy mesons within the NJL model [24, 25]. The bosonization technique was used in these studies to obtain the Lagrangian meson in HQET. Using the heavy quark propagators in the heavy quark limit, the DSE+BSE approach was also used to calculate heavy meson observables [26, 27].

In the NJL model study, the color-octet vector current interaction $(\bar{\psi}\lambda_{\rm C}^{\rm a}\gamma_\mu\psi)(\bar{\psi}\lambda_{\rm C}^{\rm a}\gamma^\mu\psi)$ is widely adopted since it is closely related to QCD interaction. In many DSE+BSE calculations, such as in Ref. [28], the interaction between two quarks was assumed to be intermediated by the gluon with a complicated effective propagator. So the color-octet vector current of the quark should be dominant. The DSE+BSE calculation using the gluon propagator was also performed in the heavy meson case [29]. If we naively treat the gluon propagator as a constant in the coordinate space, we would obtain an NJL model with color-octet vector current interaction. In the heavy quark limit where the heavy quark mass $m_{\rm Q}$ tends to infinity, we will show that the heavy quark spin symmetry is valid only for the color-octet vector current interaction.

Other contact interactions, such as the color-octet axial-vector current interaction $(\bar{\psi}\lambda_{\rm C}^{\rm a}\gamma_\mu\gamma_5\psi)(\bar{\psi}\lambda_{\rm C}^{\rm a}\gamma^\mu\gamma_5\psi)$, are needed to give a more comprehensive description of light flavor mesons such as the ρ meson [11–15]. We will show that heavy quark spin symmetry would not be

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1) E-mail: dwz@pku.edu.cn

reached if these interactions exist in the heavy quark limit. To maintain this symmetry, these interactions should be considered as higher order terms and should be $1/m_{\rm Q}$ suppressed. This is critical to extending the NJL model to include heavy quark flavors.

We will extend the NJL model to include the heavy quark flavors, and the typical DSE+BSE approach will be used to obtain the heavy meson properties. In this way, we can calculate the mass splitting between the pseudo-scalar mesons D (or B) and the vector mesons D^* (or B^*), which is the effect of finite heavy quark mass according to heavy quark expansion. Due to the fact that the heavy quark masses are far beyond the NJL cutoff scale, the usual four-dimensional cutoff is not appropriate here. We will use the three-dimensional cutoff following Refs. [6, 7, 11].

In the next section, we will generalize NJL interaction to include the heavy quark flavor and derive the mass dependence of the coupling strength parameters according to heavy quark spin symmetry. In Section 3 we will give a brief account of the DSE+BSE formalism to treat the quark and meson states. In Section 4 we will take the heavy quark limit and demonstrate the heavy quark spin symmetry, and in Section 5 a numerical calculation will be performed and the results will be compared with the empirical data. Finally we will give a brief summary.

2 NJL interaction with heavy quark symmetry

In many NJL studies, when dealing with the three light flavors, q=u,d,s, the interaction is taken to be the color current interaction

$$\mathcal{L}_4=G_{\rm V}(\bar{q}\lambda_{\rm C}^{\rm a}\gamma_\mu q)^2+G_{\rm A}(\bar{q}\lambda_{\rm C}^{\rm a}\gamma_\mu\gamma_5 q)^2. \qquad (1)$$

The interaction maintains the $U_{\rm f}(3)\otimes U_{\rm f}(3)\otimes SU_{\rm C}(3)$ symmetry. Here we will not consider the six-quark interaction which was used to deal with the $U_{\rm A}(1)$ anomaly since we are not concerned with the anomaly here and the contribution of the anomaly term is small [15]. After a Fierz transformation, we can get a Fierz invariant interaction

$$\begin{aligned}\mathcal{L}_4^F \;=\; & \frac{4}{9}G_1\sum_{i=0}^{8}\left[(\bar{q}\lambda_{\rm f}^i q)^2+(\bar{q}i\gamma_5\lambda_{\rm f}^i q)^2\right]\\ & -\frac{2}{9}G_2\sum_{i=0}^{8}\left[(\bar{q}\lambda_{\rm f}^i\gamma_\mu q)^2+(\bar{q}\lambda_{\rm f}^i\gamma_\mu\gamma_5 q)^2\right]\\ & +\text{color-octet terms}, \qquad (2)\end{aligned}$$

where

$$G_1=G_{\rm V}-G_{\rm A},\quad G_2=G_{\rm V}+G_{\rm A}. \qquad (3)$$

Here the λ_i's are the flavor Gell-Mann matrices with $\lambda_0\equiv\sqrt{\frac{2}{3}}1$. The color-octet terms do not contribute to the DSE+BSE calculation of the meson.

In Ref. [24], where the heavy flavors Q = c,b were considered, only the color-octet vector interaction $(\bar{q}\lambda_{\rm C}^{\rm a}\gamma_\mu q)(\bar{Q}\lambda_{\rm C}^{\rm a}\gamma^\mu Q)$ was considered. In Section 4, we will show that only this term in Eq. (1) respects the heavy quark spin symmetry in the heavy quark limit.

The color-octet vector interaction is, however, not enough to describe the light flavor mesons such as the vector ρ meson. We will also show that the heavy quark spin symmetry would not be reached if the axial-vector interaction exists in the heavy quark limit. To consistently describe the light sector and the heavy sector of the meson system, we assume that the NJL interaction originates in the color-octet vector current. Other currents appear as higher order correction in some series expansion, and thus should be suppressed by the $1/m_{\rm q}$ factor if the expansion is taken with respect to the constituent quark mass $m_{\rm q}$. According to this thought, we modify the NJL interaction Eq. (1) to

$$\begin{aligned}\mathcal{L}_4 \;=\; & G_{\rm V}(\bar{q}\lambda_{\rm C}^{\rm a}\gamma_\mu q)(\bar{q}'\lambda_{\rm C}^{\rm a}\gamma_\mu q')^2\\ & +\frac{h_1}{m_{\rm q}m_{\rm q'}}(\bar{q}\lambda_{\rm C}^{\rm a}\gamma_\mu q)(\bar{q}'\lambda_{\rm C}^{\rm a}\gamma_\mu q')\\ & +\frac{h_2}{m_{\rm q}m_{\rm q'}}(\bar{q}\lambda_{\rm C}^{\rm a}\gamma_\mu\gamma_5 q)(\bar{q}'\lambda_{\rm C}^{\rm a}\gamma_\mu\gamma_5 q'). \qquad (4)\end{aligned}$$

Here, we can take the light and heavy quarks into a unified frame q, q′ = u, d, s, c, b. h_1 and h_2 are dimensionless parameters. $m_{\rm q}$ and $m_{\rm q'}$ are the constituent masses of the quarks involved in the interaction.

The Fierz invariant interaction Eq. (2) has the interaction strengths

$$G_1=G_{\rm V}+\frac{h_1-h_2}{m_{\rm q}m_{\rm q'}},\quad G_2=G_{\rm V}+\frac{h_1+h_2}{m_{\rm q}m_{\rm q'}}. \qquad (5)$$

We notice that G_1 is closely related to the quark constituent mass, which is the dynamical one generated from the gap equation (see Eqs. (9) and (10) in Section 3). If G_1 depends on the constituent quark mass, then the gap equation will change radically. As shown in Fig. 1, when $h_1=h_2$, the case of an usual NJL model where $G_1=G_{\rm V}$ is independent of the constituent quark mass, the gap equation has two solutions corresponding to two chiral phases, which is believed to exist in the QCD chiral limit: a Wigner solution at $m_{\rm q}=0$ and a chiral symmetry breaking solution at $m_{\rm q}\neq 0$ when the coupling is large enough. However, when $h_1\neq h_2$, the gap equation reveals a singularity at $m_{\rm q}=0$. Hence there is no chiral phase (Wigner solution). So we must set $h_1=h_2=h$ and the

NJL interaction turns out to be

$$\mathcal{L}_4 = G_V(\bar{\psi}\lambda_C^a\gamma_\mu\psi)^2+\frac{h}{m_q m_{q'}}[(\bar{\psi}\lambda_C^a\gamma_\mu\psi)^2 +(\bar{\psi}\lambda_C^a\gamma_\mu\gamma_5\psi)^2]. \quad (6)$$

After the Fierz transformation, we obtain the relevant Fierz invariant interaction: for the light sector,

$$\mathcal{L}_4^F = \frac{4}{9}G_V\left[(\bar{q}\lambda_f^i q)^2+(\bar{q}i\gamma_5\lambda_f^i q)^2\right] -\frac{2}{9}\left(G_V+\frac{2h}{m_q m_{q'}}\right)\left[(\bar{q}\lambda_f^i\gamma_\mu q)^2+(\bar{q}\lambda_f^i\gamma_\mu\gamma_5 q)^2\right], \quad (7)$$

and for the heavy sector,

$$\mathcal{L}_4^{F\prime} = \frac{8}{9}G_V\left[(\bar{Q}q)(\bar{q}Q)+(\bar{Q}i\gamma_5 q)(\bar{q}i\gamma_5 Q)\right] -\frac{4}{9}\left(G_V+\frac{2h}{m_q m_Q}\right)[(\bar{Q}\gamma_\mu q)(\bar{q}\gamma^\mu Q) +(\bar{Q}\gamma_\mu\gamma_5 q)(\bar{q}\gamma_\mu\gamma_5 Q)]. \quad (8)$$

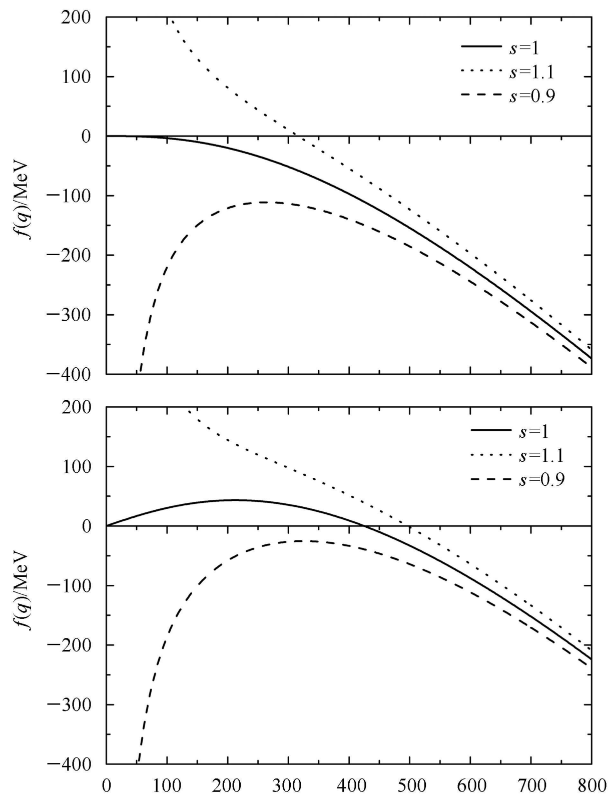


Fig. 1. The gap equation where the zero points are the quark mass solution. The cutoff is taken at Λ=750 MeV; h_2=0.65 and $h_1=sh_2$. The dimensionless parameter g_V is defined as $g_V=G_V\Lambda^2$. We show two typical situations in the figure: $g_V=gc=9\pi^2/16N_C$ where the strength is critically not enough to break the chiral symmetry; and $g_V=2.5$ with the chiral symmetry breaking solution.

Because the difference between the constituent mass and the current mass of the heavy quark should be small, we will ignore the dynamical effect on the heavy quark mass in the heavy mesons calculation.

3 The Bethe-Salpeter equation and mesons

Now we will give a brief account of the DSE and BSE method used in our calculation of meson states. Throughout this section we will use Eq. (2) as a general form of the NJL interaction.

The Dyson-Schwinger Equation (DSE) is used to obtain the dynamical quark mass m_q. The self-consistent gap equation derived from DSE reads

$$m_q=m_q^0+\Sigma_q, \quad (9)$$

where m_q^0 is the current quark mass and Σ_q is the quark self energy

$$-i\Sigma_q=i\frac{32}{9}G_1\,\mathrm{Tr}\int\frac{d^4p}{(2\pi)^4}S_q(p)=-\frac{16G_1}{9}m_q I_1(m_q), \quad (10)$$

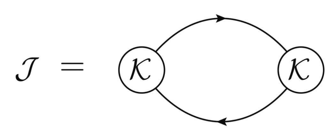
where $S_q(p)$ is the quark propagator. The expression of the integral $I_1(m_q)$ is given in Appendix A.

We use the Bethe-Salpeter equation (BSE) to obtain the meson mass and amplitude. The total quark-antiquark scattering amplitude is obtained from the ladder approximation. We decompose the amplitude into different Lorentz structures [15]. The relevant amplitudes are

$$\mathcal{T}_{ps} = T_{PP}(i\gamma_5\lambda_i\otimes i\gamma_5\lambda_j)+T_{AP}(-i\hat{\not{q}}\gamma_5\lambda_i\otimes i\gamma_5\lambda_j) +T_{PA}(i\gamma_5\lambda_i\otimes i\hat{\not{q}}\gamma_5\lambda_j)+T_{AA}^{P}(-i\hat{\not{q}}\gamma_5\lambda_i\otimes i\hat{\not{q}}\gamma_5\lambda_j), \quad (11)$$

$$\mathcal{T}_v = T_{VV}(\eta^{\mu\nu}\gamma_\mu\lambda_i\otimes\gamma_\nu\lambda_j), \quad (12)$$

where $\hat{q}^\mu=q^\mu/\sqrt{q^2}$, $\eta_{\mu\nu}=g_{\mu\nu}-\hat{q}^\mu\hat{q}^\nu$. In the ladder approximation, we only need to calculate the loop integral



which can also be decomposed to

$$\mathcal{J}_{ps}^{ij}=J_{PP}(i\gamma_5\lambda_i\otimes i\gamma_5\lambda_j)+J_{AP}(-i\hat{\not{q}}\gamma_5\lambda_i\otimes i\gamma_5\lambda_j) +J_{PA}(i\gamma_5\lambda_i\otimes i\hat{\not{q}}\gamma_5\lambda_j)+J_{AA}^{L}(-i\hat{\not{q}}\gamma_5\lambda_i\otimes i\hat{\not{q}}\gamma_5\lambda_j), \quad (13)$$

$$\mathcal{J}_v^{ij}=\eta^{\mu\nu}J_{VV}^{T}(\gamma_\mu\lambda_i\otimes\gamma_\nu\lambda_j). \quad (14)$$

Then we have

$$T=\frac{1}{1-JK}, \quad (15)$$

where

$$K_{\rm P}=\frac{16G_1}{9}({\rm i}\gamma_5\lambda_i\otimes{\rm i}\gamma_5\lambda_j),\qquad K_{\rm S}=\frac{16G_1}{9}(\lambda_i\otimes\lambda_j),$$
$$K_{\rm A}=-\frac{8G_2}{9}(\gamma_\mu\gamma_5\lambda_i\otimes\gamma_\nu\gamma_5\lambda_j),\quad K_{\rm V}=-\frac{8G_2}{9}(\gamma_\mu\lambda_i\otimes\gamma_\nu\lambda_j).\tag{16}$$

The integrals $J_{\rm AB}$ are defined in Ref. [15] and the formulae with the three-dimensional cut-off are collected in the appendix.

The meson mass $m_{\rm M}$ is determined by the pole of the amplitude,

$$\mathrm{Det}(1-JK)\big|_{q^2=m_{\rm M}^2}=0.\tag{17}$$

To calculate the weak decay constant of a pseudo-scalar meson, the quark-meson vertex is obtained by expanding the scattering amplitude near the meson pole. For a pseudo-scalar meson P, which could be π, K, D, or B, the qqP vertex reads

$$V_{\rm P}^i(p)={\rm i}\gamma_5\lambda^i\left[g_{\rm P}(p^2)-\frac{\not p}{m_{\rm q}+m_{\rm q'}}\tilde g_{\rm P}(p^2)\right],\tag{18}$$

where

$$g_{\rm P}^2=\left(\frac{{\rm d}D}{{\rm d}q^2}\right)^{-1}_{q^2=m_{\rm P}^2}K_{\rm P}(1-J_{\rm AA}K_{\rm A}),\tag{19}$$

$$\tilde g_{\rm P}=\frac{m_{\rm q}+m_{\rm q'}}{m_{\rm P}}\frac{K_{\rm A}J_{\rm PA}}{1-J_{\rm AA}K_{\rm A}}g_{\rm P},\tag{20}$$

where $D=\mathrm{Det}(1-JK)$. The pion decay constant is given by

$$\langle 0|\bar q(0)\gamma^\mu\gamma_5\frac{\lambda_i}{2}q(0)|\pi_j(p)\rangle={\rm i}f_\pi p^\mu\delta_{ij}.\tag{21}$$

A similar result holds for the kaon decay constant. In the heavy quark case the decay constant is given by

$$\langle 0|\bar q(0)\gamma^\mu\gamma_5 Q(0)|H(p)\rangle={\rm i}F_{\rm H}p^\mu,\tag{22}$$

where H could be D or B.

4 The heavy quark limit

In this section, we will discuss the heavy quark limit. After the Fierz transformation, the relevant interaction between a light quark q and a heavy quark Q in a heavy meson is written in the form

$$\mathcal{L}_4^{\rm F'}=\frac{8}{9}G_1\left[(\bar Qq)(\bar qQ)+(\bar Q{\rm i}\gamma_5 q)(\bar q{\rm i}\gamma_5 Q)\right]$$
$$-\frac{4}{9}G_2\left[(\bar Q\gamma_\mu q)(\bar q\gamma^\mu Q)+(\bar Q\gamma_\mu\gamma_5 q)(\bar q\gamma_\mu\gamma_5 Q)\right].\tag{23}$$

Considering the heavy meson at rest, $q=m_{\rm H}v$, $v=(1,0,0,0)$. In the heavy quark limit one assumes that the mass difference between $m_{\rm H}$ of the heavy meson and $m_{\rm Q}$ of the heavy quark is a small quantity l_0,

$$m_{\rm H}=m_{\rm Q}+l_0.\tag{24}$$

The heavy quark momentum p is expanded around the heavy meson momentum q as $p=q+k$, where k is assumed to be far smaller than $m_{\rm Q}$. Then the propagator of the heavy quark reduces to

$$\frac{1}{(\not k+\not q)-m_{\rm Q}}\approx\frac{\not v+1}{2(k\cdot v+l_0)}.\tag{25}$$

The expression on the right-hand side is independent of $m_{\rm Q}$. The BSE loop integrals reduce to

$$J_{\rm PP}=2{\rm i}N_{\rm C}{\rm tr}\int\frac{{\rm d}^4k}{(2\pi)^4}{\rm i}\gamma_5\frac{1}{\not k-m_{\rm q}+{\rm i}\epsilon}{\rm i}\gamma_5\frac{\not v+1}{2(k\cdot v+l_0+{\rm i}\epsilon)},\tag{26}$$

$$J_{\rm PA}=2{\rm i}N_{\rm C}{\rm tr}v^\mu\int\frac{{\rm d}^4k}{(2\pi)^4}{\rm i}\gamma_5\frac{1}{\not k-m_{\rm q}+{\rm i}\epsilon}(-{\rm i}\gamma_\mu\gamma_5)\frac{\not v+1}{2(k\cdot v+l_0+{\rm i}\epsilon)},\tag{27}$$

$$J_{\rm SS}=2{\rm i}N_{\rm C}{\rm tr}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{1}{\not k-m_{\rm q}+{\rm i}\epsilon}{\rm i}\gamma_5\frac{\not v+1}{2(k\cdot v+l_0+{\rm i}\epsilon)},\tag{28}$$

$$J_{\rm SV}=2{\rm i}N_{\rm C}{\rm tr}v^\mu\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{1}{\not k-m_{\rm q}+{\rm i}\epsilon}\gamma_\mu\frac{\not v+1}{2(k\cdot v+l_0+{\rm i}\epsilon)},\tag{29}$$

$$J_{\rm VV}^{\mu\nu}=2{\rm i}N_{\rm C}{\rm tr}\int\frac{{\rm d}^4k}{(2\pi)^4}\gamma^\mu\frac{1}{\not k-m_{\rm q}}\gamma^\nu\frac{\not v+1}{2(k\cdot v+l_0)},\tag{30}$$

$$J_{\rm AA}^{\mu\nu}=2{\rm i}N_{\rm C}{\rm tr}\int\frac{{\rm d}^4k}{(2\pi)^4}\gamma^\mu\gamma_5\frac{1}{\not k-m_{\rm q}}\gamma^\nu\gamma_5\frac{\not v+1}{2(k\cdot v+l_0)}.\tag{31}$$

We find

$$J_{\rm PP}=J_{\rm PA}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{k\cdot v-m_{\rm q}}{(k^2-m_{\rm q}^2+{\rm i}\epsilon)(v\cdot k+l_0+{\rm i}\epsilon)},\quad(32)$$

$$J_{\rm SS}=J_{\rm SV}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{k\cdot v+m_{\rm q}}{(k^2-m_{\rm q}^2+{\rm i}\epsilon)(v\cdot k+l_0+{\rm i}\epsilon)},\quad(33)$$

$$J^{\mu\nu}_{\rm VV}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{k^\mu v^\nu+v^\mu k^\nu-g^{\mu\nu}k\cdot v+g^{\mu\nu}m_{\rm q}}{(k^2-m_{\rm q}^2)(v\cdot k+l_0)},\quad(34)$$

$$J^{\mu\nu}_{\rm AA}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{k^\mu v^\nu+v^\mu k^\nu-g^{\mu\nu}k\cdot v-g^{\mu\nu}m_{\rm q}}{(k^2-m_{\rm q}^2)(v\cdot k+l_0)}.\quad(35)$$

After further decompositions $J^{\mu\nu}_{\rm VV}=J^{\rm T}_{\rm VV}(g^{\mu\nu}-v^\mu v^\nu)+J^{\rm L}_{\rm VV}v^\mu v^\nu$ and $J^{\mu\nu}_{\rm AA}=J^{\rm T}_{\rm AA}(g^{q\mu\nu}-v^\mu v^\nu)+J^{\rm L}_{\rm AA}v^\mu v^\nu$, we have

$$J^{\rm L}_{\rm VV}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{k\cdot v+m_{\rm q}}{(k^2-m_{\rm q}^2)(v\cdot k+l_0)},\quad(36)$$

$$J^{\rm T}_{\rm VV}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{-k\cdot v+m_{\rm q}}{(k^2-m_{\rm q}^2)(v\cdot k+l_0)},\quad(37)$$

$$J^{\rm L}_{\rm AA}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{k\cdot v-m_{\rm q}}{(k^2-m_{\rm q}^2+{\rm i}\epsilon)(v\cdot k+l_0+{\rm i}\epsilon)},\quad(38)$$

$$J^{\rm T}_{\rm AA}=4{\rm i}N_{\rm C}\int\frac{{\rm d}^4k}{(2\pi)^4}\frac{-k\cdot v-m_{\rm q}}{(k^2-m_{\rm q}^2+{\rm i}\epsilon)(v\cdot k+l_0+{\rm i}\epsilon)}.\quad(39)$$

Thus, in the heavy quark limit

$$J_{\rm PP}(l_0)=J_{\rm PA}(l_0)=J^{\rm L}_{\rm AA}(l_0)=-J^{\rm T}_{\rm VV}(l_0),\quad(40)$$

$$J_{\rm SS}(l_0)=J_{\rm SV}(l_0)=J^{\rm L}_{\rm VV}(l_0)=-J^{\rm T}_{\rm AA}(l_0).\quad(41)$$

For a pseudo-scalar meson, the mass equation Eq. (17) turns to be

$$(1-J_{\rm PP}(q^2)K_{\rm P})(1-J^{\rm L}_{\rm AA}(q^2)K_{\rm A})-J^2_{\rm PA}(q^2)K_{\rm P}K_{\rm A}=0,$$

which reduces to

$$1-(K_{\rm P}+K_{\rm A})J_{\rm PP}(l_0)=0,\quad(42)$$

in the heavy quark limit. The mass equation of the vector partner is $1-J^{\rm T}_{\rm VV}(q^2)K_{\rm V}=0$, which leads to

$$1+K_{\rm V}J_{\rm PP}(l_0)=0,\quad(43)$$

in the heavy quark limit. If $G_2=G_1$, then the mass equations of the pseudo-scalar meson and the vector meson are identical, and heavy quark spin symmetry is obtained. Otherwise, if $G_2\neq G_1$, then the mass of the pseudo-scalar meson differs from the mass of the vector meson. Similarly, the masses of a scalar meson and its axial-vector partner will degenerate in the heavy quark limit if and only if $G_2=G_1$.

5 Numerical results

In the NJL interaction Eq. (6), the input parameters are the current masses for light quarks and constituent masses for heavy quarks, the coupling constants and the three-dimensional cutoff. We used the light mesons' experimental data of m_π, $m_{\rm K}$, m_ρ, f_π to determine parameters $m^0_{\rm u/d}$, $m^0_{\rm s}$, $G_{\rm V}$, h and Λ. Then the experimental masses of $m_{\rm D}$ and $m_{\rm B}$ are used to determine $m_{\rm c}$ and $m_{\rm b}$. The parameters are

$$\begin{aligned}m^0_{\rm u/d}&=2.79\ {\rm MeV},\ m^0_{\rm s}=72.0\ {\rm MeV},\\ m_{\rm c}&=1.63\ {\rm GeV},\ m_{\rm b}=4.94\ {\rm GeV},\\ \Lambda&=0.8\ {\rm GeV},\quad g_{\rm V}=G_{\rm V}\Lambda^2=2.41,\ h=0.65.\end{aligned}\quad(44)$$

The resulted masses and weak decay constants are shown in the cal. I column in Table 1. We find that the meson mass spectra, both the light sector and the heavy sector, are well fitted to the experimental data. One major difficulty is that the calculated decay constant decreases with increasing meson mass, while the experimental one increases with mass. As already shown in Ref. [15], the theoretical result of $f_{\rm K}$ is smaller than the empirical data. In the case of heavy mesons, the theoretical results are smaller than the empirical ones by almost a factor of 2.

Table 1. Numerical results of the meson masses and decay constants. Cal. I column: results with the NJL interaction Eq. (6). Cal. II column: results with the interaction Eq. (1) for the light meson sector and the interaction Eq. (23) for the heavy meson sector. The experimental data are taken from Ref. [30], except for $F_{\rm B}$ and $F^*_{\rm B}$, which are taken from the lattice calculation in Ref. [31] (See also Refs. [32, 33]).

	cal. I	cal. II	exp.
$m_{\rm u}$/MeV	392	389	
$m_{\rm s}$/MeV	542	540	
m_π/MeV	139	137	135/140
$m_{\rm K}$/MeV	496	496	494/498
f_π/MeV	91.5	86.5	93.3
$f_{\rm K}$/MeV	97.9	88.6	114
m_ρ/MeV	771	775	775
$m_{\rm K^*}$/MeV	918	903	892
$m_{\rm D}$/GeV	1.87	1.86	1.86/1.87
$m_{\rm D_s}$/GeV	1.95	1.95	1.97
$m_{\rm D^*}$/GeV	1.99	2.07	2.01
$m_{\rm D^*_s}$/GeV	2.12	2.20	2.11
$m_{\rm B}$/GeV	5.28	5.28	5.28
$m_{\rm Bs}$/GeV	5.37	5.38	5.37
$m_{\rm B^*}$/GeV	5.31	5.37	5.33
$m_{\rm B^*_s}$/GeV	5.42	5.48	5.42
$F_{\rm D}$/MeV	139	125	207
$F_{\rm Ds}$/MeV	146	129	258
$F_{\rm B}$/MeV	95.6	86.2	190 (lattice)
$F_{\rm Bs}$/MeV	106	91.8	231 (lattice)

We noticed that the decay constant increases with the momentum cutoff parameter Λ. A possible explanation for this is that the momentum cutoff in the heavy sector is larger than in the light sector, which reflects the fact that the size of a heavy meson is relatively small.

The dependence of heavy meson masses on heavy quark mass is plotted in Fig. 2. We use H to represent the heavy pseudo-scalar meson and H* the heavy vector meson. One can see that when the quark mass tends to infinity, the mass splitting between the H and H* meson vanishes.

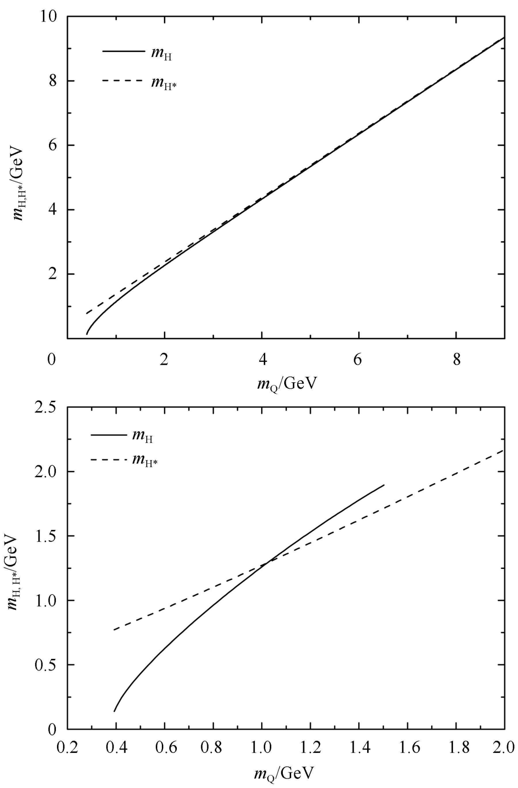


Fig. 2. The dependence of heavy-light meson masses on $m_{\rm Q}$. On the upper, with the interaction Eq. (6). On the lower, with the interaction Eq. (23). The light quark is set to u.

On the other hand, if we use the interaction in Eq. (23) and keep the parameter G_2 unchanged vs the quark masses, i.e.,

$$G_2 = G_{\rm V} + \frac{2h}{m_{\rm u}^2} = 5.41/\Lambda^2,$$

we observe a mass crossing of the H meson with the H* meson mass as the heavy quark mass increases. Beyond the crossing point, the mass relation is reverted with the H meson above the H*. The mass curve of H will further reach the mass threshold and no H bound state exists beyond this. So, a naive generalization of the NJL interaction from the light quark sector to the heavy is inappropriate.

As a comparison, we also checked the interaction of a mass independent vector interaction in the heavy meson sector, i.e. Eq. (23) with $G_1 = G_2 = g_3\Lambda^{-2}$. The interaction in the light meson sector is Eq. (1) with a different set of couplings, $G_1 = g_1\Lambda^{-2}$ and $G_2 = g_2\Lambda^{-2}$. The parameters are

$$\begin{aligned} m^0_{\rm u/d} &= 3.36\ {\rm MeV},\ m^0_{\rm s}=81.7\ {\rm MeV}, \\ m_{\rm c} &= 1.68\ {\rm GeV},\ m_{\rm b}=5.00\ {\rm GeV},\ \ \Lambda=0.7\ {\rm GeV}, \\ g_1 &= 2.52, \qquad g_2=5.82, \qquad g_3=2.53. \end{aligned} \tag{45}$$

The result is shown in the cal. II column in Table 1. We notice that the mass splitting between the heavy pseudo-scalar meson D (or B) and its vector partner D* (or B*) differs from the empirical data by roughly a factor 1.5. For the D and D*, it is 210 MeV compared to the empirical data of 150 MeV, and for the B and B* it is 80 MeV compared to 50 MeV. To reduce the mass splitting, one may decrease the coupling $g_{\rm V}$, but $g_{\rm V}$ cannot be too small, otherwise the interaction will not strongly bound the D* meson.

6 Conclusion

In this work, we have studied light and heavy mesons in a unified frame with the NJL model. We followed a traditional approach to solve the DSE and BSE, and used a three-dimensional cutoff to adequately regularize the integrals when heavy quarks are involved.

We investigated heavy quark spin symmetry in the heavy quark limit, and found that in the heavy quark limit the pseudo-scalar meson and its vector partner will have an identical mass equation only if the NJL interaction is a color-octet vector interaction which can be recognized as an approximation of a single-gluon exchange interaction.

Then we proposed an extension to the NJL interaction as in Eq. (23), which introduces the $1/m_{\rm q}$ correction to the quark current. The mass dependence suppresses the axial-vector current interaction to guarantee that heavy quark spin symmetry still holds in the heavy quark limit.

We performed numerical calculations of the light and heavy pseudo-scalar and vector mesons, both for their masses and weak decay constants. The mass spectra fit the experimental data quite well, but the weak decay constants always show a large discrepancy with the experiments. A possible explanation for this is that the momentum cutoff in the heavy sector is larger than in the light sector, which reflects the fact that the size of a heavy meson is relatively small. This issue can be studied further using some more realistic interactions than the contact ones.

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Appendix A

The current condensates in 3D cutoff

In BSE, we need to calculate the loop integral

$$J(\Gamma,\Gamma',m,m') = 2\mathrm{i}N_{\mathrm{C}}\mathrm{tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\left[\Gamma\frac{1}{\left(\not{p}+\frac{1}{2}\not{q}\right)-m+\mathrm{i}\epsilon}\right.$$
$$\left.\times\Gamma'\frac{1}{\left(\not{p}-\frac{1}{2}\not{q}\right)-m'+\mathrm{i}\epsilon}\right], \quad \text{(A1)}$$

where Γ and Γ' are the interaction vertices. For pseudo-scalar mesons, we have [15]

$$J_{\mathrm{PP}}=\frac{1}{2}\left[I_1(m)+I_1(m')\right]+\left[(m-m')^2-q^2\right]I_2(m,m',q^2), \quad \text{(A2)}$$

$$J_{\mathrm{PA},\mu}=q_\mu(m+m')\left[1-\frac{(m-m')^2}{q^2}\right]I_2(q^2,m,m')$$
$$+q_\mu\frac{m-m'}{2q^2}(I_1(m)-I_1(m')), \quad \text{(A3)}$$

$$J^{\mathrm{L}}_{\mathrm{AA}}=\frac{(m^2-m'^2)^2}{q^2}(I_2-I_2^0)-(m+m')^2I_2. \quad \text{(A4)}$$

For vector mesons, the loop integral is healed by subtracting a certain term $J^{\mathrm{T}}_{\mathrm{VV}}\rightarrow J^{\mathrm{T}}_{\mathrm{VV}}-J^{(\mathrm{T})}_{\mathrm{VV}}(q=0)+J^{(\mathrm{L})}_{\mathrm{VV}}(q=0)$ and one can obtain,

$$J^{T}_{\mathrm{VV}}=\frac{1}{3}\left[2(m^2+m'^2)(I_2-I_2^0)-[3(m-m')^2-2q^2]I_2\right.$$
$$\left.-\frac{(m^2-m'^2)^2}{q^2}(I_2-I_2^0)+4(m^2-m'^2)^2I_2^{0\prime}\right]. \quad \text{(A5)}$$

The subtracted term tends to zero when one quark mass tends to infinity. The integrations involved are,

$$I_1(m) = 8\mathrm{i}N_{\mathrm{C}}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{(p^2-m^2+\mathrm{i}\epsilon)}, \quad \text{(A6)}$$

$$I_2(m,m',q^2) = 4\mathrm{i}N_{\mathrm{C}}\int\frac{\mathrm{d}^4p}{(2\pi)^4}$$
$$\times\frac{1}{\left[\left(p+\frac{1}{2}q\right)^2-m^2+\mathrm{i}\epsilon\right]\left[\left(p-\frac{1}{2}q\right)^2-m'^2+\mathrm{i}\epsilon\right]}, \quad \text{(A7)}$$

and define, $I_2^0(m,m')\equiv I_2(m,m',0)$ and $I_2^{0\prime}=\mathrm{d}I_2/\mathrm{d}q^2|_{q^2=0}$. After a calculation, one can find, when $(m'-m)^2<q^2<(m+m')^2$,

$$I_1(m) = \frac{N_{\mathrm{C}}}{4\pi^2}\int_{4m^2}^{4(\Lambda^2+m^2)}\sqrt{1-\frac{4m^2}{\kappa^2}}\mathrm{d}\kappa^2, \quad \text{(A8)}$$

$$I_2(m,m',q^2) = -\frac{N_{\mathrm{C}}}{4\pi^2}\int_{(m+m')^2}^{\left(\sqrt{\Lambda^2+m^2}+\sqrt{\Lambda^2+m'^2}\right)^2}\mathrm{d}\kappa^2$$
$$\times\frac{\sqrt{1-2\frac{m^2+m'^2}{\kappa^2}+\left(\frac{m^2-m'^2}{\kappa^2}\right)^2}}{\kappa^2-q^2}, \quad \text{(A9)}$$

in which the Λ^2 is the three-dimensional cutoff. The same expression can be applied to the case $q^2<(m-m')^2$.

The integration involved in the 0^-, 1^- sector is,

$$j(l_0)=4\mathrm{i}N_{\mathrm{C}}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k\cdot v-m_{\mathrm{q}}}{(k^2-m_{\mathrm{q}}^2+\mathrm{i}\epsilon)(v\cdot k+l_0+\mathrm{i}\epsilon)}. \quad \text{(A10)}$$

Assuming $v=(1,\vec{0})$, and integrating out k_0 below the threshold $l_0<m_{\mathrm{q}}$, one can find,

$$j(l_0)=\frac{4N_{\mathrm{C}}}{(2\pi)^4}\pi\int\mathrm{d}^3k\frac{\sqrt{\boldsymbol{k}^2+m^2}+m_{\mathrm{q}}}{\sqrt{\boldsymbol{k}^2+m_{\mathrm{q}}^2}\left(\sqrt{\boldsymbol{k}^2+m_{\mathrm{q}}^2}-l_0\right)}. \quad \text{(A11)}$$

Introducing the 3D cutoff, one can get,

$$j(l_0)=\frac{N_{\mathrm{C}}}{(2\pi)^2}\int_{4m_{\mathrm{q}}^2}^{4(\Lambda^2+m_{\mathrm{q}}^2)}\frac{\kappa+2m_{\mathrm{q}}}{2\kappa-4l_0}\sqrt{1-\frac{4m^2}{\kappa}}\mathrm{d}\kappa^2. \quad \text{(A12)}$$

In which,

$$\kappa^2=4\boldsymbol{k}^2+4m_{\mathrm{q}}^2. \quad \text{(A13)}$$

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