Chinese Physics C Vol. 37, No. 3 (2013) 033103

Dark-matter pair production at the ILC in the littlest Higgs model with *T*-parity[*]

QIAO Qing-Peng(乔庆鹏)[1,2;1)] XU Bin(徐斌)[3]

[1] Department of Physics, Henan Normal University, Xinxiang 453003, China
[2] Department of Physics, Henan Institute of Education, Zhengzhou 450046, China
[3] Department of Mathematics and Information Sciences, North China Institute of Water Conservancy and Hydroelectric Power, Zhengzhou 450011, China

Abstract: In the littlest Higgs model with T-parity, the heavy photon ($\mathrm{A_H}$) is supposed to be a possible dark matter (DM) candidate. The direct proof of the validity of this model is to produce the heavy photon at an accelerator. In this paper, we study the production rate of $\mathrm{e^+e^-\rightarrow A_HA_H}$ at the international $\mathrm{e^+e^-}$ linear collider (ILC) in the littlest Higgs model with T-parity, and show the distributions of the transverse momenta of $\mathrm{A_H}$. The numerical results indicate that the heavy photon production rate could reach the 10^{-1}fb level at some parameter space, so this could be a good chance to observe the heavy photon via the pair production process with high luminosity at the ILC (500 $\mathrm{fb^{-1}}$). We know that DM is composed of weakly interacting massive particles, so the interactions with standard model particles are weak. How to detect heavy photons at a collider and distinguish them from other DM candidates are discussed in the final section of the paper.

Key words: littlest Higgs, T-parity, dark matter, ILC

PACS: 13.66.Fg, 13.66.Hk, 95.35.+d **DOI:** 10.1088/1674-1137/37/3/033103

1 Introduction

The standard model (SM) of particle physics, including strong and electroweak interactions, is the $SU(3)_{\mathrm{C}}\otimes SU(2)_{\mathrm{L}}\otimes U(1)_{\mathrm{Y}}$ gauge model that has been extensively tested during the past 30 years. The great success of the SM in all fields of particle physics so far shows that it is obviously a valid theory. However, because so many parameters in the theory are free and need to be phenomenologically determined, one could believe it to be only an effective theory. Besides all the questions that should tempt theorists to look for a more fundamental theory, one famous problem exists, namely the hierarchy. This is because the Higgs of the SM receives a mass through a "bare" mass term and interactions with other particles, but because of its scalar nature, the mass it receives through interactions is as large as the largest mass scale in the theory. In order to get a relatively low Higgs mass, the bare mass must be fine-tuned to several tens of an expressive place so as to accurately cancel the very large interaction terms. Meanwhile, the astrophysical observations, for example the rotation curves of galaxies [1] and the gravitational lensing effects [2], show that about 23% of the energy density of the universe is composed of dark matter (DM). One of the most fundamental problems in cosmology and particle physics today is what the nature of DM is. A flood of discussions point out that DM should be non-luminous, non-baryonic, non-relativistic, and electrically neutral [3–7]. Extensive astronomical evidence indicates that DM is a stable heavy particle that interacts with SM particles weakly. That is to say, such a particle is a weakly interacting massive particle (WIMP) [8]. The microscopic composition of DM remains a mystery and the SM does not offer an appropriate candidate to account for DM. However, many theories which extend the SM contain new particles with the correct properties to play the role of DM.

As mentioned above, new physical models are needed to solve the fine-tuning problem and provide DM candidates.

The best-known example that can solve the above-mentioned problems is a supersymmetry (SUSY) model with R-parity [9]. The minimal supersymmetric standard model (MSSM) with R-parity [10, 11] is such a model. Supersymmetric partners are introduced, and the contribution of each supersymmetric partner cancels off the contribution of each ordinary particle, so the fine-

---

Received 27 April 2012, Revised 12 September 2012

[*] Supported by National Natural Science Foundation of China (11075045), Foundation of He'nan Educational Committee (2011A140005) and Youth Foundation of Xinxiang University (12ZC10)

1) E-mail: xxqqp@mail.nankai.edu.cn

tuning doesn't exist anymore. On the other hand, a discrete gauge symmetry named R-parity is included, so the lightest neutralino, a Majorana fermion ($\tilde{\chi}_1^0$), is supposed to be the lightest stable SUSY particle and stand as the DM component, where the neutralinos ($\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$) are the lightest particles in the SUSY models. They are the fermionic SUSY partners of the neutral gauge and CP-even Higgs bosons.

A little Higgs model [12, 13] with T-parity [14, 15] provides a successful alternative to SUSY with R-parity in solving the problems of the SM mentioned above. In the initial little Higgs model (LHM), Higgs originates as a pseudo-Nambu-Goldstone boson (PNGB) of a spontaneously broken global symmetry. The global symmetry is definitely broken by the interactions of two sets, with each set preserving an unbroken subset of the global symmetry. The Higgs is an exact NGB when either set of couplings is no longer present. The hierarchy problem is explained by adopting new heavy particles at the TeV scale with the same spins as the corresponding SM particles. One-loop quadratic divergences of the Higgs boson mass are canceled by these new particles. Unfortunately, these initial models suffered from strict restrictions from the precision electroweak fits.

An ideal resolution to solve the problem is to apply a Z_2 discrete symmetry named T-parity (analogous to the R-parity in SUSY) by requiring SM particles to be even and the new heavy particles to be T-parity odd, forbidding all tree-level corrections to the electroweak precision tests. If the T-parity was conservation, the lightest T-odd particle (LTP) of an LHM would be stable and not decay, and it will play the role of the DM candidate.

The littlest Higgs model with T-parity (LHT) [16, 17] is such a type of model. It is a modification of the original littlest Higgs model. One of the unique signatures of the theory is the existence of a neutral minimum weight heavy $U(1)$ gauge boson: the heavy photon $\mathrm{A_H}$ and some other special particles. Experimentally searching them would provide direct evidence to judge the validity of the theory. There are several ways to study DM, for example, one can use the relativistic mean field theory to determine the nuclear form factor for the detection of DM [18]. Astronomical observations and restrictions may provide another way. In our previous work, we studied the heavy photon time-evolution in the LHT [19]. There is also a good opportunity to combine theoretical models with the upcoming DM direct and indirect detection experiment results from the present and future colliders in the TeV range. Because all the new particles are very heavy, they escaped detection (if they existed) in previous accelerators. By general analysis, their masses, just like the particles predicted by other models, may be in TeV regions, so one can expect to observe them at the high-energy colliders. The production of heavy photon pairs at the large hadron collider (LHC), $\mathrm{pp} \to \mathrm{A_H A_H} + \mathrm{X}$ and the associated production of $\mathrm{Q_H A_H}$ at the LHC, have been discussed in Refs. [16] and [20], respectively, where $\mathrm{Q_H}$ is the partner of the light quark. Because of the complex background, the LHC might find it difficult to confirm the theory, so the ILC will be a proper tool to better understand the properties of the LHT. In this work, we analyze the production of the heavy photon pair at the ILC.

This paper is organized as follows. The numerical results of the production rate and the distributions of the transverse momenta are presented in Section 2, where all input model parameters are explicitly listed. Our discussions on various aspects and conclusions are made in the final section.

2 Theoretical formulation and numerical results

First, let us concisely recall the relevant characteristics of the LHT. The LHT is based on a non-linear σ-model describing a global $SU(5)/SO(5)$ symmetry breaking, which takes place at an energy scale of $\Lambda \sim 4\pi f \sim 10$ TeV. The SM Higgs doublet is generally considered to be a subset of the Goldstone bosons associated with the breaking. Symmetry breaking also breaks the assumed embedded local gauge symmetry $[SU(2)\times U(1)]^2$ subgroup down to the diagonal subgroup $SU(2)_\mathrm{L}\times U(1)_\mathrm{Y}$, which is identified as the ordinary SM electroweak gauge group. The additional gauge structure leads to four extra gauge bosons at the TeV scale, $\mathrm{W_H^\pm}$, $\mathrm{Z_H}$ and $\mathrm{A_H}$. The diagonal Goldstone bosons of the matrix Π are "eaten" to become the longitudinal degrees of freedom of the heavy gauge bosons with odd T-parity at the scale f, the masses of which are

$$M(\mathrm{Z_H}) \approx M(\mathrm{W_H^\pm}) \approx gf,\ M(\mathrm{A_H}) \approx \frac{g'}{\sqrt{5}} \approx 0.16f, \tag{1}$$

where g and g' are the gauge couplings of SM $SU(2)_\mathrm{L}$ and $U(1)_\mathrm{Y}$. After electroweak symmetry breaking at the $v \ll f$ scale, the masses of the new heavy gauge bosons as well as the SM gauge boson masses receive corrections of the order v^2/f^2, and could be written as

$$\begin{aligned} M(\mathrm{Z_H}) &= M(\mathrm{W_H^\pm}) = gf\left(1-\frac{v^2}{8f^2}\right) \approx 0.65f, \\ M(\mathrm{A_H}) &= \frac{fg'}{\sqrt{5}}\left(1-\frac{5v^2}{8f^2}\right) \approx 0.16f, \end{aligned} \tag{2}$$

since the masses of the other T-odd particles are generically of level f, then the $\mathrm{A_H}$ can be assumed to be the LTP and regarded as an ideal candidate of the WIMP cold DM.

The mirror fermions acquire masses through a Yukawa-type interaction

$$\kappa f(\bar{\Psi}_2\xi\Psi'+\bar{\Psi}_1\Sigma_0\Omega\xi^{\dagger}\Omega\Psi'), \tag{3}$$

whereas Ψ_1, Ψ_2 are the fermion doublets and Ψ' is a doublet under $SU(2)_2$.

One fermion doublet $\Psi_{\rm H}=\frac{1}{\sqrt{2}}(\Psi_1+\Psi_2)$ acquires a mass κf, which is a free parameter, with the natural scale set by f. Specifically, the T-odd heavy partners of the SM leptons acquire the following masses $\sqrt{2}\kappa_l f$ [21], where κ_l is the flavor independent Yukawa coupling. The T-odd fermion mass for both lepton and quark partners will be assumed to exceed 300 GeV to avoid the colored T-odd particles from being detected in the squark searches at the Tevatron.

In the LHT model, the coupling term in the Lagrangian related to the heavy photon of our work is written as [16, 17]:

$$\mathrm{A}^{\mu}_{\rm H}\tilde{\mathrm{L}}_i\mathrm{L}_j\colon\ \mathrm{i}\frac{e}{10C_{\rm W}S_{\rm W}}\left(S_{\rm W}-5C_{\rm W}\left(\frac{v}{f}\right)^2x_{\rm h}\right)\gamma^{\mu}P_{\rm L}\delta_{ij}, \tag{4}$$

where $\tilde{\mathrm{L}}$ is the heavy lepton of the LHT and L is the SM lepton, and e is the electromagnetic coupling constant,

$$x_{\rm h}=\frac{5}{4}\frac{gg'}{5g^2-g'^2},$$

$$v=\frac{2M_{\rm W}S_{\rm W}}{e},$$

$f=1000$, $P_{\rm L}=\frac{1-\gamma_5}{2}$. We identify g and g' with the SM $SU(2)$ and $U(1)_{\rm Y}$ gauge couplings, respectively. $S_{\rm W}$ and $C_{\rm W}$ are the sine and cosine of the Weinberg angle, respectively.

The Feynman diagrams responsible for the process of $\mathrm{e}^+\mathrm{e}^-\rightarrow\mathrm{A_HA_H}$ at the tree-level are shown in Fig. 1.

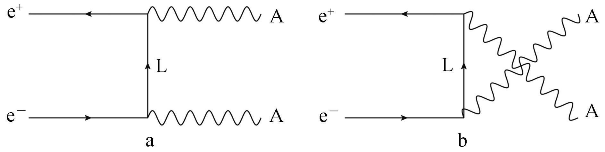

Fig. 1. The Feynman diagrams of the process $\mathrm{e}^+\mathrm{e}^-\rightarrow\mathrm{A_HA_H}$.

With the Lagrangian, the amplitude of the process can be written directly:

$$\begin{aligned}M=M_{\rm a}+M_{\rm b}\ =\ &-\mathrm{i}\left(\frac{e}{10C_{\rm W}S_{\rm W}}\left(S_{\rm W}-5C_{\rm W}\left(\frac{v}{f}\right)^2x_{\rm h}\right)\right)^2\bar{v}(P_1)\\&\times\left(\frac{1}{P_{24}^2-m_{\tilde{\rm L}}^2}\gamma^{\mu}\gamma^{\rho}\gamma^{\nu}P_{24\rho}+\frac{1}{P_{23}^2-m_{\tilde{\rm L}}^2}\gamma^{\nu}\gamma^{\rho}\gamma^{\mu}P_{23\rho}\right)P_{\rm L}u(P_2)\epsilon^*_{\mu}(P_3)\epsilon^*_{\nu}(P_4),\end{aligned} \tag{5}$$

where $P_{23}=P_2-P_3$, $P_{24}=P_2-P_4$, an ϵ is the polarization vector for the heavy photon gauge boson.

The differential cross section is given by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}=\frac{1}{2!}\frac{1}{64\pi s}\frac{1}{|\vec{p}_1|^2}\bar{\Sigma}|M|^2, \tag{6}$$

where s and t are the Mandelstam variables, $\mathrm{d}t=2|\vec{p}_1||\vec{p}_3|\mathrm{d}\cos\theta$.

With the above production amplitudes and differential cross section, we can directly obtain the production cross section of the process. For the numerical computations of the cross section, we need the electroweak fine-structure constant α to satisfy $\alpha(M_{\rm Z})=\frac{e^2}{4\pi}=\frac{1}{128}$ [22].

There are three free parameters involved in the production amplitudes: the heavy photon mass $m_{\rm AH}$, the mass of heavy lepton $m_{\tilde{\rm L}}$ and the energy of the center-of-mass frame $\sqrt{s}$. Concretely, in order to expose the possible dependence of the cross section on these parameters, we take two groups of values: $\sqrt{s}$=500, 1000 GeV, $m_{\tilde{\rm L}}$ =300, 500, 700 GeV, respectively. The $m_{\rm AH}$ takes 100 to 250 GeV in Fig. 2 and 100 to 300 GeV in Fig. 3. The numerical results of the cross sections are plotted in Fig. 2 and Fig. 3.

From these figures, we can see that the production rate decreases sharply as $m_{\rm AH}$ increases due to the constraint from the phase space of the final state involving

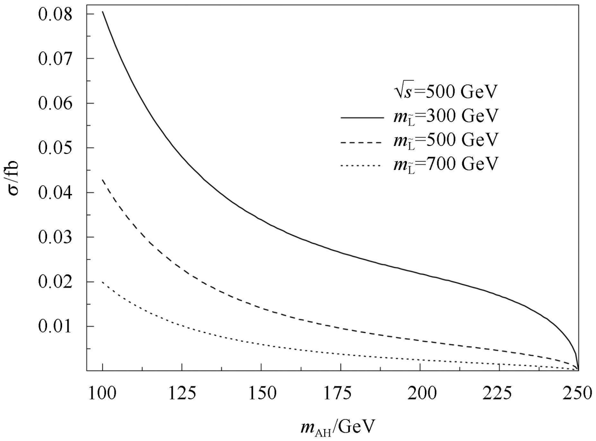

Fig. 2. The dependence of the cross section of $\mathrm{e}^+\mathrm{e}^-\rightarrow\mathrm{A_HA_H}$ on heavy photon mass $m_{\rm AH}$ (100–250 GeV) for $\sqrt{s}$=500 GeV and $m_{\tilde{\rm L}}$ =300, 500, 700 GeV at the ILC.

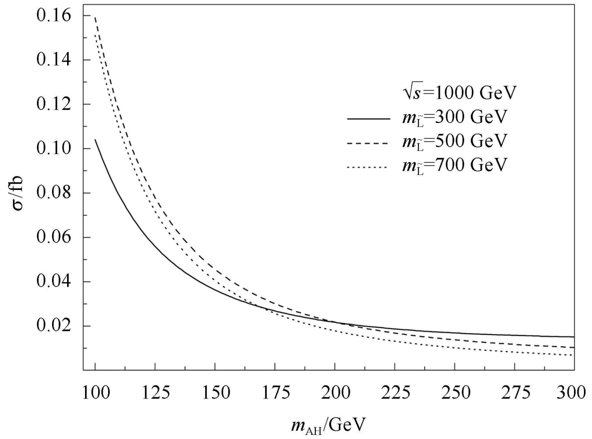

Fig. 3. The dependence of the cross section of $e^+e^- \to A_H A_H$ on heavy photon mass m_{AH} (100–300 GeV) for $\sqrt{s}$=1000 GeV and $m_{\tilde{L}}$=300, 500, 700 GeV at the ILC.

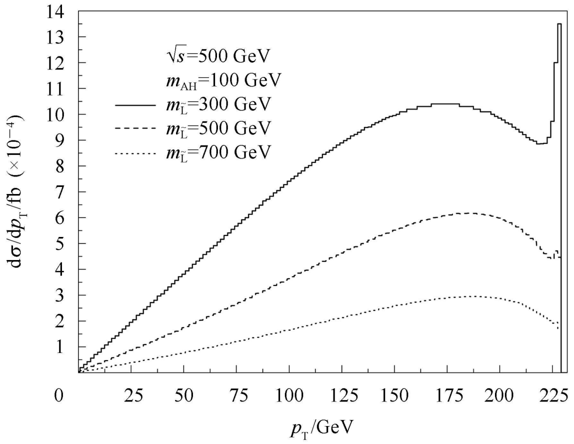

Fig. 4. The distributions of the transverse momenta of final state (p_T) for the process $e^+e^- \to A_H A_H$ with $\sqrt{s}$=500 GeV.

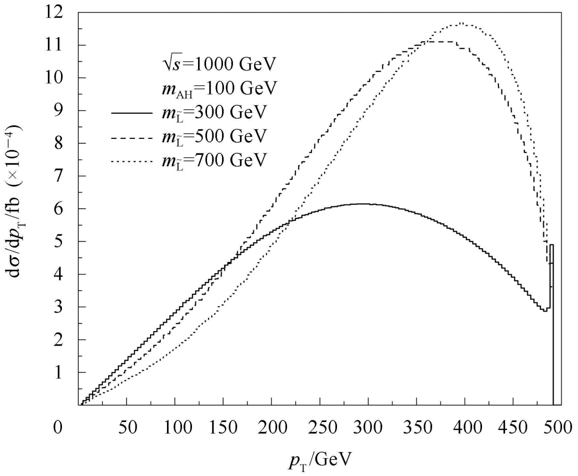

Fig. 5. The distributions of the transverse momenta of final state (p_T) for the process $e^+e^- \to A_H A_H$ with $\sqrt{s}$=1000 GeV.

the heavy photon pair. The dependence of the cross section on $\sqrt{s}$ in the range of parameters that we use is apparent: when $\sqrt{s}$ becomes large the cross section increases obviously. The relations between the cross section and $m_{\tilde{L}}$ are the other way around. When $\sqrt{s}$ is fixed as 500 GeV, $m_{\tilde{L}}$ becomes large and the cross section decreases, but when $\sqrt{s}$ takes the value of 1000 GeV, the situations are more complicated and one can see the concrete relations from the figures.

In Fig. 4 and Fig. 5, we present the transverse momentum distributions of heavy photons for $\sqrt{s}$=500 and 1000 GeV with m_{AH} equal to 100 GeV, respectively. From the figures, we can see that the differential cross section increases with the transverse momentum until the value reaches the maximum at the point that p_T equals a certain value, and then it begins to slide.

3 Discussions and conclusions

The advantages of observing the process of $e^+e^- \to A_H A_H$ at the ILC are obvious. First, astrophysical probes provide a way to study the characteristics of DM. However, astrophysical observations are unable to determine and examine the exact properties of a DM particle, and it mostly remains with collider physics to reveal the nature of DM. Second, the LHC and the ILC may well turn out to be DM factories, and the ILC background is relatively clean, so this paper is a good supplement to Refs. [16, 20]. Finally, the heavy photon is the LTP and must be produced in pairs. Constraints from the final state phase space of the above process are alleviated compared with the other processes.

The combination of cosmology and a high-energy collider seems to be a good idea [23, 24]. However, we also have some problems to solve.

The first problem is how to distinguish the A_H pair of the LHT from other DM pair production at the ILC. For example, the lightest neutralino ($\tilde{\chi}_1^0$) of SUSY with *R*-parity [10, 11]. Since different DM particles have different reaction channels and probabilities to be detected by the detectors of the ILC, this offers a way to distinguish the A_H from others. In the MSSM with *R*-parity, tree-level production cross-sections of $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ were studied systematically, and readers who are interested in these processes and want to know more about them can see the paper [25, 26]. For this reason we do not discuss this topic in detail here.

The more serious question we have to face is that unlike the production of heavy photons in the e^+e^- annihilation of six-dimensional SUSY QED [27], DM does not carry either color or an electric charge, so the direct detection of DM at a collider is very hard. In the collider experiments, DM would be similar to neutrinos and therein would escape the detector without depositing energy in the experimental devices, causing an obvious imbalance of momentum and energy in collider events. DM would manifest itself as missing energy. However, DM isn't the only source responsible for the missing energy. Limited calorimeter resolution, uninstrumented regions of the detector, and additional cosmic ray energy must all be considered thoroughly in a collider experiment. All of these factors strongly complicate the investigation of DM in collider experiments. For conservation of momentum, the sum of all momenta transverse to the beam direction must equal zero. Thus, we can ascertain the missing energy by measuring the energy deposited in each calorimeter cell of a detector. If all of the above uncertainties and the background caused by SM neutrinos have been subtracted and the vector sum of all the transverse momenta is not equal to zero, then we can claim that something invisible has been produced, and that the undetected particle(s) may be the DM candidate(s) (such as a heavy photon).

There is a better way to observe heavy photon pair production. They can be observed via radiative production $e^+e^-\rightarrow A_HA_H\gamma$. In this process the signal is a single high energetic photon and missing energy carried by the heavy photons. In our next work [28], we will discuss this problem in detail.

Although it is very hard to detect DM, $e^+e^-\rightarrow A_HA_H$ is the leading order process of DM production in the littlest Higgs model with T-parity at the ILC, so we need to predict the production rate. As a conclusion, we find that the production rate of $e^+e^-\rightarrow A_HA_H$ could reach the level of 10^{-2} fb at some small mass parameter space, which is slightly larger than the process of $e^+e^-\rightarrow A_HZ_H$ [29]. The heavy gauge bosons A_H and Z_H are produced with a cross section of 1.9 fb at the center-of-mass energy of 500 GeV, and the large mass of heavy Z_H bosons (369 GeV) suppresses the final state phase space. The production of A_H may be observable as the missing energy at the ILC thanks to its high energy and luminosity. However, for uncertain reasons the missing energy related to the A_H is difficult to be determined accurately, and so identification of DM production at the ILC is very hard. We are not currently equipped with the expertise or the ability to handle this complex analysis, but we will definitely cooperate with our experimental colleagues and experts in this field to carry out a detailed analysis.

References

1 Begeman K G, Broeils A H, Sanders R H. Mon. Not. Roy. Astron. Soc., 1991, **249**: 523
2 Massey R et al. Nature, 2007, **445**: 286
3 Roszkowski L. Phys. Lett. B, 1991 **262**: 59
4 Roszkowski L. arXiv:hepph/9903467
5 Primack J R. arXiv:astroph/0112255
6 Chen Pisin. arXiv:astroph/0303349
7 MA C P, Boylan-Kolchin M. Phys. Rev. Lett., 2004, **93**: 021301
8 Bertone G, Hooper D, Silk J. Phys. Rept., 2005, **405**: 279
9 Dreiner H K, Luhn C, Thormeier M. Phys. Rev. D, 2006, **73**: 075007
10 Goldberg H. Phys. Rev. Lett., 1983, **50**: 1419
11 Ellis J R, Hagelin J S, Nanopoulos D V et al. Nucl. Phys. B, 1984, **238**: 453
12 CHENG H C, Low I. JHEP, 2004, **0408**: 061
13 Low I. JHEP, 2004, **0410**: 067
14 Skiba W, Terning J. Phys. Rev. D, 2003, **68**: 075001
15 CHENG H C, Low I. JHEP, 2003, **0309**: 051
16 Hubisz J, Meade P. Phys. Rev. D, 2005, **71**: 035016
17 Birkedal A, Noble A, Perelstein M et al. Phys. Rev. D, 2006, **74**: 035002
18 CHEN Ya-Zheng, LUO Yan-An, LI Lei et al. Commun. Theor. Phys., 2011, **55**: 1059
19 QIAO Qing-Peng, TANG Jian, LI Xue-Qian. Commun. Theor. Phys., 2008, **50**: 1211
20 CHEN Chian-Shu, Cheung Kingman, YUAN Tzu-Chiang. Phys. Lett. B, 2007, **644**: 158
21 Cacciapaglia G, Deandrea A, Choudhury S R et al. Phys. Rev. D, 2010, **81**: 075005
22 Nakamura K et al. J. Phys. G, 2010, **37**: 075021
23 Khlopov M Y, Sakharov A S, Sudarikov A L. Grav. Cosmol., 1998, **4**: S1
24 Belotsky K M, Khlopov M Y, Sakharov A S et al. Grav. Cosmol. Suppl., 1998, **4**: 70
25 Bartl A, Fraas H, Majerotto W. Nucl. Phys. B, 1986, **278**: 1
26 Carena M S, Freitas A. Phys. Rev. D, 2006, **74**: 095004
27 Fayet P. Nucl. Phys. B, 1986, **263**: 649
28 QIAO Qing-Peng, XU Bin. arXiv:hepph/1111.6155
29 Asakawa E, Asano M, Fujii K et al. Phys. Rev. D, 2009, **79**: 075013