Influence of coupling constants on nuclear symmetry energy

LIU Bei-Bei(刘贝贝)[1)] OUYANG Fei(欧阳飞) CHEN Wei(陈伟)[2)]

Department of Physics, Jinan University, Guangzhou 510632, China

Abstract: By studying the energy of neutron star matter, we discuss the nuclear symmetry energy at different baryon densities and different coupling constants in the relativistic mean field approximation. The results show that the symmetry energy increases with baryon density at various coupling constants and incompressibilities. Furthermore, the symmetry energy at saturation density increases with increasing incompressibility at fixed d, and decreases at fixed c. Specifically, when coupling constants g_v and g_s are fixed, respectively, the symmetry energy has a little change with increasing incompressibility. It is demonstrated that the NN coupling constants have greater influences on the symmetry energy than the self-coupling constants.

Key words: symmetry energy, relativistic mean field approximation, coupling constant, incompressibility

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1 Introduction

It is essential to understand nuclear symmetry energy, which is not only related to many problems in nuclear physics, but is also relevant to a number of important issues in astrophysics, such as the pre-supernova evolution of massive stars and the cooling of neutron stars [1, 2]. The nuclear matter symmetry energy is defined as the difference of energy per nucleon between the symmetric nuclear matter and pure neutron matter, and it is an important quantity that determines the properties of the nucleus and the neutron star [3]. The thickness of neutron skin in heavy nuclei has a strong correlation with the slope parameter of nuclear symmetry energy at saturation density [4–8]. It is systematic to research the relationship in Ref. [3] with different models. The neutron star masses, radii and compositions are related to the potential part parameters of the symmetry energy, and the radius increases linearly with the derivative of symmetry energy close to the saturation density [9]. It is well known that the most efficient cooling process (Urca process) occurs in neutron star matter if the proton fraction exceeds 1/9. The proton fraction can be determined by the trend of the symmetry energy especially just above the equilibrium density. So the detailed knowledge of symmetry energy is crucial for the existence of the Urca process [9]. However, the symmetry energy is not a directly measurable quantity, we have to extract it indirectly from observables that are related to symmetry energy. The experimental determination of nuclear matter symmetry energy depends on the model that describes the experimental observable. There are two types in the experimental studies as pointed out in Ref. [10]. The Dirac-Bruecker-Hartree-Fock (DBHF), the Brueckner-Hartree-Fock (BHF), the Skyrme-Hartree-Fock (SHF) and the relativistic mean-field (RMF) model have been used to study nuclear symmetry energy, but the results are greatly different [9]. As is known, the coupling constants between nucleons and the self-coupling constants of a meson are the key factors which determine the equation of state of nuclear matter as well as its softness, and the fraction of nucleons. So it is of great interest to research the dependence of the nuclear symmetry energy on these coupling constants which still remain uncertain up to now due to the scarcity of experimental data.

In the present work, we carry out a study of the symmetry energy of neutron star matter in various conditions by RMF approximation. The RMF model, which is generally based on effective interaction Lagrangians and adjusts the parameter to fit the properties of nuclei [11], gives excellent descriptions of nuclear properties around the saturation density in the past 30 years [12–19]. As pointed out in Refs. [20–22], the nuclear symmetry energy is poorly known at supra-normal density. We consider the neutron star which consists of neutrons, protons and electrons, and this situation exists in the outer core of the neutron star.

The formulas of related quantities in the mean field approximation are given in Section 2. And Section 3

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1) E-mail: liubei880825@126.com

2) E-mail: tchenw@jnu.edu.cn

presents the definition of nuclear symmetry energy. Our results and discussions appear in Section 4.

2 The relativistic mean-field model

In the mean field approximation, the total Lagrangian density can be written as

$$L=\sum_{\mathrm{B}}\bar{\psi}_{\mathrm{B}}\left[\gamma_\mu(\mathrm{i}\partial^\mu-g_{\omega\mathrm{B}}\omega^\mu-\frac{1}{2}g_{\rho\mathrm{B}}\tau\cdot\rho^\mu)+m_{\mathrm{B}}-g_{\sigma\mathrm{B}}\sigma\right]\psi_{\mathrm{B}}$$
$$+\frac{1}{2}(\partial\sigma)^2-\frac{1}{2}m_\sigma^2\sigma^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu-U(\sigma)$$
$$-\frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu}+\frac{1}{2}m_\rho^2\rho_\mu\rho^\mu+\bar{\psi}_{\mathrm{e}}(\mathrm{i}\gamma_\mu\partial^\mu-m_{\mathrm{e}})\psi_{\mathrm{e}}, \tag{1}$$

where the field tensors $F_{\mu\nu}$ and $\rho_{\mu\nu}$ are given by

$$F_{\mu\nu}=\partial_\mu\omega_\nu-\partial_\nu\omega_\mu,\quad \rho_{\mu\nu}=\partial_\mu\rho_\nu-\partial_\nu\rho_\mu, \tag{2}$$

with

$$U(\sigma)=\frac{1}{3!}c\sigma^3+\frac{1}{4!}d\sigma^4, \tag{3}$$

where ψ_{B} (B=n, p), ψ_{e}, σ^μ, ω^μ, ρ^μ are the field operators of baryons (neutron and proton), electron, σ, ω and ρ meson, with $m_{\mathrm{B}}=938.27$ MeV, $m_{\mathrm{e}}=0.511$ MeV, $m_\sigma=550$ MeV, $m_\omega=781$ MeV and $m_\rho=770$ MeV, respectively.

In the RMF theory, the operators of the meson fields are treated as their expectation values. The meson field equations in uniform nuclear matter are given by

$$m_\sigma^2\sigma=\sum_{\mathrm{B}}g_{\sigma\mathrm{B}}\frac{\gamma}{(2\pi)^3}\int_0^{k_{F_{\mathrm{B}}}}\mathrm{d}k^3\frac{M^*}{(k^2+M^{*2})^{1/2}}-\frac{1}{2!}c\sigma^2-\frac{1}{3!}d\sigma^3, \tag{4}$$

$$m_\omega^2\omega=g_{\omega\mathrm{B}}n_{\mathrm{B}}, \tag{5}$$

$$m_\rho^2\rho=\frac{1}{2}g_\rho\rho_3^0, \tag{6}$$

$$M^*=m_{\mathrm{B}}-g_{\mathrm{s}}\sigma, \tag{7}$$

where

$$\rho_3^0=\frac{1}{3\pi^2}(k_{F_{\mathrm{p}}}^3-k_{F_{\mathrm{n}}}^3). \tag{8}$$

For neutron star matter, the compositions are required to be the charge neutral and β-equilibrium. The conditions of β-equilibrium can be expressed by

$$\mu_{\mathrm{n}}=\mu_{\mathrm{p}}+\mu_{\mathrm{e}}. \tag{9}$$

The charge neutrality can be written as

$$n_{\mathrm{p}}=n_{\mathrm{e}}, \tag{10}$$

where $n_{\mathrm{j}}=\frac{k_{F_{\mathrm{j}}}^3}{3\pi^2}$ (j=n, p, e) is the number density of neutron, proton and electron, respectively. Solving the coupled Eqs. (1)–(10) at a given baryon density n_{B}, the energy density of neutron star matter is given by

$$\varepsilon=\frac{1}{2}m_\sigma^2\sigma^2-\frac{1}{2}m_\omega^2\omega^2+U(\sigma)-\frac{1}{2}m_\rho^2\rho^2$$
$$+\sum_{\mathrm{B}}\frac{\gamma}{(2\pi)^3}\int_0^{k_{F_{\mathrm{B}}}}\mathrm{d}^3k(k^2+(m_{\mathrm{B}}-g_{\sigma\mathrm{B}}\sigma)^2)^{\frac{1}{2}}$$
$$+g_{\omega\mathrm{B}}\omega+\frac{\gamma}{(2\pi)^3}\int_0^{k_{F_{\mathrm{e}}}}\mathrm{d}^3k(k^2+m_{\mathrm{e}}^2)^{\frac{1}{2}}$$
$$+\frac{1}{3\pi^2}(k_{F,\mathrm{p}}^3-k_{F,\mathrm{n}}^3)\left(\frac{1}{2}g_{\rho\mathrm{B}}\rho_3^0\right). \tag{11}$$

3 The symmetry energy in neutron star matter

The energy per nucleon, which can be obtained by subtracting the contributions of electrons from Eq. (11), can be well approximated by

$$E(n_{\mathrm{B}},\alpha)=E(n_{\mathrm{B}},\alpha=0)+E_{\mathrm{sym}}(n_{\mathrm{B}})\alpha^2+\mathrm{O}(\alpha^4), \tag{12}$$

where the baryon density $n_{\mathrm{B}}=n_{\mathrm{n}}+n_{\mathrm{p}}$, n_{n} and n_{p} are the neutron and proton densities in the neutron star. The isospin asymmetry $\alpha=(N-Z)/A$, the term $E(n_{\mathrm{B}},\alpha=0)$ is the energy per nucleon in symmetric nuclear matter. The nuclear symmetry energy is defined as

$$E_{\mathrm{sym}}(n_{\mathrm{B}})=\frac{1}{2}\frac{\partial^2E(n_{\mathrm{B}},\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0}. \tag{13}$$

In Eq. (12), the absence of odd-order α terms is due to the exchange symmetry between protons and neutrons in nuclear matter when we neglect the coulomb interaction and assume the charge symmetry of nuclear forces. Higher-order terms in α are generally negligible for the proton abundance is about 5% at the saturation density.

The magnitude of the $O(\alpha^4)$ term has been estimated to be less than 1 MeV [3]. So the symmetry energy E_{sym} can be evaluated approximately as

$$E_{\mathrm{sym}}\approx\frac{E(n_{\mathrm{B}},\alpha)-E(n_{\mathrm{B}},\alpha=0)}{\alpha^2}. \tag{14}$$

4 Results and discussions

The coupling constants g_{s}, g_{v}, c and d, which are difficult to accurately determine by present experiments, are fitted to reproduce the saturation properties of infinite symmetric nuclear matter which have been determined by experiment. We assume that $\mathrm{B.E.}=\frac{\varepsilon}{n_{\mathrm{B}}}-m_{\mathrm{B}}$ is the binding energy of infinite symmetric nuclear matter and K_0 is its incompressibility at saturation density $n_0=0.17\ \mathrm{fm}^{-3}$. By fixing one and adjusting the others of these coupling constants, we reproduce B.E. $=-15.75$ MeV at saturation density n_0 with different K_0

(200–280 MeV). Based on these results, we study the nuclear symmetry energy in neutron star matter under various conditions as in Figs. 1–4 which show that the nuclear symmetry energy increases when the baryon density increases no matter how much K_0 is. In Fig. 1, the symmetry energy increases with the increase of incompressibility K_0 in high density region when the self-coupling constant d is fixed at 30. Also in the high density region, the symmetry energy increases with the decrease of incompressibility K_0 when the self-coupling constant $c=6000$ MeV as in Fig. 2. From Fig. 3 and Fig. 4, we can see that the symmetry energy has a little change with the incompressibility K_0 when the coupling constants are fixed ($g_s=8.7335$ and $g_v=9.4876$), respectively. These conclusions are confirmed further in Fig. 5.

Figure 5 presents the symmetry energy changing with incompressibility K_0 at the saturation density n_0, and it is in the range of 35.3–40.5 MeV. This range is higher than the empirical liquid-drop mass formula, which has

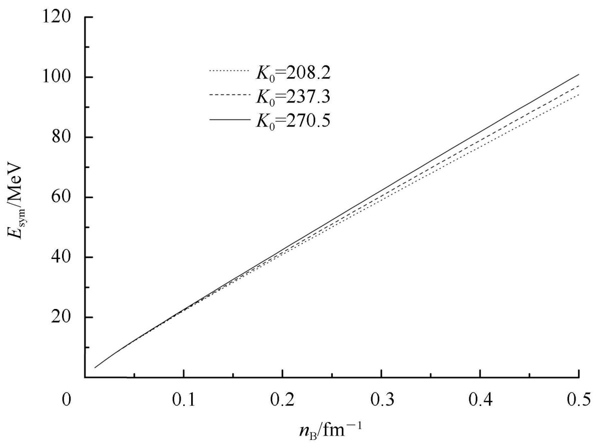

Fig. 1. Symmetry energy changes with baryon density at different K_0 when d=30.

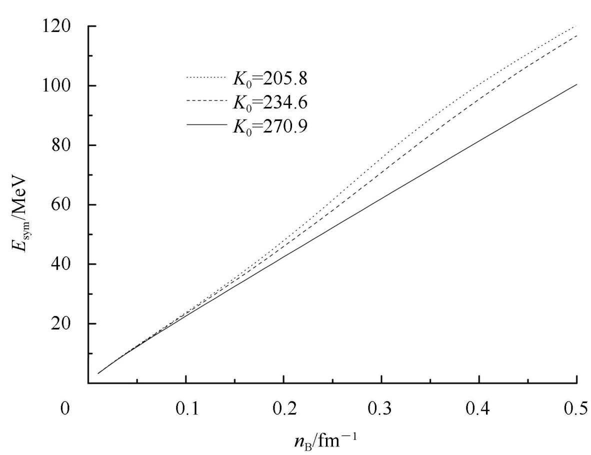

Fig. 2. Symmetry energy changes with baryon density at different K_0 when c=6000 MeV.

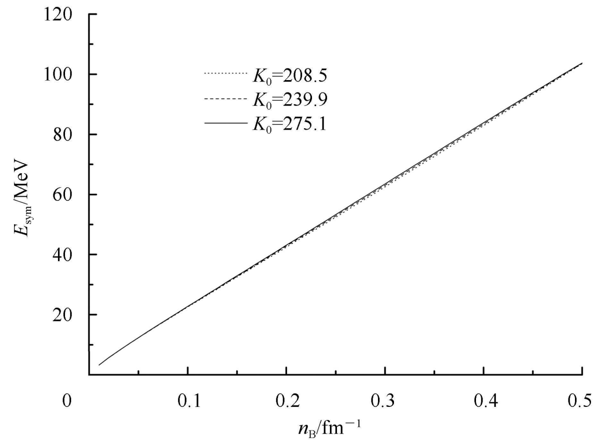

Fig. 3. Symmetry energy changes with baryon density at different K_0 when g_s=8.7335.

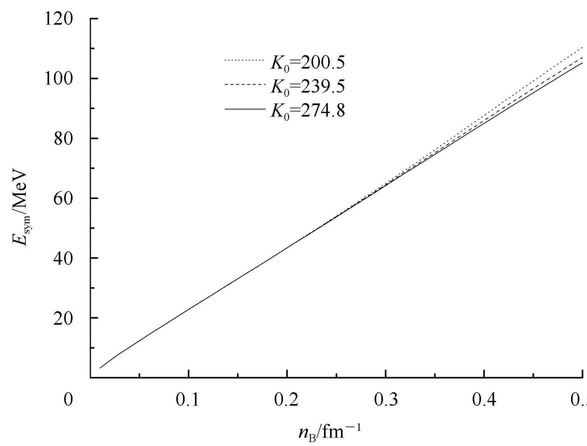

Fig. 4. Symmetry energy changes with baryon density at different K_0 when g_v=9.4876.

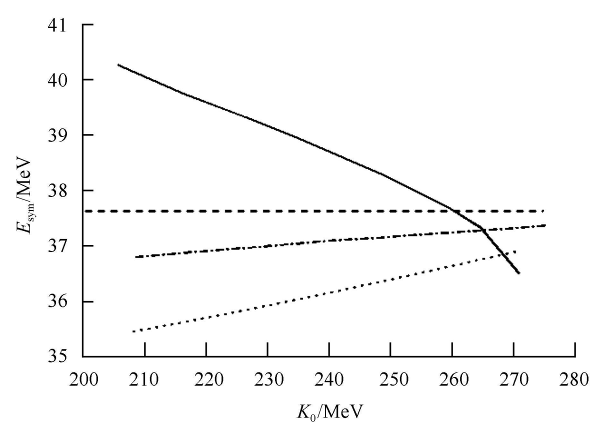

Fig. 5. Symmetry energy as a function of the incompressibility K_0 at saturation density n_0. The solid, doted, dashed and dash doted curves correspond to the situation at fixed $c=6000$ MeV, d=30, g_v=9.4876 and g_s=8.7335, respectively.

a value of $E_{\rm sym}(n_0)$ around 30 MeV [23, 24], and is similar to the values in the SHF approach[25] and other work [11] which are 26–35 MeV and 30–44 MeV, respectively. At the same time, this figure shows that the NN coupling constants $g_{\rm s}$, $g_{\rm v}$ have much more influence on the nuclear symmetry energy than the self-coupling constants d, c.

5 Summary

We have discussed the influences of NN coupling and self-coupling constants on the symmetry energy by studying neutron star matter which consists of protons, neutrons and electrons and exists in the outer core of the neutron star. We have found that the symmetry energy increases as baryon density increases under all conditions. The NN coupling constants have a greater influence on the symmetry energy than the self-coupling constants. It can be explained that $g_{\rm s}$, $g_{\rm v}$ are the most important parameters in determining the property of neutron star matter.

References

1 Li B A, Steiner A W. Phys. Lett. B, 2006, **642**: 436
2 Krastev P G, LI B A. Phys. Rev. C, 2007, **76**: 055804
3 LI B A, CHEN L W, KO C M. Phys. Rep., 2008, **464**: 113
4 Dieperink A E L, Dewulf Y, Van Neck D et al. Phys. Rev. C, 2003, **68**: 064307
5 Horowitz C J, Piekarewicz J. Phys. Rev. C, 2002, **66**: 055803
6 Typel S, Brown B A. Phys. Rev. C, 2001, **64**: 027302
7 Furnstahl R J. Nucl. Phys. A, 2002, **706**: 85
8 Karataglidis S, Amos K, Brown B A et al. Phys. Rev. C, 2002, **65**: 044306
9 Psonis V P, Moustakidis Ch C, Massen S E. Mod. Phys. Lett. A, 2007, **22**: 1233–1254
10 Shetty D V, Yennello S J. Pramana-Journal of Physics, 2010, **75**: 259–269
11 CHEN Lie-Wen, KO Che-Ming, LI Bao-An. Phys. Rev. C, 2007, **76**: 054316
12 Walecka J D. Ann. Phys. (NY), 1974, **83**: 491
13 Serot B D, Walecka J D. Adv. Nucl. Phys., 1986, **16**: 1
14 Reinhard P G. Rep. Prog. Phys., 1989, **52**: 439
15 Ring P. Prog. Part. Nucl. Phys., 1996, **37**: 193
16 Serot B D, Walecka J D. Int. J. Mod. Phys. E, 1997, **6**: 515
17 Bender M, Heenen P H, Reinhard P G. Rev. Mod. Phys., 2003, **75**: 121
18 Furnstahl R J. Lect. Notes Phys., 2004, **641**: 1
19 MENG J, Toki H, ZHOU S G et al. Prog. Part. Nucl. Phys., 2006, **57**: 470
20 LI B A, Udo Schroder W. Isospin Physics in Heavy-Ion Collisions at Intermediate Energies. New York: Nova Science Publishers, Inc., 2001
21 LI B A, KO C M, Bauer W. Int. J. Mod. Phys. E, 1998, **7**: 147
22 XU Chang, LI B A. Phys. Rev. C, 2010, **81**: 044603
23 Myers W D, Swiatecki W J. Nucl. Phys. A, 1966, **81**: 1
24 Pomorski K, Dudek J. Phys. Rev. C, 2003, **67**: 044316
25 CHEN L W, KO C M, LI B A. Phys. Rev. C, 2005, **72**: 064309