Constraint on the warped space from $\bar{B}\to X_s\gamma$ *

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Abstract: We analyze the theoretical prediction on the branching ratio of $\bar{B}\to X_s\gamma$ to order $\Lambda_{EW}^2/\Lambda_{KK}^2$ in extension of the standard model with a warped extra dimension and the custodial symmetry $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_X\times P_{LR}$, where Λ_{KK} denotes the energy scale of low-lying Kaluze-Klein excitations and Λ_{EW} denotes the electroweak energy scale. Contributions from the infinite series of Kaluze-Klein excitations are summed over through the residue theorem. The numerical result indicates that the present experimental data constrain the parameter space of the concerned model strongly.

Key words: rare decay, warped extra dimension, Kaluza-Klein mode

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1 Introduction

The Standard Model (SM) prediction on the branching ratio of $\bar{B}\to X_s\gamma$ at next to-next to-leading order [1] (NNLO) reads

$$BR(\bar{B}\to X_s\gamma)=(3.15\pm0.23)\times10^{-4}, \tag{1}$$

with the photon-energy cut-off $E_\gamma>1.6$ GeV. This result certainly coincides with the current experimental observation [2],

$$BR(\bar{B}\to X_s\gamma)=(3.55\pm0.24\pm0.09)\times10^{-4}, \tag{2}$$

with the same energy cut-off E_γ, and constrains strictly the new physics beyond the SM.

An extension of the SM with a warped dimension [3–5] provides a naturally geometrical solution to the hierarchy problem regarding the huge difference between the Planck scale and the electroweak one. The small mixing between zero modes and heavy Kaluza-Klein (KK) excitations can induce the observed fermion masses and corresponding weak mixing angles [6, 7], and suppress flavor-changing-neutral-current (FCNC) couplings [8, 9].

To accomodate light exciting KK modes with $\mathcal{O}$(1 TeV) masses, the authors of Refs. [10, 11] suggested that the gauge group in the bulk be enlarged to $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_X\times P_{LR}$. Constraints on this kind of model have been of particular interest in the past few years [12–14]. Tree-level bounds were discussed in Ref. [12], and bounds at loop-level from the flavor-changing process $\bar{B}\to X_s\gamma$ were examined in a naive dimension analysis in Ref. [13]. However, only the zero mode or the low-lying KK excitations have been considered in their work. Although the CP-violating parameter ϵ_K gives the strongest favor bounds [15–17] in the typical RS model, the experimental data on $\bar{B}\to X_s\gamma$ provide more accurate constraints on the parameter space of concerned model because the leading corrections to the relevant effective Lagrangian originate from the one-loop diagrams. Since the present experimental data all indicate the energy scale of low-lying KK excitations $\Lambda_{KK}\gg\mu_{EW}\sim v$, we sum over the infinite series of KK modes up to order v^2/Λ_{KK} through the residue theorem technique presented in Ref. [18]. Here, μ_{EW} and v denote the electroweak scale and the nonzero vacuum expectation value (VEV) of the Higgs, respectively.

With an appropriate choice of quark bulk masses, one indeed obtains agreement with the electroweak precision data in the presence of light KK excitations [19–22]. Here, we analyze the corrections from the KK modes to the branching ratio of $\bar{B}\to X_s\gamma$.

The discrete symmetry P_{LR} interchanging the local groups $SU(2)_L$ and $SU(2)_R$ implies that the five dimensional gauge couplings satisfy $g_{5L}=g_{5R}=g_5$, the local gauge group $SU(2)_L\times SU(2)_R\times U(1)_X$ is broken to the SM gauge group by the boundary conditions (BCs) on the UV brane:

$$\begin{aligned}&W_{L,\mu}^{1,2,3}(++),B_\mu(++),W_{R,\mu}^{1,2}(-+),Z_{X,\mu}(-+),(\mu=0,1,2,3),\\&W_{L,5}^{1,2,3}(--),B_5(--),W_{R,5}^{1,2}(+-),Z_{X,5}(+-).\end{aligned} \tag{3}$$

Here, the first (second) sign is the BC on the UV (IR) brane: (+) denotes a Neumann BC and (−) denotes a Dirichlet BC. In the matter field sector, the SM quarks and leptons are embedded into the bidoublets, triplets and singlets of the local gauge symmetry

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$SU(2)_{\rm L}\times SU(2)_{\rm R}$. In order to guarantee the explicit gauge invariance of electromagnetic and strong interaction at every step of the calculation, we choose the nonlinear R_ξ gauge-fixing terms [23] in the electroweak sector and the background gauge-fixing terms [24] in the strong sector respectively.

The profiles of gauge boson zero modes are flat along the fifth dimension before electroweak symmetry breaking, and the profiles of KK gauge bosons are given by

$$\chi^G_{\rm (BCs)}(y^{G(n)}_{\rm (BCs)},t)=\frac{t}{N^{\rm G}_{\rm (BCs)}}\Big(J_1(y^{G(n)}_{\rm (BCs)}t)+b^{G(n)}_{\rm (BCs)}Y_1(y^{G(n)}_{\rm (BCs)}t)\Big). \tag{4}$$

where $t=\epsilon\exp(\sigma(\phi))$, $J_1(x)$ and $Y_1(x)$ are the Bessel functions of the first and second kind, and explicit expressions for $b^{G(n)}_{\rm (BCs)}$, $N^G_{\rm (BCs)}$ can be found in Ref. [18]. Furthermore, the eigenvalues $y^{G(n)}_{\rm (BCs)}$ (n=1, 2, $\cdots$) are determined by the relevant BCs. Correspondingly, the concrete expressions of the shape functions $f^{L,c}_{\rm (BCs)}(y^{c(n)}_{\rm (BCs)},t)$, $f^{R,c}_{\rm (BCs)}(y^{c(n)}_{\rm (BCs)},t)$ for fermion KK excitations in a warped extra dimension are also given in Ref. [18].

To obtain approximately the mixing between the zero modes of charged 2/3, −1/3 quarks and corresponding KK excitations, we write the infinite dimensional column vectors for quarks in the chirality basis as [25]

$$\begin{aligned}
\varPsi_{\rm L}(2/3) &= \Big(q^{i(0)}_{\rm u_L}(++),\cdots,q^{i(n)}_{\rm u_L}(++),U^{i(n)}_{\rm L}(+-),\tilde U^{i(n)}_{\rm L}(+-),\chi^{i(n)}_{\rm d_L}(-+),u^{i(n)}_{\rm L}(--),\cdots\Big)^{\rm T},\\
\varPsi_{\rm R}(2/3) &= \Big(u^{i(0)}_{\rm R}(++),\cdots,q^{i(n)}_{\rm u_R}(--),U^{i(n)}_{\rm R}(-+),\tilde U^{i(n)}_{\rm R}(-+),\chi^{i(n)}_{\rm d_R}(+-),u^{i(n)}_{\rm R}(++),\cdots\Big)^{\rm T},\\
\varPsi_{\rm L}(-1/3) &= \Big(q^{i(0)}_{\rm d_L}(++),\cdots,q^{i(n)}_{\rm d_L}(++),D^{i(n)}_{\rm L}(+-),d^{i(n)}_{\rm L}(--),\cdots\Big)^{\rm T},\\
\varPsi_{\rm R}(-1/3) &= \Big(d^{i(0)}_{\rm R}(++),\cdots,q^{i(n)}_{\rm d_R}(--),D^{i(n)}_{\rm R}(-+),d^{i(n)}_{\rm R}(++),\cdots\Big)^{\rm T},
\end{aligned} \tag{5}$$

where i=1, 2, 3 is the index of generation, n=1, 2, $\cdots$, ∞ is the index of KK exciting modes, the signs in parentheses denote the BCs satisfied by corresponding fields on UV and IR branes, respectively. In the chirality basis Eq. (5), the mass matrices of charged 2/3, −1/3 quarks are given in Ref. [18].

We then write the mass eigenstates respectively as

$$\begin{aligned}
U_{\alpha,\rm L} &= \big[\mathcal{U}^\dagger_{\rm L}\varPsi_{\rm L}(2/3)\big]_\alpha,\\
U_{\alpha,\rm R} &= \big[\mathcal{U}^\dagger_{\rm R}\varPsi_{\rm R}(2/3)\big]_\alpha,\\
D_{\alpha,\rm L} &= \big[\mathcal{D}^\dagger_{\rm L}\varPsi_{\rm L}(-1/3)\big]_\alpha,\\
D_{\alpha,\rm R} &= \big[\mathcal{D}^\dagger_{\rm R}\varPsi_{\rm R}(-1/3)\big]_\alpha.
\end{aligned} \tag{6}$$

Here, the charged 2/3 quarks U_1, U_2, U_3 are identified as up-type quarks u, c, t, and the charged −1/3 quarks D_1, D_2, D_3 are identified as the down-type quarks d,s, b in the SM, respectively.

In the gauge sector, we can similarly express interaction eigenstates of charged and neutral electroweak gauge bosons in linear combination of the mass eigenstates as

$$\begin{aligned}
W^{(0)\pm}_{\rm L} &= (\mathcal{Z}_{\rm W})_{0,0}W^\pm+\sum_{\alpha=1}^{\infty}(\mathcal{Z}_{\rm W})_{0,\alpha}W^\pm_{\rm H_\alpha},\\
W^{(n)\pm}_{\rm L} &= (\mathcal{Z}_{\rm W})_{2n-1,0}W^\pm+\sum_{\alpha=1}^{\infty}(\mathcal{Z}_{\rm W})_{2n-1,\alpha}W^\pm_{\rm H_\alpha},\\
W^{(n)\pm}_{\rm R} &= (\mathcal{Z}_{\rm W})_{2n,0}W^\pm+\sum_{\alpha=1}^{\infty}(\mathcal{Z}_{\rm W})_{2n,\alpha}W^\pm_{\rm H_\alpha},\\
Z^{(0)} &= (\mathcal{Z}_{\rm Z})_{0,0}Z+\sum_{\alpha=1}^{\infty}(\mathcal{Z}_Z)_{0,\alpha}Z_{\rm H_\alpha},\\
Z^{(n)} &= (\mathcal{Z}_{\rm Z})_{2n-1,0}Z+\sum_{\alpha=1}^{\infty}(\mathcal{Z}_Z)_{2n-1,\alpha}Z_{\rm H_\alpha},\\
Z^{(n)}_{\rm X} &= (\mathcal{Z}_{\rm Z})_{2n,0}Z+\sum_{\alpha=1}^{\infty}(\mathcal{Z}_Z)_{2n,\alpha}Z_{\rm H_\alpha},
\end{aligned} \tag{7}$$

in which $\mathcal{Z}_{\rm W}$, $\mathcal{Z}_{\rm Z}$ respectively denote the mixing matrices for charged as well as neutral electroweak gauge bosons, and Z, $\rm W^\pm$ are identified as the corresponding gauge bosons in the SM. For infinite dimensional column vectors, we have no means of obtaining the mixing matrices $\mathcal{U}_{\rm L,R}$, $\mathcal{D}_{\rm L,R}$ exactly.

2 Constraint on the warped extra dimension from $\bar{\rm B}\rightarrow \rm X_s\gamma$

The effective Hamilton for $\bar{\rm B}\rightarrow \rm X_s\gamma$ at scales $\mu_{\rm b}=\mathcal{O}(m_{\rm b})$ is presented in Ref. [26], in which the most interesting magnetic dipole moment operators are

$$\begin{aligned}
Q_{7\gamma} &= \frac{e}{16\pi^2}m_{\rm b}\bar s_\alpha\sigma^{\mu\nu}P_{\rm R}b_\alpha F_{\mu\nu},\\
Q_{8G} &= \frac{g_s}{16\pi^2}m_{\rm b}\bar s_\alpha T^a_{\alpha\beta}\sigma^{\mu\nu}P_{\rm R}b_\beta G^a_{\mu\nu},
\end{aligned} \tag{8}$$

and tilde operators $\widetilde Q_{7\gamma}$, $\widetilde Q_{8G}$ are obtained from $Q_{7\gamma}$,Q_{8G} after interchanging the right-handed projector $P_{\rm R}=(1+\gamma_5)/2$ with the left-handed $P_{\rm L}=(1-\gamma_5)/2$. Here

α, β=1, 2, 3 denote the color indices of quarks, $F_{\mu\nu}$ and $G^a_{\mu\nu}(a=1,\cdots,8)$ are the electromagnetic and strong field strength tensors, respectively.

The magnetic penguin operators $Q_{7\gamma}$, Q_{8G}, $\widetilde{Q}_{7\gamma}$, $\widetilde{Q}_{8G}$ are induced by virtual heavy freedoms through one loop diagrams at electroweak scale, and the relevant Feynman diagrams are drawn in Fig. 1.

We present the corrections from Fig. 1(a) to the Wilson coefficients at the electroweak scale $\mu_{\rm EW}$ to order $\upsilon^2/\Lambda^2_{\rm KK}$ as:

$$\begin{aligned}
C^{(a)}_{7\gamma}(\mu_{\rm EW}) &= (1-\Delta G_{\rm F})C^{{\rm SM}(a)}_{7\gamma}(\mu_{\rm EW})+\sum_{i=1}^{3}\Big(\Upsilon^{(a)}_{i,1}\Big)_{\rm sb}x_{\rm W^\pm}F^{(a)}_{1,\gamma}(x_{{\rm u}_i},x_{\rm W^\pm})\\
&+\frac{2m^4_{\rm W}}{\mu^2_{\rm EW}\Lambda^2_{\rm KK}e^2}\sum_{i=1}^{3}\frac{m_{{\rm u}_i}}{m_{\rm b}}\Big(V^{(0)}_{\rm CKM}\Big)^\dagger_{si}(\Delta^{\rm R}_{\rm W^\pm})_{i{\rm b}}F^{(a)}_{2,\gamma}(x_{{\rm u}_i},x_{\rm W^\pm})+\mathcal{O}\Big(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\Big),\\
C^{(a)}_{8G}(\mu_{\rm EW}) &= (1-\Delta G_{\rm F})C^{{\rm SM}(a)}_{8G}(\mu_{\rm EW})+\sum_{i=1}^{3}\Big(\Upsilon^{(a)}_{i,1}\Big)_{\rm sb}x_{\rm W^\pm}F^{(a)}_{1,g}(x_{{\rm u}_i},x_{\rm W^\pm})\\
&+\frac{2m^4_{\rm W}}{\mu^2_{\rm EW}\Lambda^2_{\rm KK}e^2}\sum_{i=1}^{3}\frac{m_{{\rm u}_i}}{m_{\rm b}}\Big(V^{(0)}_{\rm CKM}\Big)^\dagger_{si}(\Delta^{R}_{\rm W^\pm})_{i{\rm b}}F^{(a)}_{2,g}(x_{{\rm u}_i},x_{\rm W^\pm})+\mathcal{O}\Big(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\Big),\\
\widetilde{C}^{(a)}_{7\gamma}(\mu_{\rm EW}) &= \frac{2m^4_{\rm W}}{\mu^2_{\rm EW}\Lambda^2_{\rm KK}e^2}\sum_{i=1}^{3}\frac{m_{{\rm u}_i}}{m_{\rm b}}(\Delta^{R}_{\rm W^\pm})^\dagger_{si}\Big(V^{(0)}_{\rm CKM}\Big)_{i{\rm b}}F^{(a)}_{2,\gamma}(x_{{\rm u}_i},x_{\rm W^\pm})+\mathcal{O}\Big(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\Big),\\
\widetilde{C}^{(a)}_{8G}(\mu_{\rm EW}) &= \frac{2m^4_{\rm W}}{\mu^2_{\rm EW}\Lambda^2_{\rm KK}e^2}\sum_{i=1}^{3}\frac{m_{{\rm u}_i}}{m_{\rm b}}(\Delta^{R}_{\rm W^\pm})^\dagger_{si}\Big(V^{(0)}_{\rm CKM}\Big)_{i{\rm b}}F^{(a)}_{2,g}(x_{{\rm u}_i},x_{\rm W^\pm})+\mathcal{O}\Big(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\Big),
\end{aligned}\tag{9}$$

with $x_i=m^2_i/\mu^2_{\rm EW}$ and

$$\begin{aligned}
C^{{\rm SM}(a)}_{7\gamma}(\mu_{\rm EW}) &= x_{\rm W^\pm}\sum_{i=1}^{3}\Big(V^{(0)}_{\rm CKM}\Big)^\dagger_{si}(V^{(0)}_{\rm CKM})_{i{\rm b}}\Big(F^{(a)}_{1,\gamma}(x_{{\rm u}_i},x_{\rm W^\pm})+\frac{23}{36}\Big),\\
C^{{\rm SM}(a)}_{8G}(\mu_{\rm EW}) &= x_{\rm W^\pm}\sum_{i=1}^{3}\Big(V^{(0)}_{\rm CKM}\Big)^\dagger_{si}\Big(V^{(0)}_{\rm CKM}\Big)_{i{\rm b}}\Big(F^{(a)}_{1,g}(x_{{\rm u}_i},x_{\rm W^\pm})+\frac{1}{3}\Big),
\end{aligned}\tag{10}$$

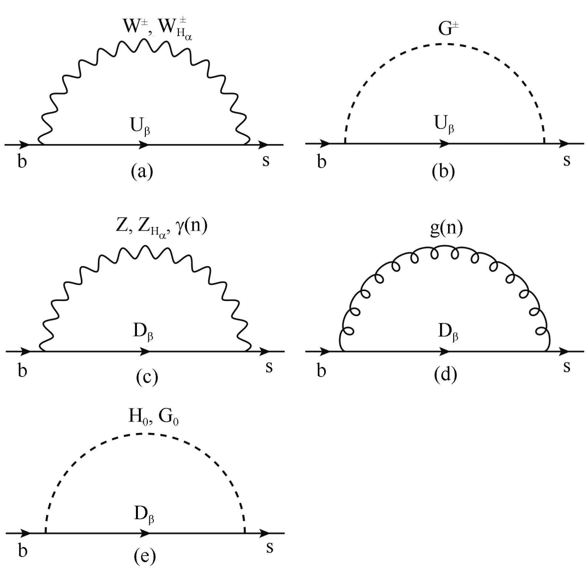

Fig. 1. The Feynman diagrams for b → sγ and b→sg in the warped extra dimension with custodial symmetry, where the photon and gluon can be attached in all possible ways.

representing the SM corrections from Fig. 1(a). The form factors are explicitly given by

$$\begin{aligned}
F^{(a)}_{1,\gamma}(x,y)&=\Big[-\frac{1}{36}\frac{\partial^3\varrho_{3,1}}{\partial y^3}-\frac{1}{4}\frac{\partial^2\varrho_{2,1}}{\partial y^2}-\frac{1}{3}\frac{\partial\varrho_{1,1}}{\partial y}\Big](x,y),\\
F^{(a)}_{2,\gamma}(x,y)&=\Big[\frac{1}{3}\frac{\partial^2\varrho_{2,1}}{\partial y^2}+\frac{4}{3}\frac{\partial\varrho_{1,1}}{\partial y}\Big](x,y),\\
F^{(a)}_{1,g}(x,y)&=\Big[\frac{1}{12}\frac{\partial^3\varrho_{3,1}}{\partial y^3}-\frac{1}{2}\frac{\partial\varrho_{1,1}}{\partial y}\Big](x,y),\\
F^{(a)}_{2,g}(x,y)&=\Big[-\frac{\partial^2\varrho_{2,1}}{\partial y^2}+2\frac{\partial\varrho_{1,1}}{\partial y}\Big](x,y).
\end{aligned}\tag{11}$$

Here, the function $\varrho_{m,n}(x,y)$ is defined through

$$\varrho_{m,n}(x,y)=\frac{x^m\ln^n x-y^m\ln^n y}{x-y}.\tag{12}$$

Adopting the 3×3 matrices $\mathcal{U}^{(0)}_{\rm L,R}$, $\mathcal{D}^{(0)}_{\rm L,R}$ denoting the rotation from chirality eigenstates to quark mass eigenstates in the absence of mixing between zero modes and corresponding KK excitations, we define the Cabibbo-Kobayashi-Maskawa (CKM) matrix as

$$(V_{\rm CKM})_{\rm tb}=(V^{(0)}_{\rm CKM})_{\rm tb}+\frac{\upsilon^2}{\Lambda^2_{\rm KK}}(\Delta^{(2)}_{\rm KK})_{\rm tb}+\mathcal{O}\left(\frac{\upsilon^3}{\Lambda^3_{\rm KK}}\right) \quad (13)$$

with $V^{(0)}_{\rm CKM}=\mathcal{U}^{(0)\dagger}_{\rm L}\mathcal{D}^{(0)}_{\rm L}$ being a unitary 3×3 matrix, and the leading order correction $\Delta^{(2)}_{\rm KK}$ together with other higher order corrections from heavy KK excitations break down the unitary property of $V_{\rm CKM}$ [22]. $\Delta G_{\rm F}$ denotes the corrections from exciting KK modes to the Fermi constant $G_{\rm F}$ extracted from the muon decay $\mu^-\to e^-\nu_\mu\bar{\nu}_e$ [27], the effective couplings $\Delta^{\rm R}_{\rm W^\pm}$, $\left(\Upsilon^{(a)}_{i,j}\right)_{\rm sb}$ $(i,j=1,\,2,\,3)$ can be found in Ref. [18].

For the Feynman diagram Fig. 1(b), we approach the corrections to the Wilson coefficients at the electroweak scale $\mu_{\rm EW}$ to order $\upsilon^2/\Lambda^2_{\rm KK}$ as:

$$\begin{aligned}
C^{\rm (b)}_{7\gamma}(\mu_{\rm EW}) =\;& (1-\Delta G_{\rm F})C^{\rm SM(b)}_{7\gamma}(\mu_{\rm EW})+\sum_{i=1}^{3}\Big\{\left(\Upsilon^{(b)}_{i,1}\right)_{\rm sb}x_{\rm W^\pm}F^{(b)}_{1,\gamma}(x_{{\rm u}_i},x_{\rm W^\pm})+\left(\Upsilon^{(b)}_{i,2}\right)_{\rm sb}x_{\rm W^\pm}F^{(b)}_{2,\gamma}(x_{{\rm u}_i},x_{\rm W^\pm})\Big\}\\
&+\frac{2m^2_{\rm W}s^2_{\rm w}}{9\Lambda^2_{\rm KK}e^2}\sum_{i,j,k=1}^{3}\left(\mathcal{D}^{(0)}_{\rm L}\right)^\dagger_{si}[f^{{\rm L},c^i_{\rm B}}_{(++)}(0,1)]\Big\{Y^{\rm u}_{ik}[\Sigma^{{\rm R},c^k_{\rm S}}_{(\mp\mp)}(1,1)]Y^{\rm u\dagger}_{kj}+2Y^{\rm d}_{ik}[\Sigma^{{\rm R},c^k_{\rm T}}_{(\pm\mp)}(1,1)]Y^{\rm d\dagger}_{kj}\Big\}\\
&\times[f^{{\rm L},c^j_{\rm B}}_{(++)}(0,1)]\left(\mathcal{D}^{(0)}_{\rm L}\right)_{j{\rm b}}+\mathcal{O}\left(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\right),\\
C^{\rm (b)}_{8G}(\mu_{\rm EW}) =\;& (1-\Delta G_{\rm F})C^{\rm SM(b)}_{8{\rm G}}(\mu_{\rm EW})+\sum_{i=1}^{3}\Big\{\left(\Upsilon^{(b)}_{i,1}\right)_{\rm sb}x_{\rm W^\pm}F^{(b)}_{1,g}(x_{{\rm u}_i},x_{\rm W^\pm})+\left(\Upsilon^{(b)}_{i,2}\right)_{\rm sb}x_{\rm W^\pm}F^{(b)}_{2,g}(x_{{\rm u}_i},x_{\rm W^\pm})\Big\}\\
&+\frac{m^2_{\rm W}s^2_{\rm w}}{12\Lambda^2_{\rm KK}e^2}\sum_{i,j,k=1}^{3}\left(\mathcal{D}^{(0)}_{\rm L}\right)^\dagger_{si}[f^{{\rm L},c^i_{\rm B}}_{(++)}(0,1)]\Big\{Y^{\rm u}_{ik}[\Sigma^{{\rm R},c^k_{\rm S}}_{(\mp\mp)}(1,1)]Y^{\rm u\dagger}_{kj}+2Y^{\rm d}_{ik}[\Sigma^{{\rm R},c^k_{\rm T}}_{(\pm\mp)}(1,1)]Y^{\rm d\dagger}_{kj}\Big\}\\
&\times[f^{{\rm L},c^j_{\rm B}}_{(++)}(0,1)]\left(\mathcal{D}^{(0)}_{\rm L}\right)_{j{\rm b}}+\mathcal{O}\left(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\right),\\
\widetilde{C}^{\rm (b)}_{7\gamma}(\mu_{\rm EW}) =\;& \sum_{i=1}^{3}\left(\Upsilon^{(b)}_{i,3}\right)_{\rm sb}x_{{\rm u}_i}F^{(b)}_{2,\gamma}(x_{{\rm u}_i},x_{\rm W^\pm})+\frac{2m^2_{\rm W}s^2_{\rm w}}{9\Lambda^2_{\rm KK}e^2}\sum_{i,j,k=1}^{3}\left(\mathcal{D}^{(0)}_{\rm R}\right)^\dagger_{si}[f^{{\rm R},c^i_{\rm T}}_{(++)}(0,1)]\\
&\times Y^{\rm d}_{ik}[\Sigma^{{\rm L},c^k_{\rm B}}_{(\pm\pm)}(1,1)]Y^{\rm d\dagger}_{kj}[f^{{\rm R},c^j_{\rm T}}_{(++)}(0,1)]\left(\mathcal{D}^{(0)}_{\rm R}\right)_{j{\rm b}}+\mathcal{O}\left(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\right),\\
\widetilde{C}^{\rm (b)}_{8G}(\mu_{\rm EW}) =\;& \sum_{i=1}^{3}\left(\Upsilon^{(b)}_{i,3}\right)_{\rm sb}x_{{\rm u}_i}F^{(b)}_{2,g}(x_{{\rm u}_i},x_{\rm W^\pm})+\frac{m^2_{\rm W}s^2_{\rm w}}{12\Lambda^2_{\rm KK}e^2}\sum_{i,j,k=1}^{3}\left(\mathcal{D}^{(0)}_{\rm R}\right)^\dagger_{si}[f^{{\rm R},c^i_{\rm T}}_{(++)}(0,1)]\\
&\times Y^{\rm d}_{ik}[\Sigma^{{\rm L},c^k_{\rm B}}_{(\pm\pm)}(1,1)]Y^{\rm d\dagger}_{kj}\times[f^{{\rm R},c^j_{\rm T}}_{(++)}(0,1)]\left(\mathcal{D}^{(0)}_{\rm R}\right)_{j{\rm b}}+\mathcal{O}\left(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\right),
\end{aligned} \quad (14)$$

where the SM corrections from Fig. 1(b) to the Wilson coefficients are written as

$$\begin{aligned}
C^{\rm SM(b)}_{7\gamma}(\mu_{\rm EW})&=\sum_{i=1}^{3}(V^{(0)}_{\rm CKM})^\dagger_{si}(V^{(0)}_{\rm CKM})_{i{\rm b}}x_{{\rm u}_i}(F^{(b)}_{1,\gamma}+F^{(b)}_{2,\gamma})(x_{{\rm u}_i},x_{\rm W^\pm}),\\
C^{\rm SM(b)}_{8G}(\mu_{\rm EW})&=\sum_{i=1}^{3}(V^{(0)}_{\rm CKM})^\dagger_{si}(V^{(0)}_{\rm CKM})_{i{\rm b}}x_{{\rm u}_i}(F^{(b)}_{1,g}+F^{(b)}_{2,g})(x_{{\rm u}_i},x_{\rm W^\pm}),
\end{aligned} \quad (15)$$

and the form factors are

$$\begin{aligned}F^{(\rm b)}_{1,\gamma}(x,y)&=\Big[-\frac{1}{72}\frac{\partial^3\varrho_{3,1}}{\partial y^3}-\frac{1}{24}\frac{\partial^2\varrho_{2,1}}{\partial y^2}+\frac{1}{6}\frac{\partial\varrho_{1,1}}{\partial y}\Big](x,y),\\ F^{(\rm b)}_{2,\gamma}(x,y)&=\Big[\frac{1}{12}\frac{\partial^2\varrho_{2,1}}{\partial y^2}-\frac{1}{6}\frac{\partial\varrho_{1,1}}{\partial y}-\frac{1}{3}\frac{\partial\varrho_{1,1}}{\partial x}\Big](x,y),\\ F^{(\rm b)}_{1,g}(x,y)&=\Big[\frac{1}{24}\frac{\partial^3\varrho_{3,1}}{\partial y^3}-\frac{1}{4}\frac{\partial^2\varrho_{2,1}}{\partial y^2}+\frac{1}{4}\frac{\partial\varrho_{1,1}}{\partial y}\Big](x,y),\\ F^{(\rm b)}_{2,g}(x,y)&=\Big[-\frac{1}{4}\frac{\partial^2\varrho_{2,1}}{\partial y^2}+\frac{1}{2}\frac{\partial\varrho_{1,1}}{\partial y}-\frac{1}{2}\frac{\partial\varrho_{1,1}}{\partial x}\Big](x,y).\end{aligned}\qquad(16)$$

Here, the concrete expressions for the effective couplings $\Delta^{\rm R}_{\rm G\pm}$, $\left(\Upsilon^{(\rm b)}_{i,j}\right)_{\rm sb}$ $(i,j=1,\,2,3)$ and $\Sigma^{\rm G}_{(\rm BCs)}(t,t')$, $\Sigma^{\rm L,c}_{(\rm BCs)}(t,t')$, $\Sigma^{\rm R,c}_{(\rm BCs)}(t,t')$ are given in Ref. [18]. Choosing $\mu_{\rm EW}=m_{\rm W}$, one can easily find that the sum (Eq. (10) and Eq. (15)) recovers the theoretical predictions on the Wilson coefficients of the dipole operators in the SM at electroweak energy scale.

For the diagram Fig. 1(c), the corrections to the relevant Wilson coefficients are analogously formulated to order $\mathcal{O}(\upsilon^2/\Lambda^2_{\rm KK})$ in Eq. (17).

$$\begin{aligned}C^{(c)}_{7\gamma}(\mu_{\rm EW})&=\Big\{\frac{(3-2s^2_{\rm W})^2}{162s^2_{\rm W}}\Big[2\left(\delta Z^{\rm d}_{\rm L}\right)^\dagger_{\rm sb}+2\left(\delta Z^{\rm d}_{\rm L}\right)_{\rm sb}+\frac{\upsilon^2}{2\Lambda^2_{\rm KK}}\left(\Delta^{\rm L}_{\rm Z}\right)_{\rm sb}+\frac{\upsilon^2}{2\Lambda^2_{\rm KK}}\left(\Delta^{\rm L}_{\rm Z}\right)^\dagger_{\rm sb}\Big]+\frac{1}{9}\left(1-\frac{2}{3}s^2_{\rm W}\right)\\&\quad\times\Big[\left(\delta Z^{\rm d}_{\rm L}\right)^\dagger_{\rm sb}+\left(\delta Z^{\rm d}_{\rm L}\right)_{\rm sb}+\frac{\upsilon^2}{2\Lambda^2_{\rm KK}}\left(\Delta^{\rm L}_{\rm Z}\right)^\dagger_{\rm sb}\Big]\Big\}+\frac{32\pi s^2_{\rm W}m^2_{\rm W}}{27\Lambda^2_{\rm KK}(kr\epsilon)^2}\sum_{i,j,k=1}^{3}\Big\{\frac{m_{{\rm d}_k}}{m_{\rm b}}\left(\mathcal{D}^{(0)}_{\rm L}\right)^\dagger_{si}\left(\mathcal{D}^{(0)}_{\rm L}\right)_{ik}\left(\mathcal{D}^{(0)}_{\rm R}\right)^\dagger_{kj}\left(\mathcal{D}^{(0)}_{\rm R}\right)_{j{\rm b}}\\&\quad\times\int_\epsilon^1{\rm d}t\int_\epsilon^1{\rm d}t'\left(\frac{25-16s^2_{\rm W}}{9c^2_{\rm W}}\Big[\Sigma^{\rm G}_{(++)}(t,t')\Big]+\frac{(3-2s^2_{\rm W})(3-4s^2_{\rm W})}{s^2_{\rm W}c^2_{\rm W}(1-2s^2_{\rm W})}\Big[\Sigma^{\rm G}_{(-+)}(t,t')\Big]\right)[f^{{\rm L},c^i_{\rm B}}_{(++)}(0,t)]^2[f^{{\rm R},c^j_{\rm T}}_{(++)}(0,t')]^2\Big\}\\&\quad+\mathcal{O}\left(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\right),\\C^{(\rm c)}_{8G}(\mu_{\rm EW})&=-3C^{(c)}_{7\gamma}(\mu_{\rm EW}),\\\widetilde{C}^{(\rm c)}_{7\gamma}(\mu_{\rm EW})&=\Big\{\frac{2}{81}s^2_{\rm W}\Big[2\left(\delta Z^{\rm d}_{\rm R}\right)^\dagger_{\rm sb}+2\left(\delta Z^{\rm d}_{\rm R}\right)_{\rm sb}+\frac{\upsilon^2}{2\Lambda^2_{\rm KK}}\left(\Delta^{\rm R}_{\rm Z}\right)_{\rm sb}+\frac{\upsilon^2}{2\Lambda^2_{\rm KK}}\left(\Delta^{\rm R}_{\rm Z}\right)^\dagger_{\rm sb}\Big]+\frac{1}{9}\left(1-\frac{2}{3}s^2_{\rm W}\right)\Big[\left(\delta Z^{\rm d}_{\rm R}\right)^\dagger_{\rm sb}\\&\quad+\left(\delta Z^{\rm d}_{\rm R}\right)_{\rm sb}+\frac{\upsilon^2}{2\Lambda^2_{\rm KK}}\left(\Delta^{\rm R}_{\rm Z}\right)^\dagger_{\rm sb}\Big]\Big\}+\frac{32\pi s^2_{\rm W}m^2_{\rm W}}{9\Lambda^2_{\rm KK}(kr\epsilon)^2}\sum_{i,j,k=1}^{3}\Big\{\frac{m_{{\rm d}_k}}{m_{\rm b}}\left(\mathcal{D}^{(0)}_{\rm R}\right)^\dagger_{si}\left(\mathcal{D}^{(0)}_{\rm R}\right)_{ik}\left(\mathcal{D}^{(0)}_{\rm L}\right)^\dagger_{kj}\left(\mathcal{D}^{(0)}_{\rm L}\right)_{j{\rm b}}\int_\epsilon^1{\rm d}t\int_\epsilon^1{\rm d}t'\\&\quad\times\sum_{n=1}^{\infty}\left(\frac{79-52s^2_{\rm W}}{27c^2_{\rm W}}\Big[\Sigma^{\rm G}_{(++)}(t,t')\Big]+\frac{(3-2s^2_{\rm W})(3-4s^2_{\rm W})^2}{s^2_{\rm W}c^2_{\rm W}(1-2s^2_{\rm W})}\Big[\Sigma^{\rm G}_{(-+)}(t,t')\Big]\right)[f^{{\rm R},c^i_{\rm T}}_{(++)}(0,t)]^2[f^{{\rm L},c^j_{\rm B}}_{(++)}(0,t')]^2\Big\}\\&\quad+\mathcal{O}\left(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\right),\\\widetilde{C}^{(c)}_{8G}(\mu_{\rm EW})&=-3\widetilde{C}^{(c)}_{7\gamma}(\mu_{\rm EW}).\end{aligned}\qquad(17)$$

The $\delta Z^{\rm d}_{\rm L,R}$ are corrections from KK exciting modes to a 3×3 mixing matrix of left- or right-handed charged $-1/3$ zero mode quarks. It is defined through $Z^{\rm d}_{\rm L,R}=1+\delta Z^{\rm d}_{\rm L,R}$ with $Z^{\rm d}_{\rm L,R}$ denoting the mixing matrices [18]. Here, $\Delta^{\rm L,R}_{\rm Z}$ represent the corrections from exciting KK modes to couplings of neutral gauge bosons and quarks similar to $\Delta_{\rm W\pm}$.

The corrections from KK exciting modes of a gluon to the Wilson coefficients of the dipole moment operators at the electroweak energy scale are given by

$$\begin{aligned}C^{(\rm d)}_{7\gamma}(\mu_{\rm EW})&=-\frac{256\pi s^2_{\rm W}m^2_{\rm W}g^2_{\rm s}}{9\Lambda^2_{\rm KK}(kr\epsilon)^2e^2}\sum_{i,j,k=1}^{3}\Big\{\frac{m_{{\rm d}_k}}{m_{\rm b}}\left(\mathcal{D}^{(0)}_{\rm L}\right)^\dagger_{si}\left(\mathcal{D}^{(0)}_{\rm L}\right)_{ik}\left(\mathcal{D}^{(0)}_{\rm R}\right)^\dagger_{kj}\left(\mathcal{D}^{(0)}_{\rm R}\right)_{j{\rm b}}\\&\quad\times\int_\epsilon^1{\rm d}t\int_\epsilon^1{\rm d}t'\Big[\Sigma^{\rm G}_{(++)}(t,t')\Big][f^{{\rm L},c^i_{\rm B}}_{(++)}(0,t)]^2[f^{{\rm R},c^j_{\rm T}}_{(++)}(0,t')]^2\Big\}+\mathcal{O}\left(\frac{\upsilon^2}{\Lambda^2_{\rm KK}}\right),\end{aligned}$$

$$C_{8G}^{(\mathrm{d})}(\mu_{\mathrm{EW}}) = \frac{256\pi s_{\mathrm{W}}^2 m_{\mathrm{W}}^2 g_{\mathrm{s}}^2}{3\Lambda_{\mathrm{KK}}^2 (kr\epsilon)^2 e^2} \sum_{i,j,k=1}^{3} \left\{ \frac{m_{\mathrm{d}_k}}{m_{\mathrm{b}}} \left(\mathcal{D}_{\mathrm{L}}^{(0)}\right)^\dagger_{si} \left(\mathcal{D}_{\mathrm{L}}^{(0)}\right)_{ik} \left(\mathcal{D}_{\mathrm{R}}^{(0)}\right)^\dagger_{kj} \left(\mathcal{D}_{\mathrm{R}}^{(0)}\right)_{j\mathrm{b}} \right.$$
$$\left. \times \int_\epsilon^1 \mathrm{d}t \int_\epsilon^1 \mathrm{d}t' \left[\Sigma_{(++)}^{\mathrm{G}}(t,t')\right] [f_{(++)}^{\mathrm{L},c_{\mathrm{B}}^i}(0,t)]^2 [f_{(++)}^{\mathrm{R},c_{\mathrm{T}}^j}(0,t')]^2 \right\} + \mathcal{O}\left(\frac{\upsilon^2}{\Lambda_{\mathrm{KK}}^2}\right),$$

$$\widetilde{C}_{7\gamma}^{(d)}(\mu_{\mathrm{EW}}) = -\frac{256\pi s_{\mathrm{W}}^2 m_{\mathrm{W}}^2 g_{\mathrm{s}}^2}{9\Lambda_{\mathrm{KK}}^2 (kr\epsilon)^2 e^2} \sum_{i,j,k=1}^{3} \left\{ \frac{m_{\mathrm{d}_k}}{m_{\mathrm{b}}} \left(\mathcal{D}_{\mathrm{R}}^{(0)}\right)^\dagger_{si} \left(\mathcal{D}_{\mathrm{R}}^{(0)}\right)_{ik} \left(\mathcal{D}_{\mathrm{L}}^{(0)}\right)^\dagger_{kj} \left(\mathcal{D}_{\mathrm{L}}^{(0)}\right)_{j\mathrm{b}} \right.$$
$$\left. \times \int_\epsilon^1 \mathrm{d}t \int_\epsilon^1 \mathrm{d}t' \left[\Sigma_{(++)}^{\mathrm{G}}(t,t')\right] [f_{(++)}^{\mathrm{R},c_{\mathrm{T}}^i}(0,t)]^2 [f_{(++)}^{\mathrm{L},c_{\mathrm{B}}^j}(0,t')]^2 \right\} + \mathcal{O}\left(\frac{\upsilon^2}{\Lambda_{\mathrm{KK}}^2}\right),$$

$$\widetilde{C}_{8G}^{(d)}(\mu_{\mathrm{EW}}) = \frac{256\pi s_{\mathrm{W}}^2 m_{\mathrm{W}}^2 g_{\mathrm{s}}^2}{3\Lambda_{\mathrm{KK}}^2 (kr\epsilon)^2 e^2} \sum_{i,j,k=1}^{3} \left\{ \frac{m_{\mathrm{d}_k}}{m_{\mathrm{b}}} \left(\mathcal{D}_{\mathrm{R}}^{(0)}\right)^\dagger_{si} \left(\mathcal{D}_{\mathrm{R}}^{(0)}\right)_{ik} \left(\mathcal{D}_{\mathrm{L}}^{(0)}\right)^\dagger_{kj} \left(\mathcal{D}_{\mathrm{L}}^{(0)}\right)_{j\mathrm{b}} \right.$$
$$\left. \times \int_\epsilon^1 \mathrm{d}t \int_\epsilon^1 \mathrm{d}t' \left[\Sigma_{(++)}^{\mathrm{G}}(t,t')\right] [f_{(++)}^{\mathrm{R},c_{\mathrm{T}}^i}(0,t)]^2 [f_{(++)}^{\mathrm{L},c_{\mathrm{B}}^j}(0,t')]^2 \right\} + \mathcal{O}\left(\frac{\upsilon^2}{\Lambda_{\mathrm{KK}}^2}\right). \quad (18)$$

As the coupling among the neutral Higgs/Goldstone, the standard charged $-1/3$ quark and the corresponding KK excitation of a charged $-1/3$ quark is proportional to quark mass (m_{s} or m_{b}), the corresponding corrections to the Wilson coefficients of dipole moment operators from Fig. 1(e) contain an additional suppression factor $m_{\mathrm{b}}m_{\mathrm{s}}/m_{\mathrm{W}}^2$ besides the global suppression factor $\upsilon^2/\Lambda_{\mathrm{KK}}^2$, and can be ignored safely [18].

With the elements of the evolution matrices presented in Ref. [28], the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ at hadron scale is written as

$$Br(\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma)=R\Big(|C_{7\gamma}(\mu_{\mathrm{b}})|^2+|\widetilde{C}_{7\gamma}(\mu_{\mathrm{b}})|^2+N(E_\gamma)\Big), \quad (19)$$

where the overall factor $R=2.47\times10^{-3}$, and the nonperturbative contribution $N(E_\gamma)=(3.6\pm0.6)\times10^{-3}$ [28]. In our numerical analysis, we choose the hadron scale μ_{b}=2.5 GeV, and include the SM contribution at NNLO level $C_{7\gamma}(\mu_{\mathrm{b}})=-0.3523$ [1]. Meanwhile we approach the corrections from the KK excitations in the leading-order approximation.

To apply the warped Froggatt-Nielsen mechanism [29], we adopt the ansatz for the hierarchical structures of the profiles of zero modes on the IR brane presented in Refs. [18, 27]. Without losing generality, we choose the Yukawa couplings $Y_{ij}^{\mathrm{u}}=0.01$ ($i\neq j$, $i,j=1$, 2, 3), $Y_{21}^{\mathrm{d}}=Y_{31}^{\mathrm{d}}=Y_{32}^{\mathrm{d}}=0.01$ and $\Lambda_{\mathrm{KK}}=1$ TeV. Fixing the bulk masses c_{B}^i, c_{S}^i, c_{T}^i, we derive the other elements of Yukawa couplings numerically through the warped Froggatt-Nielsen mechanism.

Because the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ is quite insensitive to the bulk masses c_{S}^i ($i=1$, 2, 3), we choose $c_{\mathrm{S}}^1=-0.75$, $c_{\mathrm{S}}^2=-0.55$, $c_{\mathrm{S}}^3=-0.35$ in our numerical analysis. In addition, we also assume $c_{\mathrm{B}}^3\leqslant c_{\mathrm{B}}^2\leqslant c_{\mathrm{B}}^1$ and $c_{\mathrm{T}}^3\geqslant c_{\mathrm{T}}^2\geqslant c_{\mathrm{T}}^1$ to guarantee the profiles of zero modes on the IR brane to satisfy the hierarchical structures.

In Fig. 2(a), we present the constraint on the $c_{\mathrm{B}}^1-c_{\mathrm{T}}^1$ plane from the experimental data when $c_{\mathrm{B}}^3=c_{\mathrm{B}}^2-0.1=c_{\mathrm{B}}^1-0.2$, and $c_{\mathrm{T}}^3=c_{\mathrm{T}}^2+0.1=c_{\mathrm{T}}^1+0.2$. The solid line represents the theoretical prediction on the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ by fitting the central value of the present experimental data $BR(\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma)=3.55\times10^{-4}$, the gray region represents the difference between the theoretical prediction and the central value of experimental data lying in one standard derivation, the gray slashed region represents the difference lying in two standard derivations, the gray meshed region represents the difference lying in three standard derivations, respectively. In Fig. 2(b), we plot the dependence of the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ on the bulk masses c_{B}^1, c_{T}^1. Besides the global suppression factor $\upsilon^2/\Lambda_{\mathrm{KK}}^2$, the dominating corrections from KK excitations to the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ depend on the bulk masses c_{B}^i ($i=1$, 2, 3) in terms of $[f_{(++)}^{\mathrm{L},c_{\mathrm{B}}^i}(0,t)][f_{(++)}^{\mathrm{L},c_{\mathrm{B}}^j}(0,t)]$. Because of this reason, the contributions from new physics to the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ decrease quickly as $c_{\mathrm{B}}^1\geqslant1$, and can be neglected safely compared with the contributions from the SM to the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$. Actually, the function $[f_{(++)}^{\mathrm{L},c}(0,1)]$ tends to zero steeply as $c>0.5$.

In Fig. 3(a), we present similarly the constraint on the $c_{\mathrm{B}}^1-c_{\mathrm{T}}^1$ plane from the experimental data as $c_{\mathrm{B}}^3/c_{\mathrm{B}}^2=c_{\mathrm{B}}^2/c_{\mathrm{B}}^1=(2/3)^{\mathrm{sgn}(c_{\mathrm{B}}^1)}$, $c_{\mathrm{T}}^3/c_{\mathrm{T}}^2=c_{\mathrm{T}}^2/c_{\mathrm{T}}^1=(2/3)^{-\mathrm{sgn}(c_{\mathrm{T}}^1)}$. In Fig. 3(b), we plot the dependence of the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ on the bulk masses c_{B}^1, c_{T}^1. Because the function $[f_{(++)}^{\mathrm{L},c}(0,1)]$ drops to zero steeply as $c>1$, the contributions from new physics to the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$ decrease quickly as $c_{\mathrm{B}}^1\geqslant1$, and can be neglected safely compared with the contributions from the SM to the branching ratio of $\bar{\mathrm{B}}\rightarrow \mathrm{X_s}\gamma$.

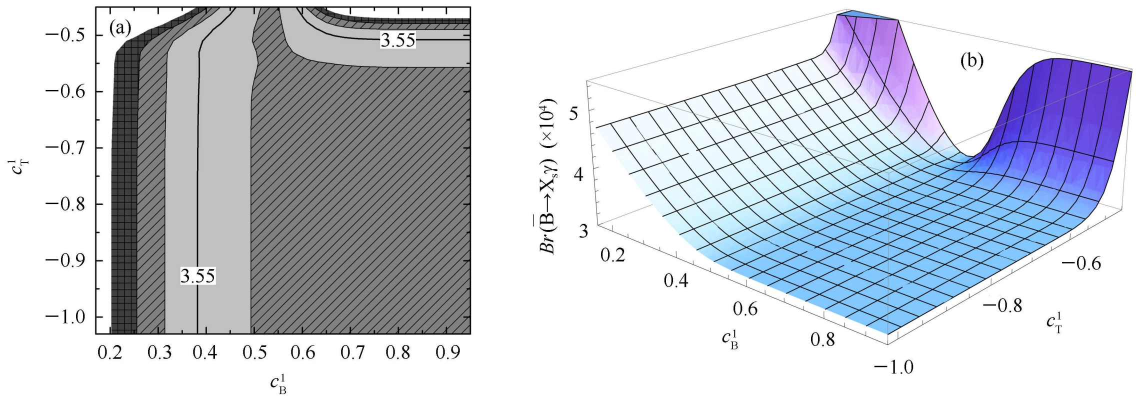

Fig. 2. As $c_{\rm B}^3=c_{\rm B}^2-0.1=c_{\rm B}^1-0.2$, and $c_{\rm T}^3=c_{\rm T}^2+0.1=c_{\rm T}^1+0.2$, (a) the constraint on $c_{\rm B}^1$-$c_{\rm T}^1$ plane from $\bar{\rm B}\to {\rm X_s}\gamma$; (b) the dependence of the branching ratio of $\bar{\rm B}\to {\rm X_s}\gamma$ on the bulk masses $c_{\rm B}^1$, $c_{\rm T}^1$.

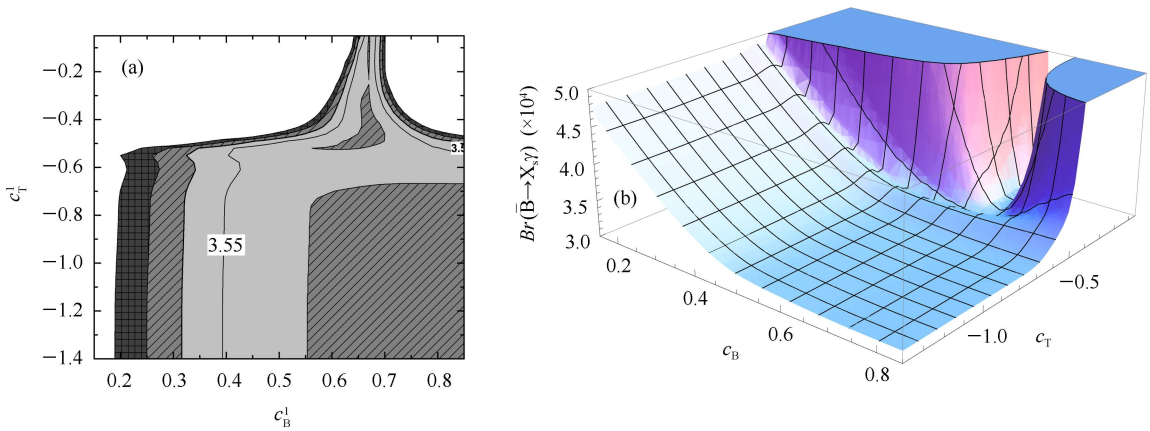

Fig. 3. As $c_{\rm B}^3/c_{\rm B}^2=c_{\rm B}^2/c_{\rm B}^1=(2/3)^{{\rm sgn}(c_{\rm B}^1)}$, and $c_{\rm T}^3/c_{\rm T}^2=c_{\rm T}^2/c_{\rm T}^1=(2/3)^{-{\rm sgn}(c_{\rm T}^1)}$, (a) the constraint on $c_{\rm B}^1$-$c_{\rm T}^1$ plane from $\bar{\rm B}\to {\rm X_s}\gamma$; (b) the dependence of the branching ratio of $\bar{\rm B}\to {\rm X_s}\gamma$ on the bulk masses $c_{\rm B}^1$, $c_{\rm T}^1$.

In Fig. 4, we present the branching ratio of $\bar{\rm B}\to {\rm X_s}\gamma$ varying with $Y_{21}^{\rm d}$ at different energy scales of low-lying Kaluza-Klein excitations. The bulk masses are chosen as $c_{\rm S}^i=\{-0.75,\ -0.55,\ -0.35\}$, $c_{\rm B}^i=\{0.65,\ 0.55,\ 0.45\}$ and $c_{\rm T}^i=\{-0.5,\ -0.4,\ -0.3\}$, and the imagined part of the Yukawa entries are set to be zero for simplification. The gray band in this figure denotes the experimental data with 3σ deviation. The solid line represents the energy scale of low-lying KK mode $\Lambda_{\rm KK}=1$ TeV, the dotted line represents $\Lambda_{\rm KK}=1.5$ TeV, the dash-dot line represents $\Lambda_{\rm KK}=2$ TeV, the dashed line represents $\Lambda_{\rm KK}=2.5$ TeV, and the dash-dot-dot line represents $\Lambda_{\rm KK}=3$ TeV. As the branching ratio is suppressed by $\Lambda_{\rm KK}^{-2}$, the contributions from new physics become smaller with $\Lambda_{\rm KK}$ increasing. If the other entries of Yukawa coupling are set to 0.01, the result shows that the branching ratio of $\bar{\rm B}\to {\rm X_s}\gamma$ increases when $Y_{21}^{\rm d}$ runs from 0.0001 to 0.25. From the upper boundary of gray region, we can obtain the constraints on $Y_{21}^{\rm d}$. Branching ratio varying with other entries can also be obtained similarly, i.e., $Y_{31}^{\rm d}$ and $Y_{32}^{\rm d}$, etc.

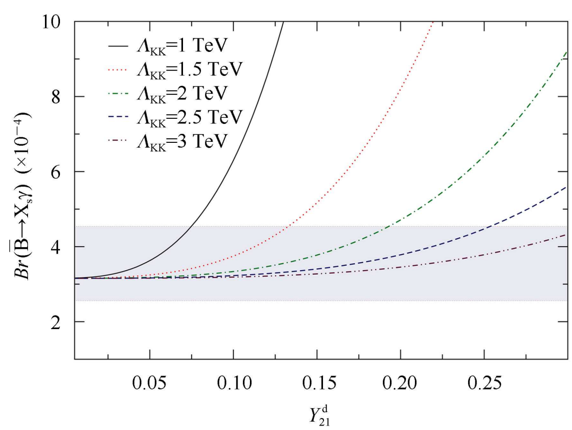

Fig. 4. (color online). Assuming the Yukawa entries $Y_{i,j}^{\rm u}=Y_{31}^{\rm d}=Y_{32}^{\rm d}=0.01$, $(i\neq j,\ i,j=1,\ 2,\ 3)$, we give the branching ratio of $\bar{\rm B}\to {\rm X_s}\gamma$ varying with $Y_{21}^{\rm d}$ at $\Lambda_{\rm KK}=\{1,\ 1.5,\ 2,\ 2.5,\ 3\}$.

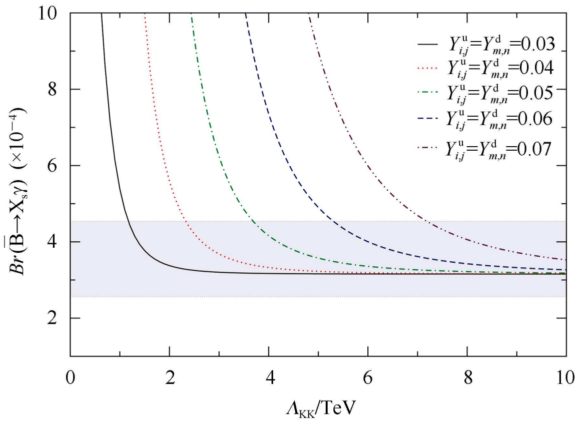

Fig. 5. (color online) The dependence of the branching ratio of $\bar{B}\to X_s\gamma$ on the energy scale of low-lying Kaluza-Klein excitations Λ_{KK} with different Yukawa entries.

As an important model parameter, the new physics energy scale provides a suppression factor Λ_{KK}^{-2}. We show the branching ratio varying with Λ_{KK} in Fig. 5 when all the entries of Yukawa take the same value. Constraints on Λ_{KK} can be obtained from the experimental band with 3σ deviation. The contributions from new physics drop to zero at limit $\Lambda_{KK}\to\infty$, then the branching ratio decreases quickly to SM prediction. So the decoupling theorem is satisfied within the framework with a warped extra dimension and custodial symmetry. The five lines displayed in this figure denote different values of Yukawa entries. The solid line shows the result when $Y^u_{i,j}=Y^d_{m,n}=0.03$, $(i\neq j,\ m>n)$, the dotted line corresponds to $Y^u_{i,j}=Y^d_{m,n}=0.04$, the dash-dot line gives the result corresponding to $Y^u_{i,j}=Y^d_{m,n}=0.05$, the dashed line corresponds to $Y^u_{i,j}=Y^d_{m,n}=0.06$, and the dash-dot-dot line corresponds to $Y^u_{i,j}=Y^d_{m,n}=0.07$.

3 Summary

In this work, we present the radiative correction to the rare decay $\bar{B}\to X_s\gamma$ in the SM extension with the warped extra dimension and the custodial symmetry. Applying the effective field theory, we approximate the radiative corrections to order $\upsilon^2/\Lambda_{KK}^2$, and sum over all the contributions originating from virtual KK excitations. Under the limit $\Lambda_{KK}\to\infty$, we recover the SM theoretical predictions on the Wilson coefficients of the dipole operators presented in literature exactly. In addition, we also analyze the possible constraint on the parameter space of new physics from experimental observation of the branching ratio of $\bar{B}\to X_s\gamma$.

References

1 Misiak M et al. Phys. Rev. Lett., 2007, **98**: 022002
2 Nakamura K et al. (Paricle Data Group collaboration). J. Phys. G, 2010, **37**: 075021
3 Randall L, Sundrum R. Phys. Rev. Lett., 1999, **83**: 3370; Randall L, Sundrum R. Phys. Rev. Lett., 1999, **83**: 4690
4 CHANG S et al. Phys. Rev. D, 2000, **62**: 084025
5 Csaki C, Hubisz J, Meade P. arXiv: 0510275 (hep-ph)
6 Grossman Y, Neubert M. Phys. Lett. B, 2000, **474**: 361
7 Gherghetta T, Pomarol A. Nucl. Phys. B, 2000 **586**: 141
8 Huber S J. Nucl. Phys. B, 2003, **666**: 269
9 Azatov A, Toharia M, ZHU L. Phys. Rev. D, 2009, **80**: 035016
10 Agashe K et al. Phys. Lett. B, 2006, **641**: 62
11 Carena M S et al. Phys. Rev. D, 2007, **76**: 035006
12 Blanke M, Buras A J, Duling B, Gori S, Weiler A. JHEP, 2009, **0903**: 001
13 Agashe K, Perez G, Soni A. Phys. Rev. D, 2005, **71**: 016002
14 Blanke M, Shakya B, Tanedo P, Tsai Y. arXiv:1203.6650 (hep-ph)
15 Csaki C, Falkowski A, Weiler A. JHEP, 2008, **0809**: 008
16 Gedalia O, Isidori G, Perez G. Phys.Lett. B, 2009, **682**: 200
17 Bauer M, Malm R, Neubert M. Phys. Rev. Lett., 2012, **108**: 081603
18 FENG T F et al. Phys. Rev. D, 2011, **84**: 096012
19 Djouadi A, Moreau G, Richard F. Nucl. Phys. B, 2007, **773**: 43
20 Bouchart C, Moreau G. Nucl. Phys. B, 2009, **810**: 66
21 Blanke M, Buras A J et al. JHEP, 2009, **0903**: 108
22 Buras A J, Duling B, Gori S. JHEP, 2009, **0909**: 076
23 Abbott L F. Nucl. Phys. B, 1981, **185**: 189; Gavela M B, Girardi G, Malleville C, Sorba P. Nucl. Phys. B, 1981, **193**: 257; Deshpande N G, Nazerimonfared M. Nucl. Phys. B, 1983, **213**: 390
24 Randall L, Schwartz M D. JHEP, 2001, **0111**: 003; Phys. Rev. Lett., 2002, **88**: 081801; Contino R, Pomarol A. JHEP, 2004, **0411**: 058; ZHU H S, LI C S, DAI L, GAO J, WANG J, YUAN C P. JHEP, 2011, **1109**: 043
25 Albrecht M E, Blanke M, Buras A J, Duling B, Gemmler K. JHEP, 2009, **0909**: 064
26 Buchalla G, Buras A J, Lautenbacher M E. Rev. Mod. Phys., 1996, **68**: 1125
27 Casagrande S, Goertz F, Haisch U, Neubert M, Prof T. JHEP, 2008, **0810**: 094
28 Buras A J, Merlo L, Stamou E. JHEP, 2011, **1108**: 124
29 Froggatt C D, Nielsen H B. Nucl. Phys. B, 1979, **147**: 277