Effect of magnetic field decay on the chemical heating of cooling neutron stars*

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Abstract: The effect of magnetic field decay on the chemical heating and thermal evolution of neutron stars is discussed in this paper. Our main goal is to study how the chemical heating mechanism and thermal evolution are changed by the field decay and how the magnetic field decay is modified by the thermal evolution. We compare stars cooling with chemical heating with one without chemical heating and find that the decay of the magnetic field is delayed significantly by the chemical heating. We find that the effect of chemical heating has been suppressed through the decaying magnetic field by the spin-down of the stars at a later stage. Compared with typical chemical heating, we find the decay of the magnetic field can even cause the surface temperature to turn down at an older age. When we discuss the cooling of neutron stars, we should consider the coupling effect of the magnetic field and the rotational evolution of neutron stars on the heating mechanisms.

Key words: neutron star, thermal evolution, chemical heating, magnetic field decay

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1 Introduction

Many neutron stars are detected as pulsars, whose regular pulsations in the radio, X-ray, and/or optical bands are produced by a strong magnetic field turning around during the stellar rotation period. It is widely accepted that a rapidly spinning neutron star loses its rotational energy by magnetic dipole radiation. The rotational evolution of isolated neutron stars should be determined by the evolution of their magnetic fields [1, 2].

Neutron stars also are very hot during birth, with temperatures well above 10^{10}K. This heat is radiated away mainly by neutrinos from the interior during the first million years or so (the neutrino cooling era) and, later, the emission of photons from the surface dominates the cooling of the star (the photon cooling era). This photon luminosity and its change with time depend on the properties of dense matter in the interior of neutron stars and its magnetic field [1, 2]. So, observations of thermal radiation can provide important information about the state of matter above and below nuclear density as well as for the magnetic field [3–5].

With the development of X-ray detectors, the thermal radiation from old and close neutron stars with surface temperature 10^5K has become possible. Becker and Trümper [6] detected several middle-aged and old neutron stars in the soft X-ray band. The upper limit of surface temperature for some millisecond pulsars was also found [7]. The Hubble Space Telescope also detected the thermal emission from a few relatively old pulsars in the optical and UV bands [8, 9]. The corresponding surface temperatures turn out to be surprisingly high compared with the predictions of the standard cooling model (cooling through modified URCA neutrino emission processes without any heating mechanisms) [10]. Such high temperatures can be understood only if some additional heating mechanisms operate in relatively old neutron stars, such as chemical heating [11–13], compositional transitions in the crust [14], crust cracking [15], r-mode dissipation heating [16, 17], and deconfinement heating [4, 5, 18]. The energy resources of these heating mechanisms are intimately related to the rotation evolution of neutron stars. Some of them are directly related to the conversion of rotational energy. For example, the heating energy of chemical heating comes from the rotational energy which is converted into heating by storing rotat-

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ional energy in terms of chemical energy.

Magnetic field decay has been studied previously. Goldreich & Reisenegger [19] discussed the processes which promote the dissipation of magnetic energy in neutron stars. Heyl et al. studied the decaying magnetic field of a magnetar and how this decay affects the cooling of stars [20–22]. In addition, many works have shown observational evidence for the heating from magnetic field decay in neutron stars [23–26].

In summary, the rotational, magnetic field and thermal evolution of neutron stars are coupled together, and the three may influence each other. For example, the decay of the magnetic field can influence the spin-down of neutron stars and the departure from chemical equilibrium. Furthermore, the departure from chemical equilibrium can heat the stars and change the thermal evolution behaviors. The decay of the magnetic field is affected by the temperature and rotational evolution of neutron stars. Moreover, the released magnetic field energy can also affect the thermal evolution. The main goal of this work is to discuss the coupling evolution of neutron stars including the rotating, thermal and magnetic field, especially the effect of the decaying magnetic field on the chemical heating of cooling neutron stars.

The paper is organized as follows: the models we used will be introduced in Section 2. In Section 3 the numerical results will be presented and analyzed. Discussions and conclusions are illustrated in Section 4.

2 The models

2.1 Equations of chemical evolution

Chemical heating is a heating mechanism which is associated with reactions produced by density changes. When neutron stars are spinning down, their internal density will be increased. This density variation can change the chemical equilibrium state throughout the core. If the finite departure from the chemical equilibrium which modifies the reaction is large enough, the net effect of the reactions will increase the chemical energy at the expense of the stored rotational energy. This is a channel through which a neutron star's rotational energy can be converted into heat [13].

$\delta\mu$ denotes the departure from chemical equilibrium, T denotes the internal temperature of the stars. Following the work done by Reisenegger [13], we assume that the chemical and thermal state of the star described by $\delta\mu$ and T, which are assumed to be independent of position within the star.

Defining $u=\delta\mu/(\pi k_{\rm B}T)$, the chemical energy released per unit time by the modified URCA processes under chemical non-equilibrium and the neutrino emissive rate are [13]:

$$\mathit{\Gamma}(T,\delta\mu)\delta\mu=\epsilon(T,0)\frac{14680u^2+7560u^4+860u^6+24u^8}{11513}, \quad (1)$$

and

$$\epsilon(T,\delta\mu)=\epsilon(T,0)\left(1+\frac{22020u^2+5670u^4+420u^6+u^8}{11513}\right), \quad (2)$$

where $\epsilon(T,0)$ is the neutrino emissivity rate of modified URCA processes at equilibrium:

$$\epsilon(T,0)\approx3.5\times10^{13}\left(\frac{x_{\rm eq}n}{n_0}\right)^{1/3}T_8^8 \text{ ergs cm}^{-3}\text{ s}^{-1}, \quad (3)$$

where $n_0=0.16$ fm^{-3} is the normal nuclear baryon density.

The equation for the evolution of $\delta\mu$ can be written as [13]:

$$\frac{{\rm d}\delta\mu}{{\rm d}t}=-E_{xx}\left(\alpha n\frac{E_{nx}}{E_{xx}}\frac{\mathit{\Omega}\dot{\mathit{\Omega}}}{G\rho_{\rm c}}+\frac{\mathit{\Gamma}}{n}\right), \quad (4)$$

for a given equation of state we can evaluate the second-order derivatives of the energy density, E_{nx} and E_{xx}.

2.2 Equation of magnetic field decay

Goldreich & Reisenegger [19] studied several physical mechanisms for the magnetic field decay in neutron stars: ohmic decay, ambipolar diffusion and Hall drift. Depending on the strength of magnetic field, each of these processes may dominate the evolution. The time scales for ohmic decay, ambipolar diffusion and Hall drift can be written as [19, 21]:

$$t_{\rm ohmic}\sim2\times10^{11}\frac{L_5^2}{T_8^2}\left(\frac{n}{n_0}\right)^3\text{ yr}, \quad (5)$$

$$t_{\rm ambip}^{s}\sim3\times10^9\frac{L_5^2T_8^2}{B_{12}^2}\text{ yr}, \quad (6)$$

$$t_{\rm ambip}^{\rm irr}\sim\frac{5\times10^{15}}{T_8^6B_{12}^2}\text{ yr}+t_{\rm ambip}^{\rm s}, \quad (7)$$

$$t_{\rm Hall}\sim5\times10^8\frac{L_5^2T_8^2}{B_{12}}\left(\frac{n}{n_0}\right)\text{ yr}, \quad (8)$$

where L_5 is a characteristic length scale of the flux loops through the outer core in unit of 10^5 cm, T_8 is the core temperature in unit of 10^8 K, and B_{12} is the field strength in unit of 10^{12} G.

Heyl et al. studied how the decaying magnetic field affects the thermal evolution of neutron stars [20–22]. Assuming the magnetic field is a dipole outside the star and in the insulating envelope and that the neutron star crust is thin relative to the radius of the star, we get the total energy released in the crust by the decay of the magnetic field. The total energy contained in the neutron star's magnetic field can be written as [21]:

$$E_{\rm B}=\frac{1}{12}B_{\rm p}^2R^3, \tag{9}$$

where $B_{\rm p}$ is the strength of the magnetic field at the pole on the surface and R is the radius of the neutron star. Substantial magnetic energy may be contained in small structures below the envelope; consequently, this is a lower bound on the total energy of the neutron star's magnetic field. The energy released by the decay of the field is [21]:

$$\dot{E}_{\rm B}=-\frac{1}{6}\frac{{\rm d}B_{\rm p}}{{\rm d}t}B_{\rm p}R^3. \tag{10}$$

In this paper, the magnetic energy is supposed to be dissipated below the region of the crust which supplies the bulk of the opacity; and the isothermal core of the neutrons star is heated by the field decay. Then, the equation for the field decay is of the form [21]:

$$\frac{{\rm d}B_{\rm p}}{{\rm d}t}=-B_{\rm p}\left(\frac{1}{t_{\rm ohmic}}+\frac{1}{t_{\rm ambip}}+\frac{1}{t_{\rm Hall}}\right). \tag{11}$$

2.3 Equation of thermal evolution

The thermal evolution of neutron stars is governed by general relativistic equations of heat diffusion inside the star, and is mainly caused by neutrino emission from the core and by the photon emission from the surface of the star [10, 27, 28]. After thermal relaxation, the core temperature becomes constant throughout the stellar interior, it is conventional to divide the neutron stars into two parts, including the interior and the outer heat blanketing envelope [29]. As discussed in [10, 13, 30], this assumption has been shown to be effective for $t>10^3$ yr. In this work, we consider a simple model of a normal neutron star core composed of neutrons, protons, electrons (npe matter), and ignore the potential presence of exotic particles. We assume that the star loses its thermal energy by the modified URCA processes: $\rm n{+}n\to n{+}p{+}e^-{+}\bar{\nu}_e$ and $\rm n{+}p{+}e^-\to n{+}n{+}\nu_e$.

The equation of thermal evolution can be written as:

$$C_{\rm v}\frac{{\rm d}T}{{\rm d}t}=\frac{\Gamma\delta\mu}{n}-\frac{\dot{E}_{\nu}}{n}-\dot{E}_{\gamma}+\frac{m_{\rm n}}{M}\dot{E}_{\rm B}, \tag{12}$$

where $C_{\rm v}$ is the specific heat, $\Gamma\delta\mu$ is the chemical energy released by the reactions, $\dot{E}_{\nu}$ is the neutrino emission rate which is determined by the state of matter of the core, $\dot{E}_{\gamma}$ is the energy loss rate from blackbody radiation from the surface and $\dot{E}_{\rm B}$ is the magnetic field decay heating rate, $m_{\rm n}$ and M are the mass of a neutron and the star.

As the envelope serves as a heat blanket, there exists a temperature gradient from core temperature to surface temperature. Since we do not discuss the ultra-high magnetic field, we apply a formula which is demonstrated by [29] which is an effective approximation while $B<10^{13}$ G. It reads: $T_{\rm s}=3.08\times10^6\ g_{\rm s,14}^{1/4}\ T_9^{0.5495}$, where $g_{\rm s,14}$ is the proper surface gravity of the star in units of 10^{14} cm s^{-2}. Using the relation between the surface and interior temperature, we get the blackbody radiation energy loss rate: $\dot{E}_{\gamma}=\frac{m_{\rm n}}{M}L_{\gamma}$, where $L_{\gamma}=4\pi R^2\sigma T_{\rm s}^4$ is the surface photon luminosity. Once the star model is set, we can get its corresponding specific heat, neutrino emission rate and surface photon luminosity.

2.4 Equation of rotational evolution

Assume the spin-down of the star is caused by the magnetic dipole radiation, then the equation governing the evolution of the angular velocity is:

$$\frac{{\rm d}\Omega}{{\rm d}t}=-\frac{2R^6}{3c^3I}\Omega^3(t)B^2(t), \tag{13}$$

where I is the total rotational inertia, $I=\frac{8\pi}{3}\int_0^R\rho r^4{\rm d}r$.

3 Numerical results

In our calculation, we take the equation of state named AV14+UVII [31]. A $1.4M_{\odot}$ neutron star mainly cooling through modified URCA processes, such that the equation of state and the corresponding parameters are: $\rho_{\rm c}=1.2\times10^{15}$ g·cm^{-3}, $n=0.56$ fm^{-3}, $x_{\rm eq}=0.07$, $R=10.4$ km. Then, $C_{\rm V}$, E_{nx} and E_{xx} are calculated. With the given initial spin period, temperature and magnetic field, we can solve Eqs. (4), (10), (12) and (13) numerically. In this work, we assume the initial period $P_0=1$ ms and the initial core temperature $T_0=10^{10}$ K.

Figure 1 shows the time evolution of the decaying magnetic field with chemical heating and magnetic field decay heating (solid lines). For comparison, we also give the curves for decaying fields which are cooling through the modified URCA processes without any heating effects (dotted lines). Obviously, the evolving heating mechanism will make the magnetic field decay slower during the photon cooling era ($t>10^6$ yr). The smaller the initial magnetic field strength, the slower the magnetic field decays. The evolution of the magnetic field depends on the thermal evolution procedure. Without heating mechanisms, the magnetic field decays rapidly when $t>10^6$ yr. When the heating effects are considered, the magnetic field decay will be delayed.

In Fig. 2, the evolution of $|\delta\mu|/(\pi k_{\rm B})$ for different initial magnetic fields is shown. $\delta\mu$ denotes the departure from chemical equilibrium. Compared with the fixed magnetic field, $|\delta\mu|/(\pi k_{\rm B})$ becomes smaller with the decaying field for the lower magnetic field ($B<10^{11}$ G) in the photon cooling era. Meanwhile, the evolution of the surface temperature for different initial magnetic fields is shown in Fig. 3. Considering the coupling evolution, the surface temperature becomes even lower at a later stage, especially for the intermediate field (10^9 G, 10^{10} G). That coincides with the variation of $|\delta\mu|/(\pi k_B)$.

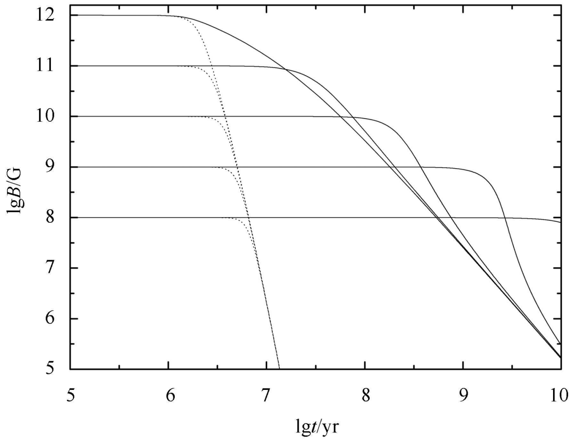

Fig. 1. Time evolution of the magnetic field. The initial magnetic fields are $B_i = 10^8$ G, 10^9 G, 10^{10} G, 10^{11} G, 10^{12} G from top to bottom. The solid lines are for the decaying fields with chemical heating and magnetic field decay heating, the dotted lines are for the decaying field of the standard cooling model.

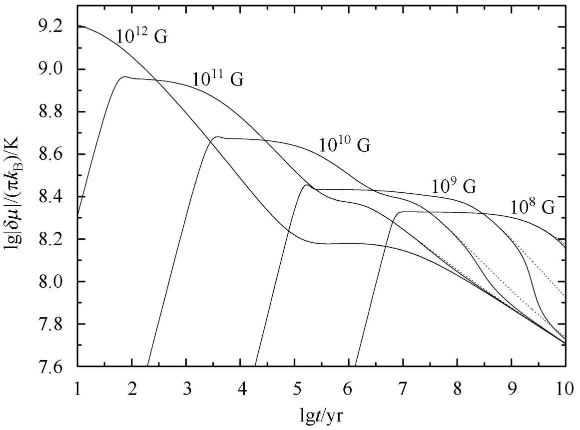

Fig. 2. Time evolution of the $\delta\mu/(\pi k_B)$ with different initial magnetic fields. The solid lines are for the decaying magnetic fields with chemical heating and magnetic field decay heating, the dotted lines are for the typical chemical heating.

Figure 4 plots the dependence of the surface temperature on B. Since neutrino emissivity and photon luminosity make the characteristic cooling time significantly shorter than the one for magnetic field decay, the heating effect is negligible and the magnetic field decay is much slower than the thermal one during the first 10^6 yr, so we can find that the surface temperature decreases while the magnetic field decays at early stage. This phase of evolution lasts while the total heating rate is smaller than the luminosity. When the heating plays a dominating role in the thermal evolution, the characteristic cooling time scale becomes comparable with the decay time scale of the magnetic field. When considering the heating mechanisms, we find that the two time scales become comparable earlier.

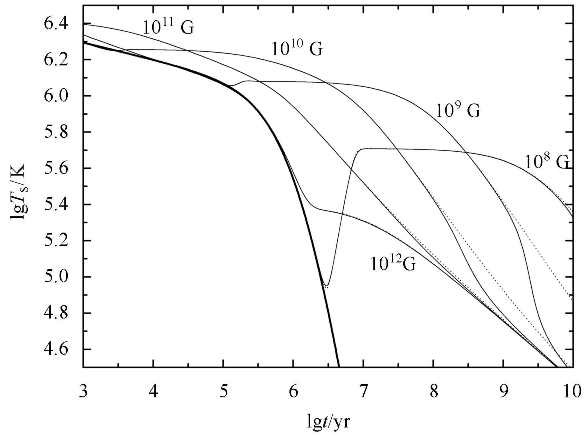

Fig. 3. Evolution of the surface temperature with initial magnetic fields $B_i=10^8$ G, 10^9 G, 10^{10} G, 10^{11} G, 10^{12} G from top to bottom. The solid lines are for the decaying fields with chemical heating and magnetic field decay heating, the dotted lines are for the typical chemical heating model, the bold line is the temperature evolution for the standard cooling.

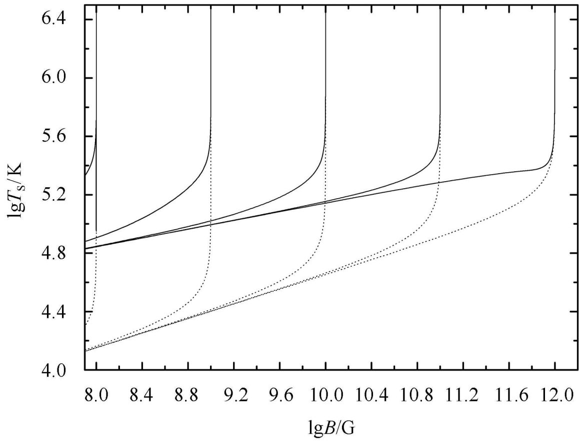

Fig. 4. Dependence of the surface effect temperature T_s^∞ on the magnetic field B. The initial magnetic field is $B_i=10^8$G, 10^9G, 10^{10}G, 10^{11}G, 10^{12}G from left to right. The solid lines are for the decaying fields with chemical heating and magnetic field decay heating, the dotted lines are for the decaying magnetic field of the standard cooling model.

We plot the time evolution of angular velocity in units of initial angular velocity for different initial magnetic fields in Fig. 5. The solid lines are for decaying fields with chemical heating and magnetic field decay heating.

The dotted lines are for typical chemical heating with a standard magnetic dipole braking model. The spin-down is delayed obviously in the photon cooling era. The magnetic field of a pulsar decays significantly over a time span of a few million years, then its real age must be considerably smaller than the spin-down time scale. In such case, it is incorrect to write the age of the pulsar in the form of $|\Omega/2\dot{\Omega}|$.

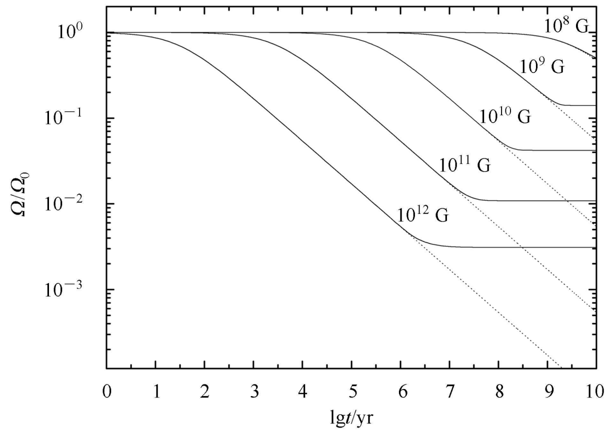

Fig. 5. The time evolution of angular velocity in units of initial angular velocity with different initial magnetic fields. The solid lines are for decaying fields with chemical heating and magnetic field decay heating, the dotted lines are for typical chemical heating with the standard magnetic braking model.

4 Discussions and conclusions

Our work shows that: the effect of chemical heating has been suppressed through a decaying magnetic field by the spin-down of the stars at a later stage; the thermal evolution delays the decay of the magnetic field through heating effects; while the decaying magnetic field even makes the surface temperature become lower in the photon cooling era through rotation and chemical evolution. In Eq. (5), it is obvious that the evolution of $\delta\mu$ is closely connected with the rotation evolution (through angular velocity directly or indirectly). $\delta\mu$ becomes smaller in the photon cooling era since the spin-down of the stars has been delayed by the decaying magnetic field. Meanwhile, the surface temperature of the star also becomes lower in the same era. These show that the effect of chemical heating has been suppressed by the spin-down of the stars through decaying magnetic field at a later stage. When we discuss the thermal evolution of neutron stars, we should consider the coupling effect of magnetic field evolution and the spin-down of neutron stars on the heating mechanisms.

From radio observations, we get the period, the period derivation and the magnetic field. Meanwhile, we obtain the surface temperature of certain pulsars from X-ray and optical/UV band observations. So, it may be possible to derive certain constraints on the age, the initial magnetic field and the period of the pulsar from multi-band (optical/UV, radio, X-ray) observations.

We have considered the case of the cooling of neutron stars with the modified URCA processes. When the proton abundance in the nuclear matter exceeds 1/9, direct URCA processes can process in the interior of neutron stars [10]. Furthermore, nuclear matter in the interior of neutron stars can be in the states of a meson (π or K) condensation state or another exotic state [10]. Such status will lead to rapid cooling of neutron stars, thus slower magnetic field decay and interaction between rapid cooling and magnetic field decay are not obvious.

We are aware that our model is very simple and can be improved in many respects. Firstly, in order to explain the observational data and show the quantitative effect of coupling evolution, a more realistic neutron star model should be considered carefully, including the particle distribution and the spatial structure of the star as done by Fernández et al. [12]. Secondly, a more elaborate model of the magnetic field evolution may be used, instead of a simple model formulated in Section 3, in order to take into account the variation of the magnetic field of the stars. This could be supplemented by a more realistic simulation of the cooling processes and star model. Thirdly, the reaction rates in superfluid neutron stars are suppressed at low temperatures [3] and superfluidity makes ambipolar diffusion inefficient [32]. These arguments suggest that the coupling evolution of a superfluid neutron star should be studied thoroughly in future work.

In spite of some shortcomings, our work shows the qualitative importance of the coupling evolution of thermal, rotational and magnetic field of neutron stars. We do not try to explain the observational data precisely. Future investigations will consider a consistent evolution including the geometry of the magnetic field, the thermal structure of neutron stars with magnetized envelopes, and comparisons with the thermal emission observation data.

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