Chinese Physics C Vol. 38, No. 2 (2014) 025101

Thermodynamics of noncommutative geometry inspired black holes based on Maxwell-Boltzmann smeared mass distribution *

LIANG Jun(梁钧)[1;1)] LIU Yan-Chun(刘艳春)[2] ZHU Qiao(朱桥)[1]

[1] College of Science, Shannxi University of Science and Technology, Xi'an 710021, China
[2] Library, Shannxi University of Science and Technology, Xi'an 710021, China

Abstract: In order to further explore the effects of non-Gaussian smeared mass distribution on the thermodynamical properties of noncommutative black holes, we consider noncommutative black holes based on Maxwell-Boltzmann smeared mass distribution in (2+1)-dimensional spacetime. The thermodynamical properties of the black holes are investigated, including Hawking temperature, heat capacity, entropy and free energy. We find that multiple black holes with the same temperature do not exist, while there exists a possible decay of the noncommutative black hole based on Maxwell-Boltzmann smeared mass distribution into the rotating (commutative) BTZ black hole.

Key words: physics of black holes, black hole thermodynamics, noncommutative geometry

PACS: 04.70.-s, 04.70.Dy, 02.40.Gh **DOI:** 10.1088/1674-1137/38/2/025101

1 Introduction

The idea of noncommutative spacetimes was originally introduced by Snyder to cure the divergences in relativistic quantum field theory [1, 2]; in recent years, the renewed interest in noncommutative spacetimes has mainly been due to its relevance in quantum gravity research [3].

Black hole physics plays a key role in the development of quantization gravity. Ever since Hawking proved the thermal radiation of a collapsing black hole using the techniques of quantum field theory in a curved spacetime background [4, 5], extensive studies on black hole physics have been done from theoretical view points.

A few years ago, Nicolini et al. first found a noncommutative inspired Schwarzschild black hole solution in four dimensions [6–8]; subsequently, the model was extended to include the electric charge [9] and extra-spatial dimensions [10, 11]. The remarkable property of the noncommutative inspired black hole solution is that there exists an extreme mass M_0 under which no horizon can be formed; thus, there will be a remnant after the Hawking radiation, which could probably solve the so-called paradox of black hole information loss [12]. From then on, noncommutative geometry inspired black holes aroused great interest among researchers. Kim et al. investigated thermodynamical similarity between the noncommutative Schwarzschild black hole and the Reissner-Nordström black hole [13]; Nozari and Mehdipour investigated Parikh-Wilczek tunneling of noncommutative black holes [14–17]; Mann et al. investigated the pair creation of noncommutative black holes in a background with a positive cosmological constant [18]; Giri found out and calculated the asymptotic quasinormal modes of a noncommutative geometry inspired Schwarzschild black hole [19]; Ding et al. studied the influence of the spacetime noncommutative parameter on the strong field gravitational lensing in the Reissner-Nordström black hole spacetime [20]; Mureika et al. analyzed comprehensively a noncommutative (1+1)-dimensional black hole [21]; Myung et al. [22, 23] and Tejeiro et al. [24] studied black hole thermodynamics in noncommutative spaces (for a comprehensive review see Ref. [3]).

In most current works, the noncommutative smearing is mathematically introduced to replace the point-like source term with a Gaussian distribution; however, Park pointed out in Ref. [25] that the Gaussianity need not always be required, although non-Gaussian smeared mass distribution has not been studied much so far, and showed that noncommutative black hole solutions based on non-Gaussian smeared mass distribution do exist in three-dimensional de Sitter spacetime. He gave the matter density as

$$\rho(r)=\frac{M}{\pi\Gamma\left(\dfrac{n}{2}+1\right)}\frac{r^n}{L^{n+2}}\mathrm{e}^{-\frac{r^2}{L^2}},$$

where M denotes the total mass, L is a characteris-

Received 25 March 2013, Revised 7 May 2013

* Supported by Natural Science Foundation of Education Department of Shannxi Provincial Government (12JK0954) and Doctorial Scientific Research Starting Fund of Shannxi University of Science and Technology (BJ12-02)

1) E-mail: liangjunbeijing@163.com

tic length scale of the matter distribution and $n = 1$ and $n = 2$ correspond to a Reyleigh distribution and a Maxwell-Boltzmann distribution, respectly. The Reyleigh smeared mass distribution was first introduced by Myung et al. in Ref. [23]; they found that there may exist a phase transition between the noncommutative black hole based on a Reyleigh distribution and the non-rotating BTZ black hole[1)]. In order to further explore the effects of non-Gaussian smeared mass distribution on the thermodynamical properties of noncommutative black holes, we consider Maxwell-Boltzmann smeared mass distribution in this paper. We deduce and discuss the thermodynamical quantities of black holes, including Hawking temperature, heat capacity, entropy and free energy, and analyze the influence of parameters θ, l (or the cosmological constant Λ, l is related to the cosmological constant by $\Lambda=-\dfrac{1}{l^2}$, see below) and angular momentum J on the black hole. We find that there do not exist multiple black holes with the same temperature, while there exists a possible decay of the noncommutative black hole based on Maxwell-Boltzmann smeared mass distribution into the rotating (commutative) BTZ black hole.

The outline of this paper is as follows: in Section 2, we introduce a noncommutative inspired black hole solution of Einstein equations in AdS_3 spacetime using an anisotropic perfect fluid. Then in Section 3, we study the thermodynamics of this noncommutative black hole. The final section is for the conclusions.

2 Noncommutative black hole solution in (2+1)-dimensions

The noncommutative effect can be incorporated in gravity in two main ways [29–36]. One is to directly take the spacetime as noncommutative $[\hat{x}^\mu,\hat{x}^\nu]=\mathrm{i}\theta^{\mu\nu}$, where $\hat{x}^\mu$ are the spacetime coordinate operators and $\theta^{\mu\nu}$ is a real-valued antisymmetric constant tensor of dimension length squared, which determines the fundamental cell discretization of spacetime manifold in the same way as the Planck constant $\hbar$ discretizes the phase space. Then use the Seiberg-Witten map to recast the gravitational theory in noncommutative space in terms of the corresponding theory in commutative space. This leads to correction terms involving powers of $\theta^{\mu\nu}$ in the metric. The Moyal star product, defined in terms of $\theta^{\mu\nu}$ as $(f\star g)\equiv\exp\left[\dfrac{\mathrm{i}}{2}\theta^{\mu\nu}\dfrac{\partial}{\partial x^\mu}\partial x^\nu\right]f(x)g(y)|_{x=y}$, where f and g are infinitely differentiable arbitrary functions, is used in this method [37–39]. The other is called the coordinate coherent states approach (the noncommutative black hole obtained by the coordinate coherent states approach is usually called the noncommutative (geometry) inspired black hole), which is proposed from a different point of view for the study of noncommutative quantum mechanics and quantum field theory. The main idea of this approach is to incorporate the effect of noncommutativity in the mass term of the gravitational source. More specifically, the substitution rule is to replace the pointlike mass density described by a δ-function with a Gaussian distribution, this is equivalent to saying that the only modification occurs at the level of the energy-moment tensor, while Einstein tensor is formally left unchanged in the Einstein equation (see below) [3, 6–10, 13–21, 24, 36]. However, as we have mentioned above, Park further pointed out that the Gaussianity need not be required always and gave the matter density as

$$\rho(r)=\frac{M}{\pi\Gamma\left(\dfrac{n}{2}+1\right)}\frac{r^n}{L^{n+2}}\mathrm{e}^{-\frac{r^2}{L^2}}$$

[25]. Setting $n=2$ and defining $L=2\sqrt{\theta}$, we get the following Maxwell-Boltzmann smeared mass density

$$\rho_\theta(r)=\frac{Mr^2}{16\pi\theta^2}\mathrm{e}^{-\frac{r^2}{4\theta}}. \tag{1}$$

The smeared mass distribution is now given by

$$M_\theta(r)=\int_0^r 2\pi r'\rho_\theta(r')\mathrm{d}r'=M\left[1-\left(\frac{r^2}{4\theta}+1\right)\mathrm{e}^{-\frac{r^2}{4\theta}}\right]. \tag{2}$$

In the limit of $\dfrac{r}{\sqrt{\theta}}\rightarrow\infty$, we get $M_\theta(r)\rightarrow M$.

In order to find a black hole solution in AdS_3 spacetime, we introduce the Einstein equations (in $c=G=1$ unit)

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=8\pi T_{\mu\nu}+\frac{1}{l^2}g_{\mu\nu}, \tag{3}$$

where l is related with the cosmological constant by $\Lambda=-\dfrac{1}{l^2}$, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric of spacetime and $T_{\mu\nu}$ is the energy-momentum tensor. These quantities are defined by

$$R_{\mu\nu}\equiv R^\lambda_{\mu\nu\lambda}, \tag{4}$$

$$R\equiv R^\mu{}_\mu=g^{\mu\nu}R_{\nu\mu}, \tag{5}$$

$$R^\lambda_{\mu\nu\rho}\equiv\partial_\nu\Gamma^\lambda_{\mu\rho}-\partial_\rho\Gamma^\lambda_{\mu\nu}+\Gamma^\sigma_{\mu\rho}\Gamma^\lambda_{\sigma\nu}-\Gamma^\sigma_{\mu\nu}\Gamma^\lambda_{\sigma\rho}, \tag{6}$$

$$\Gamma^\lambda_{\mu\nu}\equiv\frac{1}{2}g^{\lambda\sigma}(\partial_\mu g_{\nu\sigma}+\partial_\nu g_{\sigma\mu}-\partial_\sigma g_{\mu\nu}), \tag{7}$$

$$\mathrm{d}s^2\equiv g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu, \tag{8}$$

$$g_{\mu\sigma}g^{\sigma\nu}=\delta^\nu_\mu, \tag{9}$$

1) The discovery of the (2+1)-dimensional BTZ black hole [26] is considered to be one of the important progresses in general relativity in recent years [27]. Although a toy model in some aspects, it has aroused significant interest by virtue of its connections with certain string theories and its role in understanding the holographic description of asymptotically anti-de Sitter spacetimes [28]. In addition, this black hole model has proven to be an especially useful laboratory for studying the thermodynamical properties of black holes.

where $R^{\lambda}_{\mu\nu\rho}$ is the Riemann-Christoffel tensor and $\Gamma^{\lambda}_{\mu\nu}$ is the Christoffel symbol. The conservation law $T^{\mu\nu}{}_{;\nu} \equiv \partial_\nu T^{\mu\nu} + \Gamma^{\mu}_{\rho\nu}T^{\rho\nu} + \Gamma^{\nu}_{\rho\nu}T^{\mu\rho} = 0$ tells us that the energy-momentum tensor takes an anisotropic form $T^{\mu}_{\nu} = \mathrm{diag}(-\rho_\theta, p_r, p_\perp)$, where the radial and tangential pressure are given by $p_r = -\rho_\theta$ and $p_\perp = -\rho_\theta - r\partial_r\rho_\theta$, respectively. Then solving the Einstein equations of motion, we obtain the line element as

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + f^{-1}(r)\mathrm{d}r^2 + r^2(\mathrm{d}\varphi + N^{\varphi}\mathrm{d}t^2), \tag{10}$$

where

$$f(r) = -8M\left[1 - \left(\frac{r^2}{4\theta}+1\right)\mathrm{e}^{-\frac{r^2}{4\theta}}\right] + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \tag{11}$$

and

$$N^{\varphi} = -\frac{J}{2r^2}. \tag{12}$$

Noticing that in the limit of $\frac{r}{\sqrt{\theta}} \to \infty$, this solution reduces to the well known rotating BTZ black hole solution with angular momentum J and total mass M,

$$\mathrm{d}s^2 = -\left(8M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)\mathrm{d}t^2 + \left(8M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\varphi + N^{\varphi}\mathrm{d}t^2); \tag{13}$$

therefore, for convenience, from now on, we would call this black hole the NCBTZ black hole. The horizons of the element (10) can be obtained from the equation $f(r) = 0$. f as a function of r for different mass M is plotted in Fig. 1.

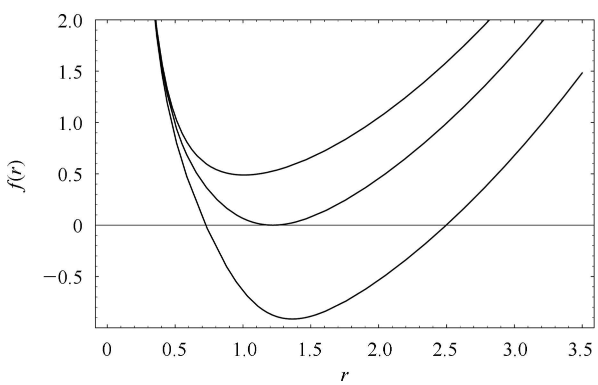

Fig. 1. f vs r for θ=0.1, l=2, J=1 and different values of M. Curves are for M = 0.002, 0.076, 0.200 from top to bottom. We obtain $r_0 \approx 1.22$ and $M_0 \approx 0.076$.

From Fig. 1, we see that, for this NCBTZ black hole, there are two horizons, i.e., the inner(Cauchy) horizon r_{C} and outer(event) horizon r_{H}; the distance between the horizons will increase by increasing the black hole mass M. Fig. 1 also shows that there exists an extremal mass of $M = M_0$ below which no black hole can be found. At the extremal mass M_0, the inner and outer horizons meet at the extreme horizon r_0 ($r_{\mathrm{C}} \leqslant r_0 \leqslant r_{\mathrm{H}}$). The horizon radius of the extreme black hole is determined by the condition of $f=0$ and $\partial_r f = 0$, which gives

$$r_0^4 = \frac{J^2 l^2}{4} + \frac{r_0^6 l^2\left(\frac{r_0^2}{l^2} + \frac{J^2}{4r_0^2}\right)\mathrm{e}^{-\frac{r_0^2}{4\theta}}}{16\theta^2\left[1 - \left(\frac{r_0^2}{4\theta}+1\right)\mathrm{e}^{-\frac{r_0^2}{4\theta}}\right]}. \tag{14}$$

Here the second term on the right side of the equation is the noncommutativity correction.

Then, the mass of the extremal black hole can be written as

$$M_0 = \frac{\frac{r_0^2}{l^2} + \frac{J^2}{4r_0^2}}{8\left[1 - \left(\frac{r_0^2}{4\theta}+1\right)\mathrm{e}^{-\frac{r_0^2}{4\theta}}\right]}. \tag{15}$$

As an example, we obtain $r_0 \approx 1.22$ and $M_0 \approx 0.076$ numerically for θ=0.1, l=2 and J=1.

3 Thermodynamics of the NCBTZ black hole

One of the most interesting aspects of black hole physics is the thermodynamical properties. Let us now consider the Hawking temperature of the NCBTZ black hole, which is calculated as

$$T_{\mathrm{H}} = \frac{1}{4\pi}\partial_r f|_{r_{\mathrm{H}}} = \frac{r_{\mathrm{H}}}{2\pi l^2}\left\{1 - \frac{J^2 l^2}{4r_{\mathrm{H}}^4} - \frac{\left(r_{\mathrm{H}}^2 + \frac{J^2 l^2}{4r_{\mathrm{H}}^2}\right)r_{\mathrm{H}}^2\mathrm{e}^{-\frac{r_{\mathrm{H}}^2}{4\theta}}}{16\theta^2\left[1 - \left(\frac{r_{\mathrm{H}}^2}{4\theta}+1\right)\mathrm{e}^{-\frac{r_{\mathrm{H}}^2}{4\theta}}\right]}\right\}. \tag{16}$$

Here, the last term inside the curly brackets on the right of the equation is the noncommutativity correction.

In Fig. 2, we plot the Hawking temperature of the NCBTZ black hole as a function of r_{H} (from (16)); we find that the case here is different from that of the regular black hole with the Gaussian matter source in AdS_5 background [40], T_{H} is a monotonically increasing function of r_{H}; therefore, there do not exist multiple black holes with the same temperature. The noncommutativity leads to the existence of an extremal horizon r_0, as r_{H} goes to r_0, the black hole approaches to a zero temperature. By setting T_{H}=0, for given θ, l and J, we can obtain the value of the extremal horizon r_0 numerically, which is the same as that obtained from the condition of $f=0$ and $\partial_r f = 0$; this shows the consistency of the method. From Fig. 2(a), we see that by increasing the parameter θ, the value of r_0 increases and, for the pur-

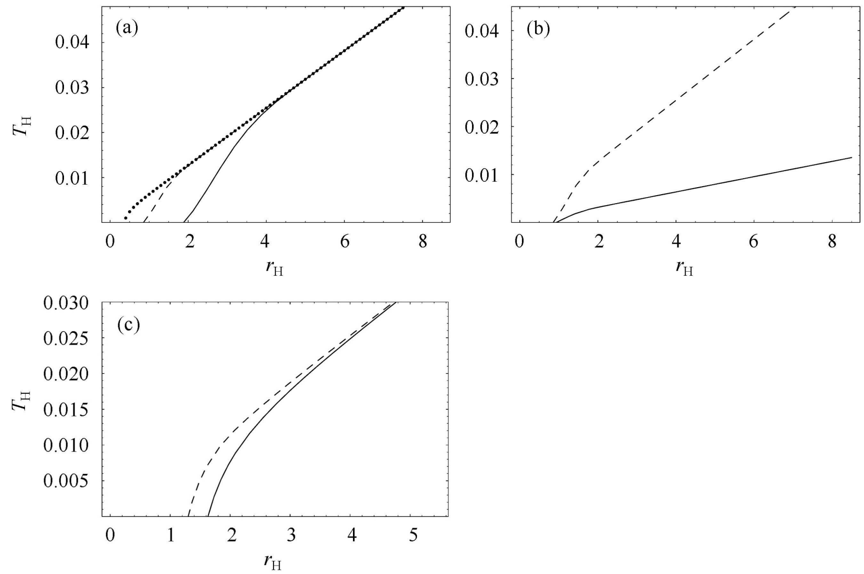

Fig. 2. T_H as a function of r_H. (a) The solid and dashed lines are for the NCBTZ black hole with θ=0.5 and θ=0.1, respectively, while the dotted line is for the rotating BTZ black hole. We have taken l=5 and J=0.05 in all three cases; (b) the solid and dashed lines are for the NCBTZ black hole with l=10 and l=5, respectively. In both cases, we have taken θ=0.1 and J=0.05; (c) the solid and dashed lines are for the NCBTZ black hole with J=1 and J=0.5, respectively. In both cases, we have taken θ=0.1 and l=5.

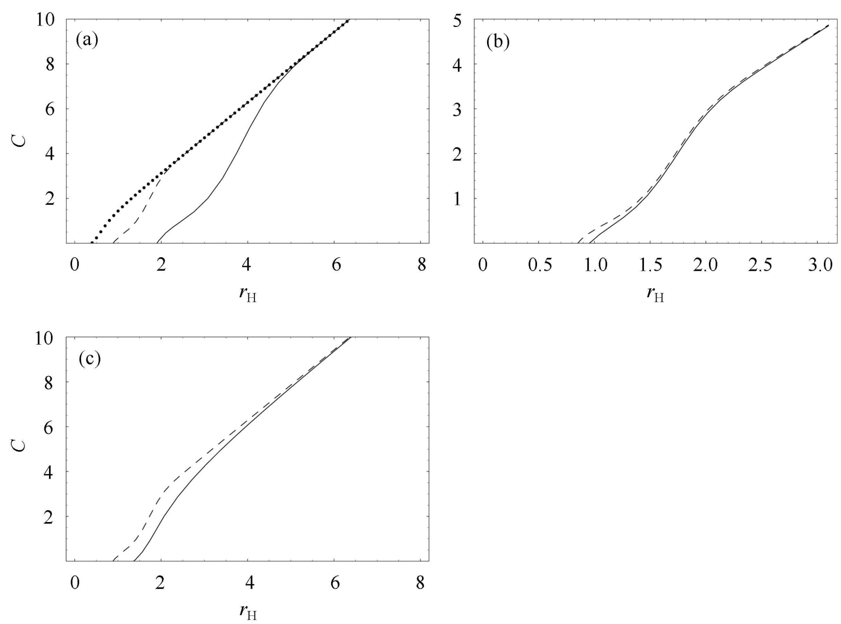

Fig. 3. The heat capacity as a function of r_H. (a) The solid and dashed lines are for the NCBTZ black hole with θ=0.5 and θ=0.1, respectively, while the dotted line is for the rotating BTZ black hole. We have taken l=10 and J=0.03 in all three cases; (b) the solid and dashed lines are for the NCBTZ black hole with l=20 and l=2, respectively. In both cases, we have taken θ=0.1 and J=0.03; (c) the solid and dashed lines are for the NCBTZ black hole with J=0.3 and J=0.03, respectively. In both cases, we have taken θ=0.1 and l=10.

pose of comparison, we also plot the Hawking temperature of a rotating BTZ black hole as a function of the horizon radius. Fig. 2(b) shows that the value of r_0 is insensitive to the change in the value of l. From Fig. 2(c), we see that the value of r_0 becomes larger as J increases.

In order to check the stability of the NCBTZ black hole, we calculate the heat capacity

$$C=\frac{\partial M}{\partial T_{\rm H}}=\frac{\partial M}{\partial r_{\rm H}}\frac{1}{\dfrac{\partial T_{\rm H}}{\partial r_{\rm H}}}. \tag{17}$$

The heat capacity as a function of $r_{\rm H}$ is shown in Fig. 3. For instance, our numerical calculation shows that the heat capacity vanishes at the extremal horizon $r_0\approx0.885$ (for $\theta=0.1$, $l=10$ and $J=0.03$) and $r_0\approx1.898$ (for $\theta=0.5$, $l=10$ and $J=0.03$), respectively (see Fig. 3(a)). The capacity is positive for $r_{\rm H}>r_0$, therefore, this NCBTZ black hole is stable. (The heat capacity will become negative for $r_{\rm H}<r_0$. However, this region is unphysical, it is not allowed by the definition of the horizon radii.) For the purpose of comparison, in Fig. 3(a), we also plot the heat capacity of a rotating BTZ black hole as a function of the horizon radius. Fig. 3(b) shows that the heat capacity of the NCBTZ black hole is insensitive to the change in the value of l. The behavior of the heat capacity of the NCBTZ black hole for different values of J is shown in Fig. 3(c).

Our next step is to calculate the entropy of the NCBTZ black hole. We require that the first law be satisfied with the NCBTZ black hole, $\mathrm{d}M=T_{\rm H}\mathrm{d}S_{\rm H}+\Omega\mathrm{d}J$, where Ω is the angular velocity of the black hole. On the other hand, as physical quantities are evaluated on the event horizon, M can be expressed as $M=M(r_{\rm H},J)$ and $\mathrm{d}M$ can be expressed as $\mathrm{d}M=\dfrac{\partial M}{\partial_{r_{\rm H}}}\mathrm{d}r_{\rm H}+\dfrac{\partial M}{\partial J}\mathrm{d}J$. From the two different forms of $\mathrm{d}M$ one finds the expression for black hole entropy $\mathrm{d}S_{\rm H}=\dfrac{1}{T_{\rm H}}\dfrac{\partial M}{\partial r_{\rm H}}\mathrm{d}r_{\rm H}$. Defining the entropy of the NCBTZ black hole as follows [41]:

$$S=\int_{r_0}^{r_{\rm H}}\frac{1}{T_{\rm H}}\frac{\partial M}{\partial r_{\rm H}}\mathrm{d}r_{\rm H}, \tag{18}$$

where we find $S=0$ for the extreme horizon radius r_0, which is a reasonable choice because this choice automatically guarantees vanishing thermodynamical entropy at absolute zero, as it is required by the third law of thermodynamics. Finally, we get

$$S=\frac{\pi}{2}\int_{r_0}^{r_{\rm H}}\frac{\mathrm{d}r_{\rm H}}{1-\left(\dfrac{r_{\rm H}^2}{4\theta}+1\right)\mathrm{e}^{-\frac{r_{\rm H}^2}{4\theta}}}. \tag{19}$$

Obviously, this entropy does not satisfy the area law.

As is well known, phase transition is one of the important aspects in the study of the thermodynamics of black holes and the free energy plays a crucial role to test the phase transition. In order to analyze the possibility of phase transition, let us define the on-shell free energy as [23]

$$F(r_{\rm H},\theta)=M(r_{\rm H},\theta)-M_0(\theta)-T_{\rm H}(r_{\rm H},\theta)S(r_{\rm H},\theta). \tag{20}$$

Here, the extreme black hole is used as the ground state [42].

As an example, we show the free energy of the NCBTZ black hole with $\theta=0.1$, $l=10$ and $J=0.03$ as well as the free energy of the rotating BTZ black hole with $l=10$ and $J=0.03$ in Fig. 4. Clearly, the free energy of the NCBTZ black hole is higher than that of the rotating BTZ black hole at any value of $r_{\rm H}$. Therefore, there exists a possible decay of the NCBTZ black hole based on the Maxwell-Boltzmann distribution into the rotating BTZ black hole.

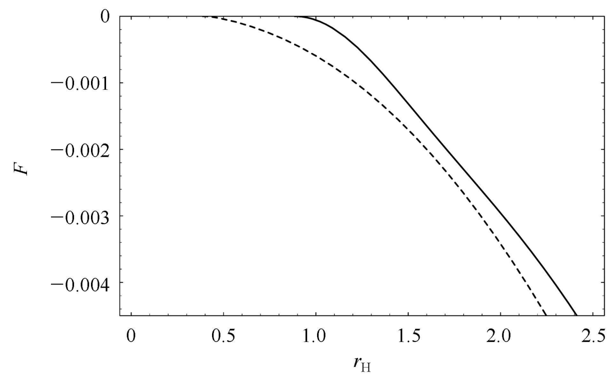

Fig. 4. Free energy F vs $r_{\rm H}$. The solid and dashed lines show the free energy of the NCBTZ black hole with $\theta=0.1$ and that of the rotating BTZ black hole, respectively. In both cases, we have taken $l=10$ and $J=0.03$.

4 Conclusions

In this paper, we investigate the Hawking temperature, heat capacity, entropy and free energy of noncommutative geometry inspired black holes based on a Maxwell-Boltzmann smeared mass distribution in (2+1)-dimensional spacetime. The noncommutativity leads to the existence of an extreme horizon radius, at which the black hole approaches to an absolute zero temperature. The temperature is a monotonically increasing function of the event horizon, which indicates that there do not exist multiple black holes with the same temperature. We use the first law of thermodynamics to derive the entropy, which further confirms the incompatibility of the first law of thermodynamics with Bekenstein-Hawking entropy for the noncommutative inspired black hole [43]. Because the heat capacity is positive, this black hole is stable. In addition, our numerical calculation shows that there exists a possible decay of the NCBTZ black hole into the rotating BTZ black hole.

We would like to thank the anonymous referee for his/her helpful comments, which improve this paper greatly.

References

1 Snyder H S. Phys. Rev., 1947, **71**: 38
2 Snyder H S. Phys. Rev., 1947, **72**: 68
3 Nicolini P. Int. J. Mod. Phys. A, 2009, **24**: 1229
4 Hawking S W. Nature, 1974, **248**: 30
5 Hawking S W. Commun. Math. Phys., 1975, **43**: 199
6 Nicolini P, Smailagic A, Spallucci E. Phys. Lett. B, 2006, **632**: 547
7 Nicolini P, Smailagic A, Spallucci E. arXiv: hep-th/0507226
8 Nicolini P. J. Phys. A, 2005, **38**: L631
9 Ansoldi S et al. Phys. Lett. B, 2007, **645**: 261
10 Rizzo T G. J. High Energy Phys., 2006, **0609**: 021
11 Spallucci E, Nicolini P, Smailagic A. Phys. Lett. B, 2009, **670**: 449
12 Hawking S W. Phys. Rev. D, 2005, **72**: 084013
13 Kim W, Son E J, Yoon M. J. High Energy Phys., 2008, **0804**: 042
14 Nozari K, Mehdipour S H. Class. Quantum Grav., 2008, **25**: 175015
15 Nozari K, Mehdipour S H. J. High Energy Phys., 2009, **0903**: 061
16 Mehdipour S H. Mod. Phys. Lett. A, 2010, **25**: 5543
17 Mehdipour S H. Phys. Rev. D, 2010, **81**: 124049
18 Mann B, Nicolini P. Phys. Rev. D, 2011, **84**: 064014
19 Giri P R. Int. J. Mod. Phys. A, 2007, **22**: 2047
20 DING C, JING J. J. High Energy Phys., 2011, **1110**: 052
21 Mureika J R, Nicolini P. Phys. Rev. D, 2011, **84**: 044020
22 Myung Y S, Kim Y W, Park Y. J. High Energy Phys., 2007, **0702**: 012
23 Myung Y S, Yoon M. Eur. Phys. J. C, 2009, **62**: 405
24 Tejeiro J M, Larranage A. Pramana J. Phys., 2012, **78**: 1
25 Park M I. Phys. Rev. D, 2009, **80**: 084026
26 Bañados M, Teitelboim C, Zanelli J. Phys. Rev. Lett., 1992, **69**: 1849
27 CAI R G, LU Z J, ZHANG Y Z. Phys. Rev. D, 1997, **55**: 853
28 Aharony O, Gubser S S, Maldacena J et al. Phys. Rep., 2000, **323**: 183
29 Rivelles V. Phys. Lett. B, 2003, **558**: 191
30 Aschieri P, Blohman C, Dimitrijeric M et al. Class. Quantum Grav., 2005, **22**: 3511
31 Aschieri P, Dimitrijeric M, Meyer F et al. Lett. Math. Phys., 2006, **78**: 61
32 Lopez-Dominguez J C, Obregon O, Sabido M. Phys. Rev. D, 2006, **74**: 084024
33 Chryssomalakos C, Okon E. J. High Energy Phys., 2007, **0708**: 012
34 Chaichian M, Tureanu A, Zet G. Phys. Lett. B, 2008, **660**: 573
35 Mukherjee P, Saha A. Phys. Rev. D, 2008, **77**: 064014
36 Banerjee R, Majhi B R, Modak S K. Class. Quantum Grav., 2009, **26**: 085010
37 Szabo R J. Phys. Rep., 2003, **378**: 207
38 Chakraborty B, Gangopadhyay S, Saha A. Phys. Rev. D, 2004, **70**: 107707
39 Scholtz F G, Chakraborty B, Gangopadhyay S et al. Phys. Rev. D, 2005, **71**: 085005
40 Smailagic A, Spallucci E. arXiv: 1212.5044
41 Nozari K, Mehdipour S H. Commun. Theor. Phys., 2010, **53**: 503
42 Chamblin A, Emparan R, Johnson C V et al. Phys. Rev. D, 1999, **60**: 064018
43 LIANG J, LIU B. Europhys. Lett., 2012, **100**: 30001