ρ-ω mixing in J/ψ→VP decays*

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Abstract: The study of ρ-ω mixing has mainly focused on vector meson decays with isospin $I=1$, namely the $\rho(\omega)\to\pi^+\pi^-$ process. In this paper, we present a study of ρ-ω mixing in $\rho(\omega)\to\pi^+\pi^-\pi^0$ ($I=0$) using a flavor parameterization model for the $J/\psi\to VP$ process. By fitting a theoretical framework to PDG data, we obtain the $SU(3)$-breaking effect parameters $s_V=0.03\pm0.12$, $s_P=0.17\pm0.17$ and the ρ-ω mixing polarization operator $\Pi_{\rho\omega}=(0.006\pm0.011)$ GeV^2. New values are found for the branching ratios when the mixing effect is incorporated: $Br(J/\psi\to\omega\pi^0)=(3.64\pm0.37)\times10^{-4}$, $Br(J/\psi\to\omega\eta)=(1.48\pm0.17)\times10^{-3}$, $Br(J/\psi\to\omega\eta')=(1.55\pm0.56)\times10^{-4}$, these are different from the corresponding PDG2012 values by 19%, 15% and 15%, respectively.

Key words: ρ-ω mixing, branching ratio, J/ψ→VP decay

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1 Introduction

In 1961, Glashow suggested that electromagnetic transitions lead to ρ-ω mixing [1]. Eight years later, direct experimental evidence for ρ-ω mixing was observed [2], which was followed a year later by Willemsen's study [3]. In the following thirty years, along with the development of the vector meson dominance (VMD) model [4–8], many theories have been proposed to understand ρ-ω mixing, such as: charge symmetry violation (CVS) [9–12], quantum chromodynamics sum rules (QCDSR) [13, 14], chiral perturbation theory (ChPT) [15, 16] and hidden local symmetry (HLS) [17–19].

The isospin of 2π and 3π systems can be 0, 1, 2 and 0, 1, 2, 3, respectively. Considering the contribution of phase space, ρ mainly decays into 2π (I=1) and ω decays to 3π (I=0), and the direct decays $\omega\to2\pi$ (I=0), $\rho\to3\pi$ (I=1) and other decays ($I\neq0,1$) are suppressed. However, the $\omega\to2\pi$ (I=1) decay has also been observed, as listed in PDG2012, which may imply isospin violation; that is, ρ-ω mixing. ρ could also decay into 3π (I=0) through ρ-ω mixing.

Up to the present day, most ρ-ω mixing studies have been based on vector meson decays with isospin $I=1$; namely, $\rho(\omega)\to\pi\pi$. These processes have been well studied, both theoretically and experimentally [7, 20, 21]. On the other hand, the mixing in $\rho(\omega)\to3\pi$ decays with isospin $I=0$ is not yet so well understood. It is difficult to measure the process directly from experiment because $\Gamma_\rho\gg(m_\omega-m_\rho)$ and $Br(\omega\to\pi^+\pi^-\pi^0)\gg Br(\rho^0\to\pi^+\pi^-\pi^0)$ [22].

However, a study of $\rho(\omega)\to3\pi$ interference with $J/\psi\to VP$ decays has been made using a flavor parameterization method [20, 22]. With J/ψ decays, the small value of $Br(\rho^0\to\pi^+\pi^-\pi^0)/Br(\omega\to\pi^+\pi^-\pi^0)$ can be compensated for, to some extent, by the large value of $Br(J/\psi\to\rho^0\pi^0)/Br(J/\psi\to\omega\pi^0)$, which provides a new insight into ρ-ω mixing. The parameterization of the $J/\psi\to VP$ process has been developed with single and double Okubo-Zweig-Iziuka (SOZI, DOZI) rules [23–27] and is widely used; for example, in Ref. [28]. The SND group has taken the ρ-ω mixing effect into account with the VMD model in their study of $e^+e^-\to3\pi$ decays below 0.98 GeV [29]. Their theoretical model for $e^+e^-\to3\pi$ may also be used in our study of $V\to3\pi$. The mixing phenomenon between ρ and ω in J/ψ decays will serve as an important probe for the testing of various theoretical models and of G-parity violation.

The main purpose of this paper is to study ρ-ω mixing in the processes $e^+e^-\to J/\psi\to V\pi^0\to\pi^+\pi^-\pi^0\pi^0$ with the VMD model of $V\to3\pi$ [29, 30] and the flavor parameterization method of $J/\psi\to VP$ decays [23, 24]. By doing a fit, we expect to derive the mixing parameter $\Pi_{\rho\omega}$, and to modulate the measured $Br(J/\psi\to\omega\pi^0)$

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and $Br(\mathrm{J}/\psi\to\omega\eta(\eta'))$ according to the mixing value. Once the above theoretical framework is developed and validated, we can study ρ-ω mixing experimentally using the world's largest $\mathrm{e^+e^-}\to\mathrm{J}/\psi$ data sample collected by the BESIII (Beijing Spectrometer) experiment [31–34].

The contents of our paper are organized as follows. In Section 2, using the VMD model, as applied to the $\mathrm{e^+e^-}\to 3\pi$ process [29] and taking into account the $\omega\to\pi^+\pi^-\pi^0$ contact term [30], we describe the process $\mathrm{e^+e^-}\to\mathrm{J}/\psi\to\mathrm{V}\pi^0\to\pi^+\pi^-\pi^0\pi^0$ and give its cross section. In Section 3, by using the flavor parameterization method [23, 24], we perform a fit to the existing branching ratios within the theoretical framework of $\mathrm{J}/\psi\to\mathrm{VP}$ decays. The conclusions and the interpretation of the results are given in Section 4. The appendices give the detailed notations of the mixing formulae.

2 Theoretical framework for the $\mathrm{e^+e^-}\to\mathrm{J}/\psi\to\mathrm{V}\pi^0\to\pi^+\pi^-\pi^0\pi^0$ process

The SND result [29] and related branching ratios in PDG2012 [35] indicate that the decay channels $\mathrm{J}/\psi\to\rho'\pi^0$, $\mathrm{J}/\psi\to\rho''\pi^0$, $\mathrm{J}/\psi\to\omega'\pi^0$, $\mathrm{J}/\psi\to\omega''\pi^0$ have little contribution to our process of interest. We will omit these channels and calculate only the $\mathrm{e^+e^-}\to\mathrm{J}/\psi\to\mathrm{V}\pi^0\to\pi^+\pi^-\pi^0\pi^0$ (V=ρ, ω, φ) process in this paper. The framework used by SND [29, 36–38] is adopted in the calculation, and the $\omega\to\pi^+\pi^-\pi^0$ contact term is taken into account [30, 39–43].

The cross section of the $\mathrm{e^+e^-}\to\mathrm{J}/\psi\to\mathrm{V}\pi^0\to\pi^+\pi^-\pi^0\pi^0$ process is

$$\frac{\mathrm{d}\sigma(s,m_0,m_+)}{\mathrm{d}m_0\mathrm{d}m_+}=\frac{1}{s^{3/2}}\frac{\left|\vec{p}_+\times\vec{p}_-\right|^2}{12\pi^2\sqrt{s}}m_0m_+|F|^2, \qquad (1)$$
$$F=F_{\rho\pi}(s)+F_{\omega\pi}(s)+F_{3\pi}(s).$$

Here s is the invariant mass of the $\pi^+\pi^-\pi^0$ system, $\vec{p}_+$ and $\vec{p}_-$ are the momenta of π^+ and π^- mesons in the 3π system rest frame, and m_+ and m_0 are the invariant masses of $\pi^+\pi^0$ and $\pi^+\pi^-$.

$F_{\rho\pi}(s)$ ($F_{\omega\pi}(s)$, $F_{3\pi}(s)$) in Eq. (1) is the form factor for the vector mesons decays through $\mathrm{V}\to\rho\pi$ ($\mathrm{V}\to\omega\pi$, $\mathrm{V}\to 3\pi$) channel, taking into account the transition described in Fig. 1(a, b, c) (Fig. 1(d), Fig. 1(e)). The form factors have the forms

$$F_{\rho\pi}(s)=\left[a_{3\pi}+\sum_{\mathrm{i}=+,0,-}\frac{g_{\rho^{\mathrm{i}}\pi\pi}}{D_{\rho^{\mathrm{i}}}(m_{\mathrm{i}})Z(m_{\mathrm{i}})}\right]$$
$$\times\left\{2g_{\omega\rho\pi}(s)\left[\frac{A_{\psi\omega\pi}(s)}{D_\omega(s)}-\frac{\Pi_{\rho\omega}A_{\psi\rho\pi}(s)}{D_\omega(s)D_\rho(s)}\right]\right.$$
$$\left.+\frac{2A_{\psi\phi\pi}(s)g_{\phi\rho\pi}\mathrm{e}^{\mathrm{i}\Phi_{\omega\phi}}}{D_\phi(s)}\right\},$$

$$F_{\omega\pi}(s)=\frac{-\Pi_{\rho\omega}g_{\rho^0\pi\pi}}{D_\omega(m_0)D_\rho(m_0)}\frac{2A_{\psi\rho\pi}(s)g_{\rho\omega\pi}}{D_\rho(s)},$$

$$F_{3\pi}(s)=6g_{\omega3\pi}\left[\frac{A_{\psi\omega\pi}(s)}{D_\omega(s)}-\frac{\Pi_{\rho\omega}A_{\psi\rho\pi}(s)}{D_\omega(s)D_\rho(s)}\right]. \qquad (2)$$

m_- is the invariant mass of $\pi^-\pi^0$ and satisfies

$$m_-=\sqrt{s+m_{\pi^0}^2+2m_\pi^2-m_0^2-m_+^2}. \qquad (3)$$

$A_{\psi\mathrm{V}\pi}(s)$ is the amplitude of the $\mathrm{e^+e^-}\to\mathrm{J}/\psi\to\mathrm{V}\pi^0$ process:

$$A_{\psi\mathrm{V}\pi}(s)\equiv g\sqrt{Br\left(\mathrm{J}/\psi\to\mathrm{V}\pi^0\right)}$$
$$\times\sqrt{\frac{q^3(m_\psi,\sqrt{s},m_{\pi^0})\mathrm{e}^{-q^2\left(m_\psi,\sqrt{s},m_{\pi^0}\right)/8\beta^2}}{q^3(m_\psi,m_\mathrm{V},m_{\pi^0})\mathrm{e}^{-q^2\left(m_\psi,m_\mathrm{V},m_{\pi^0}\right)/8\beta^2}}}, \qquad (4)$$

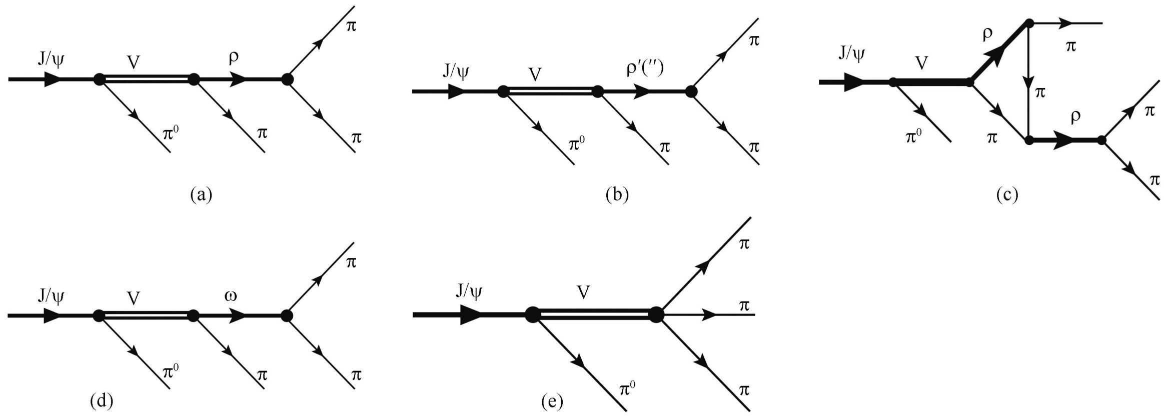

Fig. 1. $\mathrm{J}/\psi\to\mathrm{V}\pi^0\to\pi^+\pi^-\pi^0\pi^0$ processes. (a) ρπ channel contributions; (b) possible transition $\mathrm{V}\to\rho'^{('')}\pi\to\pi^+\pi^-\pi^0$; (c) interaction of ρ and π mesons in the final state; (d) ωπ channel contributions; and, (e) contact term for higher order contributions, which requires the same space-time point for all particles when decay happens.

where g is a factor with dimension GeV^2, which includes the coupling constant of the decay $e^+e^-\to J/\psi$, and q is the momentum defined as

$$q(M,m_1,m_2)=\frac{\sqrt{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}}{2M}. \quad (5)$$

$g_{V\rho(\omega)\pi}$ is the coupling constant for the decay $V\to\rho(\omega)\pi$. $g_{\omega\pi\pi}$ and $g_{\rho\pi\pi}$ are the coupling constants for the decays $\omega\to\pi\pi$ and $\rho\to\pi\pi$, respectively. $g_{\omega 3\pi}$ is the coupling constant for the contact term $\omega\to 3\pi$. The values of these coupling constants were calculated according to Refs. [29, 30, 36] as: $g_{\rho^0\pi\pi}=5.975$, $g_{\rho^\pm\pi\pi}=5.989$, $g_{\rho\omega\pi}=16.8\ \mathrm{GeV}^{-1}$, $g_{\omega\rho\pi}=15.0\ \mathrm{GeV}^{-1}$, $g_{\phi\rho\pi}=0.827\ \mathrm{GeV}^{-1}$, and $g_{\omega 3\pi}=-47.0\ \mathrm{GeV}^{-3}$.

$\phi_{\rho V}(\phi_{\omega V})$ is a relative interference phase between $\rho(\omega)$ and vector mesons V; thus, $\phi_{\rho\rho}=0$, $\phi_{\omega\omega}=0$. We adopt the value $\phi_{\omega\phi}=(163\pm3\pm6)^\circ$ obtained by SND [29], which takes into account the ϕ-ω mixing and is consistent with the theoretical prediction [44].

$D_V(s)$ is the propagator function, which is defined as

$$D_V(s)=m_V^2-s-i\sqrt{s}\Gamma_V(s), \quad (6)$$

where the s-dependent widths of vector mesons $\Gamma_V(s)$ are defined in SND [29].

$a_{3\pi}=(0.1\pm2.3\pm2.5)\ \mathrm{GeV}^{-2}$ represents the contribution from the $V\to\rho'(\rho'')\pi\to\pi^+\pi^-\pi^0$ processes [29]. The factor $Z(m_i,s)$ is defined as $Z(m,s)=1-is_1\Phi(m,s)$ [45], where $s_1=0.3\pm0.3\pm0.3$ [29].

Here, the ρ-ω mixing in $J/\psi\to(\rho^0,\omega)\pi^0\to\rho\pi\pi$ $(\omega\pi\pi)$ and $V\to(\rho^0,\omega)\pi^0\to3\pi$ decays is considered (refer to Eqs. (A2), (A3), (A4), (A5) in Appendix A). It is a well-known fact that the real part of the coupling constant of the direct transition $\omega\to\pi^+\pi^-$ has no contribution to the amplitude of $\omega\to\pi^+\pi^-$ decays [45]. Therefore, we have ignored the term $g^{(0)}_{\omega\pi\pi}$, as well as the terms $g^{(0)}_{\rho\rho\pi}$ and $g^{(0)}_{\rho3\pi}$. $\Pi_{\rho\omega}$ is a polarization operator, and it is speculated that $\Pi_{\rho\omega}$ satisfies $\mathrm{Im}(\Pi_{\rho\omega})\ll\mathrm{Re}(\Pi_{\rho\omega})$ [29, 45, 46]; thus, we only consider the real part of $\Pi_{\rho\omega}$. Its value should be positive because it is extracted from the modulus of the amplitude.

The cross section $\sigma(s)$ is defined as the integral of $\sigma(s,\ m_0,\ m_+)$ over m_0 and m_+:

$$\sigma(s)=\iint\frac{1}{s^{3/2}}\frac{\left|\vec{p}_+\times\vec{p}_-\right|^2}{12\pi^2\sqrt{s}}m_0m_+|F|^2\mathrm{d}m_0\mathrm{d}m_+. \quad (7)$$

3 Fitting of mixing parameters

3.1 Strategy for the fit

The flavor parameterization method used in Ref. [23] is applied here to study the $J/\psi\to VP$ process [24–27]. The decays proceed through strong and electromagnetic interactions, where the effects of double Okubo-Zweig-Iziuka (DOZI) rule-violation and $SU(3)$ flavor symmetry breaking should be taken into account.

The general parameterization of the amplitudes is written in Table 1, where the terms X_η, Y_η, Z_η and $X_{\eta'}$, $Y_{\eta'}$, $Z_{\eta'}$ include the η-η' mixing (Eq. (A9)); ϕ_P is the η-η' mixing angle, and $\phi_{\eta'G}$ weights the amount of gluonium in η'. The terms ω_q, ϕ_s indicate that ω-ϕ mixing is considered as in Eq. (A10). Then, the amplitudes of the decays including ω or ϕ are rewritten as

$$\begin{aligned}M_\omega&=\cos\theta_{\omega\phi}M_{\omega_q}-\sin\theta_{\omega\phi}M_{\phi_s},\\ M_\phi&=\sin\theta_{\omega\phi}M_{\omega_q}+\cos\theta_{\omega\phi}M_{\phi_s},\end{aligned} \quad (8)$$

where $\theta_{\omega\phi}$ is the mixing angle of ρ and ω. The value of $\theta_{\omega\phi}$ can be set to 0 if this mixing is ignored. Here, we take the values $\theta_{\omega\phi}=(3.2\pm0.1)^\circ$ and $s_e=0.19\pm0.05$, as in Refs. [24, 47, 48].

Table 1. General parametrization of amplitudes for $J/\psi\to VP$.

process	amplitude (M_i)
$\rho^+\pi^-$, $\rho^0\pi^0$, $\rho^-\pi^+$	$g+eE^{i\theta}$
$K^{*+}K^-$, $K^{*-}K^+$	$g(1-s)+eE^{i\theta}(1+s_e)$
$K^{*0}\bar{K}^0$, $\bar{K}^{*0}K^0$	$g(1-s)-eE^{i\theta}(2-s_e)$
$\omega_q\eta$	$(g+eE^{i\theta})X_\eta+\sqrt{2}rg\left[\sqrt{2}X_\eta+(1-s_P)Y_\eta\right]+\sqrt{2}r'gZ_\eta$
$\omega_q\eta'$	$(g+eE^{i\theta})X_{\eta'}+\sqrt{2}rg\left[\sqrt{2}X_{\eta'}+(1-s_P)Y_{\eta'}\right]+\sqrt{2}r'gZ_{\eta'}$
$\phi_s\eta$	$\left[g(1-2s)-2eE^{i\theta}(1-s_e)\right]Y_\eta+rg(1-s_V)\left[\sqrt{2}X_\eta+(1-s_P)Y_\eta\right]+r'g(1-s_V)Z_\eta$
$\phi_s\eta'$	$\left[g(1-2s)-2eE^{i\theta}(1-s_e)\right]Y_{\eta'}+rg(1-s_V)\left[\sqrt{2}X_{\eta'}+(1-s_P)Y_{\eta'}\right]+r'g(1-s_V)Z_{\eta'}$
$\rho\eta$	$3eE^{i\theta}X_\eta$
$\rho\eta'$	$3eE^{i\theta}X_{\eta'}$
$\omega_q\pi^0$	$3eE^{i\theta}$
$\phi_s\pi^0$	0

Similar to Refs. [26, 27, 49], the branching ratio is given by

$$Br_{\rm cor}(\mathrm{J}/\psi\to\omega\pi)=\frac{Br(\mathrm{J}/\psi\to\omega\pi\to4\pi)}{Br(\omega\to3\pi)}=|M_{\psi\omega\pi^0}|^2q^3\mathrm{e}^{-q^2/8\beta^2}, \quad (9)$$

where β is a scale of the energy and the value β=0.5 GeV is commonly adopted [26, 27, 49].

Differing from the ideal branching ratio given above, the actual measured ratio can be written as

$$Br_{\rm uncor}(\mathrm{J}/\psi\to\omega\pi)=\frac{Br(\mathrm{J}/\psi\to\mathrm{V}\pi\to4\pi)}{Br(\omega\to3\pi)}=f\cdot\sigma_{\pi^0}, \quad (10)$$

where σ_{π^0} is the integral of $\sigma(s)$ (Eq. (7)), in which π^0 indicates $\mathrm{J}/\psi\to4\pi$ via $\mathrm{V}\pi^0$, instead of via $\mathrm{V}\eta$ or $\mathrm{V}\eta'$. The integrating range is $\sqrt{s}\in$ [0.6, 1.0] GeV. f is a constant factor with dimension GeV^2 that absorbs the factor of g in Eq. (4). Eq. (4) can then be redefined as

$$A_{\psi\mathrm{V}\pi}(s)\equiv\sqrt{Br\left(\mathrm{J}/\psi\to\mathrm{V}\pi^0\right)}\times\sqrt{\frac{q^3(m_\psi,\sqrt{s},m_{\pi^0})\mathrm{e}^{-q^2\left(m_\psi,\sqrt{s},m_{\pi^0}\right)/8\beta^2}}{q^3(m_\psi,m_{\mathrm{V}},m_{\pi^0})\mathrm{e}^{-q^2\left(m_\psi,m_{\mathrm{V}},m_{\pi^0}\right)/8\beta^2}}}. \quad (11)$$

The values of $Br(\mathrm{J}/\psi\to\omega\eta)$ and $Br(\mathrm{J}/\psi\to\omega\eta')$ can be calculated in a similar way to the $Br(\mathrm{J}/\psi\to\omega\pi^0)$ case. Although the branching ratios have the same form as Eq. (10), Eq. (11) is slightly different:

$$A_{\psi\mathrm{V}\eta^{(\prime)}}(s)\equiv\sqrt{Br(\mathrm{J}/\psi\to\mathrm{V}\eta^{(\prime)})}\times\sqrt{\frac{q^3(m_\psi,\sqrt{s},m_{\eta^{(\prime)}})\mathrm{e}^{-q^2(m_\psi,\sqrt{s},m_{\eta^{(\prime)}})/8\beta^2}}{q^3(m_\psi,m_{\mathrm{V}},m_{\eta^{(\prime)}})\mathrm{e}^{-q^2(m_\psi,m_{\mathrm{V}},m_{\eta^{(\prime)}})/8\beta^2}}}. \quad (12)$$

The branching ratios reported in PDG2012 [35] are listed in the third column of Table 2, where the subscript "cor" and "uncor" indicate without and with the contribution from the mixing effect, respectively.

In total, 12 parameters appear in Table 1 and Table 2: g, e, r, s, s_{V}, s_{P}, θ, ϕ_{P}, $\phi_{\eta'\mathrm{G}}$, r', f and $\mathit{\Pi}_{\rho\omega}$. However, there are 11 branching ratios in Table 2. By fixing some parameters to the expected values [24, 27], we may fit the remaining parameters by minimizing

$$\chi^2=\frac{1}{N}\sum_{\mathrm{i}}\frac{(Br_{\mathrm{i}}^{\mathrm{vis}}-Br_{\mathrm{i}}^{\mathrm{th}})^2}{\Delta_{\mathrm{i}}^2}, \quad (13)$$

where $Br_{\mathrm{i}}^{\mathrm{vis}}$ and Δ_{i} are the $\mathrm{J}/\psi\to\mathrm{VP}$ branching ratios and corresponding errors given by PDG2012 [35]; and $Br_{\mathrm{i}}^{\mathrm{th}}$ is calculated by Eq. (9), except for $Br(\mathrm{J}/\psi\to\omega\pi^0(\eta,\eta'))$, which is calculated by Eq. (10). N is the number of branching ratios used.

The fit is performed according to the following configuration. We mark all items as "tag" and each item "tag[i]" is described below:

1) tag[1]: defines whether ρ-ω mixing is taken into account in the fit. If ρ-ω mixing is not included, we just need to fit with Table 1 and Eq. (9), which is similar to Ref. [24]. "tag[1] "=1 or 2 refers to without or with mixing in the fit, respectively.

2) tag[2]: defines the initial values and step-width. "tag[2] "=1 or 2 refers to using the reference values from [24, 27] as initial values and 0.01% of these values as step-widths, or set to "0" or "1" as initial values and 10^{-6} as step-widths, respectively.

3) tag[3]: defines whether or not the parameters are limited within a physical range. "tag[3] "=1 or 2 refers to no limit or with a limit, respectively.

4) tag[4]: defines how to deal with the effects of the $SU(3)$-breaking contributions s_{V} and s_{P}. "tag[4] "=1, 2 or 3 means a free fit, fixed to 0, or set to the reference values from [24], respectively.

5) tag[5]: defines how to deal with the contributions from gluonium $\phi_{\eta'\mathrm{G}}$ and r'. "tag[5]"=1, 2 or 3 means a free, fixed to 0, or set to the reference values from [24], respectively.

6) tag[6]: defines whether the values of parameters g, e, r, s, s_{V}, s_{P}, θ, ϕ_{P}, $\phi_{\eta'\mathrm{G}}$ and r' are fixed to the values in Refs. [24, 27], before fitting f and $\mathit{\Pi}_{\rho\omega}$. "tag[6]"=1 or 2 refer to these parameters not being fixed, or fixed, respectively.

The fit configuration is represented by the setting of these tag numbers. For example, "tag=121211" means no ρ-ω mixing; "0" or "1" set as the initial values, with 10^{-6} as the step-width; no limits on the parameters; $s_{\mathrm{V}}=0$, $s_{\mathrm{P}}=0$; $\phi_{\eta'\mathrm{G}}$ and r' are free; parameters are not fixed in the fit.

3.2 Results of the fit

Two models have been used in the fit: with the J/ψ form factor (i.e. β=0.5 GeV) and without the J/ψ form factor (i.e. $\beta=10^{10}$ GeV). If a fit result does not satisfy $g>0$, $e>0$, $|r|<1$, $|s|<1$, $|s_{\mathrm{V}}|<1$, $|s_{\mathrm{P}}|<1$, $|r'|<1$ and $\mathit{\Pi}_{\rho\omega}>0$, it has no physical meaning and is marked as "Invalid". A fit with χ^2/d.o.f<1.5 is acknowledged as a good fit. The results of good fits with valid physical meaning are studied carefully.

The detailed analysis described in the next section shows that it is much more reasonable to take ρ-ω mixing and the J/ψ form factor (i.e. β=0.5 GeV) into account in the fit. The corresponding fit results are listed in Table 3.

3.3 Discussion

We have made the following observations from the fit results of good fits with valid physical meaning:

Table 2. The branching ratios $J/\psi \to VP$ ($\times 10^{-3}$) from PDG2012 and from our fit. "Fit 1" and "Fit 2" indicate the two different fit parameter configurations described in the text.

No.	process	PDG2012 [35]	Fit 1 (χ^2/d.o.f.=0.022/1)	Fit 2 (χ^2/d.o.f.=1.61/3)
1	$\rho^+\pi^-+\rho^0\pi^0+\rho^-\pi^+$	16.9±1.5	16.9 ± 1.2	15.93 ± 0.82
2	$K^{*+}K^-+K^{*-}K^+$	5.12±0.30	5.12 ± 0.21	5.25 ± 0.14
3	$K^{*0}\bar{K}^0+\bar{K}^{*0}K^0$	4.39±0.31	4.39 ± 0.19	4.54 ± 0.25
4	$(\omega\eta)_{\rm cor}$	—	1.279 ± 0.050	1.48 ± 0.17
5	$(\omega\eta')_{\rm cor}$	—	0.13 ± 0.26	0.155 ± 0.056
6	$\phi\eta$	0.75±0.08	0.86 ± 0.13	0.79 ± 0.10
7	$\phi\eta'$	0.40±0.07	0.38 ± 0.21	0.370 ± 0.066
8	$\rho\eta$	0.193±0.023	0.1930 ± 0.0043	0.1968 ± 0.0040
9	$\rho\eta'$	0.105±0.018	0.105 ± 0.024	0.100 ± 0.018
10	$(\omega\pi^0)_{\rm cor}$	—	0.320 ± 0.032	0.364 ± 0.037
11	$\phi\pi^0$	$<6.4\times10^{-3}$(C.L.90%)	0.00095 ± 0.00020	0.00108 ± 0.00021
12	$(\omega\pi^0)_{\rm uncor}$	0.45±0.05	0.45 ± 0.93	0.45 ± 0.25
13	$(\omega\eta)_{\rm uncor}$	1.74±0.20	1.74 ± 0.45	1.72 ± 0.41
14	$(\omega\eta')_{\rm uncor}$	0.182±0.021	0.18 ± 0.18	0.184 ± 0.036

Table 3. Result of fit with ρ-ω mixing and J/ψ form factor effects, that is β=0.5 GeV (χ^2/d.o.f<1.5). The index of the fits (in the first column) marked with "*" means the fit results have large differences from the values in the references (listed in the first row). "Dif" is defined as Dif$=\sum(|x_{\rm fit}-x_{\rm Ref}|/\Delta(x)_{\rm Ref})$, where $x_{\rm fit}$ and $x_{\rm Ref}$ are the values of parameters (g, e, r, s, $s_{\rm V}$, $s_{\rm P}$, θ, $\phi_{\rm P}$, $\phi_{\eta'{\rm G}}$ and r') from the fit or from the reference, respectively, and $\Delta(x)_{\rm Ref}$ is the error from the reference.

No.	tag/Dif	g	e	r	s	$s_{\rm V}$	$s_{\rm P}$	$s_{\rm e}$	θ	$\phi_{\rm P}$	$\phi_{\eta'{\rm G}}$	$r_{\rm P}$	$\Pi_{\rho\omega}$/GeV^2	f/GeV^2
	χ^2/(d.o.f)	err	err	err	err	err	err	err	err	err	err	err	err	err
	Ref [24, 27]/0	2.11	0.213	−0.43	0.27	−0.03	−0.08	0.19	1.34	44.6	32	−0.04		
	2.6 /3	0.10	0.012	0.08	0.03	0.09	0.10	0.05	0.12	4.1	11	0.20		
1*	211221/11.45	2.200	0.1800	−0.350	0.300	0	0	0.19	1.30	38.0	0	0	0.0140	0.00410
	1.7 /3	0.073	0.0090	0.012	0.022	0	0	0	0.13	2.6	0	0	0.0063	0.00050
2	211231/5.375	2.200	0.2000	−0.390	0.290	0	0	0.19	1.30	42.0	32	−0.04	0.006	0.00450
	1.61 /3	0.077	0.0099	0.014	0.025	0	0	0	0.12	2.7	0	0	0.011	0.00053
3	211321/9.779	2.200	0.1800	−0.340	0.290	−0.03	−0.08	0.19	1.30	38.0	0	0	0.0170	0.00380
	3.02 /3	0.073	0.0090	0.012	0.023	0	0	0	0.13	2.6	0	0	0.0059	0.00047
4	211331/3.742	2.200	0.190	−0.380	0.280	−0.03	−0.08	0.19	1.30	41.0	32	−0.04	0.0110	0.00410
	3.04 /3	0.078	0.010	0.014	0.025	0	0	0	0.12	2.8	0	0	0.0083	0.00049
5*	212121/14.55	2.20	0.1800	−0.360	0.320	0.03	0.17	0.19	1.30	38.0	0	0	0.004	0.0052
	0.022 /1	0.10	0.0090	0.029	0.036	0.12	0.17	0	0.13	2.9	0	0	0.023	0.0014
6	212131/7.811	2.200	0.1900	−0.400	0.310	0.022	0.11	0.19	1.30	42.0	32	−0.04	0.000	0.00500
	0.454 /1	0.099	0.0091	0.020	0.032	0.089	0.10	0	0.12	2.8	0	0	0.041	0.00062
7	212231/5.375	2.200	0.200	−0.390	0.290	0	0	0.19	1.30	42.0	32	−0.04	0.006	0.00450
	1.61 /3	0.077	0.010	0.013	0.024	0	0	0	0.12	2.7	0	0	0.011	0.00054
8	212321/9.779	2.200	0.1800	−0.340	0.290	−0.03	−0.08	0.19	1.30	38.0	0	0	0.0170	0.00380
	3.02 /3	0.073	0.0090	0.012	0.023	0	0	0	0.13	2.6	0	0	0.0058	0.00046
9	212331/3.742	2.200	0.190	−0.380	0.280	−0.03	−0.08	0.19	1.30	41.0	32	−0.04	0.0110	0.00410
	3.04 /3	0.078	0.010	0.014	0.025	0	0	0	0.12	2.8	0	0	0.0083	0.00049
10*	221131/37.6	2.20	0.2000	−0.690	0.320	0.500	0.17	0.19	1.30	−42.0	32	−0.04	0.035	0.0019
	0.029 /1	0.10	0.0099	0.062	0.036	0.056	0.18	0	0.12	2.9	0	0	0.018	0.0010
11	222331/3.742	2.200	0.1900	−0.380	0.280	−0.03	−0.08	0.19	1.30	41.0	32	−0.04	0.0110	0.00410
	3.04 /3	0.077	0.0099	0.014	0.025	0	0	0	0.12	2.7	0	0	0.0082	0.00049

1) Regardless of whether the J/ψ form factor is considered or not, about half of the 77 fit configurations give a result with a reasonable χ^2 value (χ^2/d.o.f<1.5);

2) Regardless of whether the mixing is included or not, most of the fit results are consistent with the results in Refs. [24, 27];

3) The fitted $SU(3)$-breaking contributions are very small, with significant error; that is, $s_{\rm V}$=0.03±0.12, $s_{\rm P}$= 0.17±0.17;

4) The gluonium contribution has little effect on the

fit. If it is considered, the fit results are consistent with the values in Ref. [24] of $\phi_{\eta' G}=32\pm11$, $r'=-0.04\pm0.20$, especially when ρ-ω mixing is included;

5) The fit does not depend on whether or not a physics range limit is applied to the parameters; and,

6) In the case where ρ-ω mixing is ignored, there is no difference between setting the initial values for the fit to the references values [24, 27] or to generally used values ("0" or "1").

From the comparison between the cases $\beta=0.5$ GeV and $\beta=10^{10}$ GeV we note that by taking ρ-ω mixing into account the fit can succeed both when the initial fit values are set to those in Ref. [24, 27] and when they are set to the generally used values ("0 or 1") when the J/ψ form factor (i.e. $\beta=0.5$ GeV) is considered. Otherwise (i.e. $\beta=10^{10}$ GeV) the initial values have to be set to the reference values [24, 27] to ensure a good fit.

If $\beta=0.5$ GeV, it should also be pointed out that the χ^2 of the fit is better when ρ-ω mixing is considered than not, although the obtained parameters may differ a little from Ref. [24]. However, if $\beta=10^{10}$ GeV, we see that the χ^2 of the fits are worse when ρ-ω mixing is included, although the parameters obtained are similar to those in Ref. [24].

In summary, about half of the fit configurations give stable, consistent, and reasonable (χ^2/d.o.f<1.5) fit results. The effects of the $SU(3)$-breaking contributions is small ($s_V=0.031\pm0.12$, $s_P=0.17\pm0.17$). The contribution of gluonium has a negligible effect on the fit, and when included the results are consistent with Ref. [24] ($\phi_{\eta' G}=32\pm11$, $r'=-0.04\pm0.20$). It is preferable to include ρ-ω mixing and J/ψ form factor effects, which leads to a reasonable and stable result. The fit configurations of "tag=211231" and "tag=212121" (the second and fifth row in Table 3) are accepted, and the branching ratios calculated according to the two sets of fitted parameters listed in the fifth ("Fit 2") and fourth ("Fit 1") column in Table 2, respectively. Their errors are evaluated by assuming that the fitted parameters follow Gaussian distributions, then randomly picking 1000000 points to calculate the deviation from the observed branching ratios. Taking errors into account, the "tag=211231" configuration is preferred.

4 Conclusion

From the global fit to PDG data according to our theoretical framework describing J/ψ → VP processes, we have obtained parameters for the flavor parameterization model, as listed in Table 3. It turns out that whether or not the contribution of gluonium is considered has little effect on the fit. If it is considered, the fit gives results consistent with the values in Ref. [24] ($\phi_{\eta' G}=32\pm11$, $r'=-0.04\pm0.20$). The effects of the $SU(3)$-breaking contributions are also negligible:

$$s_V=0.03\pm0.12,\quad s_P=0.17\pm0.17. \tag{14}$$

Including the mixing effect in the fit, we obtained new values for the branching ratios of $Br(J/\psi\to\omega\pi^0(\eta,\eta'))$, as listed in Table 2. It should be noted that a difference of about 19% (15%, 15%) is observed in the branching ratios compared with the PDG2012 values [35], when mixing effects are incorporated:

$$\begin{aligned} Br(J/\psi\to\omega\pi^0)&=(3.64\pm0.37)\times10^{-4},\\ Br(J/\psi\to\omega\eta)&=(1.48\pm0.17)\times10^{-3},\\ Br(J/\psi\to\omega\eta')&=(1.55\pm0.56)\times10^{-4}. \end{aligned} \tag{15}$$

The value of the ρ-ω mixing polarization operator is also obtained:

$$\Pi_{\rho\omega}=(0.006\pm0.011)\ \mathrm{GeV}^2. \tag{16}$$

The significance of $\Pi_{\rho\omega}$ is 0.36, which means that it has a large probability of being zero. This value is comparable with the value calculated by the formula given in Ref. [29]:

$$\Pi_{\rho\omega}=\mathrm{Re}(\Pi_{\rho\omega})=\sqrt{\frac{\Gamma_\omega}{\Gamma_{\rho^0}(m_\omega)}Br(\omega\to\pi^+\pi^-)}\times\left|\left(m_\omega^2-m_{\rho^0}^2\right)-\mathrm{i}m_\omega\left(\Gamma_\omega-\Gamma_{\rho^0}(m_\omega)\right)\right|. \tag{17}$$

The $\Pi_{\rho\omega}$ value is 0.0042 GeV^2 or 0.0033 GeV^2 when parameters from SND [29] or PDG2012 [35] are used, respectively.

Figure 2 shows the ratio between the cross sections with ($\Pi_{\rho\omega}=0.006$ GeV^2) and without ($\Pi_{\rho\omega}=0$ GeV^2) ρ-ω mixing ($\sigma(s)_{\mathrm{mix}}/\sigma(s)_{\mathrm{nomix}}$) as a function of the

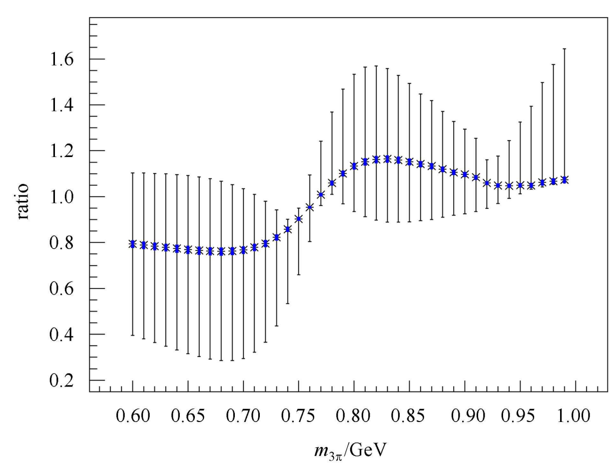

Fig. 2. The ratio of $\sigma(s)_{\mathrm{mix}}$ (with ρ-ω mixing, $\Pi_{\rho\omega}=0.006$ GeV^2) to $\sigma(s)_{\mathrm{nomix}}$ (without ρ-ω mixing, $\Pi_{\rho\omega}=0$ GeV^2).

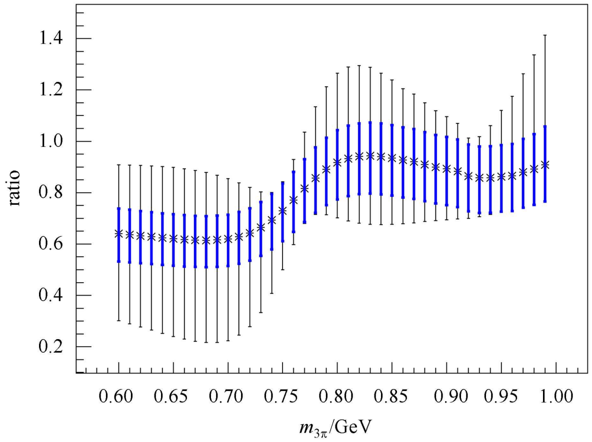

Fig. 3. The ratio of $\sigma(s)_{\rm new}$ (with our corrected branching ratios and mixing) to $\sigma(s)_{\rm old}$ (with PDG2012's branching ratios and no mixing). The thick blue lines (color online) represent the errors calculated by ignoring the error in $\Pi_{\rho\omega}$.

invariant mass of the 3π system, where the corrected branching ratios are used. It can be seen clearly that the ρ-ω mixing has a significant effect on the shape of the $m_{3\pi}$ spectrum, with the variance reaching a maximum of about ±20% above or below the nominal ω mass.

Figure 3 shows the ratio between the cross sections with our corrected branching ratios (and with mixing) and with PDG2012's branching ratios (and no mixing) ($\sigma(s)_{\rm new}/\sigma(s)_{\rm old}$), as a function of the invariant mass of the 3π system. It can also be clearly observed that our derivation has a significant effect on the shape of the $m_{3\pi}$ spectrum, with the variance reaching about 40% at its largest and about 20% near the nominal ω mass.

The errors in Figs. 2 and 3 are caused mainly by the uncertainty in $\Pi_{\rho\omega}$, which has a limited significance. Further checks are expected by experiment. This work on the cross section of $e^+e^-\to J/\psi\to V\pi^0\to\pi^+\pi^-\pi^0\pi^0$ can be used in the analysis of J/ψ data at BESIII.

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Appendix A

Notation for ρ-ω mixing

The mechanism of ρ-ω mixing has been reviewed in many previous works [7, 29, 36–38, 50–52]. The wave-functions of unmixed ω and ρ states are given as [53]:

$$
\begin{aligned}
|\omega^{(0)}\rangle &\equiv \frac{1}{\sqrt{2}}|u\overline{u}+d\overline{d}\rangle,\\
|\rho^{(0)}\rangle &\equiv \frac{1}{\sqrt{2}}|u\overline{u}-d\overline{d}\rangle,
\end{aligned} \tag{A1}
$$

while the wave-functions of physical states ω and ρ under the pole approximation assumption can be written in general as:

$$
\begin{aligned}
|\omega\rangle &= |\omega^{(0)}\rangle+\varepsilon|\rho^{(0)}\rangle,\\
|\rho\rangle &= |\rho^{(0)}\rangle-\varepsilon|\omega^{(0)}\rangle,
\end{aligned} \tag{A2}
$$

where the superscript (0) denotes the coupling constants of the pure, unmixed states. Here

$$
\begin{aligned}
\varepsilon &= \frac{\Pi_{\rho\omega}}{D_\omega(s)-D_\rho(s)},\\
D_{\rm V}(s) &= m_{\rm V}^2-s-{\rm i}\sqrt{s}\Gamma_{\rm V}(s).
\end{aligned} \tag{A3}
$$

$D_{\rm V}(s)$ is the propagator function; $\Gamma_{\rm V}(s)$ is the width of the vector meason; and $\Pi_{\rho\omega}\equiv\langle\rho^{(0)}|W|\omega^{(0)}\rangle$ [7, 50] is the polarization operator for the mixing. Note that ε is not a real number, hence the transfer matrix from isospin basis to physical basis is not unitary. In Ref. [29] ε is negative with the same expression.

Under this framework, the coupling constants for $\omega(\rho)\to\pi^+\pi^-(\rho\pi,3\pi)$ decays can be determined as follows:

$$
\begin{aligned}
g_{\omega\pi\pi} &= g^{(0)}_{\omega\pi\pi}+\varepsilon g^{(0)}_{\rho\pi\pi}, & g_{\rho\pi\pi} &= g^{(0)}_{\rho\pi\pi}-\varepsilon g^{(0)}_{\omega\pi\pi},\\
g_{\omega\rho\pi} &= g^{(0)}_{\omega\rho\pi}+\varepsilon g^{(0)}_{\rho\rho\pi}, & g_{\rho\rho\pi} &= g^{(0)}_{\rho\rho\pi}-\varepsilon g^{(0)}_{\omega\rho\pi},\\
g_{\omega 3\pi} &= g^{(0)}_{\omega 3\pi}+\varepsilon g^{(0)}_{\rho 3\pi}, & g_{\rho 3\pi} &= g^{(0)}_{\rho 3\pi}-\varepsilon g^{(0)}_{\omega 3\pi},
\end{aligned} \tag{A4}
$$

and for $J/\psi\to(\rho^0,\omega)\pi^0$:

$$
\begin{aligned}
A_{\psi\omega\pi}(s) &= A^{(0)}_{\psi\omega\pi}(s)+\varepsilon(s)A^{(0)}_{\psi\rho\pi}(s),\\
A_{\psi\rho\pi}(s) &= A^{(0)}_{\psi\rho\pi}(s)-\varepsilon(s)A^{(0)}_{\psi\omega\pi}(s),
\end{aligned} \tag{A5}
$$

where "g" and "A" are defined as Section 2.

Notation for η-η′ mixing

The wave-functions of the physical states η and η′ can be written in general as [24, 25, 27, 54]:

$$
\begin{aligned}
|\eta\rangle &= X_\eta|\eta_q\rangle+Y_\eta|\eta_s\rangle+Z_\eta|G\rangle,\\
|\eta'\rangle &= X_{\eta'}|\eta_q\rangle+Y_{\eta'}|\eta_s\rangle+Z_{\eta'}|G\rangle,
\end{aligned} \tag{A6}
$$

where

$$
\begin{aligned}
&|\eta_q\rangle \equiv \frac{1}{\sqrt{2}}|u\overline{u}+d\overline{d}\rangle, \quad |\eta_s\rangle\equiv|s\overline{s}\rangle,\\
&|G\rangle = |\text{gluonium}\rangle,
\end{aligned} \tag{A7}
$$

and

$$X^2_{\eta(\eta')}+Y^2_{\eta(\eta')}+Z^2_{\eta(\eta')}=1. \tag{A8}$$

Assuming no gluonium content in η, the mixing can be parameterized in terms of two angles [24, 27]:

$$\begin{aligned} X_\eta&=\cos\phi_{\rm P}, & X_{\eta'}&=\sin\phi_{\rm P}\cos\phi_{\eta'{\rm G}},\\ Y_\eta&=-\sin\phi_{\rm P}, & Y_{\eta'}&=\cos\phi_{\rm P}\cos\phi_{\eta'{\rm G}},\\ Z_\eta&=0, & Z_{\eta'}&=-\sin\phi_{\eta'{\rm G}}, \end{aligned} \tag{A9}$$

where $\phi_{\rm P}$ is the η-η′ mixing angle, and $\phi_{\eta'{\rm G}}$ weights the amount of gluonium in η′.

The dimensions in Eqs. (A7) are absorbed into the state expression for the intuitive impression, which is frequently done in the literature. However, Thorsten Feldmann [55] thinks that these formulae are at best using a very sloppy notation, and one has to carefully distinguish between partonic Fock states in some factorization formulae, and physical states of the QCD Hamiltonian. More details and stricter formulae are given in Ref. [55].

Notation for ω-ϕ mixing

Similar to η-η′ mixing, a relatively simple expression for ω−ϕ mixing is used [24, 27]:

$$\begin{aligned} |\omega\rangle&=\cos\phi_{\omega\phi}|\omega_{\rm q}\rangle-\sin\phi_{\omega\phi}|\phi_{\rm s}\rangle,\\ |\phi\rangle&=\sin\phi_{\omega\phi}|\omega_{\rm q}\rangle+\cos\phi_{\omega\phi}|\phi_{\rm s}\rangle, \end{aligned} \tag{A10}$$

where $|\omega_{\rm q}\rangle$ and $|\phi_{\rm s}\rangle$ are the analog non-strange and strange states of $|\eta_{\rm q}\rangle$ and $|\eta_{\rm s}\rangle$ respectively, and $\phi_{\omega\phi}$ is the mixing angle between ρ and ω.

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