Loschmidt Echo of a central spin coupled to an XY spin chain: The role of the Dzyaloshinsky-Moriya interaction *

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Abstract: By introducing the Dzyaloshinsky-Moriya (DM) interaction, the Loschmidt Echo (LE) of a quantum system consisting of a central spin and its surrounding environment characterized by an XY spin chain was investigated analytically and numerically. At the critical points of the magnetic field, the LE presents an obvious decay. The decay amplitude can be tuned by the DM interaction. In some specific intervals the DM interaction can remarkably delay the decay of the LE. On the other hand, the DM interaction can change the effects of the anisotropy parameter on the LE.

Key words: Loschmidt Echo, Dzyaloshinsky-Moriya interaction, dynamical evolution behavior

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1 Introduction

The Loschmidt Echo (LE), quantifying irreversibility and chaos in quantum mechanics [1–3], has been extensively studied in recent years [4–8]. It measures the distance between two identically prepared states evolving under slightly different Hamiltonians and was introduced to describe the hypersensitivity of the time evolution to the perturbations experienced by the environmental system [9–11]. In particular, the effects of environmental quantum criticality on the dynamical behaviors of the LE have been studied in many environmental systems, including spin chain systems [12, 13] and Tonks-Girardeau gas [14], Dicke model [15] and so on. Besides the LE, the other kinds of quantum correlation have attracted considerable attention, for example: entanglement [16–18], discord [19], and fidelity [20]. In experiment, Yu et al. have studied the quantum correlation of the single atoms in the ring lattice [21].

Among the research studies of the dynamical behaviors of the LE in spin chain, most researchers have mainly focused on the spin-spin interaction [12, 13, 22]. However, the spin-orbit coupling, that is, Dzyaloshinsky-Moriya (DM) interaction, often presents in the models of many low-dimensional magnetic materials. It has been reported that the DM interaction can obviously change the spectroscopic modes of multiferroic materials [23, 24]. The fidelity of quantum cloning in a spin network can also been improved by the DM interaction [25]. It can also influence the quantum phase transition (QPT) of the system and enhance the entanglement [26, 27]. Thus, the DM coupling is very important in spin systems and the effects of the DM coupling on properties of the systems should be considered.

Recently, we have discussed the critical behaviors of an XY spin chain with different interactions in terms of the LE, including the three-site interaction [28–30] and DM interaction [31]. In the work of Ref. [31], we mainly focused on critical behavior of XY spin chain with DM interaction. But the effects of DM interaction on the dynamical behaviors of the LE are still unclear. In this work, we further explore the dynamical behaviors of the LE with DM interaction in detail and find that DM interaction is crucial in determining the decay behaviors of the LE.

2 Results and discussion

For the quantum system of a central spin transversely coupled to an environmental spin chain described by the XY model with the DM interaction in the presence of an external magnetic field, we have deduced the exact expression of the LE in the given initial conditions [32]. It is assumed that the central spin is initially in the superposition state of the ground $|g\rangle$ and excited state $|e\rangle$. In this case, the evolution of the environmental spin chain is driven by the two different effective Hamiltonians H_α ($\alpha = g,\ e$). At the same time, the environmental spin

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chain is assumed to be prepared initially in the ground state of H_g. Then, the LE obeys the following expression [32]:

$$LE(t)=\prod_{k=1}^{M}[1-\sin^2(2\varPhi_k)\sin^2(\varLambda_{k,e}t)]=\prod_{k=1}^{M}LE_k(t). \quad (1)$$

Here, k =1, 2⋯, M; $M=N/2$ for N even and N is environmental spin chain size; t is evolution time; $\varPhi_k=(\theta_{k,e}-\theta_{k,g})/2$ is angular distance and $\theta_{k,g}=\arctan[\gamma\sin(x_k)/(\lambda+\beta-\cos(x_k))]$, $\theta_{k,e}=\arctan[\gamma\sin(x_k)/(\lambda-\beta-\cos(x_k))]$; $\varLambda_{k,e}=2[\varepsilon_{k,e}+2D\sin(x_k)]$ is the energy spectrum of the effective Hamiltonian H_e and $\varepsilon_{k,e}=[(\lambda-\beta-\cos(x_k))^2+(\gamma\sin(x_k))^2]^{0.5}$; γ is the anisotropy in the in-plane interaction of the environmental spin chain; λ is the intensity of the external magnetic field; β characters the interaction between the system and the environment; and, D is the intensity of DM interaction and $x_k=2\pi k/N$.

Following Ref. [33], we first carry out a heuristic analysis of the features of the $LE(t)$. By introducing a cutoff frequency k_c, we define the partial product for the $LE(t)$, as follows

$$LE_c\equiv\prod_{k=1}^{k_c}LE_k\geqslant LE(t). \quad (2)$$

The corresponding partial sum can be readily obtained as

$$S(t)=\ln LE_c=-\sum_{k=1}^{k_c}|\ln LE_k|. \quad (3)$$

For the case of small k and large N, one has

$$\varLambda_{k,\alpha}=2|\lambda+\kappa_\alpha\beta-1|+4Dx_k,\ \ (\kappa_g=1,\ \kappa_e=-1), \quad (4)$$

$$\sin(2\varPhi_k)\approx\frac{4\pi\gamma\beta k}{N|(\lambda-\beta-1)(\lambda+\beta-1)|}. \quad (5)$$

As a result, if N is large enough and k is small, the approximation of $S(t)$ can be obtained as

$$S(t)\approx-\sum_{k=1}^{k_c}\frac{16\pi^2\gamma^2\beta^2k^2\sin^2(\varLambda_{k,e}t)}{N^2(\lambda-\beta-1)^2(\lambda+\beta-1)^2}. \quad (6)$$

In the derivation of the above equation, we employ the formula $\ln(1-x)\approx-x$ for small x.

In the weak coupling regime $g\ll1$, when $\lambda\to1$ and in a short time t, one has

$$LE_c(t)\approx\exp[-(\tau_1+\tau_2+\tau_3)t^2]. \quad (7)$$

Where

$$\tau_1=\frac{16\gamma^2\beta^4E(k)}{(\lambda-\beta-1)^2(\lambda+\beta-1)^2},\quad E(k)=4\pi^2\sum_{k=1}^{k_c}\frac{k^2}{N^2}; \quad (8)$$

$$\tau_2=\frac{64D\gamma^2\beta^3F(k)}{(\lambda-\beta-1)^2(\lambda+\beta-1)^2},\quad F(k)=8\pi^3\sum_{k=1}^{k_c}\frac{k^3}{N^3}; \quad (9)$$

$$\tau_3=\frac{64D^2\gamma^2\beta^2G(k)}{(\lambda-\beta-1)^2(\lambda+\beta-1)^2},\quad G(k)=16\pi^4\sum_{k=1}^{k_c}\frac{k^4}{N^4}. \quad (10)$$

From the above analysis one can find that when λ is adjusted to the value of 1, the LE will exponentially decay in the second power of time. In addition, the decay amplitude is mainly determined by the expression $\tau_1+\tau_2+\tau_3$, which is a second-order polynomial against variable D. For the case of D=0, the decay amplitude is mainly determined by τ_1, which has been studied in Ref. [33].

Figure 1(a) shows the variations of the LE with changing magnetic field intensity (λ) and time (t) for the case of D=1.0 and N=100. The dip appears during the evolution of the LE. The position of the dip is exactly the critical point λ_c=1. Similar behaviors can be observed in large size spin chain, as shown in Fig. 1(b–c). The changes of the LE as functions of λ and D with t=5 are shown in Fig. 1(d). It is easy to see that the position of the dip cannot shift and the depth changes with changing the value of D. There exists a special value that can delay the decay of the LE deduced by the QPT. These results are in accord with the theoretical analysis.

The changes of the LE as the functions of D ant t are shown in Fig. 2(a). At the critical point λ_c=1, the LE decays more sharply with increasing D in the region $D\geqslant0$. However, in the region $D<0$, the trend is not monotonous and there exists an extreme of the LE. The decay of the LE can be delayed remarkably with increasing D along the negative direction. Once D passes through a certain value D_s, the decay amplitude of the LE increases with increasing D. In order to obtain the certain values D_s, the LE as a function of D for λ_c=1.0 are shown in Fig. 2(b–c). It can be seen that D_s is about -0.5 which is independent on the evolution time t (Fig. 2(b)) and the spin chain size (Fig. 2(c)). The results show that in some specific intervals the DM interaction can remarkably delay the decay of the LE at the critical points. In the preceding heuristic analysis, it has been shown in Eqs. (7–10) that the expression $\tau_1+\tau_2+\tau_3$ determining the decay amplitude is a second-order polynomial against variable D. And so there exists an extremum point at which the expression $\tau_1+\tau_2+\tau_3$ gets its minimum value and the decay behavior of the LE can be delayed remarkably. The values of $D=-0.5$ is the corresponding extremum point in the case of λ_c=1.0.

Figure 3 gives the effects of the spin chain size N on the dynamical behaviors of the LE with different DM interactions for the case of λ=1. The typical values of D are chosen as 0, 0.5 and −1, respectively, as shown in Fig. 3(a, b, d). It can be seen that for small N, such as N=50, 100 and 200, the LE first decays, followed by the oscillatory behavior, and the decay amplitude of the LE increases with increasing N. For large N, such as N=300, the oscillatory behavior becomes increasingly weaker, and the LE decays from unity to zero in a short time. The results can be explained from the Eq. (1): because $LE_k(t)$ is less than unity, the larger that N becomes, the smaller that $LE(t)$ will be. Fig. 3(c) shows the dynamical behaviors of the LE with D=−0.5. Compared with the other three figs, the decay of the LE is obviously delayed with the same N. These results show that the special strength of the DM interaction can delay the decay of the LE, as deduced by the increase of the spin chain size N.

The effects of the anisotropy parameters γ on the LE

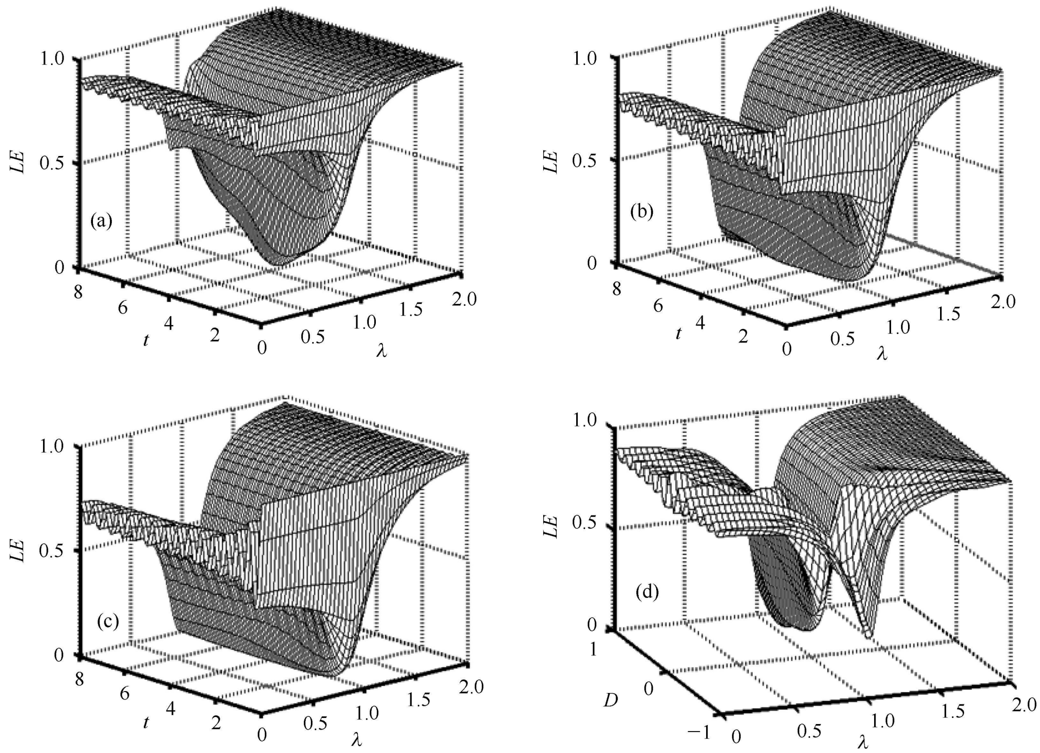

Fig. 1. (a) LE as functions of λ and t for the case of D=1.0 with N=100; (b) the same with (a) expect for N=200; (c) the same with (a) expect for N=300; (d) the LE as the functions of λ and D for the case of t=5.0 with N=100. The other parameters for plots are γ=1.0 and β=0.05.

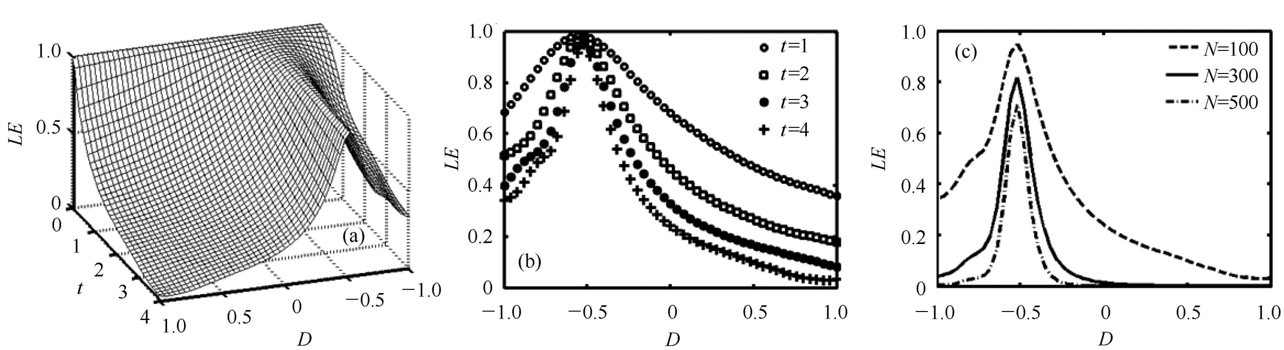

Fig. 2. (a) LE as the functions of D and t for N=100; (b) LE as a function of D for N=100 with different t; (c) LE as the function of D for t=1 with different N. the other parameters for plots are λ=1, γ=1.0, β=0.05.

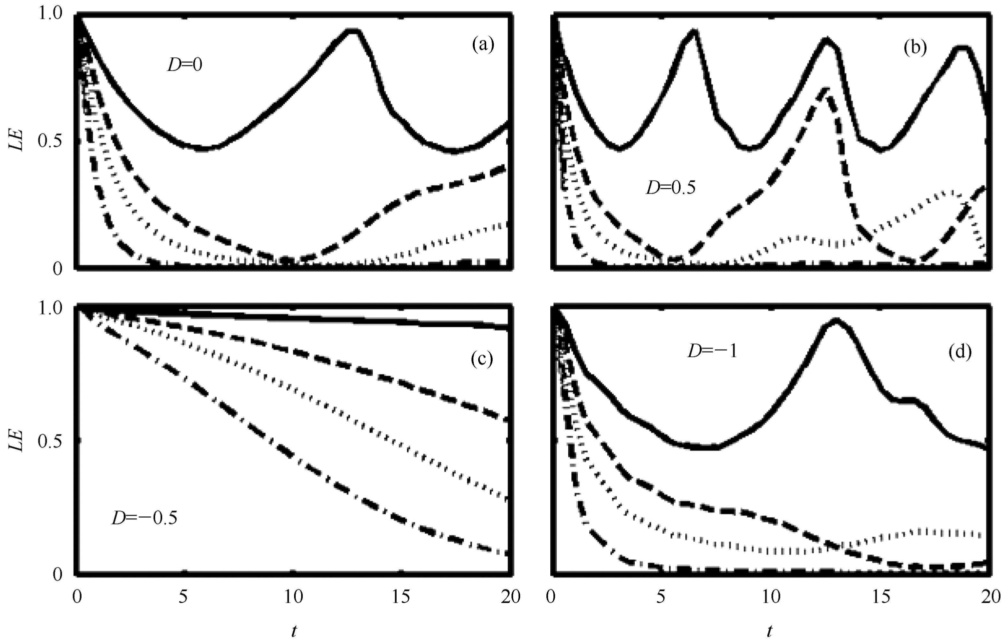

Fig. 3. LE as a function of t with different N for the case of D=0 (a), D=0.5 (b), D=−0.5 (c), D=−1 (d). Here, λ=1, γ=1.0, β=0.05 and N=50 (solid lines), N=100 (dashed lines), N=150(dotted lines), N=300 (dotted-dashed lines).

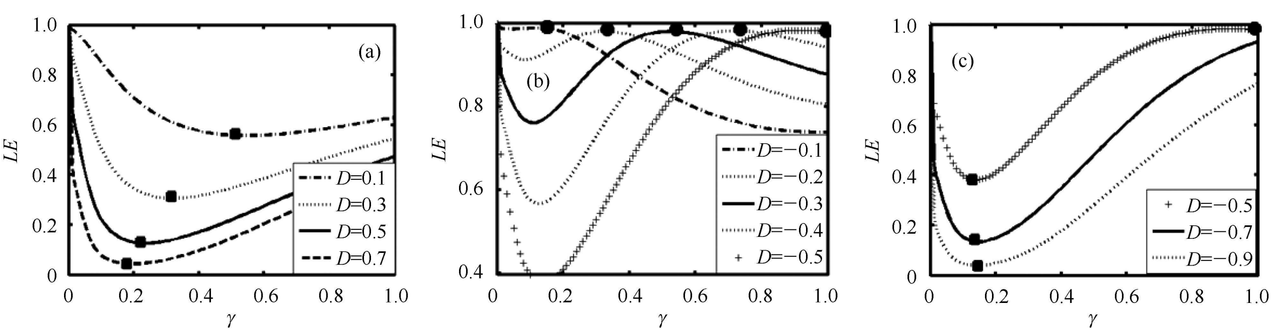

Fig. 4. LE as a function of γ with different D at the given evolution time t=1. The box (dot) marks the minimum (maximum) of the LE in each curve and the corresponding value of γ is $\gamma_{\min}$ ($\gamma_{\max}$).

with different DM interactions are shown in Fig. 4. For the special case of γ=0, LE is almost in unity. While for the case of $\gamma\neq0$, both the direction and the strength of the DM interaction can affect the function of $LE(\gamma)$. In the region of $D\geqslant0$, the change of the LE with γ is not monotonous. There exists a minimum value of LE in each curve and the corresponding γ value is $\gamma_{\min}$, which can been seen in Fig. 4(a). For $\gamma\leqslant\gamma_{\min}$, the decay amplitude of LE increases with increasing γ. For $\gamma\geqslant\gamma_{\min}$, the decay of LE weakens with increasing γ. The value of the $\gamma_{\min}$ is dependent on the strength of the DM interaction. For example, $\gamma_{\min}$ are 0.51, 0.32, 0.22 and 0.18, respectively, when D changes 0.1 to 0.7. Fig. 4(b–c) gives the change of LE with γ in the region of $D<0$. For $|D|\leqslant0.5$, there exists an extreme point $\gamma_{\max}$ in each curve, which can delay the decay of the LE remarkably, as shown in Fig. 4(b). The value of $\gamma_{\max}$ shifts towards the direction of higher γ values with increasing D and with D=−0.5, $\gamma_{\max}\rightarrow\gamma$=1. For $|D|$>0.5, with increasing γ the LE first decays to the minimum and then increases gradually, as shown in Fig. 4(c). It can be found that the value of D has a significant effect on the curve of $LE(\gamma)$.

3 Conclusions

The dynamical behaviors of the LE of a quantum system consisting of a central spin and its surrounding environment characterized by an XY spin chain with the DM interaction have been studied. It can be seen that there is a deep valley at the critical line during the evolution of the *LE*. There exists a special value of $D=-0.5$ that can remarkably delay the decay of the *LE* deduced by the QPT and the increase of the spin chain size N. On the other hand, the DM interaction can change the effects of the anisotropy parameter γ on the *LE*.

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