Lepton number violation in D meson decay *

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Abstract: The lepton number violation (LNV) process can be induced by introducing a fourth generation heavy Majorana neutrino, which is coupled to the charged leptons of the Standard Model (SM). There have been many previous studies on the leptonic number violating decay processes with this mechanism. We follow the trend to study the process: $\mathrm{D}\to\mathrm{Kll}\pi$ with the same-sign dilepton final states. We restrict ourselves to certain neutrino mass regions, in which the heavy neutrino could be on-shell and the dominant contribution to the branching fraction comes from the resonance enhanced effect. Applying the narrow width approximation (NWA), we found that the upper limit for the branching fractions for $\mathrm{D}^0\to\mathrm{K}^-\mathrm{l}^+\mathrm{l}^+\pi^-$ are generally at the order of 10^{-12} to 10^{-9}, if we take the most stringent upper limit bound currently available in the literature for the mixing matrix elements. We also provide the constraints, which is competitive compared to the LNV B decays, on the mixing matrix element $|V_{\mathrm{eN}}|^2$ based on the upper limit of $\mathrm{D}^0\to\mathrm{K}^-\mathrm{e}^+\mathrm{e}^+\pi^-$ estimated from the Monte-Carlo (MC) study at BESIII. Although the constraints are worse than the ones from $(0\nu\beta\beta)$ decay in the literature, the future experiment at the charm factory may yield more stringent constraints.

Key words: Majorana neutrino, D meson decay, lepton number violating, mixing matrix element

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1 Introduction

The discovery of neutrino oscillations [1–3] and the observation of unexpected large θ_{13} [4] have convincingly shown that neutrinos have finite mass and that lepton flavor is violated in neutrino propagation. The generation of neutrino masses is still one of the fundamental puzzles in particle physics. To obtain the non-vanishing mass of neutrinos, one can make a minimal extension of the Standard Model (SM) by including the right-handed neutrinos. Once the consensus that the neutrino is a massive fermion has been reached, another urgent task is to figure out whether the neutrinos are Dirac or Majorana particles; the latter case is characterized by being their own antiparticles.

The fact is, at present, the Majorana neutrino is one of the favorite choices for most theories, since the masses of the observed light neutrinos could be naturally derived from heavy neutrinos via the so-called 'see-saw' mechanism [5–11]. Owing to the new heavy neutrino's Majorana nature, it is its own antiparticle, which allows processes that violate lepton-number conservation by two units. Consequently, searches for Majorana neutrinos are of fundamental interest.

There is one promising method to probe the Majorana nature of neutrinos, i.e. neutrinoless double beta $(0\nu\beta\beta)$ decay [12–24]. As we have mentioned above, the Majorana nature of the new heavy neutrinos can induce the lepton number violation (LNV) by exchange of virtual Majorana neutrinos between two associated beta decays. Although the first double-beta decay was proposed as early as 1935 by Goepper-Mayer, it was not until four years later that Furry first calculated the $(0\nu\beta\beta)$ decay based on the Majorana theory [25]. These early explorations give an impetus to many years of experimental and theoretical research. It is an interesting question whether the moderately heavy sterile neutrino with a mass from a few hundred MeV to a few GeV exists. If such a neutrino exists, the decay rates of these processes can be substantially enhanced by the neutrino-resonance

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effect which is induced by taking the mass of virtual Majorana neutrino as a sterile neutrino mass. As a consequence, we can measure such LNV processes, or set up the upper bounds on sterile neutrino mass and mixing matrix elements. Due to their fundamental interests and possibilities of measurements, these LNV processes have been extensively studied in both theory [26] and experiment [27–34].

In recent years, most of the studies of these LNV processes focus on the three-body and four-body $\Delta L=2$ decays of K, D and B mesons, as well as tau lepton decays. The decay processes: K^+, D^+, $\mathrm{B}^+\to \mathrm{l}^+\mathrm{l}^+\mathrm{M}^-$, where M denotes a vector or pseudoscalar meson and l=e, μ, τ, have been extensively studied in Refs. [26, 35–39]. Meanwhile, the tau lepton decay $\tau^-\to \mathrm{l}^+\mathrm{M}_1^-\mathrm{M}_2^-$ has also been discussed in Refs. [37, 40–43]. Compared to the three-body LNV processes, the studies of four-body LNV decays are relatively rare. The $\mathrm{B}\to\mathrm{Dll}\pi$ four-body decay has been recently calculated in Ref. [44], and the four-body decays $\tau^-\to \mathrm{M}^+\mathrm{l}^-\mathrm{l}'^-\nu_\tau$ are investigated in Refs. [45, 46]. Recently, more and more attention is directed to the four-body LNV decay, the advantage of some of the four-body decays in comparison with the corresponding three-body ones is absence of the Cabibbo-Kobayashi-Maskawa (CKM) suppression. In this paper, we will study the four-body LNV decays $\mathrm{D}\to\mathrm{K}\pi\mathrm{l}^+\mathrm{l}^+$.

In the Particle Data Book (PDG) [34], the upper limits for the branching fractions of three-body $\mathrm{D}\to\mathrm{l}^+\mathrm{l}^+\mathrm{M}^-$ decays are at the level of 10^{-6} to 10^{-4}. For the four-body LNV D decays, we had less information until recently. In Ref. [47], the branching fractions of D meson decay $\mathrm{D}^0\to\mathrm{M}_1^-\mathrm{M}_2^-\mathrm{l}^+\mathrm{l}^+$ were reported to be of the order 10^{-5}–10^{-4}. From the experimental point of view, the BESIII experiment is taking data at open-charm threshold, and an integrated luminosity of 2.9 fb^{-1} data sample has been collected at ψ(3770) by the BESIII detector [48, 49]. Estimated from the Monte-Carlo (MC) sample of the same luminosity, the sensitivity of $\mathrm{D}\to\mathrm{K}\pi\mathrm{l}^+\mathrm{l}^+$ can reach the level of 10^{-9} [50]. It is interesting to investigate $\mathrm{D}\to\mathrm{K}\pi\mathrm{l}^+\mathrm{l}^+$ decays mediated by an on-shell Majorana neutrino.

The paper is organized as follows: in Section 2, we give a brief introduction to the theoretical framework involving the LNV decays; in Section 3, we sketch some techniques in our computations, like phase space parametrization and narrow width approximation (NWA), etc; in Section 4, we give our numerical results for the branching fractions of LNV D meson decays; and finally comes the summary. We also provide the analytical results of the squared amplitude in appendix A.

2 Theoretical framework

We consider the LNV four-body decay of D meson:

$$\mathrm{D}(p)\to\mathrm{K}(p_1)+\mathrm{l}^+(p_2)+\mathrm{l}^+(p_3)+\pi^-(p_4), \tag{1}$$

where D with momentum p and K with momentum p_1 can be charged or neutral, two l^+ are charged leptons with momenta p_2 and p_3, respectively, and the charged pion π^- has momentum p_4.

Following the previous studies [26, 51], such an LNV process can be induced through a Majorana neutrino N coupled to the charged leptons l. Such gauge interactions are described by the following vertex in the Lagrangian:

$$\mathcal{L}=-\frac{\mathrm{g}}{\sqrt{2}}W_\mu^+\sum_{\mathrm{l=e}}^{\tau}V_{\mathrm{lN}}^*\overline{N^c}\gamma^\mu P_{\mathrm{L}}\mathrm{l}+\mathrm{h.c.}, \tag{2}$$

where $P_{\mathrm{L}}=\frac{1}{2}(1-\gamma_5)$, N is the mass eigenstate of the fourth generation Majorana neutrino, V_{lN} is the mixing matrix between the charged lepton l neutrino ν_{l} and heavy Majorana neutrino N, their restrictive bounds are reported in Ref. [52].

$$|V_{\mathrm{eN}}|^2<3\times10^{-3},\ |V_{\mu\mathrm{N}}|^2<3\times10^{-3},\ |V_{\tau\mathrm{N}}|^2<6\times10^{-3}. \tag{3}$$

At leading order, the Feynman diagrams are displayed in Fig. 1 for the charged and neutral D meson decays, the diagrams with the charged leptons exchanged are also included. If the neutrino mass is from a few hundred MeV to several GeV, the neutrino propagator in diagrams (a), (c), (e) and (f) in Fig. 1 could be on-shell, i.e. the neutrino becomes a resonance, and its contribution to the decay can be much enhanced due to such a neutrino-resonance effect. Therefore the contributions from other diagrams are negligible compared to such enhanced diagrams, since the neutrino cannot become a resonance in those diagrams due to the kinematic restrictions. In addition, we note that diagrams (c) and (f) are suppressed with respect to diagrams (a) and (e) in Fig. 1 due to smaller CKM factors ($|V_{\mathrm{cd}}V_{\mathrm{us}}/V_{\mathrm{cs}}V_{\mathrm{ud}}|\sim0.05$). Therefore, we only keep diagram (a) and (e) in Fig. 1 as the dominant contributions.

The transition amplitude for such an LNV process can be written as:

$$\begin{aligned}\mathcal{M} =&\ 2G_{\mathrm{F}}^2V_{\mathrm{cs}}V_{\mathrm{ud}}\langle K|\bar{s}\gamma^\mu(1-\gamma_5)c|D\rangle\langle\pi|\bar{u}\gamma^\nu(1-\gamma_5)d|0\rangle\\ &\times\left[V_{\mathrm{lN}}^2m_{\mathrm{N}}\bar{u}(p_2)\left(\frac{\gamma^\mu\gamma^\nu}{q_{\mathrm{N}}^2-m_{\mathrm{N}}^2+\mathrm{i}\Gamma_{\mathrm{N}}m_{\mathrm{N}}}\right.\right.\\ &\left.\left.+\frac{\gamma^\nu\gamma^\mu}{q_{\mathrm{N}}'^2-m_{\mathrm{N}}^2+\mathrm{i}\Gamma_{\mathrm{N}}m_{\mathrm{N}}}\right)P_{\mathrm{R}}v(p_3)\right],\end{aligned} \tag{4}$$

where $P_{\mathrm{R}}=\frac{1}{2}(1+\gamma_5)$, $G_{\mathrm{F}}=1.16639\times10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant, and V_{cs} and V_{ud} are the CKM matrix elements. We have already replaced the neutrino propagator with its resonant type:

$$\frac{1}{q_{\mathrm{N}}^2-m_{\mathrm{N}}^2}\to\frac{1}{q_{\mathrm{N}}^2-m_{\mathrm{N}}^2+\mathrm{i}\Gamma_{\mathrm{N}}m_{\mathrm{N}}}, \tag{5}$$

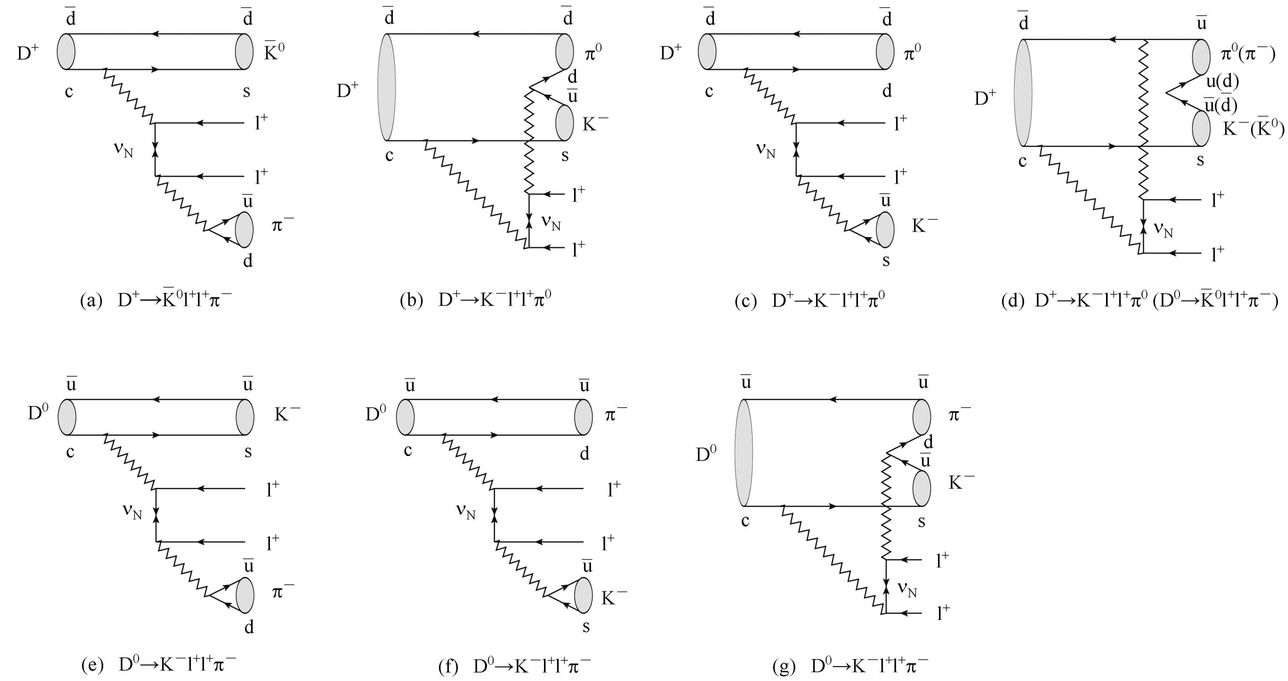

Fig. 1. Leading order Feynman diagrams for ΔL=2 charged and neutral D meson decays.

and $q_{\rm N}$ is the momentum of heavy Majorana neutrino, $q'_{\rm N}$ is the same except with two charged leptons exchanged, $m_{\rm N}$ is the mass of such a heavy Majorana neutrino, $\Gamma_{\rm N}$ is the total decay width of the heavy Majorana neutrino.

The decay width of the heavy Majorana neutrino can be obtained by adding up all contributions of neutrino decay channels which can be opened up at the mass $m_{\rm N}$ [26]:

$$\Gamma_{\rm N}(m_{\rm N})=\sum_{\rm s}\Gamma({\rm N}\rightarrow\text{final states})\theta(m_{\rm N}-\sum_{\rm s}m_{\rm s}), \quad (6)$$

where $m_{\rm s}$ in argument of Heaviside θ function are the masses of final state particles in the corresponding decay channel. All expressions for these decay widths can be found in appendix C of Ref. [26].

The matrix element involving π in Eq. (4) is related to the decay constant of charged π by:

$$\langle\pi(p_4)|\bar{u}\gamma^{\nu}(1-\gamma_5)d|0\rangle={\rm i}f_{\pi}p_4^{\nu}, \quad (7)$$

where f_{π} is the decay constant of the charged π. Then the squared amplitude can be obtained as:

$$\begin{aligned}|\mathcal{M}|^2 = {} & G_{\rm F}^4|V_{\rm cs}|^2|V_{\rm ud}|^2\langle K|\bar{s}\gamma^{\mu}(1-\gamma_5)c|D\rangle \\ & \times\langle K|\bar{s}\gamma^{\rho}(1-\gamma_5)c|D\rangle^*({\rm i}f_{\pi}p_4^{\nu})({\rm i}f_{\pi}p_4^{\sigma})^* \\ & \times{\rm Tr}\left[\left(PN_2\gamma_{\mu}\gamma_{\nu}+PN_3\gamma_{\nu}\gamma_{\mu}\right)(1+\gamma_5)(\not{p}_3-m_3)\right. \\ & \left.\times(1-\gamma_5)\left(PN_2^*\gamma_{\sigma}\gamma_{\rho}+PN_3^*\gamma_{\mu1}\gamma_{\nu1}\right)(\not{p}_2+m_2)\right],\end{aligned} \quad (8)$$

where the factors PN_1 and PN_2 are defined as:

$$PN_i=\frac{(V_{\rm lN})^2m_{\rm N}}{(p-p_1-p_i)^2-m_{\rm N}^2+{\rm i}m_{\rm N}\Gamma_{\rm N}}. \quad (9)$$

The complete analytical expression for the squared amplitude can be found in appendix A.

3 Computation techniques

3.1 Kinematics for four-body decay

The kinematics of the four-body decay $\rm D\rightarrow Kl^+l^+\pi^-$ can be described in terms of five independent variables: $\{s_{12}, s_{34}, \theta_{12}, \theta_{34}, \phi\}$, which have the following geometrical definitions [53] (see Fig. 2):

(i) $s_{12}\equiv(p_1+p_2)^2$ is the square of invariant-mass of the Kl system;

(ii) $s_{34}\equiv(p_3+p_4)^2$ is the square of invariant-mass of the πl system;

(iii) θ_{12} is the angle between the K three-momentum in the Kl rest frame and the line of flight of the Kl in the D rest frame;

(iv) θ_{34} is the angle between the π three-momentum in the πl rest frame and the line of flight of the πl in the D rest frame;

(v) ϕ is the angle between the normals to the planes defined in the D rest frame by the Kl pair and the πl pair.

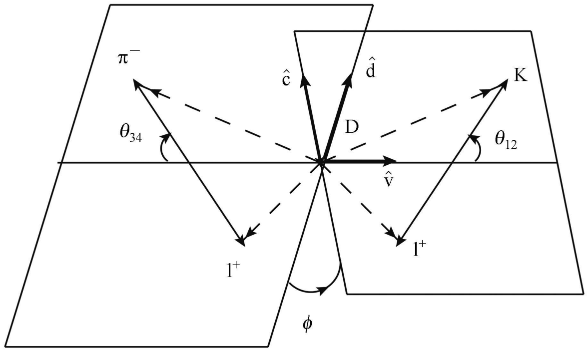

Fig. 2. Kinematics of four-body decays: $D \to Kl^{+}l^{+}\pi^{-}$ in the rest frame of D meson.

The angular variables are shown in Fig. 2, where $\boldsymbol{K}$ is the K three-momentum in the Kl center-of-mass (CM) frame and $\boldsymbol{\pi}$ is the three-momentum of the π in the πl CM frame. Let $\hat{\boldsymbol{v}}$ be the unit vector along the Kl direction in the D rest frame, $\hat{\boldsymbol{c}}$ the unit vector along the projection of $\boldsymbol{K}$ perpendicular to $\hat{\boldsymbol{v}}$, and $\hat{\boldsymbol{d}}$ the unit vector along the projection of $\boldsymbol{\pi}$ perpendicular to $\hat{\boldsymbol{v}}$. We have

$$\begin{aligned}\cos\theta_{12} &\equiv \frac{\hat{\boldsymbol{v}}\cdot\boldsymbol{K}}{|\boldsymbol{K}|}, \cos\theta_{34}\equiv-\frac{\hat{\boldsymbol{v}}\cdot\boldsymbol{\pi}}{|\boldsymbol{\pi}|},\\ \cos\phi &\equiv \hat{\boldsymbol{c}}\cdot\hat{\boldsymbol{d}}, \sin\phi\equiv(\hat{\boldsymbol{c}}\times\hat{\boldsymbol{v}})\cdot\hat{\boldsymbol{d}}.\end{aligned} \tag{10}$$

The Lorentz invariant phase space for the four-body decay is defined as

$$\begin{aligned}\int\mathrm{d}\Phi_4 &= \int\prod_{i=1}^{4}\frac{\mathrm{d}^3p_i}{(2\pi)^3 2E_i}(2\pi)^4\delta(p-p_1-p_2-p_3-p_4)\\ &= \int\frac{\mathrm{d}s_{12}}{2\pi}\frac{\mathrm{d}s_{34}}{2\pi}\mathrm{d}\Phi_2(q_{12},q_{34})\mathrm{d}\Phi_2(p_1,p_2)\mathrm{d}\Phi_2(p_3,p_4)\\ &= \int\frac{\mathrm{d}s_{12}}{2\pi}\frac{\mathrm{d}s_{34}}{2\pi}\left(\frac{\bar{\beta}}{8\pi}\int\frac{\mathrm{d}\cos\theta}{2}\frac{\mathrm{d}\phi}{2\pi}\right)\\ &\quad\times\left(\frac{\bar{\beta}_{12}}{8\pi}\frac{\mathrm{d}\cos\theta_{12}}{2}\frac{\mathrm{d}\phi_{12}}{2\pi}\right)\left(\int\frac{\bar{\beta}_{34}}{8\pi}\frac{\mathrm{d}\cos\theta_{34}}{2}\frac{\mathrm{d}\phi_{34}}{2\pi}\right),\end{aligned} \tag{11}$$

where $q_{ij}=p_i+p_j$ and $\bar{\beta}$ and $\bar{\beta}_{ij}$ are defined as:

$$\begin{aligned}\bar{\beta}&=\sqrt{1-\frac{2(s_{12}+s_{34})}{s}+\frac{(s_{12}-s_{34})^2}{s^2}},\\ \bar{\beta}_{ij}&=\sqrt{1-\frac{2(m_i^2+m_j^2)}{s_{ij}}+\frac{(m_i^2-m_j^2)^2}{s_{ij}^2}}.\end{aligned} \tag{12}$$

Here we decompose a four-body phase space integral into a product of two-body phase space integrals. This is very useful if one considers a production of two particles, each of which subsequently decays into a two-body state.

3.2 NWA

With the squared amplitude in Eq. (8) and phase space parametrization in Eq. (11), we are ready to obtain the decay rate for $D\to Kll\pi$ using the decay rate formula:

$$\Gamma(D\to Kll\pi)=\frac{1}{2!}\frac{1}{2m_D}\int\mathrm{d}\Phi_4|\mathcal{M}|^2. \tag{13}$$

Let us look at the Eq. (4) again before we start to perform the phase space integral. In general, $q_N\neq q'_N$, and it is convenient to split up the individual resonant contributions by the single-diagram-enhanced (SDE) multi-channel integration method [54]. To do this, we define the functions

$$f_i=\frac{|\mathcal{M}_i|^2}{\sum_i|\mathcal{M}_i|^2}|\sum_i\mathcal{M}_i|^2, \tag{14}$$

where each $\mathcal{M}_i$ corresponds to the amplitude for a single diagram, then the total amplitude squared is given by

$$|\mathcal{M}|^2=|\sum_i\mathcal{M}_i|^2=\sum_i f_i. \tag{15}$$

The amplitude squared now splits up into the functions f_i defined above and the phase space integration can be done for each f_i separately. Moreover the peak structure of each f_i is the same as of the single squared amplitude $|\mathcal{M}_i|^2$. When the width Γ_N of the heavy Majorana neutrino is very small compared to the neutrino mass m_N, we can apply NWA:

$$\int\frac{\mathrm{d}m_i^2}{(m_i^2-m_N^2)^2+\Gamma_N^2m_N^2}\bigg|_{\Gamma_N\to0}=\int\mathrm{d}m_i^2\delta(m_i^2-m_N^2)\frac{\pi}{\Gamma_N m_N}. \tag{16}$$

Applying the NWA and SDE multi-channel integration method, we can make convenient simplification for the phase space integration and the computation can be carried out in parallel. The contribution from each f_i can be added up after phase space integration.

4 Numerical results

The matrix element $\langle K|\bar{s}\gamma^\mu(1-\gamma_5)c|D\rangle$ for D to K can be parameterized as:

$$\begin{aligned}&\langle K(p_1)|\bar{s}\gamma^\mu(1-\gamma_5)c|D(p)\rangle\\ &=\left(p_1^\mu+p^\mu-q^\mu\frac{m_D^2-m_K^2}{q^2}\right)f_+(q^2)\\ &\quad+\frac{m_D^2-m_K^2}{q^2}q^\mu f_0(q^2),\end{aligned} \tag{17}$$

where $q=p-p_1$, and f_+ and f_0 are two form factors. When we take the lepton mass as zero, the $f_0(q^2)$ will not contribute, the strong interaction dynamics can be described by a single form factor $f_+(q^2)$. We use the modified pole (MP) [55] ansatz to parameterize the form factor $f_+(q^2)$:

$$f_+(q^2)=\frac{f_+(0)}{\left(1-\frac{q^2}{m_{\rm pole}^2}\right)\left(1-\alpha_{\rm pole}\frac{q^2}{m_{\rm pole}^2}\right)}, \tag{18}$$

where $m_{\rm pole}$ is the pole mass which is predicted to be the $\rm D^{*-}$ mass, and $\alpha_{\rm pole}$ is a free parameter. We take the parameters from the CLEO-c measurement [56] as follows:

$$f_+(0)=0.739,\ m_{\rm pole}=1.91\ \text{GeV},\ \alpha_{\rm pole}=0.30.$$

Table 1. The masses of mesons and leptons (all the values are taken from PDG) [34]

meson	$\rm D^+$	$\rm D^0$	$\rm K^0$	$\rm K^-$	$\rm e^-$	μ^-	π^-
mass/MeV	1869.62	1864.86	497.614	493.677	0.511	105.658	139.57

Table 2. Other input parameters used in our case (all the values are taken from PDG) [34]

parameter	$G_{\rm F}$	$\lvert V_{\rm ud}\rvert$	$\lvert V_{\rm cs}\rvert$	f_π
value	1.166×10^{-5} GeV^{-2}	0.974	1.006	130.41 MeV

We adopt the following values for other input parameters in Table 1 and 2 in our numerical evaluation.

As for the neutrino mixing matrix elements, we take the most stringent upper limit currently available in the literature [26, 43] in our numerical computation, the upper limits are extracted from the plots of Fig. 3 and Fig. 4 in Refs. [26, 43]. We take the most stringent upper limit for $|V_{\rm eN}|^2$ as the derivation from a reanalysis of neutrinoless double beta decay experimental data [57], and as for $|V_{\mu\rm N}|^2$ we do not include the contour labeled with PS 191, since its 90% confidence level curve was obtained from various additional assumptions, touching upon both data processing and their theoretical interpretations, making the corresponding limits not too firm to compete with the $(0\nu\beta\beta)$-limits. These values are listed in Table 3 for both $|V_{\rm eN}|^2$ and $|V_{\mu\rm N}|^2$.

Since $|V_{\tau\rm N}|^2$ is not controlled by experimental limits due to the absence of experimental data on the LNV or lepton flavor violation processes involving two τ-leptons, we use an ad hoc assumption:

$$|V_{\rm eN}|^2\sim|V_{\mu\rm N}|^2\sim|V_{\tau\rm N}|^2=|V_{\rm lN}|^2. \tag{19}$$

frequently used in the literature in our computation for the total decay width of heavy majorana neutrino $\Gamma_{\rm N}$.

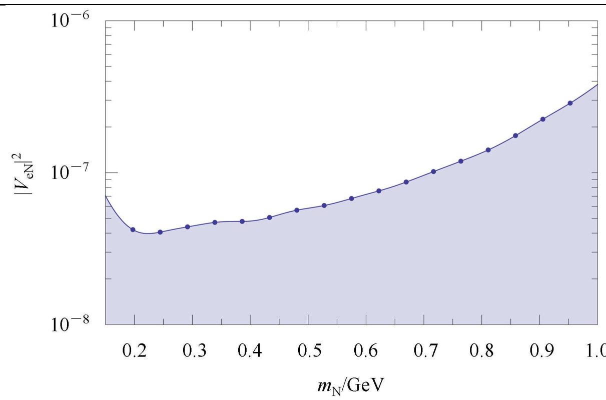

Fig. 3. Upper limits on $|V_{\rm eN}|^2$ at 90% confidence level as a function of the Majorana neutrino mass from the $\rm D^0\to K^-e^+e^+\pi^-$ estimated from the MC study at BESIII.

To make the heavy Majorana neutrino resonant, the neutrino mass $m_{\rm N}$ should satisfy the following kinematical restrictions:

$$m_{\rm l}+m_\pi\leqslant m_{\rm N}\leqslant m_{\rm D}-m_{\rm l}-m_{\rm K}. \tag{20}$$

We take the neutrino mass $m_{\rm N}$ from 150 MeV to 1000 MeV, the numerical results for the upper limits of branching fractions are listed in Tables 4 and 5. We find that the magnitude of the branching fraction for the $\rm D^0\to K^-e^+e^+\pi^-$ and $\rm D^0\to K^-\mu^+\mu^+\pi^-$ are generally at the order of 10^{-12} to 10^{-9} with the most stringent mix matrix elements; and the branching fractions for the charged D meson decay are generally a few times larger

Table 3. The most stringent upper limit for the neutrino mixing matrix at different majorana neutrino mass $m_{\rm N}$, the upper limits are extracted from the plots of Fig. 3 and Fig. 4 in Refs. [26, 43], we have taken the most stringent upper limit for $|V_{\rm eN}|^2$ as the derivation from a reanalysis of neutrinoless double beta decay experimental data [57], and as for $|V_{\mu\rm N}|^2$ we do not include the contour labeled with PS 191.

$m_{\rm N}$/MeV	150	200	250	300	350	400	450	500	550
$\lvert V_{\rm eN}\rvert^2(\times10^{-8})$	4.7	3.4	2.8	2.4	2.1	1.7	1.7	1.6	1.5
$\lvert V_{\mu\rm N}\rvert^2(\times10^{-7})$	19.3	9.3	8.1	2.9	0.9	9.2	7.2	5.6	4.6
$m_{\rm N}$/MeV	600	650	700	750	800	850	900	950	1000
$\lvert V_{\rm eN}\rvert^2(\times10^{-8})$	1.4	1.4	1.4	1.4	1.3	1.3	1.3	1.2	1.2
$\lvert V_{\mu\rm N}\rvert^2(\times10^{-7})$	3.8	3.3	3.0	2.6	2.3	2.0	1.6	1.5	1.3

Table 4. Upper limits on the branching fractions for $D^0 \to K^- + l^+ + l^+ + \pi^-$ and $D^0 \to K^- + \mu^+ + \mu^+ + \pi^-$. The masses of heavy Majorana neutrino are in units of MeV, and the total decay width of D^0 is $\Gamma_{\rm tot} = 1.605\times10^{-9}$ MeV.

$D^0 \to K^- e^+ e^+ \pi^-$				$D^0 \to K^- \mu^+ \mu^+ \pi^-$			
m_N	$\Gamma/\Gamma_{\rm tot}$	m_N	$\Gamma/\Gamma_{\rm tot}$	m_N	$\Gamma/\Gamma_{\rm tot}$	m_N	$\Gamma/\Gamma_{\rm tot}$
150	4.5×10^{-10}	600	4.0×10^{-11}	150	–	600	9.0×10^{-10}
200	6.7×10^{-10}	650	2.8×10^{-11}	200	–	650	5.9×10^{-10}
250	4.7×10^{-10}	700	2.0×10^{-11}	250	1.6×10^{-9}	700	3.8×10^{-10}
300	2.9×10^{-10}	750	1.4×10^{-11}	300	1.7×10^{-9}	750	2.3×10^{-10}
350	2.0×10^{-10}	800	9.5×10^{-12}	350	5.4×10^{-10}	800	1.4×10^{-10}
400	1.3×10^{-10}	850	6.1×10^{-12}	400	5.0×10^{-9}	850	7.7×10^{-11}
450	1.0×10^{-10}	900	3.5×10^{-12}	450	3.3×10^{-9}	900	3.6×10^{-11}
500	7.5×10^{-11}	950	1.9×10^{-12}	500	2.2×10^{-9}	950	1.7×10^{-11}
550	5.4×10^{-11}	1000	9.4×10^{-13}	550	1.4×10^{-9}	1000	7.4×10^{-12}

Table 5. Upper limits on the branching fractions for $D^+ \to \bar{K}^0 + l^+ + l^+ + \pi^-$ and $D^+ \to \bar{K}^0 + \mu^+ + \mu^+ + \pi^-$. The masses of heavy Majorana neutrino are in units of MeV, and the total decay width of D^+ is $\Gamma_{\rm tot} = 6.329\times10^{-10}$MeV.

$D^+ \to \bar{K}^0 e^+ e^+ \pi^-$				$D^+ \to \bar{K}^0 \mu^+ \mu^+ \pi^-$			
m_N	$\Gamma/\Gamma_{\rm tot}$	m_N	$\Gamma/\Gamma_{\rm tot}$	m_N	$\Gamma/\Gamma_{\rm tot}$	m_N	$\Gamma/\Gamma_{\rm tot}$
150	1.1×10^{-9}	600	1.0×10^{-10}	150	–	600	2.3×10^{-9}
200	1.7×10^{-9}	650	7.2×10^{-11}	200	–	650	1.5×10^{-9}
250	1.2×10^{-9}	700	5.1×10^{-11}	250	4.1×10^{-9}	700	9.8×10^{-10}
300	7.3×10^{-10}	750	3.7×10^{-11}	300	4.2×10^{-9}	750	5.9×10^{-10}
350	5.1×10^{-10}	800	2.4×10^{-11}	350	1.4×10^{-9}	800	3.6×10^{-10}
400	3.3×10^{-10}	850	1.6×10^{-11}	400	1.3×10^{-8}	850	2.0×10^{-10}
450	2.6×10^{-10}	900	9.1×10^{-12}	450	8.5×10^{-9}	900	9.3×10^{-11}
500	1.9×10^{-10}	950	4.8×10^{-12}	500	5.5×10^{-9}	950	4.4×10^{-11}
550	1.4×10^{-10}	1000	2.4×10^{-12}	550	3.5×10^{-9}	1000	1.9×10^{-11}

than the one for the neutral D meson decay. We note that upper limits of the branching fractions generally decrease when the neutrino mass increases.

At the BESIII experiment, estimated from 2.9 fb^{-1} MC sample, the upper limit for the decay of $D^0 \to K^- e^+ e^+ \pi^-$ obtained in Ref. [50] is about 1.0×10^{-9} at 90% confidence level. It almost approaches the upper limit with the most stringent mixing matrix elements available in the literature for the low Majorana neutrino mass.

We use the following relation between decay rates Γ of $D \to K\pi ll$ with different mixing matrix elements to derive the upper limits bound of $|V_{eN}|^2$:

$$\frac{\Gamma(m_N, V_{eN}(m_N))}{\Gamma(m_N, V'_{eN}(m_N))} = \frac{|V_{eN}(m_N)|^4}{|V'_{eN}(m_N)|^4}. \tag{21}$$

In Fig. 3, we plot the exclusion regions provided by this MC study. From such estimation, we expect the BESIII experiment can provide competitive constraints on the mixing matrix element $|V_{eN}|^2$ compared to the LNV B decays [33]. We can see that the constraint is worse than the one from $(0\nu\beta\beta)$ decay currently available in the literature [26, 43]; however it is a different kind of channel from the charmed meson decay. In the future, at the charm factory, about 1 ab^{-1} integrated luminosity will be collected per year [58], and an improvement by three orders of magnitude on the branching fractions would yield more stringent constraint on both the mass of Majorana neutrino and the mixing matrix elements.

5 Summary

We have investigated the LNV four-body decay of D meson by introducing a fourth generation heavy Majorana neutrino, which is coupled to the charged leptons from the SM. Taking the most stringent limit bound currently available in the literature for the mixing matrix element, we found that the upper limit of the branching fractions are generally at the order of 10^{-12} to 10^{-9}. New constraints on the mixing matrix element $|V_{eN}|^2$ based on the upper limit of $D^0 \to K^- e^+ e^+ \pi^-$ estimated from the MC study at BESIII are also provided. Although the constraints are worse than the ones from $(0\nu\beta\beta)$ decay in the literature, the future experiment at the charm factory may yield more stringent constraints.

Note added: After the calculation was completed and while we were preparing the draft, a related work recently appeared in arXiv [59], which also investigated the lepton-number violating D meson four-body decay processes. Aside from the different strategy in parameterizing the $D \to K$ form factor (the authors of [59] used the Bethe-Salpeter approach to estimate those form factors, while we use the MP ansatz whose parameters are directly extracted from the latest published CLEO-c data [56]), our numerical predictions for the branching ratios are in agreement with theirs in magnitude if we take the same mixing matrix elements.

Appendix A

Squared Amplitude

We give the analytical expression of the squared amplitude in this appendix.

$$\begin{aligned}
\mathcal{C}_1 =\ & f_\pi^2|PN_2|^2G_{\rm F}^4|V_{\rm cs}|^2|V_{\rm ud}|^2\Big\{8D_0^2(f_0-f_1)^2m_{\rm K}^4(-2m_\pi^2((p_{23}+p_{24}-p_{34})m_{\rm l}^2+m_{\rm l}^4-2p_{23}p_{34})\\
&+4p_{34}^2(m_{\rm l}^2+p_{23})+m_\pi^4p_{23})+m_{\rm K}^2(16f_1(-D_0(f_0-f_1)(-2m_\pi^2((p_{12}+p_{13}+p_{23}+p_{24}-p_{34})m_{\rm l}^2\\
&+m_{\rm l}^4-p_{14}p_{23}+p_{13}p_{24}-p_{12}p_{34}-2p_{23}p_{34})+4p_{34}((p_{14}+p_{34})m_{\rm l}^2+p_{14}p_{23}-p_{13}p_{24}+p_{12}p_{34}\\
&+p_{23}p_{34})+m_\pi^4p_{23})+2f_1m_\pi^2p_{23}-4f_1p_{24}p_{34})-16D_0^2(f_0-f_1)^2m_{\rm D}^2(-2m_\pi^2((p_{23}+p_{24}-p_{34})m_{\rm l}^2\\
&+m_{\rm l}^4-2p_{23}p_{34})+4p_{34}^2(m_{\rm l}^2+p_{23})+m_\pi^4p_{23}))+8D_0^2(f_0-f_1)^2m_{\rm D}^4(-2m_\pi^2((p_{23}+p_{24}-p_{34})m_{\rm l}^2\\
&+m_{\rm l}^4-2p_{23}p_{34})+4p_{34}^2(m_{\rm l}^2+p_{23})+m_\pi^4p_{23})+16D_0(f_0-f_1)f_1m_{\rm D}^2(-2m_\pi^2((p_{12}+p_{13}+p_{23}\\
&+p_{24}-p_{34})m_{\rm l}^2+m_{\rm l}^4-p_{14}p_{23}+p_{13}p_{24}-p_{12}p_{34}-2p_{23}p_{34})+4p_{34}((p_{14}+p_{34})m_{\rm l}^2+p_{14}p_{23}\\
&-p_{13}p_{24}+p_{12}p_{34}+p_{23}p_{34})+m_\pi^4p_{23})+8f_1^2(-2m_\pi^2((2p_{12}+2p_{13}+p_{23}+p_{24}-p_{34})m_{\rm l}^2+m_{\rm l}^4\\
&+4p_{12}p_{13}-2p_{14}p_{23}+2p_{13}p_{24}-2p_{12}p_{34}-2p_{23}p_{34})+4p_{34}((2p_{14}+p_{34})m_{\rm l}^2+4p_{12}p_{14}+2p_{14}p_{23}\\
&-2p_{13}p_{24}+2p_{12}p_{34}+p_{23}p_{34})+m_\pi^4p_{23})\Big\} \qquad\qquad (A1)
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_2 =\ & f_\pi^2|PN_3|^2G_{\rm F}^4|V_{\rm cs}|^2|V_{\rm ud}|^2\Big\{8D_0^2(f_0-f_1)^2m_{\rm K}^4(-2m_\pi^2((p_{23}-p_{24}+p_{34})m_{\rm l}^2+m_{\rm l}^4-2p_{23}p_{24})\\
&+4p_{24}^2(m_{\rm l}^2+p_{23})+m_\pi^4p_{23})+m_{\rm K}^2(16f_1(-D_0(f_0-f_1)(-2m_\pi^2((p_{12}+p_{13}+p_{23}-p_{24}+p_{34})m_{\rm l}^2\\
&+m_{\rm l}^4-p_{14}p_{23}-p_{13}p_{24}-2p_{23}p_{24}+p_{12}p_{34})+4p_{24}((p_{14}+p_{24})m_{\rm l}^2+p_{14}p_{23}+p_{13}p_{24}+p_{23}p_{24}\\
&-p_{12}p_{34})+m_\pi^4p_{23})+2f_1m_\pi^2p_{23}-4f_1p_{24}p_{34})-16D_0^2(f_0-f_1)^2m_{\rm D}^2(-2m_\pi^2((p_{23}-p_{24}+p_{34})m_{\rm l}^2\\
&+m_{\rm l}^4-2p_{23}p_{24})+4p_{24}^2(m_{\rm l}^2+p_{23})+m_\pi^4p_{23}))+8D_0^2(f_0-f_1)^2m_{\rm D}^4(-2m_\pi^2((p_{23}-p_{24}+p_{34})m_{\rm l}^2\\
&+m_{\rm l}^4-2p_{23}p_{24})+4p_{24}^2(m_{\rm l}^2+p_{23})+m_\pi^4p_{23})+16D_0(f_0-f_1)f_1m_{\rm D}^2(-2m_\pi^2((p_{12}+p_{13}+p_{23}\\
&-p_{24}+p_{34})m_{\rm l}^2+m_{\rm l}^4-p_{14}p_{23}-p_{13}p_{24}-2p_{23}p_{24}+p_{12}p_{34})+4p_{24}((p_{14}+p_{24})m_{\rm l}^2+p_{14}p_{23}\\
&+p_{13}p_{24}+p_{23}p_{24}-p_{12}p_{34})+m_\pi^4p_{23})+8f_1^2(-2m_\pi^2((2p_{12}+2p_{13}+p_{23}-p_{24}+p_{34})m_{\rm l}^2+m_{\rm l}^4\\
&-2(p_{14}p_{23}+(p_{13}+p_{23})p_{24})+2p_{12}(2p_{13}+p_{34}))+4p_{24}((2p_{14}+p_{24})m_{\rm l}^2+2p_{14}p_{23}+p_{23}p_{24}\\
&+2p_{13}(2p_{14}+p_{24})-2p_{12}p_{34})+m_\pi^4p_{23})\Big\} \qquad\qquad (A2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{C}_3 =\ & f_\pi^2PN_2^*PN_3G_{\rm F}^4|V_{\rm cs}|^2|V_{\rm ud}|^2\Big\{8D_0^2(f_0-f_1)^2(p_{23}m_\pi^4+2(m_{\rm l}^4+p_{23}m_{\rm l}^2+p_{23}(p_{24}+p_{34}))m_\pi^2\\
&-2(m_{\rm l}^2(p_{24}^2+p_{34}^2)-2p_{23}p_{24}p_{34}))m_{\rm D}^4+16D_0(f_0-f_1)f_1(p_{23}m_\pi^4+2(m_{\rm l}^4+(p_{12}+p_{13}+p_{23})m_{\rm l}^2\\
&+p_{23}(p_{14}+p_{24}+p_{34}))m_\pi^2-2((p_{24}^2+p_{34}^2+p_{14}(p_{24}+p_{34}))m_{\rm l}^2+p_{13}p_{24}^2+p_{12}p_{34}^2-p_{12}p_{24}p_{34}\\
&-p_{13}p_{24}p_{34}-2p_{23}p_{24}p_{34}-p_{14}p_{23}(p_{24}+p_{34})))m_{\rm D}^2+\zeta(-32{\rm i}(m_\pi^2+2p_{14}+p_{24}+p_{34})f_1^2\\
&-32iD_0(f_0-f_1)m_{\rm D}^2(m_\pi^2+p_{24}+p_{34})f_1+32iD_0(f_0-f_1)m_{\rm K}^2(m_\pi^2+p_{24}+p_{34})f_1)\\
&+8D_0^2(f_0-f_1)^2m_{\rm K}^4(p_{23}m_\pi^4+2(m_{\rm l}^4+p_{23}m_{\rm l}^2+p_{23}(p_{24}+p_{34}))m_\pi^2-2(m_{\rm l}^2(p_{24}^2+p_{34}^2)-2p_{23}p_{24}p_{34}))\\
&+8f_1^2(p_{23}m_\pi^4+2(m_{\rm l}^4+(2p_{12}+2p_{13}+p_{23})m_{\rm l}^2+4p_{12}p_{13}+p_{23}(2p_{14}+p_{24}+p_{34}))m_\pi^2\\
&-2((p_{24}^2+p_{34}^2)m_{\rm l}^2-4p_{14}^2p_{23}+2p_{14}((p_{24}+p_{34})m_{\rm l}^2+2p_{13}p_{24}-p_{23}p_{24}+2p_{12}p_{34}-p_{23}p_{34})\\
&-2(p_{13}(p_{34}-p_{24})p_{24}+p_{34}(p_{12}p_{24}+p_{23}p_{24}-p_{12}p_{34}))))+m_{\rm K}^2(-16D_0^2(f_0-f_1)^2(p_{23}m_\pi^4\\
&+2(m_{\rm l}^4+p_{23}m_{\rm l}^2+p_{23}(p_{24}+p_{34}))m_\pi^2-2(m_{\rm l}^2(p_{24}^2+p_{34}^2)-2p_{23}p_{24}p_{34}))m_{\rm D}^2\\
&-16D_0f_1(f_0(p_{23}m_\pi^4+2(m_{\rm l}^4+(p_{12}+p_{13}+p_{23})m_{\rm l}^2+p_{23}(p_{14}+p_{24}+p_{34}))m_\pi^2\\
&-2((p_{24}^2+p_{34}^2+p_{14}(p_{24}+p_{34}))m_{\rm l}^2+p_{13}p_{24}^2+p_{12}p_{34}^2-p_{12}p_{24}p_{34}-p_{13}p_{24}p_{34}-2p_{23}p_{24}p_{34}\\
&-p_{14}p_{23}(p_{24}+p_{34})))+f_1(p_{23}m_\pi^4-2(m_{\rm l}^4+(p_{12}+p_{13}-p_{23})m_{\rm l}^2-2p_{23}^2+p_{14}p_{23}-p_{23}p_{24}\\
&-p_{23}p_{34}+2p_{24}p_{34})m_\pi^2+2((p_{24}^2-4p_{34}p_{24}+p_{34}^2+p_{14}(p_{24}+p_{34}))m_{\rm l}^2+p_{13}p_{24}^2+p_{12}p_{34}^2-4p_{24}p_{34}^2\\
&-4p_{24}^2p_{34}-p_{12}p_{24}p_{34}-p_{13}p_{24}p_{34}-6p_{23}p_{24}p_{34}-p_{14}p_{23}(p_{24}+p_{34})))))\Big\}+{\rm h.c.} \qquad\qquad (A3)
\end{aligned}$$

where $p_{ij}=p_i\cdot p_j$, $\zeta=\varepsilon^{\mu\nu\rho\sigma}p_{1\mu}p_{2\nu}p_{3\rho}p_{4\sigma}$ (we adopt the convention $\varepsilon^{0123}=1$ for the Levi-Civita tensor), and D_0 are defined as:

$$D_0 = \frac{1}{m_\pi^2+2(m_{\mathrm{K}}^2+p_{23}+p_{24}+p_{34})} \tag{A4}$$

and the amplitude squared is

$$|\mathcal{M}|^2=\mathcal{C}_1+\mathcal{C}_2+\mathcal{C}_3. \tag{A5}$$

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