Beam breakup simulation study for a high energy ERL *

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Abstract: The maximum beam current that can be accelerated in an energy recovery linac (ERL) can be severely limited by the transverse multi-pass beam breakup instability (BBU), especially in future ERL light sources with multi-GeV high energy beam energy and more than 100 mA average current. In this paper, the multi-pass BBU of such a high energy ERL is studied based on the simulation of a 3-GeV ERL light source that is proposed by KEK. This work is expected to provide a reference for future high energy ERL projects.

Key words: energy recovery linac, beam breakup, higher order mode

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1 Introduction

Energy recovery linacs (ERLs) are expected to provide high current electron beams with an RF power supply that is much lower than traditional linacs. At the same time, the excellent beam qualities of linear accelerator, such as low emittance, small energy spread and short bunch length, are able to be maintained compared with those of storage rings. These characteristics make ERLs very suitable for future ultra-brightening synchrotron light sources, free electron lasers, nuclear physics research, and so on.

Multi-pass transverse beam breakup is known to be one of the key issues of ERLs. It is primarily caused by a positive feedback between the recirculated bunch with transverse offset and insufficiently damped dipole higher order modes in superconducting cavity. The average current of an ERL can be severely limited by this effect. Studies on the multi-pass beam breakup instability (BBU) of small-scale ERLs with several tens MeV and average current around 10 mA have been done before [1, 2]. For ERL based synchrotron light sources with the energy of a few GeV, hundreds of cavities will be used and the electron bunches have much more complicated beam dynamics than in small-scale ERLs with just few cavities. Furthermore, in order to get higher brightness synchrotron radiations, the required average current of ERL light source, typically over 100 mA, is much higher than small-scale ERLs. In this case, multi-pass BBU is a significant issue and should be analyzed carefully.

Several high energy ERL light sources are proposed [3]. One of these sources is a synchrotron X-ray light source based on a 3 GeV ERL at KEK, which is expect to be a successor of the existing synchrotron light sources of Photon Factory in KEK. A preliminary design report of this project was published in 2012 [4, 5]. Recently, we performed a study of multi-pass beam breakup for this facility. In this paper, the BBU simulation results of the KEK 3 GeV ERL are presented. Some features of the BBU of high energy ERLs are then discussed, based on the simulation results.

2 Multi-pass beam breakup

In ERLs, the dipole HOM in a cavity imposes a transverse kick to an electron bunch on the first pass and gives it a transverse momentum. On the second or higher passes, this electron bunch comes back to the same cavity with a transverse offset. The electron bunch with a transverse offset can constructively or destructively interact with the HOM, which deflected it on the previous pass. Therefore, there exists a feedback between the HOM field and the recirculating bunch. The enhanced feedback by a series of bunches can cause an exponential increase of the HOM power if the HOM is not sufficiently damped. The HOM transverse kick will become strong enough so

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that the beam strikes the cavity wall and becomes lost. This process is called multi-pass beam breakup.

A two-dimensional analytical formula for the multi-pass BBU threshold current is [6]

$$I_{\mathrm{th}}=-\frac{2pc}{e\left(\frac{\omega}{c}\right)\left(\frac{R_{\mathrm{d}}}{Q}\right)Q_{\mathrm{ext}}M_{12}^{*}\sin(\omega T_{\mathrm{r}})}, \qquad (1)$$

where (R_{d}/Q) is the shunt impedance of the dipole mode in the cavity, Q_{ext} is the external quality factor, ω is the HOM frequency, T_{r} is the bunch recirculating time, and

$$M_{12}^{*}=T_{12}\cos^2\theta+\frac{1}{2}(T_{14}+T_{23})\sin 2\theta+T_{34}\sin^2\theta,$$

where T_{ij} are the elements of the pass-to-pass transport matrix and θ is the polarization angle of the dipole HOM.

Eq. (1) shows the main determinants of multi-pass BBU instability in an ERL. But it is only valid in the case of a single cavity, single HOM and $M_{12}^{*}\sin(\omega T_{\mathrm{r}})<0$. In real cases, the situation is more complicated. It is necessary to use simulation codes to compute the BBU threshold current. In this paper, the particle tracking code bi developed by I. Bazarov [7] at Cornell University is used in the simulation.

3 BBU simulation

3.1 KEK 3-GeV ERL light source

Several linac configurations have been proposed. In this paper, we are referring two of them. One of the linac configurations consists of 28 cryomodules with 8 cavities in each cryomodule. The cavity accelerating gradient is about 13.4 MV/m and the final energy after acceleration is about 3.01 GeV [4]. The betatron function and dispersion of the this linac configuration are shown in Fig. 1. Another configuration consists of 34 cryomodules with 8 cavities in each cryomodule. The cavity accelerating gradient is about 12.5 MV/m and the final energy after acceleration is about 3.41 GeV for this configuration [8].

To improve the dipole HOM damping, a 9-cell KEK-ERL mode-2 cavity (shown in Fig. (2)), which has a large

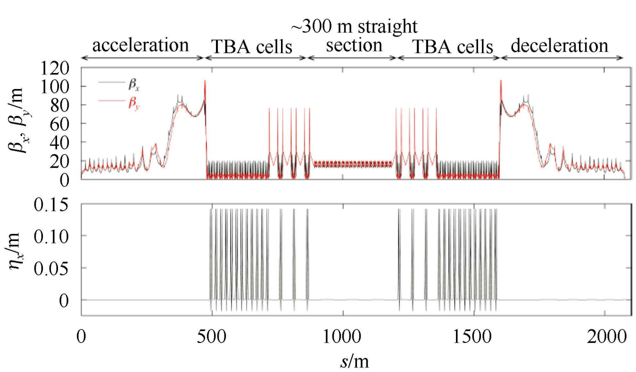

Fig. 1. Betatron function (upper) and dispersion (lower) of 3-GeV ERL light source.

iris with a diameter of 80 mm and two large beam pipes with diameters of 100 and 123 mm has been developed [9]. A previous work shows that a BBU threshold current of more than 600 mA can be achieved when applying this type of cavity to a 5-GeV ERL configuration [10]. Several major HOMs in the mode-2 cavity are listed in Table 1.

Table 1. Major HOMs in KEK-ERL 9-cell cavity.

f/GHz	Q_{e}	R/Q/ (Ω/cm^2)	$(R/Q)Q_{\mathrm{e}}/f$/ (Ω/cm^2/GHz)
1.835	1.1010×10^3	8.087	4852
1.856	1.6980×10^3	7.312	6691
2.428	1.6890×10^3	6.801	4732
3.002	2.9990×10^4	0.325	3246
4.011	1.1410×10^4	3.210	9135
4.330	6.0680×10^5	0.018	2522

3.2 Influence of betatron phase advance on the BBU threshold current

As can be seen in Eq. (1), the BBU threshold current is a function of M_{12}^{*}. For simplicity, we assume that there is no x-y coupling in the recirculating loop. Each dipole HOM has two different directions of polarizations (x ($\theta=0°$) and y ($\theta=90°$)) and the two polarized HOMs have the same value of frequency, R/Q and Q_{ext}. In this case, the value of M_{12}^{*} is only a function of T_{12} or T_{34} for the two independent polarizations, respectively. The value of $T_{12}(T_{34})$ for the transport from the region with momentum p_i to the region with momentum p_{f} can be written in terms of β-function and phase advance $\Delta\psi$ as

$$T_{12}(T_{34})(\mathrm{i}\rightarrow\mathrm{f})=\sqrt{\frac{\beta_{\mathrm{i}}\beta_{\mathrm{f}}}{p_{\mathrm{i}}p_{\mathrm{f}}}}\sin\Delta\psi. \qquad (2)$$

In order to simulate the transverse dynamics correctly, it is important to include the focusing effect of the RF field in the superconducting cavity. In this work, Rosenzweig's form of the transport matrix for a pure π-mode standing-wave cavity [11] is applied in the simulation; that is,

$$M_{\mathrm{cav}}=\begin{pmatrix}\cos\alpha-\sqrt{2}\sin\alpha & \sqrt{8}\frac{\gamma_{\mathrm{i}}}{\gamma'}\sin\alpha\\ -\frac{3}{\sqrt{8}}\frac{\gamma'}{\gamma_{\mathrm{f}}}\sin\alpha & \frac{\gamma_{\mathrm{i}}}{\gamma_{\mathrm{f}}}\left[\cos\alpha+\sqrt{2}\sin\alpha\right]\end{pmatrix}, \qquad (3)$$

where $\alpha=\frac{1}{\sqrt{8}}\ln\frac{\gamma_{\mathrm{f}}}{\gamma_{\mathrm{i}}}$, $\gamma_{\mathrm{i(f)}}$ is the initial (final) relativistic factor of the particle, $\gamma'=qE_0\cos(\Delta\phi)/m_0c^2$, where E_0 is the maximum particle energy gain from the RF cavity and $\Delta\phi$ is the phase of acceleration.

For a higher BBU threshold current, one has to make the pass-to-pass value of T_{12} (T_{34}) as small as possible. As shown in Eq. (2), by adjusting the betatron phase advances to make its value an integer of multiple of π throughout the whole recirculating loop, $T_{12}(T_{34}) \sim 0$ can be achieved. Consequently, an extremely large BBU threshold current, up to infinite, is obtained in the single cavity case. In real ERL configurations with more than one cavity, the ideal condition of $\Delta\psi = 0$ cannot be satisfied for every cavity in the linac. Yet, we can still scan the betatron phase advance of the return loop to search for the optimized value of BBU threshold current.

Fig. 2. 1.3 GHz 9-cell KEK-ERL mode-2 cavity.

Figure 3 shows the BBU threshold current as a function of the betatron phase advance for both the 3.01 GeV and the 3.41 GeV linac configurations. The HOM parameters shown in Table 1 are used in the calculation. The maximum BBU threshold current is found to be about 342 mA for the 3.01 GeV configuration and 300 mA for the 3.41 GeV configuration. The minimum BBU threshold current is 270 mA for the 3.01 GeV configuration and 220 mA for the 3.41 GeV configuration. The BBU threshold currents of both configurations meet the requirement for a 100 mA average current.

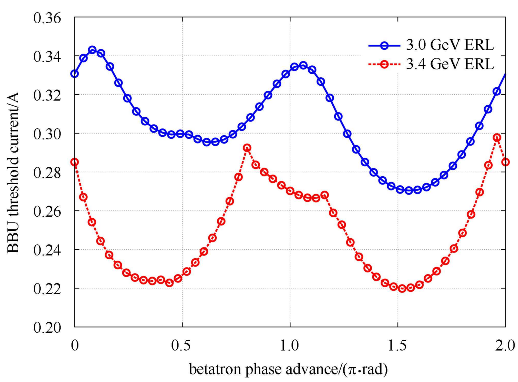

Fig. 3. (color online) BBU threshold current of two existing design of lattice. Blue: 3.0 GeV ERL configuration. Red: 3.4 GeV ERL configuration.

3.3 HOM randomization

The previous simulation is based on the assumption that all cavities have the same HOM parameters. However, according to the simulation and experimental measurement [12], the randomization of both HOM frequency and external quality factor ($Q_{\rm ext}$) due to cavity shape error are naturally introduced in the fabrication and tuning process of superconducting cavities. The frequency randomization reduces the coherent excitation of dipole modes in the cavity string and consequently improves the BBU threshold current. In order to simulate the influence of HOM frequency randomization, we assume the frequency randomization of the same type of HOM in different cavities in the linac to be a Gaussian distribution with desired RMS frequency spread width $\sigma_{\rm f}$. We generate 1000 different sets of the HOM data with $\sigma_{\rm f} = 1$ MHz in the linac cavities of the 3.0 GeV ERL scheme, and calculate the BBU threshold current of each set of HOM data. The statistical histogram of BBU threshold current distribution of this simulation is shown in Fig. 4.

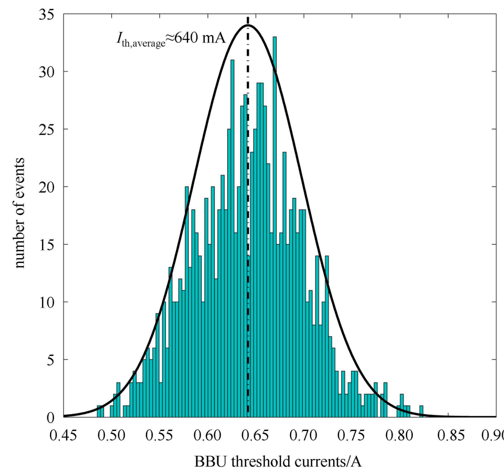

Fig. 4. Average BBU threshold current at different value of frequency spread.

Due to the limited cavity number, the BBU threshold current with HOM frequency randomization shows an obvious statistical fluctuation. Therefore, we usually calculate the BBU threshold current for the same value of $\sigma_{\rm f}$ many times and employ the mean value of the BBU threshold current and its standard deviation to represent the BBU feature of such a condition. We calculate the BBU threshold current 50 times with different HOM random seeds at each value of RMS frequency spreads. Fig. 5 shows that the average BBU threshold currents can be significantly improved along with the RMS frequency spread $\sigma_{\rm f}$ increases, reaching about 940 mA when $\sigma_{\rm f} = 2$ MHz.

Similar to the HOM frequency spread, the external quality factor of different cavities also shows a statistical distribution. As shown in Eq. (1), the value of $Q_{\rm ext}$ plays an essential role in the BBU instability. Therefore, the randomization of $Q_{\rm ext}$ may impose a remarkable influence on the BBU threshold current. To investigate the influence, we assume the distribution of $Q_{\rm ext}$ to be a uniform distribution from 0.1 to 10 times the nominal value listed in Table. 1. At the same time, a Gaussian frequency distribution of $\sigma_{\rm f} = 2$ MHz is also applied to the HOMs in order to make the simulation close to the real situation. The BBU simulation is performed 100 times. The statistical distribution of the BBU threshold

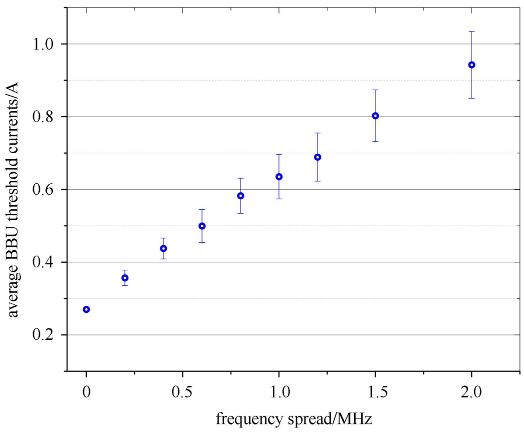

Fig. 5. Average BBU threshold current at different value of frequency spread.

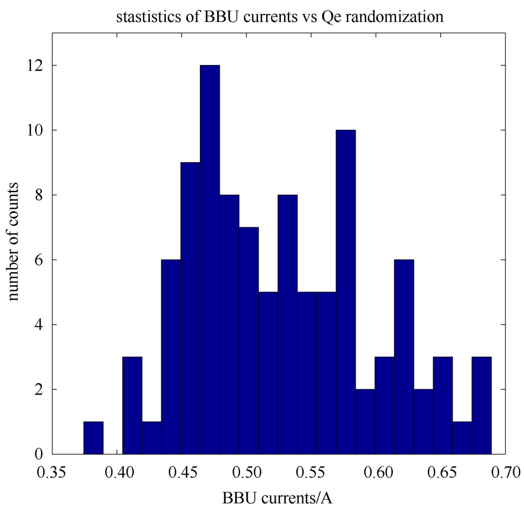

Fig. 6. Statistical distribution of the BBU threshold current with $Q_{\rm ext}$ randomization.

currents for the 3.01 GeV configuration is shown in Fig. 6. The result shows a broad distribution of the BBU threshold current due to the $Q_{\rm ext}$ randomization.

3.4 Return loop length

BBU threshold current is also a function of the recirculating loop length. The variation of $T_{\rm r}$ (in Eq. (1)) affects the HOM phase that the electron bunch experiences in the second pass through the linac. Fig. 7 shows the BBU threshold current versus the recirculating loop length variation, where $\Delta T/T_0$ represents the return loop length variation in terms of the relative recirculating time variation. In the case of $\sigma_{\rm f}=0$, the BBU threshold current shows a quasi-periodic oscillation, which is determined by the most threatening HOM in the KEK-ERL mode-2 cavity, as shown in Table. 1; that is, the HOM with the frequency $f=4.011$ GHz. In the case of $\sigma_{\rm f}=1$ MHz this oscillation is smeared because the coherent excitation of this HOM is disturbed by the frequency randomization.

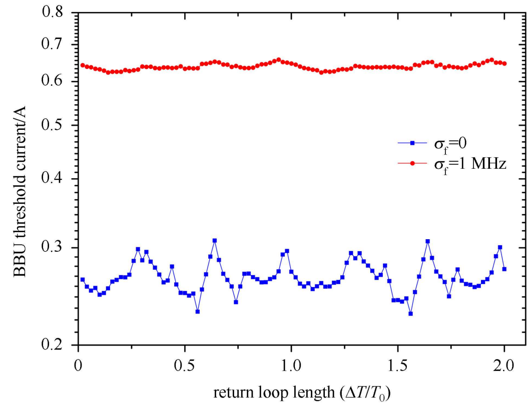

Fig. 7. (color online) BBU threshold current v.s. recirculating loop length. Blue: HOM frequency spread $\sigma_{\rm f}=0$. Red: HOM frequency spread $\sigma_{\rm f}=1$ MHz.

4 Discussion

As discussed in Section 2, the multi-pass BBU mainly evolves from the process where a particle experiences a transverse kick of the dipole HOMs when it passes through the superconducting cavity. The kick angle can be evaluated as

$$x'(y')=\frac{V_{\perp}}{V_{\rm p}}, \tag{4}$$

where $V_{\perp}$ is the transverse voltage of dipole HOM, which is determined by the value of $(R/Q)Q_{\rm ext}$, and $V_{\rm p}=pc/e$, where p is the beam momentum in the cavity. Thus, the HOM damping of superconducting cavity is of fundamental importance in high-energy and high-current ERLs. A previous study gives an empirical criterion of the HOM properties to achieve 100 mA operation in an ERL [13]

$$(R/Q)Q_{\rm ext}/f<1.4\times10^5\ (\Omega/{\rm cm}^2/{\rm GHz}),$$

as listed in Table. 1, all HOMs in the KEK-ERL mode-2 cavity satisfy this criterion so that a sufficiently high BBU threshold current can be expected by applying this type of cavity. The simulation results shown above can explicitly prove this empirical judgement. In order to suppress HOMs to meet this criterion, various types of superconducting cavity have been developed internationally, such as the 7-cell cavity developed for the ERL based X-ray light source at Cornell University [14], the 5-cell cavity developed for the ERL based e-cooling project at BNL [15], etc..

It can also be inferred from Eq. (1) and Eq. (4) that the cavities at low energy sections (i.e., the cavities at the start and the end of the linac) contribute more to the BBU. We calculated the BBU threshold current of each single cryomodule in the linac of the 3.41 GeV configuration. The results are shown in Fig. 8. From the figure we can see the BBU threshold currents of the first and last cryomodules are much smaller than the cryomodules in the middle of the linac. As shown in Fig. 3, one can increase the BBU threshold current by adjusting the betatron phase advance of the return loop. In fact, the higher BBU threshold current occurs when the betatron phase advance makes the T_{12} (T_{34}) value of the low energy cavities smaller. To mitigate the instability, it is also advisable to make sure that the low energy cavities have smaller Q_{ext} so that its contribution to the BBU can be smaller.

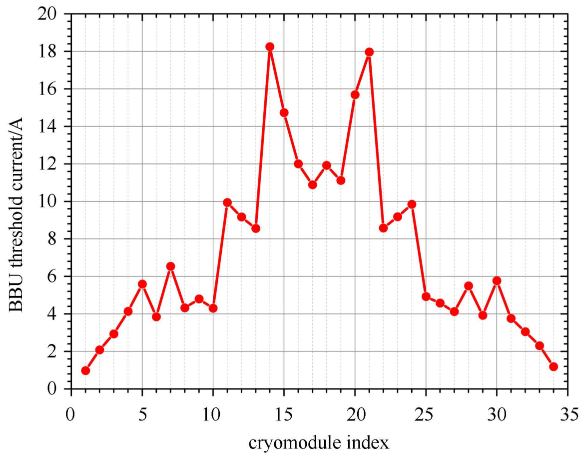

Fig. 8. BBU threshold current of each single cryomodule in the linac.

From Eq. (4), we can also infer that an obvious approach to increase the BBU threshold current is to increase the accelerating gradient of the cavity. Fig. 9 shows the BBU simulation of five ERL layouts with the same linac configuration but different accelerating gradients. A distinct increase of the BBU threshold current can be observed in the figure as the accelerating gradient increases. One can also expect a linear dependency of the BBU threshold current on the gradient of the cavity.

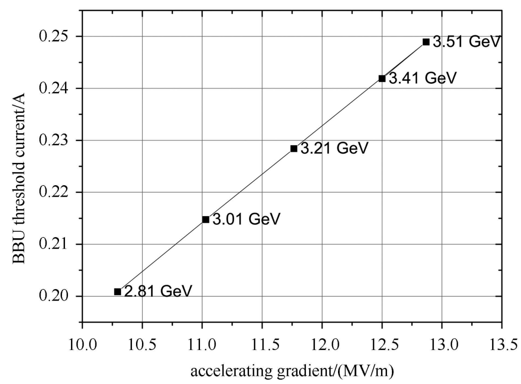

Fig. 9. BBU threshold current v.s. accelerating gradient.

5 Summary

The transverse multi-pass BBU instability for a high energy ERL has been investigated in this paper. In particular, we analyzed the BBU of the KEK 3 GeV ERL light source by numerical simulation. It can be inferred from the results that the designed average current of 100 mA or more is a promising goal using the 9-cell KEK-ERL mode-2 cavity and the existing designs of linac optics. The BBU threshold current with the randomization of both HOM frequency and external quality factor are also investigated based on the simulation results. This shows that the BBU threshold current can be significantly influenced by the HOM randomization. The BBU threshold current dependence on beam energy and cavity accelerating gradient was then discussed. The results indicate that, by improving the cavity accelerating gradient, the BBU threshold current can be improved distinctly.

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