# Description of the evolution of mixed-symmetry states in the $N$=78 isotonic chain with IBM2 *

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**Abstract:** The characteristics of the lowest mixed-symmetry states $2^+_{\rm ms}$ and $1^+_{\rm ms}$ for $^{132}$Xe, $^{134}$Ba and $^{136}$Ce in the even-even $N=78$ isotones are investigated within the framework of the IBM2 model. The lowest mixed-symmetry state $2^+_{\rm ms}$ levels for both a single isolated state in $^{132}$Xe and $^{136}$Ce and a fragmented state in $^{134}$Ba are reproduced by the predictions. The agreement between the IBM2 calculation and the experimental values is good for the $B$(E2) and $B$(M1) transition probabilities both quantitatively and qualitatively. The predicted summed $B$(M1) strength follows the experimental data, remaining nearly constant as a function of proton number along the chain of the $N$=78 isotones.

**Key words:** $N$=78 isotone, mixed-symmetry states, IBM2

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## 1 Introduction

Nuclear shape phase transitions are a current topic in nuclear structure physics [1–7]. Theoretical studies have typically been based on phenomenological geometric models of nuclear shapes, and algebraic models of nuclear structure [7]. In the algebraic models, the interacting boson model (IBM) [8] which provides direct correspondence between the nuclear shape and the dynamic symmetries, has been widely used to study the shape phase transitions. Moreover, the proton-neutron interacting boson model (IBM2) [8], which takes into account separately the proton and neutron degrees of freedom, has been used to describe the phase structure of two-fluid bosonic systems. This model leads to a new type of excitation called a mixed-symmetry state (MSS), a special class of collective states in which protons and neutrons are not fully in phase, interpreted geometrically as an excitation where protons and neutrons are moving out of phase with respect to each other. In order to establish the properties of such states, the $F$-spin quantum number was introduced [9]. Similar to the fermionic isospin, projections of the $F$-spin are $F_z=+1/2$ for proton and $F_z=-1/2$ for neutron bosons. For a system of $N_\pi$ proton bosons and $N_\nu$ neutron bosons, the maximum $F$-spin is $F_{\max}=(N_\pi+N_\nu)/2$ and $F_{\min}=(N_\pi-N_\nu)/2$. States with maximal $F$-spin are called full-symmetric states (FSSs) contrary to states with a $F$-spin $F_{\min}\leqslant F<F_{\max}$, which are called MSSs.

The most prominent mixed-symmetry state is the $1^+$ scissors mode, which was discovered by Bohle et al. in the 1980s [10]. Iachello et al. predicted an energetically lower $2^+_{1,\rm ms}$ state, as the fundamental one-phonon mixed-symmetry state in spherical vibrational nuclei [8, 11]. In recent years, a series of experimental and theoretical systematic studies have been devoted to nuclei around the doubly magic $^{132}$Sn$_{82}$ through its dynamic symmetries and decay properties [12, 13]. A large number of the mixed-symmetry states $2^+_{\rm ms}$ have been identified in the $A\sim130$ mass region. For the even-even $N=78$ isotones, however, the lowest MSS $2^+_{\rm ms}$ have been identified in $^{136}$Ce [14], $^{134}$Ba [15,16] and $^{132}$Xe [17]. Meanwhile, $1^+_{\rm ms}$ states have also been found in $^{134}$Ba [18] and $^{132}$Xe [19]. On the other hand, the first experimental evidence of the E(5) symmetry [20] has been in $^{134}$Ba [21] and possibly in $^{132}$Xe [22] and $^{136}$Ce [23]. The E(5) symmetry applies to nuclei at the critical point of phase transition from spherical vibrators to deformed $\gamma$-soft nuclei. It is well known that the nuclear structure variation is dependent on not only the neutron number $N$, but also the proton number $Z$. Therefore, the lowest MSS $2^+_{\rm ms}$ in $^{136}$Ce, $^{134}$Ba and $^{132}$Xe have provided a possibility to study the evolution of the MSS structure and the influence of the E(5) shape or shape transition on low-lying

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M1 strength distributions along the $N = 78$ isotonic chain. However, there is no systematic production by the IBM2 model for the mixing-symmetry states along the isotonic chain in this mass region [24]. In this paper, we mainly study the characteristics of the MSSs $2^+_{\rm ms}$, $1^+_{\rm ms}$ along the even-even $N=78$ isotones ranging from $^{132}$Xe to $^{136}$Ce within the framework of the IBM2, with special attention paid to the electromagnetic transition strength and the structure of the lowest five excitation $2^+$ states, among which the MSSs are observed.

In the next section the Hamiltonian, E2 and M1 operators used in this work are briefly described. In Section 3, the criteria adopted for the determination of the model parameters, and the comparison of the numerical results with the experimental data are presented. Finally, a summary and some remarks are given in Section 4.

## 2 Model and method

In the IBM2 model, the physical dominant interaction contained in the Hamiltonian [8] is

$$\hat{H}=\varepsilon_{\rm d}(\hat{n}_{{\rm d}\pi}+\hat{n}_{{\rm d}\nu})+\kappa\hat{Q}_\pi\cdot\hat{Q}_\nu+\omega_{\pi\pi}\hat{L_\pi}\cdot\hat{L_\pi}+\hat{M}_{\pi\nu}, \qquad (1)$$

where $\hat{n}_{{\rm d}\rho}=d^\dagger_\rho\cdot\tilde{d}_\rho$ and $\hat{Q}_\rho=(s^\dagger_\rho\tilde{d}_\rho+d^\dagger_\rho s_\rho)^{(2)}+\chi_\rho(d^\dagger_\rho\tilde{d}_\rho)^{(2)}$ represent the d-boson number operator and quadrupole operator for the proton ($\rho=\pi$) and neutron ($\rho=\nu$), respectively. The parameter $\chi_\rho$, which appears in the quadrupole operator, determines the type of the deformation. The third term on the right-hand side of Eq. (1), stands for the dipole proton-proton interaction, and $L_\pi$ is the angular momentum operator with $\hat{L_\pi}=\sqrt{10}[d^\dagger_\pi\tilde{d_\pi}]^{(1)}$. The fourth term, which represents the Majorana interaction with its strength, is given by $\hat{M}_{\pi\nu}=\lambda_2(s^\dagger_\pi d^\dagger_\nu-s^\dagger_\nu d^\dagger_\pi)^{(2)}\cdot(s_\pi\tilde{d}_\nu-s_\nu\tilde{d}_\pi)^{(2)}+\Sigma_{k=1,3}\lambda_k(d^\dagger_\pi d^\dagger_\nu)^{(k)}\cdot(\tilde{d}_\pi\tilde{d}_\nu)^{(k)}$.

$\varepsilon_{\rm d}$ is the energy of the d-boson, $\kappa$ the strength of the quadrupole-quadrupole interaction between neutron-bosons and proton-bosons, $\omega_{\pi\pi}$ the strength of the dipole proton-proton interaction, and $\lambda_k$ ($k$=1, 2, 3) the strength of the Majorana interaction. The properties of mixed-symmetry states can test parts of the IBM2 Hamiltonian, to which the fully-symmetry states are insensitive to, e.g., the Majorana operator.

In the IBM2, the E2 and M1 operators are given by

$$\hat{T}^{(\rm E2)}=e_\pi\hat{Q}_\pi+e_\nu\hat{Q}_\nu, \qquad (2)$$

$$\hat{T}^{(\rm M1)}=\sqrt{3/4\pi}(g_\nu\hat{L_\nu}+g_\pi\hat{L_\pi}), \qquad (3)$$

where the term $Q_\rho$ is the same as in Eq. (1), and for consistency we choose the same value for $\chi_\rho$ as in the Hamiltonian. The $e_\rho$ and $g_\rho$ are the effective quadrupole charges and the gyromagnetic ratios, respectively.

## 3 Results and discussion

The valence nucleons (holes) of the $^{136}$Ce, $^{134}$Ba and $^{132}$Xe isotones are in the 50–82 proton major shell. For the $N=78$ isotones with $A$ = 136 to 132, there are two hole-like neutron bosons, and four to two particle-like proton bosons. In principle, the quantities $\varepsilon_{\rm d}$, $\kappa$, $\chi_\pi$, $\chi_\nu$, and $\omega_{\pi\pi}$, which determine the structure of the energy spectrum, depend on both proton and neutron boson numbers. In order to reduce the number of free parameters and increase the coincidence with the microscopic calculations, we maintain $\kappa=-0.11$ MeV for all isotones. Meanwhile, $\chi_\nu=0.800$ is kept constant, and $\chi_\pi$ changes with the proton boson number. Due to the small number of valence neutrons, the effect of the dipole interacting neutron-neutron bosons for a set of $N=78$ isotones is ignored, while the dipole interacting proton-proton boson strength $\omega_{\pi\pi}$ also changes with the proton boson number. The parameter $\varepsilon_{\rm d}$ was determined so as to reproduce as closely as possible the excitation energies of the ground state bands in each isotone, and the $\omega_{\pi\pi}$ influences the relative position of the energy levels for the $2^+_1$, $2^+_2$ and other states in the γ-band. As for the Majorana parameters, since they not only influence all fully symmetric states but also differently influence states of mixed symmetry, the parameters $\lambda_1,\lambda_2$ and $\lambda_3$ were adjusted by fitting to the experimentally known mixed-symmetry $2^+_{\rm ms}$ and $1^+_{\rm ms}$ states. In this work, we are interested mostly in the lowest five excited $2^+$ levels, which are located below 2.8 MeV in all studied nuclei. By a least-squares fit to the energies of the experimental data, the best fit IBM2 parameters were calculated and are listed in Table 1. The calculated results for the relevant levels are compared with the experimental data in Table 2.

Table 1. Parameters used to calculate energy spectra of $N=78$ isotones. All parameters are given in MeV except $\chi_\pi$($\kappa=-0.110$ MeV, $\chi_\nu$=0.800).

| | $\epsilon$ | $\chi_\pi$ | $\omega_{\pi\pi}$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ |
|---|---|---|---|---|---|---|
| $^{132}$Xe | 0.850 | −0.800 | −0.006 | 0.500 | 0.450 | −0.340 |
| $^{134}$Ba | 0.770 | −0.900 | 0.008 | 0.280 | 0.410 | −0.420 |
| $^{136}$Ce | 0.740 | −1.000 | 0.013 | 0.200 | 0.470 | −0.400 |

Using Eq. (2), we calculated the $B$(E2) transition probability. In general, the E2 transition results are not sensitive to whether $e_\pi=e_\nu$ or not. Only transitions between the symmetry and mixed symmetry states are sensitive to such a choice, and the $B$(E2) transitions between states with different $F$-spin states and different d-boson are as a rule proportional to $(e_\pi-e_\nu)^2$. In this calculation, we make the effective charge $e_\pi=2e_\nu$. By fitting to the experimental value of $B({\rm E2},\ 2^+_1\rightarrow0^+_1)$ in each isotone, we obtain $e_\pi=2e_\nu$=0.172, 0.180 and 0.160 eb for $^{132}$Xe, $^{134}$Ba and $^{136}$Ce, respectively, which are similar to the charges being used for its neighboring $N=80$ isotones in Refs. [25, 26]. For the magnetic transitions, we set the g factors to the same as in Refs. [27, 28] $g_\nu=-0.10$ and $g_\pi=1.25$. All the calculations were performed for each nucleus using the code NPBOS [29]. The theoretical predictions of the relevant transition strengths $B$(E2) and $B$(M1) for $^{132}$Xe, $^{134}$Ba and $^{136}$Ce are compared with the corresponding available experimental data as shown in Table 3. Only transitions for which experimental values are known are shown.

From Table 1, one may notice that the evolution of these parameters follows a smooth trend, according to the gradual changes in nuclear structure. $\epsilon$ decreases slightly as the proton number linearly increases, which is consistent with the energy of the $2_1$ level, which decreases as one moves to higher proton numbers in the $N=78$ isotone chain [14]. The modulus of the structure parameter $\chi_\pi$ increases slowly and linearly from $^{132}$Xe to $^{136}$Ce, while the sum of $\chi_\pi+\chi_\nu$ changes linearly from zero to $-0.2$. At the same time the total boson number increases. The combined effect of these two features causes the trend from the $\gamma$-unstable triaxial towards the weakly axial symmetry, showing consistency with results of recent studies [30, 31]. The variation of the dipole interacting proton-proton boson strength $\omega_{\pi\pi}$ fits with the increasing proton number in the $N=78$ isotones, although the absolute value for all nuclei is very small. $\lambda_1$ shows a similar tendency to the already observed mixed symmetry $1_1^+$ states in $^{132}$Xe and $^{134}$Ba, and $\lambda_2$, $\lambda_3$ have a similar trend for the isotonic chain. In general, the overall behaviors of the parameters are consistent with existing phenomenological fitting calculations [32] and the mapping of the constrained self-consistent mean-field calculations with a Skyrme energy density function onto the appropriate boson system [33]. However, the magnitude of $\kappa$ in the present case is slightly small in comparison to other previous studies [32, 33].

As shown in Table 2, the agreement of the experimental and calculated excitation energies is satisfactory. The calculated results of the $2_1^+$, $4_1^+$ and $6_1^+$ states in the ground state band for all nuclei reproduce the experimental data precisely. The $4_2^+$ state in $^{132}$Xe is well reproduced by the calculations. Although our theoretical energy for the $3_1^+$ state in $^{134}$Ba is higher than the experimental data, the relative position of the $3_1^+$ state to the ground states are correctly described. Meanwhile, the calculation results correctly predict that the energy of the $2_1^+$ level decreases as one moves to a higher proton boson number. In contrast to the $2_1^+$ state, the lowest MSS $2_{\rm ms}^+$ increases in energy as one goes to larger values of proton number in the $N=78$ isotones. The lowest MSS in $^{132}$Xe is the $2_3^+$ state at 1.985 MeV, while the $2_4^+$ state at 2.155 MeV has been identified as the dominant fragment of the MSS in $^{136}$Ce. They are quite nicely reproduced by the corresponding predications of the $2_3^+$ state at energy 1.950 MeV and the $2_4^+$ state at energy 2.114 MeV having the MS in $^{132}$Xe and $^{136}$Ce, respectively. In $^{134}$Ba the 2.029 and 2.088 MeV levels for the $2_3^+$ and $2_4^+$ states, respectively, appear to share the properties of the lowest MSS $2_{\rm ms}^+$. The calculated level for the $2_3^+$ state in $^{134}$Ba is slightly lower than the experimental data, while the calculated level for the $2_4^+$ state is slightly higher than the experimental data. The theoretical level spacing between the $2_3^+$ and $2_4^+$ states is wider than the measured data by about 150 keV. However, the weighted average energies of the $2_{\rm ms}^+$ states in $^{134}$Ba is reproduced well. More importantly, the tendency that the energy of the lowest $2_{\rm ms}^+$ state or the average of the fragments of the lowest $2_{\rm ms}^+$ state increases from $^{132}$Xe to $^{136}$Ce is reproduced accurately. For the first scissor mode, the IBM2 analysis gives 2.705 MeV in $^{132}$Xe and 2.565 MeV in $^{134}$Ba, which correspond to the previously observed values of 2.714 and 2.571 MeV in $^{132}$Xe and $^{134}$Ba, respectively. Additionally, the other states, $2_3^+$ in $^{134}$Ba and $^{136}$Ce, $2_5^+$ in the whole isotonic chain and $2_6^+$ in $^{136}$Ce, agree overall with the experimentally observed values.

Table 2. The calculated results of the relevant levels (in MeV) for $^{132}$Xe, $^{134}$Ba and $^{136}$Ce compared with the experimental data. The experimental data are from Refs. [14–19].

| $L_{\rm i}^+$ | $^{132}$Xe | | $^{134}$Ba | | $^{136}$Ce | |
|---|---|---|---|---|---|---|
| | Exp. | Cal. | Exp. | Cal. | Exp. | Cal. |
| $2_1^+$ | 0.667 | 0.665 | 0.605 | 0.603 | 0.552 | 0.552 |
| $2_2^+$ | 1.297 | 1.426 | 1.168 | 1.280 | 1.092 | 1.169 |
| $4_1^+$ | 1.440 | 1.464 | 1.401 | 1.379 | 1.314 | 1.282 |
| $3_1^+$ | 1.803 | 1.748 | 1.643 | 1.732 | | |
| $4_2^+$ | 1.963 | 2.132 | | | | |
| $2_3^+$ | 1.985 | 1.949 | 2.029 | 1.938 | 2.067 | 2.066 |
| $6_1^+$ | 2.112 | 2.286 | 2.211 | 2.258 | 2.214 | 2.221 |
| $2_4^+$ | 2.187 | 2.258 | 2.088 | 2.189 | 2.155 | 2.114 |
| $2_5^+$ | 2.555 | 2.465 | 2.371 | 2.297 | 2.275 | 2.412 |
| $2_6^+$ | | | | | 2.451 | 2.579 |
| $1_1^+$ | 2.714 | 2.705 | 2.571 | 2.565 | | |

From Table 3, we know that the computed $B$(E2) transition probabilities are in good agreement with the experimental data. In more detail, $B(\text{E2},2_1^+\rightarrow 0_1^+)$, $B(\text{E2},4_1^+\rightarrow 2_1^+)$ and $B(\text{E2},2_2^+\rightarrow 2_1^+)$ are all within the experimental uncertainties, and most of the calculated transition probability deviations from the experimental values are smaller than $10^{-1}$. The calculated results are in good agreement with the tendency for transition strength to smoothly increase with increasing proton number. In general the transition from $2_2^+$ to the ground state $B(\text{E2},2_2^+\rightarrow 0_1^+)$ is weak, being 1/10 or even less of the strong E2 transitions. This feature is well described by the predicted values of the transition probabilities, which are 0.192, 0.239 and 0.188 W.u. in $^{132}$Xe, $^{134}$Ba and $^{136}$Ce, respectively. Meanwhile, the calculated values $B(\mathrm{E2},2_3^+\to 0_1^+)$ in $^{132}$Xe, $B(\mathrm{E2},2_4^+\to 0_1^+)$ in $^{136}$Ce and $B(\mathrm{E2},2_3^+\to 0_1^+)$, $B(\mathrm{E2},2_4^+\to 0_1^+)$ in $^{134}$Ba are of the same order of magnitude as $B(\mathrm{E2},2_2^+\to 0_1^+)$, showing the typical weak collective E2 transitions. Furthermore, the signature for MSS decays with a weak E2 transition to the ground state have been fairly well described.

Table 3. The calculated and experimental values of the $^{132}$Xe, $^{134}$Ba and $^{136}$Ce for the relevant transition probabilities $B$(E2)W.u and $B(\mathrm{M1})\mu_\mathrm{N}^2$. The experimental data are from Refs. [14–19].

$^{132}$Xe

| $I_\mathrm{i}\to I_\mathrm{f}$ | experiment | | theory | |
|---|---|---|---|---|
| | $B$(E2) | $B$(M1) | $B$(E2) | $B$(M1) |
| $2_1\to 0_1$ | 23.0(15) | | 22.92 | |
| $2_2\to 0_1$ | 0.056(7) | | 0.192 | |
| $2_2\to 2_1$ | 29.4(46) | ⩽0.015(1) | 33.4 | 0.024 |
| $4_1\to 2_1$ | 29.5(45) | | 30.84 | |
| $4_1\to 2_2$ | | | 0.060 | |
| $3_1\to 2_1$ | | | 0.494 | 0.001 |
| $3_1\to 2_2$ | | | 25.85 | 0.237 |
| $3_1\to 4_1$ | | | 6.763 | 0.0179 |
| $4_2\to 2_1$ | | | 0.001 | |
| $4_2\to 2_2$ | | | 1.686 | |
| $4_2\to 4_1$ | | | 2.450 | 0.002 |
| $2_3\to 0_1$ | 0.67(18) | | 0.508 | |
| $2_3\to 2_1$ | 1.14(73) | 0.22(6) | 0.254 | 0.261 |
| $2_4\to 0_1$ | 0.20(3) | | 0.233 | |
| $2_4\to 2_1$ | ⩽3.1(9) | | 0.254 | 0.071 |
| $2_4\to 2_2$ | ⩽32(13) | | 0.431 | |
| $2_5\to 2_1$ | | | 0.239 | |
| $2_5\to 4_1$ | | | 2.500 | |
| $2_5\to 2_3$ | | | 10.62 | 0.009 |
| $1_1\to 0_1$ | | 0.087(7) | | 0.029 |
| $1_1\to 2_2$ | | 0.180 | | 0.489 |

$^{134}$Ba

| $I_\mathrm{i}\to I_\mathrm{f}$ | experiment | | theory | |
|---|---|---|---|---|
| | $B$(E2) | $B$(M1) | $B$(E2) | $B$(M1) |
| $2_1\to 0_1$ | 33(1) | | 32.52 | |
| $2_2\to 0_1$ | 0.4(1) | | 0.239 | |
| $2_2\to 2_1$ | 49(6) | 0.0003(1) | 48.78 | 0.01106 |
| $4_1\to 2_1$ | 52(6) | | 46.63 | |
| $3_1\to 2_1$ | 0.024(1) | 0.00011(4) | 0.120 | 0.00013 |
| $3_1\to 2_2$ | 4.30(1) | <0.00004 | 19.91 | 0.21045 |
| $2_3\to 0_1$ | 0.42(1) | | 0.430 | |
| $2_3\to 2_1$ | 0.96(1) | 0.062(8) | 0.875 | 0.06472 |
| $2_3\to 2_2$ | ⩽20.5 | ⩽0.01 | 4.065 | 0.00484 |
| $2_4\to 0_1$ | 1.43(1) | | 0.478 | |
| $2_4\to 2_1$ | ⩽0.097 | 0.137(12) | 0.001 | 0.10455 |
| $2_4\to 2_2$ | 0.717(1) | | 0.956 | 0.00047 |
| $0_3\to 2_1$ | 14(4) | | 0.173 | |
| $2_5\to 0_1$ | 0.048(1) | 0.001 | 0.932 | 0.0019 |
| $2_5\to 2_1$ | 1.43(1) | | 0.299 | 0.00199 |
| $1_1\to 0_1$ | | 0.027(4) | | 0.06353 |
| $1_1\to 2_1$ | | 0.101 | | 0.00454 |
| $1_1\to 2_2$ | 0.239(1) | 0.096(18) | 0.582 | 0.64619 |

Table 3 continued.

$^{136}$Ce

| $I_\mathrm{i}\to I_\mathrm{f}$ | experiment | | theory | |
|---|---|---|---|---|
| | $B$(E2) | $B$(M1) | $B$(E2) | $B$(M1) |
| $2_1\to 0_1$ | 39(4) | | 39.00 | |
| $2_2\to 0_1$ | 0.55(9) | | 0.188 | |
| $2_2\to 2_1$ | 48(7) | 0.0010(9) | 59.42 | 0.01204 |
| $4_1\to 2_1$ | 56(10) | | 57.82 | |
| $2_3\to 0_1$ | 1.2(6) | | 0.01 | |
| $2_3\to 2_1$ | 0.79(4) | 0.025(2) | 0.004 | 0.01473 |
| $2_3\to 2_2$ | ⩽7(2) | ⩽0.02(1) | 5.13 | 0.00659 |
| $2_4\to 0_1$ | 0.56(3) | | 0.54 | |
| $2_4\to 2_1$ | 4.0(3) | 0.16(3) | 0.18 | 0.13212 |
| $2_4\to 2_2$ | ⩽11(2) | ⩽0.036(7) | 1.96 | 0.00096 |
| $2_5\to 0_1$ | 0.57(4) | | 0.0034 | |
| $2_5\to 2_1$ | ⩽0.6(2) | ⩽0.0052(4) | 0.00 | 0.00183 |
| $2_6\to 0_1$ | 0.26(3) | | 0.33 | |
| $2_6\to 2_1$ | ⩽0.9(2) | ⩽0.0097(3) | 0.08 | 0.07913 |
| $2_6\to 2_2$ | ⩽6(2) | ⩽0.04(2) | 0.001 | 0.00033 |

The calculated M1 transition strengths for $^{132}$Xe, $^{134}$Ba and $^{136}$Ce in comparison to available experimental data, are also shown in Table 3. As can be seen, the fragmentation and magnitudes of M1 transition strengths are reproduced accurately. According to IBM2, the lowest MSS is characterized by a large M1 transition strength to the first excited $2_1^+$ state. In the presented calculations, it appears to be dominant in the third excited state $2_3^+$ for $^{132}$Xe, and the fourth excited states $2_4^+$ for $^{134}$Ba and $^{136}$Ce. In $^{132}$Xe, the $2_3^+$ state with the largest value $B(\mathrm{M1},2_3^+\to 2_1^+)=0.22(6)\mu_\mathrm{N}^2$ as the single of the MMS $2_\mathrm{ms}^+$ has been observed. The calculated value $B(\mathrm{M1},2_3^+\to 2_1^+)=0.26\mu_\mathrm{N}^2$ for $^{132}$Xe is in good agreement, within the experimental uncertainty. For the $^{136}$Ce nucleus, the calculated value $B(\mathrm{M1},2_4^+\to 2_1^+)=0.132\mu_\mathrm{N}^2$ is very close to the measurement M1 transition strength of the $2_4^+$ level, which was found to be the dominant fragment of the lowest MMS $2_\mathrm{ms}^+$. In contrast to the $^{132}$Xe and $^{136}$Ce, the $^{134}$Ba MMS is shared by the $2_4^+$ states, and to a lesser extent the $2_3^+$ states. Table 3 also shows that the agreement between the IBM2 calculated values and the experimental values is good for $^{134}$Ba both quantitatively and qualitatively. Especially, the predicted value of the summed strength $\Sigma_k B(\mathrm{M1},2_k^+\to 2_1^+)\cong 0.19\mu_\mathrm{N}^2$ for $^{134}$Ba is very close to the experimental value $0.20\mu_\mathrm{N}^2$, and fits better to the experimental data than the theoretical predictions reported in Refs. [16, 34]. Moreover, the summed $B$(M1) strength of the calculation follows the experimental data, which remains nearly constant as a function of proton number along the chain of the $N=78$ isotones. At the same time, the experimental $B(\mathrm{M1},1_1^+\to 0_1^+)$ data in $^{132}$Xe and $^{134}$Ba are reproduced by the calculations, although theory and experiment do not agree within the errors. The $1_1^+$ MMS shows strong decay branching to the second $2_2^+$ state in $^{132}$Xe and $^{134}$Ba, and $B(\mathrm{M1},1_1^+\rightarrow 2_2^+)$ is, indeed, much larger than $B(\mathrm{M1},1_1^+\rightarrow 0_1^+)$. The calculated results for the two nuclei are consistent with the experimental behavior, and agree within an order of magnitude with the experimental ones. In addition, as well as the $1_1^+\rightarrow 2_2^+$ transition, the M1 transition $1_1^+\rightarrow 2_1^+$, which is forbidden within the IBM framework, was observed in $^{134}$Ba. This transition was explained as an $F$-spin E2 transition in IBM2 [35]. Further investigations show that $^{134}$Ba has an E(5) symmetry [21], which is located at the critical point of phase transition from spherical vibration to deformed $\gamma$-soft nuclei. Nuclei at the critical point should exhibit, in comparison to their neighboring isotopes (isotones), dramatic changes in their structure and hence of the experimental observable [19, 36]. This indicated that the $O(6)$ symmetry in $^{134}$Ba is severely broken. The electromagnetic transition properties are dominated by the ubiquity of strong M1 transitions, arising from the mixing of $U(5)$ states of different $F$-spin [37]. However the calculated $B(\mathrm{M1},1_1^+\rightarrow 2_1^+)$ is $0.005\mu_\mathrm{N}^2$. This value is about 20 times too small to explain the observed strength as a pure M1 transition with a similar previous study [32]. In contrast to the $B(\mathrm{M1},1_1^+\rightarrow 2_1^+)$ transition, the $2_2^+$ state $B(\mathrm{M1};2_2^+\rightarrow 2_1^+)$ value observed in $^{134}$Ba, has $B(\mathrm{M1})=0.0003(1)\mu_\mathrm{N}^2$. This is much less than the corresponding data observed in $^{132}$Xe and $^{136}$Ce, where $B(\mathrm{M1})=0.015(1)$ and $0.0010(9)\mu_\mathrm{N}^2$, respectively, which are of the order of magnitude of the typical transition strength in this region. The calculated result $B(\mathrm{M1};2_2^+\rightarrow 2_1^+)$ for $^{134}$Ba is similar to those in $^{132}$Xe and $^{136}$Ce and is considerably larger than the experimental value.

## 4 Conclusion

In summary, we have investigated the characteristics of the mixed-symmetry states $2_\mathrm{ms}^+$ and $1_\mathrm{ms}^+$ for $^{132}$Xe, $^{134}$Ba and $^{136}$Ce three nuclei in the even-even $N=78$ isotones, within the framework of the IBM2. The agreement of the calculated excitation energies and the experimental values is satisfactory. The calculation results correctly predict that the energy of the first excitation $2_1^+$ level decreases as one moves to a higher proton boson number. The lowest MSS in $^{132}$Xe and the dominant fragment of the MSS in $^{136}$Ce are quite nicely reproduced by the corresponding predication. In $^{134}$Ba the $2_3^+$ and $2_4^+$ states share the properties of the lowest MSS $2_\mathrm{ms}^+$, and the agreement of calculated levels with experiments is satisfactory, although the theoretical level spacing between the $2_3^+$ and $2_4^+$ states is slightly wider than in the experiment. Overall, the tendency for the energy of the lowest $2_\mathrm{ms}^+$ state or the average of the fragments of the lowest $2_\mathrm{ms}^+$ state to increases from $^{132}$Xe to $^{136}$Ce is reproduced accurately. Meanwhile the $1_\mathrm{ms}^+$ level for $^{132}$Xe and $^{134}$Ba has been described nicely.

Both the computed $B$(E2) and M1 transition probabilities are in good agreement with the experimental data. Most of the $B(\mathrm{E2},2_1\rightarrow 0_1)$, $B(\mathrm{E2},4_1\rightarrow 2_1)$ and $B(\mathrm{E2},2_2\rightarrow 2_1)$ results are within the experimental uncertainties. The calculated results are consistent with the tendency for transition strength to smoothly increase with increasing proton number. Furthermore the signature for MSS decays with a weak E2 transition to the ground state have been fairly described. The fragmentation and magnitudes of M1 transition strengths for $^{132}$Xe, $^{134}$Ba and $^{136}$Ce are reproduced accurately. The calculated value $B(\mathrm{M1};2_3^+\rightarrow 2_1^+)=0.26\mu_\mathrm{N}^2$ for $^{132}$Xe is in good agreement within the experimental uncertainty, and the calculated value $B(\mathrm{M1};2_4^+\rightarrow 2_1^+)=0.132\mu_\mathrm{N}^2$ is close to the measured M1 transition strength for the dominant fragment of the lowest MSS $2_4^+$ in $^{136}$Ce. For the $^{134}$Ba MSS sharing by the $2_4^+$ and $2_3^+$ states, the agreement between the IBM2 calculated values and the experimental values are good quantitatively and qualitatively. Moreover, the summed $B$(M1) strength of the calculation follows the experimental data, remaining nearly constant as a function of proton number along the chain of $N=78$ isotones. At the same time, the experimental $B(\mathrm{M1},1_1^+\rightarrow 0_1^+)$ data in $^{132}$Xe and $^{134}$Ba are reproduced by the calculations, too. The calculated results of the $B(\mathrm{M1},1_1^+\rightarrow 2_2^+)$ for $^{132}$Xe and $^{134}$Ba are also consistent with the experimental behavior. However, the calculated $B(\mathrm{M1},1_1^+\rightarrow 2_1^+)$ in $^{134}$Ba is too small to explain the observed strength as a pure M1 transition. More experimental and theoretical investigations are needed to explain this aspect.

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