I=1/2 low-lying mesons in lattice QCD *

FU Zi-Wen(傅子文)[1)]

Key Laboratory of Radiation Physics and Technology (Sichuan University), Ministry of Education; Institute of Nuclear Science and Technology, College of Physical Science and Technology, Sichuan University, Chengdu 610064, China

Abstract: Using a conventional constituent-quark model, $I=1/2$ scalar κ, vector K*(892), and axial vector K_1 mesons are studied in the asqtad-improved staggered fermion with wall-source and point-sink interpolators. The mass ratio of $m_{\kappa}/m_{K^*(892)}$ is numerically confirmed to apparently vary with quark mass, and the experimental ordering $m_{K^*(892)}>m_{\kappa}$ holds elegantly when the light u/d quark masses are sufficiently small, while the valence strange quark mass is fixed to its physical value. We also get reasonable signals for the K_1 meson suggested by the SCALAR Collaboration from lattice QCD. The computations are conducted with the MILC N_f=3 flavor gauge configurations at three lattice spacings: a≈0.15, 0.12, and 0.09 fm.

Key words: scalar meson, vector meson, axial vector meson

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1 Introduction

In modern hadron physics, chiral symmetry plays an important role, and the σ meson is a meaningful ingredient. The κ meson can be naturally regarded as a member of nonet scalar states of the chiral $SU(3)\bigotimes SU(3)$ symmetry due to the existence of the σ meson. At present, the σ and κ mesons are both listed by the Particle Data Group (PDG) [1]. Moreover, the presence of the κ meson is strongly indicated by recent experiments [2–5].

The K*(892) meson is a well-established $q\bar{q}$ state belonging to the $SU(3)$ octet, and some experimental analyses have precisely measured its mass [6]. Moreover, there are a few lattice studies on the K*(892) meson [7–10]. Using the quenched approximation, the SCALAR group explored the κ meson $\left(\text{i.e., } I=\frac{1}{2} \text{ scalar } \bar{q}q\right)$ with relatively large quark mass, and found that the mass ratio of κ mass to K*(892) mass (i.e., m_{κ}/m_{K^*}) is about 2.0 with rather large statistical errors [11–14], which is not consistent with the experimental ordering $m_{\kappa}<m_{K^*(892)}$.

Note that the point-source and point-sink interpolators of $I=\frac{1}{2}$ low-lying mesons unavoidably lead to large statistical errors due to the mixture of excited states; moreover, the lattice cutoff was not properly selected to house large meson masses for the rather small lattice size. Consequently, the SCALAR-determined κ masses should be only regarded as upper limits, as commented by the SCALAR authors in Refs. [11–14].

Using dynamical simulations and small quark mass, the SCALAR group recently presented a preliminary result based on the variational method for the mass ratio of κ mass to ρ(770) mass at a single lattice ensemble, i.e., m_{κ}/m_{ρ}=1.29(5) [15], which indicates that the mass ratio of m_{κ}/m_{K^*} should be smaller than 1.29. All the SCALAR results indicate that the mass ratio m_{κ}/m_{K^*} is definitely not constant, and depends on the quark mass. This is actually in agreement with Nebreda and Pelaez's brilliant expositions that κ mass has a much stronger dependence on quark mass than the K*(892) meson does; in fact, it grows faster by a factor of 3 [16].

In our previous works, we follow the SCALAR group's work to exploit the point-source and point-sink interpolators to study K*(892) decay width [10] and the κ meson [17]. Nonetheless, as expected, we found that the statistical errors are not quite satisfactory, and the plateaus in the mass plots are not quite clear.

The noise-to-signal ratio in the lattice computation of a meson mass is generally proportional to $e^{(m_M-m_{\pi})t}$, where m_M is the desired meson mass. Hence, the most efficient way to improve statistics is to use smaller quark mass and finer lattice space. Lattice studies with staggered fermions are more economical than those with

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1) E-mail: fuziwen@scu.edu.cn

other discretizations, which permits lattice studies with larger lattice spatial dimension (L) or smaller quark mass. For these reasons, we carry out a lattice calculation on $N_{\rm f}=3$ flavor MILC gauge configurations in the asqtad-improved staggered sea quarks. In this simulation, the valence strange quark is fixed to its physical mass, and we use a broad range of light valence quark mass with the largest lattice dimension $40^3\times 96$, and the lowest pion mass 240 MeV, which allows us to further explore the chiral limit.

Additionally, in order to efficiently reduce the overlaps to the excited states of $I=1/2$ low-lying mesons we use the wall-source and point-sink interpolators, which are used by the MILC Collaboration to calculate the hadron spectrum [7, 18]. As anticipated, the statistical errors are significantly reduced as compared with our previous works [10, 17]. Moreover, we found that the mass ratio of $m_\kappa/m_{\rm K^*}$ is quantitatively dependent on quark mass, and is indeed larger than 1 for the larger quark mass. The by-product of this work is that the K*(892) mass turns out to be larger than the κ mass when the light u quark mass is sufficiently small, thus, the experimental ordering $m_{\rm K^*(892)}>m_\kappa$ elegantly holds even when we study them with the simple constituent-quark model.

The high-statistics WA3 experiment established the existence of the K$_1$ meson [19]. The axial vector K$_1$ meson is now listed by PDG with $I(J^P)=\frac{1}{2}(1^+)$ [1], and pioneering lattice studies of the K$_1$ meson have been investigated by the SCALAR group [11–14]. For completeness of the study on $I=\frac{1}{2}$ low-lying mesons, we also attempt the K$_1$ meson, which is a possible candidate for the oscillating parity partner of the K*(892) meson with staggered fermions [10]. In this work, the signals of the K$_1$ meson are found to be acceptable.

2 Correlator

As explained earlier, to efficiently reduce the overlaps of excited states we use the wall-source and point-sink interpolators for the $I=1/2$ low-lying mesons [7, 18]. In the present study, we use staggered fermions, and the local lattice operators for the vector K*(892) meson, scalar κ meson, and axial vector K$_1$ meson are $\gamma_i\otimes\gamma_i$, $I\otimes I$, and $\gamma_i\gamma_5\otimes\gamma_i\gamma_5$, respectively [20–23].

It is important to note that the κ, K* and K$_1$ propagators only involve a connected diagram and contain no disconnected part, which is difficult to calculate [11, 24]. Moreover, the κ meson is a flavor non-singlet state, which cannot mix with the glueball state. As a result, the signals of the κ propagators should be relatively clear, and lattice studies on the $I=1/2$ low-lying mesons will be an important way to shed some light on the mysterious structure of the scalar meson.

The staggered fermion meson propagators, in general, contain the single-particle form [23],

$$\mathcal{C}(t) = \sum_i A_i \cosh\left[m_i\left(t-\frac{T}{2}\right)\right] + \sum_i A_i'(-1)^{t+1}\cosh\left[m_i'\left(t-\frac{T}{2}\right)\right], \qquad (1)$$

where desired- and opposite-parity states are included, and the opposite-parity states maintain the temporally oscillating prefactor $(-1)^{t+1}$ [23].

For the K*(892) correlator, only one mass is taken with each parity [7, 18], and the oscillating parity partner is a meson with $J^P=1^+$. The K$_1$ meson also has $J^P=1^+$, and it is thus the possible parity partner of the K*(892) meson. Nevertheless, the states with $J^P=1^+$ can be multihadron states as well [10]. As MILC fits the a$_1$ meson, for the axial vector K$_1$ correlator, we can, in principle, use a three-state form to get an acceptable fitting quality [7]. However, in practice, we can also obtain satisfactory fitting quality with only the two-state form. In fact, this is partially physical, since the splittings between the ground and excited states are generally larger for the K$_1$ meson. Furthermore, the parity partner of the κ meson is the K$_{\rm A}$ meson [23]. As practised in our previous work [17], κ correlators should fit with the consideration of the bubble contribution.

We should remark at this point that our lattice investigation for the K$_1$ meson is absolutely an ideal case [13], since the K$_1$ meson is actually from the mixing between $I=1/2$ $J^{PC}=1^{++}$ and 1^{+-} states [13], which are of pseudoscalar-vector character [25].

3 Simulations and results

In the present study, we use the MILC $N_{\rm f}=3$ flavor gauge configurations with asqtad-improved staggered sea quarks [7, 26, 27]. The simulation parameters of the MILC gauge configurations are listed in Table 1, where a broad range of quark mass is included to investigate the mass ratios of $m_\kappa/m_{\rm K^*}$, especially at small quark masses, which are not explored by the SCALAR collaboration yet. The gauge configurations were gauge-fixed to the Coulomb gauge before calculating the propagators. For simple notation, it is convenient to use $(am_{\rm l}, am_{\rm s})$ to identify the MILC lattice ensembles. Moreover, it is MILC's convention to call lattice ensembles "fine" for spatial lattice spacing $a\approx 0.09$ fm, "coarse" for $a\approx 0.12$ fm, and "medium-coarse" for $a\approx 0.15$ fm.

The standard conjugate gradient method is utilized to acquire the required matrix elements of the inverse

Table 1. Lattice dimensions are described with spatial (L) and temporal (T) size. The gauge coupling is given by $\beta{=}10/g^2$. The bare masses of the light and strange quark masses are written in terms of am'_l and am'_s, respectively. $L{=}aN_L$ is the lattice spatial dimension in fm, and pion masses are given in MeV. The physical strange-quark mass is indicated by am_s [27]. The number of gauge configurations used in this work is given by N_{cfg}, and the number of time slices calculated the propagators for each configuration is shown by N_{slice}. The last Column gives our lattice-measured ρ mass in lattice units.

$L^3\times T$	β	am'_l/am'_s	L/fm	m_π/MeV	am'_s	r_1/a	N_{cfg}	N_{slice}	am_ρ
				$a{\approx}0.09$ fm					
$40^3\times96$	7.08	0.0031/0.031	3.3	246	0.0252	3.695(4)	100	48	0.3571(39)
$32^3\times96$	7.085	0.00465/0.031	2.8	301	0.0252	3.697(3)	200	48	0.3721(43)
$28^3\times96$	7.09	0.0062/0.031	2.4	347	0.0252	3.699(3)	200	48	0.3892(40)
$28^3\times96$	7.10	0.0093/0.031	2.4	423	0.0252	3.705(3)	500	16	0.4051(21)
$28^3\times96$	7.11	0.0124/0.031	2.4	487	0.0252	3.712(4)	480	16	0.4133(20)
$28^3\times96$	7.18	0.031/0.031	2.4	756	0.0252	3.822(10)	484	16	0.4742(32)
				$a{\approx}0.12$ fm					
$32^3\times64$	6.715	0.005/0.005	3.7	275	0.0344	2.697(5)	696	32	0.4984(24)
$24^3\times64$	6.76	0.005/0.05	2.9	268	0.0344	2.647(3)	517	32	0.5309(35)
$20^3\times64$	6.76	0.007/0.05	2.4	316	0.0344	2.635(3)	251	32	0.5560(30)
$20^3\times64$	6.76	0.01/0.05	2.4	372	0.0344	2.618(3)	210	4	0.5724(74)
$20^3\times64$	6.79	0.02/0.05	2.4	523	0.0344	2.644(2)	264	4	0.6617(28)
$20^3\times64$	6.81	0.03/0.05	2.4	638	0.0344	2.650(4)	564	4	0.6483(20)
$20^3\times64$	6.83	0.04/0.05	2.4	733	0.0344	2.664(5)	350	4	0.6848(30)
$20^3\times64$	6.85	0.05/0.05	2.4	818	0.0344	2.686(8)	424	4	0.7154(16)
$20^3\times64$	6.96	0.1/0.1	2.4	1155	0.0344	2.687(0)	340	4	0.8621(9)
				$a{\approx}0.15$ fm					
$20^3\times48$	6.566	0.00484/0.0484	2.9	240	0.0426	2.162(5)	604	24	0.6695(78)
$16^3\times48$	6.572	0.0097/0.0484	2.3	334	0.0426	2.152(5)	631	24	0.6968(59)
$16^3\times48$	6.586	0.0194/0.0484	2.3	463	0.0426	2.138(4)	621	24	0.7542(53)
$16^3\times48$	6.600	0.0290/0.0484	2.3	559	0.0426	2.129(5)	576	24	0.7884(37)
$16^3\times48$	6.628	0.0484/0.0484	2.3	716	0.0426	2.124(6)	618	24	0.8659(18)

Dirac fermion matrices. All the numerical calculations are evaluated in double precision to avoid potential roundoff errors, and the conjugate gradient residual is selected to be 1.0×10^{-5}, which is generally smaller than that for generating the MILC gauge configurations [7]. Moreover, in order to improve the statistics, all the propagators are calculated from a given number of time slices (N_{slice}) which are indicated in the ninth column of Table 1. The time slices are evenly spread through the lattice, namely, only one source time slice was chosen at a time. At the end of the evaluation, we gather all the propagators. It is worth stressing that we can adjust the values of N_{slice} for each lattice ensemble to guarantee the extraction of relevant masses with the desired precision.

The valence u/d quark masses are set to their dynamical quark masses for all lattice ensembles, while the valence strange quark is fixed to its physical mass, which was determined by the MILC Collaboration [27]. In the usual manner, we extracted π, K, and fictitious $s\bar{s}$ masses [7]. These pseudoscalar masses are used to evaluate the bubble contribution to κ correlators [17], where three low energy couplings (μ, δ_A, and δ_V) are fixed to the MILC-determinated values [28]. After neatly removing the unwanted bubble terms from κ propagators, the remaining κ propagators have clean information, and we then fit them with the physical model in Eq. (1).

As practised in Ref. [7], the propagators of the $I{=}1/2$ low-lying mesons are commonly fit by changing the minimum fitting distances D_{min} and putting the maximum distances D_{max} either at $T/2$ or where the fractional statistical errors for two consecutive time slices are roughly beyond 30%. The mean value and statistical error at each time slice are computed by the jackknife technique. The masses of κ, K*(892) and K_1 mesons are secured from the effective mass plots, and cautiously picked up by the overall assessment of the plateau in the mass as the function of D_{min}, good fit quality, and D_{min} large enough to suppress the excited states [7, 17].

For the MILC fine, coarse, and medium-coarse lattice ensembles, we give an example of effective mass plots of κ, K*(892) and K_1 mesons, which are exhibited in Fig. 1. We found that the plateaus for the K*(892) meson are clear, and the effective mass plots commonly have small uncertainties within a broad minimum time distance region for the fine (0.0031, 0.031) ensemble, coarse (0.005, 0.05) ensemble, and medium-coarse (0.00484, 0.0484) ensemble. Note that the statistics of the K*(892) meson are significantly improved as compared with our previous study for the (0.00484, 0.0484) ensemble [10], where the point-source and point-sink interpolators are used.

From Fig. 1, we notice that the plateaus for the κ meson are obvious, and are remarkably enhanced as compared with our previous studies [17], where the point-source interpolator is used, and the plateaus are often quite short [17]. It is interesting and important to note that the plateaus of the κ meson are clearly below those of the $K^*(892)$ meson for all three lattice ensembles, and the effective mass plots commonly have small errors within a relatively broad minimum time distance region: $9 < D_{\min} < 18$ for the (0.0031, 0.031) ensemble, $4 < D_{\min} < 15$ for the (0.005, 0.05) ensemble, and $4 < D_{\min} < 9$ for the (0.00484, 0.0484) ensemble.

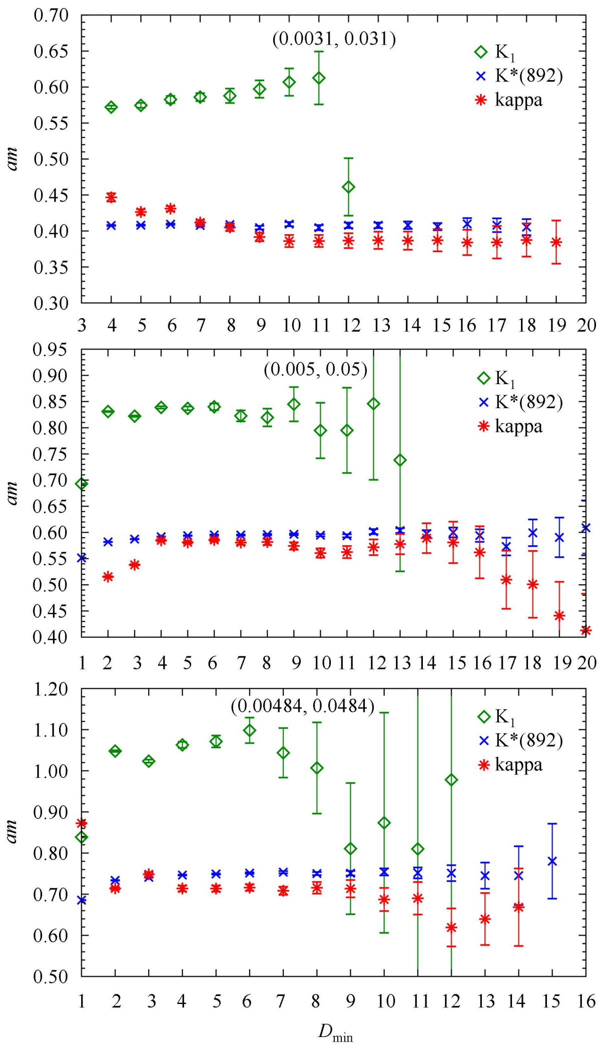

Fig. 1. (color online) Mass plots of κ, $K^*(892)$, and K_1 mesons as a function of $D_{\min}$ for MILC fine (0.0031, 0.031) ensemble (top panel), coarse (0.005, 0.05) ensemble (middle panel), and medium-coarse (0.00484, 0.0484) ensemble (bottom panel).

Fitting the K_1 meson is challenging due to its large mass. From Fig. 1, we note that the plateaus in the effective mass plots are often short. This is not surprising for us since the K_1 meson is a p-wave meson [7]. In practice, we have to select a small enough $D_{\min}$ to get acceptable fits, and only the time range $4 \leqslant D_{\min} \leqslant 6$ is considered.

The extracted masses of the κ, $K^*(892)$ and K_1 mesons, along with fitting ranges and fitting qualities, are summarized in Table 2. The last column gives the mass ratios m_κ/m_{K^*}, where the errors are inherited from the statistical errors of both κ and $K^*(892)$ mesons. It is worth mentioning that the κ mass and K^* mass have only small statistical errors, and the errors of the K_1 mass are also reasonable. This reveals that the usage of wall-source and point-sink interpolators is a key technique in this work. Of course, the summations over a sufficient

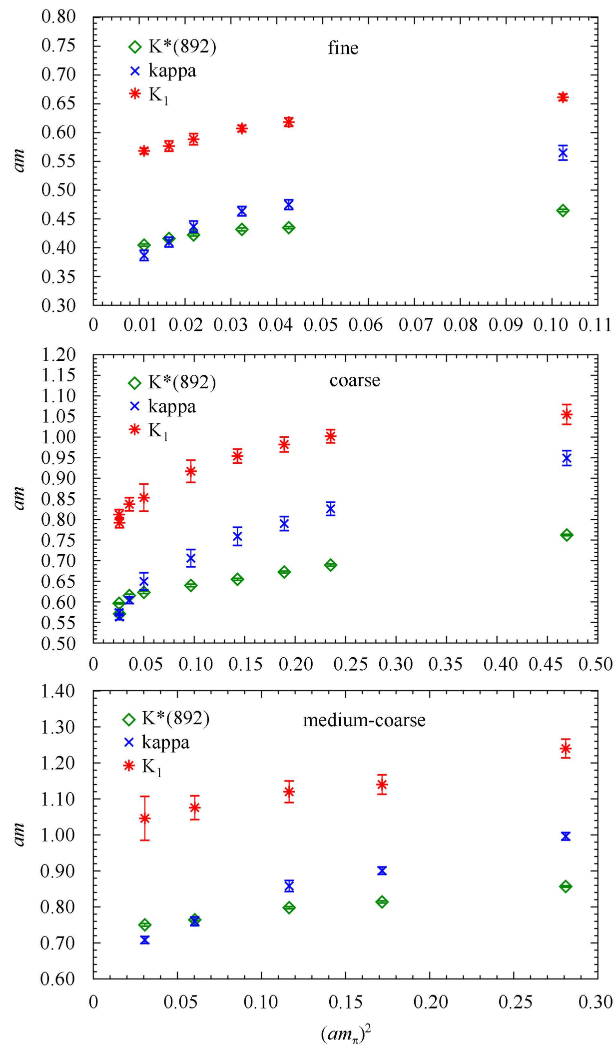

Fig. 2. (color online) The masses of κ, $K^*(892)$, and K_1 mesons as a function of $(am_\pi)^2$ for the MILC fine (top panel), coarse (middle panel) and medium-coarse (bottom panel) lattice ensembles.

Table 2. Summaries of $I=1/2$ low-lying meson masses. The second, third and fourth columns show the fitted value of m_{K^*}, its fit range, and fit quality: χ^2 divided by the number of degrees of freedom, respectively. The fifth, sixth, and seventh columns give the fitted value of m_κ, its fit range, and fit quality, respectively. The eighth, ninth, and tenth columns give the fitted value of m_{K_1}, its fit range, and fit quality, respectively. The last column gives the mass ratios of $m_\kappa/m_{\mathrm{K}^*}$.

ensemble	am_{K^*}	range	χ^2/D	am_κ	range	χ^2/D	am_{K_1}	range	χ^2/D	$m_\kappa/m_{\mathrm{K}^*}$
(0.0031, 0.031)	0.4045(19)	14–48	42.2/31	0.3866(89)	12–32	19.3/17	0.5682(99)	8–16	3.6/5	0.956(22)
(0.00465, 0.031)	0.4160(20)	15–48	27.5/30	0.4097(84)	12–40	36.4/25	0.5766(90)	7–24	16.1/14	1.006(21)
(0.0062, 0.031)	0.4223(19)	15–48	37.1/30	0.4367(96)	12–48	23.1/33	0.5885(97)	6–16	15.3/7	1.034(23)
(0.0093, 0.031)	0.4318(25)	17–30	14.6/10	0.4634(81)	12–48	33.2/33	0.6071(54)	6–16	11.6/7	1.073(11)
(0.0124, 0.031)	0.4348(18)	15–48	23.7/30	0.4748(85)	12–48	34.7/33	0.6184(69)	6–16	9.3/7	1.092(20)
(0.031, 0.031)	0.4646(22)	17–48	22.5/28	0.5649(126)	12–48	42.6/33	0.6614(56)	6–16	4.1/7	1.216(28)
(0.005, 0.005)	0.5708(23)	10–32	17.5/19	0.5630(65)	9–32	28.6/20	0.792(12)	7–32	26.7/22	0.986(12)
(0.005, 0.05)	0.5965(16)	9–32	26.4/20	0.5742(71)	9–32	20.7/20	0.812(12)	7–32	9.8/22	0.963(12)
(0.007, 0.05)	0.6152(36)	10–32	10.3/19	0.6041(82)	9–32	25.3/20	0.837(16)	6–16	5.4/7	0.982(15)
(0.01, 0.05)	0.6231(29)	10–18	3.8/5	0.650(21)	7–32	29.9/22	0.853(33)	6–16	4.2/7	1.043(33)
(0.02, 0.05)	0.6399(30)	8–32	18.3/21	0.706(21)	8–16	6.8/5	0.917(27)	6–16	5.7/7	1.110(44)
(0.03, 0.05)	0.6547(25)	9–32	22.6/20	0.759(22)	8–18	6.3/7	0.954(17)	6–20	6.5/11	1.163(33)
(0.04, 0.05)	0.6723(24)	9–32	22.4/20	0.790(17)	7–32	28.0/22	0.982(18)	6–18	7.9/9	1.174(23)
(0.05, 0.05)	0.6893(26)	10–32	4.5/19	0.826(16)	7–32	25.7/22	1.002(16)	6–11	0.6/2	1.199(23)
(0.1, 0.1)	0.7623(17)	10–32	18.8/19	0.949(18)	7–13	6.0/3	1.055(24)	7–20	12.2/10	1.225(23)
(0.00484, 0.0484)	0.7502(38)	8–24	3.9/13	0.7083(97)	7–24	14.2/14	1.046(61)	7–16	3.6/6	0.944(14)
(0.0097, 0.0484)	0.7642(30)	7–24	17.5/14	0.7601(121)	7–24	17.1/14	1.076(33)	6–16	9.6/7	0.995(16)
(0.0194, 0.0484)	0.7979(30)	8–24	18.6/13	0.8582(153)	7–24	11.4/14	1.120(30)	6–16	3.2/7	1.076(20)
(0.0290, 0.0484)	0.8138(28)	8–24	24.6/13	0.9007(102)	6–24	16.8/15	1.140(27)	6–16	5.0/7	1.107(13)
(0.0484, 0.0484)	0.8567(20)	8–24	17.0/13	0.9961(104)	6–24	21.7/15	1.240(26)	6–16	11.5/7	1.163(12)

number of time slices for propagators also play an important role. In order to intuitively see the pion mass dependence of κ, $\mathrm{K}^*(892)$ and K_1 mesons, we plot these masses as a function of $(am_\pi)^2$ in Fig. 2.

We should remark at this point that we cannot directly compare our results for the $\mathrm{K}^*(892)$ meson with those of the MILC determinations [7, 18] since the valence strange quark masses are usually set to be equal to its sea quark by the MILC collaboration [7, 18], while we fix the valence strange quark to be its physical mass in the present study. To fairly compare our calculation, we list our lattice-measured ρ meson in the last column of Table 1, which is in good agreement with the MILC determinations [7, 18]. Moreover, our lattice-measured pion masses, listed in the fifth column of Table 1, also agree perfectly with the MILC determinations [7, 18].

From Table 2 and Fig. 2, we clearly find that the mass ratios of $m_\kappa/m_{\mathrm{K}^*}$ definitely vary with quark mass, especially at lower quark masses. Note that it is not a crude linear form on the whole, but for large quark masses, we can roughly approximate it in a linear form. For small quark masses, the chiral logarithms are clearly seen. To make our reports more intuitive, these mass ratios are presented graphically in Fig. 3 as a function of pion mass. Note that the mass ratios of $m_\kappa/m_{\mathrm{K}^*}$ are dimensionless quantities, and we calculate these ratios at three MILC lattice spacings, consequently it is appropriate to exhibit our results in terms of dimensionless quantities, and pion masses are then scaled with the MILC scale r_1 [7]. The definition of r_1 and the benefits of using r_1 are discussed in Ref. [7].

As expected, we indeed reproduce the relatively large mass ratios of $m_\kappa/m_{\mathrm{K}^*}$ for large quark masses, which is fairly consistent with the SCALAR group's recent tentative results, where the mass ratios of $m_\kappa/m_{\mathrm{K}^*}$ are indicated to be smaller than 1.29(5) for a single lattice ensemble with the hopping parameter $h_{\mathrm{u/d}}=0.1390$ [15]. Note that the mass ratio of $m_\kappa/m_{\mathrm{K}^*}$ for the SCALAR group's early lattice results [11–13] has a rather large statistical error (about 30%); this is partially due to the usage of large quark masses, and the point-source and point-sink interpolators. Our previous works about the κ meson also suffered from huge statistical errors due to the usage of the point-source and point-sink interpolators [10, 17].

It is interesting and important to note that mass ratios of $m_\kappa/m_{\mathrm{K}^*}$ are smaller than one when the quark masses are small enough. This is not at all surprising. It is well-known that the vector ρ meson grows slower than the scalar σ meson with the variation of pion mass [29]. Likewise, the vector $\mathrm{K}^*(892)$ should grow slower than the scalar κ meson. Our lattice simulation indeed shows that the $K^*(892)$ meson grows slower than the κ meson when the pion mass is varied (the valence strange quark mass is fixed to its physical value), which is qualitatively consistent with Nakamura's lattice results in Fig. 6 of Ref. [14]. Moreover, these features are also qualitatively consistent with Nebreda and Pelaez's statements that the κ mass has a much stronger dependence on the quark mass than $K^*(892)$ meson does [16]. Consequently, the $K^*(892)$ mass is larger than the κ mass when the pion mass is sufficiently small[1)], therefore, the experimental ordering $m_{K^*(892)} > m_\kappa$ is nicely kept even when we study them with the simple constituent-quark model. This is an encouraging and exciting result.

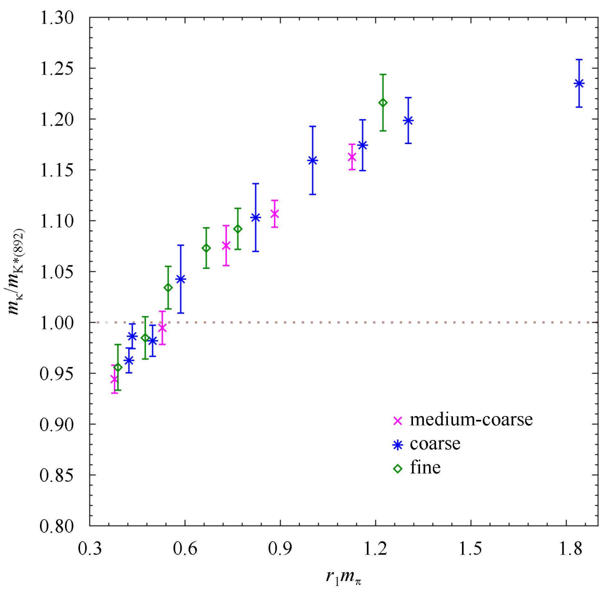

Fig. 3. (color online) Mass ratios of m_κ/m_{K^*} as a function of $r_1 m_\pi$ with the MILC scale r_1 [7] for MILC "fine", "coarse" and "medium-coarse" lattice ensembles. The brown dotted line indicates the position of $m_\kappa = m_{K^*}$.

4 Summary and outlook

In this work, we employ the simple constituent quark model to study $I=1/2$ low-lying κ, K^* and K_1 mesons using the MILC $N_f=3$ flavors gauge configurations with asqtad-improved staggered sea quarks. We employ the wall-source and point-sink interpolators for κ, K^* and K_1 mesons, and the signals for κ and K^* mesons are found to be significantly improved as compared with those using point-source and point-sink interpolators [11–14, 17], and are also comparable with recent results from the SCALAR group with the variational method [15].

We should remember that the effective mass plots of the κ meson still suffer from large errors at large $D_{\min}$. Moreover, the signals of the K_1 meson are not satisfactory. Our lattice attempts of the K_1 meson are absolutely in the preliminary stage. This is not surprising since the K_1 meson is a mixture of two $SU(3)$ eigenstates, which have pseudoscalar-vector characters [25]. Additionally, the K_1 meson is unstable and usually leads to new levels from the decay products [30]. These characteristics should be appropriately incorporated in the K_1 interpolators for more sophisticated lattice simulation.

In this paper, the κ mass and $K^*(892)$ mass are secured with acceptable quality, and consequently the mass ratio of m_κ/m_{K^*} is obtained with relatively small errors. As anticipated, the mass ratio of m_κ/m_{K^*} is numerically confirmed to vary with pion mass, and the experimental ordering $m_{K^*(892)} > m_\kappa$ holds nicely for small enough light quark mass. This will somewhat aid researchers in examining the intrinsic attributes of scalar mesons.

Admittedly, since the strange quark mass m_s is larger than the light up quark mass m_u, the observed mass ordering $m_{a_0(980)} > m_\kappa$ cannot be reconciled with the conventional $\bar{u}d$ and $\bar{u}s$ states. The tetraquark interpretations of scalar mesons can realize this experimental ordering [31–37]. It is worth stressing that the properties of the scalar resonance $a_0(980)$, whose mass is in the vicinity of the $K\bar{K}$ threshold, should consider $K\bar{K}$ scattering, which is suggested by the ETM Collaboration in Ref. [34]. Recently, there has been a lattice attempt at $K\bar{K}$ scattering [38]. Nonetheless, robust calculations of $K\bar{K}$ scattering should adopt the finer lattice ensemble and small light quark masses [38], a task which would need astronomical computer resources; we therefore reserve this ambitious work for the future.

Additionally, from Table 2 and Fig. 2, we discern that the signals of κ and K_1 for lattice ensembles with small pion mass are usually better. Moreover, the bubble contribution to the κ operator should be further suppressed for enough lattice spatial extent L. For these purposes, we are preparing a lattice study with the MILC superfine lattice ensembles. Furthermore, we are also planning to use the variational method, which includes the traditional quark-antiquark operators and four-quark structure, to study the scalar mesons [15, 37]. We expect the signals of the relevant propagators will be significantly further improved.

The reliable extraction of κ and $K^*(892)$ masses from two-point correlation functions will inspire us to further investigate the resonance masses of $K^*(892)$ and κ [10, 39] since both $K^*(892)$ and κ mesons are resonances. Moreover, this will stimulate us to study the σ meson from lattice QCD [24]. We will unceasingly and undauntedly appeal for computer resources to accomplish these fascinating enterprises.

1) We observe the rule of thumb that the mass ratios of m_κ/m_{K^*} are smaller than one when the pion masses are roughly smaller than 300 MeV, where the valence strange quarks are fixed to its physical ones.

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