Inflation in de Sitter spacetime and CMB large scale anomaly *

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Abstract: The influence of cosmological constant-type dark energy in the early universe is investigated. This is accommodated by a new dispersion relation in de Sitter spacetime. We perform a global fit to explore the cosmological parameter space by using the CosmoMC package with the recently released Planck TT and WMAP polarization datasets. Using the results from the global fit, we compute a new CMB temperature–temperature (TT) spectrum. The obtained TT spectrum has lower power compared with that based on the ΛCDM model at large scales.

Key words: de Sitter spacetime, dark energy, inflation, anisotropy

PACS: 98.80.Cq **DOI:** 10.1088/1674-1137/39/9/095101

1 Introduction

The anisotropy of Cosmic Microwave Background radiation (CMBR) was first discovered by NASA's Cosmic Background Explorer (COBE) satellite in the 1990s [1]. The results were later confirmed by the balloon experiments and the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [2]. The observation data can be well fitted by the ΛCDM (cold dark matter plus dark energy in a form of the cosmological constant Λ) model. With the help of the Planck data [3], one can now place an unprecedentedly precise constraint on six cosmological parameters (with accuracy down to the 10% level) [4].

However, for the CMBR power spectrum, the observed values of C_ℓ for low ℓ, especially for the quadrupole component $\ell=2$, are smaller than those predicted by the standard cosmological model. H. Liu and T.-P. Li [5, 6] proposed that the CMB quadrupole in the WMAP data is artificial and the corresponding values should actually be near zero. C. Bennett et al. [7], on the other hand, carefully re-examined the WMAP 7-year data and reported that the quadrupole amplitude value is consistent with that predicted by the the ΛCDM model at a 95% confidence level and shows no anomalies.

In a previous paper, we proposed that inflation in a de Sitter spacetime could possibly alleviate this controversy [8]. In fact, a de Sitter universe is a cosmological solution of Einstein's field equations in general relativity with a positive cosmological constant Λ. We considered the possible effects of dark energy in the form of a cosmological constant during the inflationary period. In this scenario, the cosmological constant-type dark energy was once predominant in the early universe. The dynamics of the universe are accommodated by a new dispersion relation—the dispersion relation in de Sitter spacetime. We found that for certain cosmological parameter values, the modified inflation model gives a CMB temperature–temperature (TT) power spectrum with lower power at large scales, alleviating the low-ℓ multipole issue.

In this paper, we use the Markov Chain Monte Carlo sampler (CosmoMC) [9] to explore the cosmological parameter space. The recently released Planck TT [3] and WMAP polarization [2] datasets are used in our global fit. More stringent constraints are presented on the cosmological parameters. We obtain values of the cosmological parameters from global fitting and compute a new TT spectrum with the Code for Anisotropies in the Microwave Background (CAMB) [10].

The rest of the paper is organized as follows. Section 2 is devoted to the setup of inflation in de Sitter spacetime. In Section 3, we present a global fit with the combined datasets and obtain numerical results. Discussion and conclusions are given in Section 4.

2 Inflation in de Sitter spacetime

De Sitter spacetime is a vacuum solution of Einstein's field equations with a positive cosmological constant. It can be realized as a four-dimensional pseudo-sphere embedded in a five dimensional Minkowski flat space ($\eta_{\rm AB}$=diag(1, −1, −1, −1, −1)) [11]

$$\eta_{AB}\xi^A\xi^B = -\frac{1}{K}=-R^2,$$

Received 19 December 2014, Revised 28 March 2015

∗ Supported by National Natural Science Foundation of China (11375203)

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$$\mathrm{d}s^2 = \eta_{AB}\mathrm{d}\xi^A\mathrm{d}\xi^B, \qquad A,B=0,1,2,3,4, \tag{1}$$

where K is the Riemannian curvature and R is the radius of de Sitter spacetime.

For a free particle with mass m_0, the five dimensional angular momentum M^{AB} is defined as

$$M^{AB} \equiv m_0\left(\xi^A\frac{\mathrm{d}\xi^B}{\mathrm{d}s}-\xi^B\frac{\mathrm{d}\xi^A}{\mathrm{d}s}\right), \tag{2}$$

where s is the affine parameter. For the rest of the article, the Greek indices (i.e. μ, ν, α, β, etc.) run from 0 to 3.

The momentum of a free particle in de Sitter spacetime is defined as

$$P^\mu \equiv R^{-1}M^{4\mu}. \tag{3}$$

The angular momentum $J^{\mu\nu}$ in de Sitter spacetime is defined as

$$J^{\mu\nu} \equiv M^{\mu\nu} = x^\mu P^\nu - x^\nu P^\mu. \tag{4}$$

An invariant can be constructed in terms of the angular momentum M^{AB},

$$\begin{aligned} m_0^2 &= -\frac{K}{2}M^{AB}M_{AB} = E^2 - \boldsymbol{P}^2 - \frac{K}{2}J^{\mu\nu}J_{\mu\nu}, \\ E &= P^0, \quad \boldsymbol{P} = (P^1,\ P^2,\ P^3). \end{aligned} \tag{5}$$

The generators $\hat{M}_{AB}$ of de Sitter group $SO(1{,}4)$ are of the form

$$\hat{M}_{AB} \equiv \frac{1}{\mathrm{i}}\left(\xi_A\frac{\partial}{\partial\xi^B}-\xi_B\frac{\partial}{\partial\xi^A}\right). \tag{6}$$

The Klein–Gordon equation for a free scalar can be written as

$$\left(-\frac{K}{2}\hat{M}_{AB}\hat{M}^{AB}-m_0^2\right)\phi(x)=0. \tag{7}$$

Equation (7) can be solved analytically [12]. The dispersion relation for a free scalar in de Sitter spacetime is

$$E^2 = m_0^2 + k^2 + K(2n+l)(2n+l+2), \tag{8}$$

where n and l respectively denote the radial and the angular quantum number of the system. For massless particles like photons, the above dispersion relation becomes

$$\begin{aligned} \omega^2 &= k^2 + \varepsilon_\gamma^{*2}, \\ \varepsilon_\gamma^* &\equiv \sqrt{K(2n_\gamma+l_\gamma)(2n_\gamma+l_\gamma+2)}, \end{aligned} \tag{9}$$

where ω and k are the frequency and wavenumber of the photon.

Quantum field theory in de Sitter spacetime is an interesting topic, in which progress has been made in recent years [13–15]. However, there is still not a general framework of quantum field theory in de Sitter spacetime.

Now, we calculate the primordial power spectrum $\mathcal{P}_{\delta\phi}(k)$ in de Sitter spacetime. The inflation field perturbation is denoted by $\delta\phi(\boldsymbol{x},t)$. In the momentum representation, the evolution equation of the primordial perturbation is given by [16]

$$\delta\sigma_{\boldsymbol{k}}'' + \left(\omega^2 - \frac{2}{\tau^2}\right)\delta\sigma_{\boldsymbol{k}} = 0, \qquad \omega^2 \equiv k^2 + \varepsilon_\gamma^{*2}, \tag{10}$$

where

$$\delta\sigma_{\boldsymbol{k}} \equiv a\delta\phi_{\boldsymbol{k}}\ . \tag{11}$$

A prime represents differentiation with respect to the conformal time τ [17]. The above equation has an exact particular solution

$$\begin{aligned} \delta\sigma_{\boldsymbol{k}} &= \frac{\mathrm{e}^{-\mathrm{i}\omega\tau}}{\sqrt{2\omega}}\left(1+\frac{\mathrm{i}}{\omega\tau}\right) \\ &= \frac{\mathrm{e}^{-\mathrm{i}\sqrt{k^2+\varepsilon_\gamma^{*2}}\ \tau}}{\sqrt{2}\left(k^2+\varepsilon_\gamma^{*2}\right)^{1/4}}\left(1+\frac{\mathrm{i}}{\sqrt{k^2+\varepsilon_\gamma^{*2}}\ \tau}\right). \end{aligned} \tag{12}$$

The power spectrum $\mathcal{P}_{\delta\phi}(k)$ is defined as [16]

$$\mathcal{P}_{\delta\phi}(k) \equiv \frac{k^3}{2\pi^2}\left|\frac{1}{a}\delta\phi_{\boldsymbol{k}}\right|^2. \tag{13}$$

Considering the super-horizon criterion, one has

$$-k\tau = \frac{k}{aH} \ll 1, \tag{14}$$

where $a=-1/H\tau$ (H is the Hubble parameter) [16]. By making use of (12), (13) and (14), we obtain the primordial power spectrum [8]

$$\mathcal{P}_{\delta\phi}(k) = \frac{H^2}{4\pi^2}\cdot\frac{k^3}{\left(k^2+\varepsilon_\gamma^{*2}\right)^{3/2}}. \tag{15}$$

For perturbations on small scales, we obtain the usual scale-invariant primordial power spectrum

$$\mathcal{P}_{\delta\phi}(k) \simeq k^0. \tag{16}$$

For large scales, we have

$$\mathcal{P}_{\delta\phi}(k) \simeq \frac{H^2}{4\pi^2}\cdot k^3. \tag{17}$$

The power spectrum of the comoving curvature perturbation $\mathcal{R}$ is usually parameterized as [17]

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{\dot{\phi}^2}\ \mathcal{P}_{\delta\phi}(k) \equiv A_{\mathrm{s}}^2\left(\frac{k}{k_{\mathrm{p}}}\right)^{n_{\mathrm{s}}-1}\cdot\frac{(k/k_{\mathrm{p}})^3}{[(k/k_{\mathrm{p}})^2+\varepsilon^{*2}]^{3/2}}, \tag{18}$$

where $k_{\mathrm{p}}=0.05\ \mathrm{Mpc}^{-1}$ and $\varepsilon^*=\varepsilon_\gamma^*/k_{\mathrm{p}}$. A_{s} is the power amplitude and n_{s} is the scalar spectral index.

We plot the primordial spectrum (18) in Fig. 1.

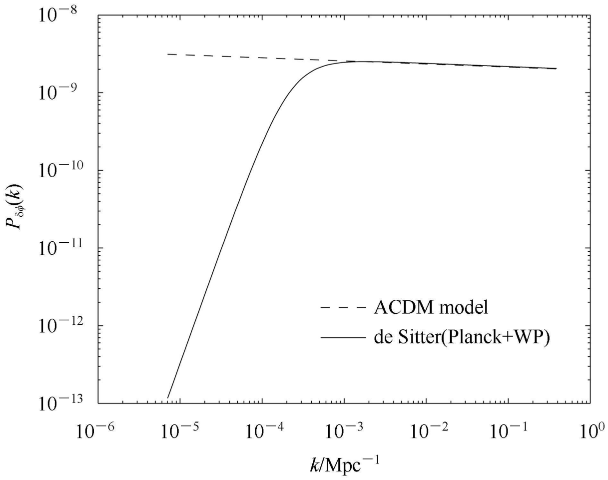

Fig. 1. The primordial power spectrum $P_{\delta\phi}(k)$. The dashed line stands for the $P_{\delta\phi}(k)$ in standard ΛCDM model and the solid line represents the $P_{\delta\phi}(k)$ in de Sitter spacetime. One can see that there is a cutoff at large scales. This should lead to a suppressed TT spectrum when ℓ is small.

3 Numerical results

We use the Markov Chain Monte Carlo sampler (CosmoMC) to perform a global fitting of the cosmological parameters. The Planck TT [3] and WMAP polarization [2] datasets will be used in the global fitting. There are six parameters which come from the ΛCDM model and one extra parameter ε^*. The results are shown in Table 1 and Fig. 1.

Table 1. The 68% limits for the cosmological parameters originating in the ΛCDM and de Sitter models with data combination Planck+WP.

parameter	ΛCDM model	de Sitter
$\Omega_{\rm b}h^2$	0.02205±0.00028	0.02203±0.00028
$\Omega_{\rm c}h^2$	0.1199±0.0027	0.1200±0.0026
$100\theta_{\rm MC}$	1.04131±0.00063	1.04121±0.00063
τ	$0.089^{+0.012}_{-0.014}$	0.098±0.015
$n_{\rm s}$	0.9603±0.0073	0.9585±0.0073
$\ln(10^{10}A^{\rm s})$	$3.089^{+0.024}_{-0.027}$	3.106±0.030
$100\varepsilon^*$	—	0.4266±0.1945

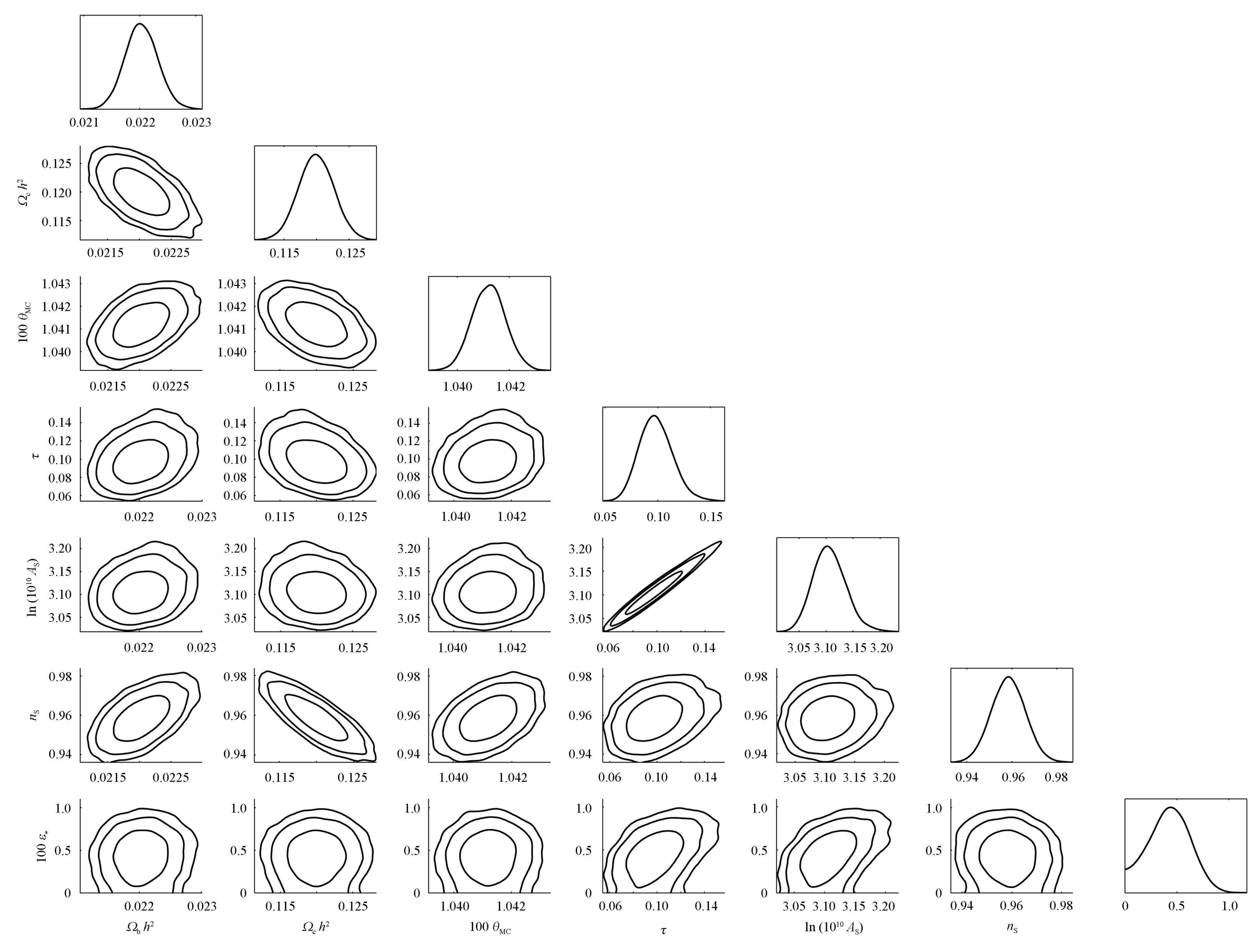

Fig. 2. Contour plots and likelihood distributions of the seven cosmological parameters with data combination Planck+WP in de Sitter spacetime.

In Table 1, one can see that the values of the six basic parameters in de Sitter spacetime are similar to those based on the standard cosmological model [3]. We are more interested in the new parameter ε_γ^*. With $\varepsilon_\gamma^*=2.13\times10^{-4}$ Mpc^{-1}, we can calculate the wavelength related to ε_γ^*, i.e. $\lambda=4.69\times10^3$ Mpc. The wavelength is comparable to the Hubble horizon. From Fig. 2, one can see that the combined datasets favour the new parameter ε_γ^* being non-zero at around 1σ confidence level.

Finally, we use the modified version of CAMB to compute the new CMB temperature-temperature spectrum in de Sitter spacetime with the global fitting results. The obtained TT spectrum is obviously suppressed at large scales.

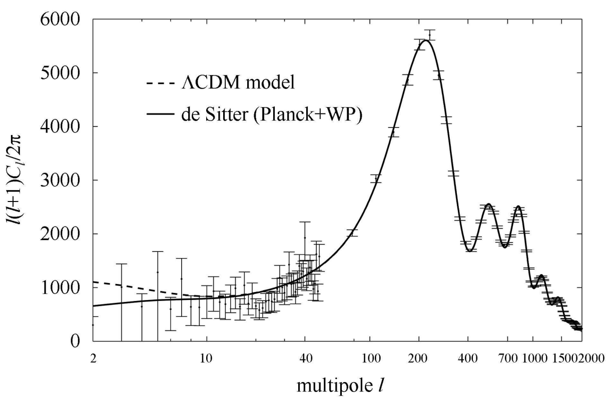

Fig. 3. TT spectra with data combination Planck+WP. The dashed line shows the TT spectrum based on the standard ΛCDM model and the solid line indicates the TT spectrum in de Sitter spacetime.

In Fig. 3, the dashed line indicates the best-fit CMB TT spectrum in the standard ΛCDM model and the solid line represents the TT spectrum in de Sitter spacetime. Compared to the TT spectrum in the ΛCDM model, the spectrum in de Sitter spacetime is obviously suppressed when $\ell < 20$. At the quadrupole in particular, the spectrum drops to its lowest point, almost half the value of the standard TT spectrum.

4 Conclusions

In this paper, we analyzed an inflation model in de Sitter spacetime and got a modified primordial spectrum. Possible effects of cosmological constant-type dark energy were considered during the inflationary period. We got a lower TT spectrum at large scales, which refers to the low-ℓ multipole anomaly. The new scenario has a new parameter, namely ε_γ^*. To constrain the cosmological parameters, we used the CosmoMC package to perform a global fit with Planck TT and WMAP polarization datasets. We found that the results of global fitting in de Sitter spacetime are similar to those based on the standard ΛCDM model. With $\varepsilon_\gamma^* = 2.13\times10^{-4}$ Mpc^{-1}, we calculated the wavelength related to ε_γ^*, i.e. $\lambda = 4.69\times10^3$ Mpc. The wavelength is comparable to the Hubble horizon. The combined datasets favour that the new parameter ε_γ^* is non-zero at about 1σ confidence level. The new TT spectrum shows lower energy at large scales, which is just as would be expected.

We are grateful to Dr. Hai-Nan Lin, Si-Yu Li, Sai Wang and Yu Sang for useful discussions.

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