Mass spectra and Regge trajectories of Λ_c^+, Σ_c^0, Ξ_c^0 and Ω_c^0 baryons *

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Abstract: We calculate the mass spectra of the singly charmed baryons (Λ_c^+, Σ_c^0, Ξ_c^0 and Ω_c^0) using the hypercentral constituent quark model (hCQM). The hyper color Coulomb plus linear potential is used to calculate the masses of positive (up to $J^p = \frac{7}{2}^+$) and negative (up to $J^p = \frac{9}{2}^-$) parity excited states. The spin-spin, spin-orbital and tensor interaction terms are also incorporated for mass spectra. We have compared our results with other theoretical and lattice QCD predictions for each baryon. Moreover, the known experimental results are also reasonably close to our predicted masses. By using the radial and orbital excitation, we construct Regge trajectories for the baryons in the (n, M^2) plane and find their slopes and intercepts. Other properties of these baryons, like magnetic moments, radiative transitions and radiative decay widths, are also calculated successfully.

Keywords: baryons, potential model, Regge trajectories

PACS: 12.39.pn, 12.40.Yx, 14.20.-c **DOI:** 10.1088/1674-1137/40/12/123102

1 Introduction

The measurement and calculation of baryonic excited states are an important area of activity for worldwide experimental facilities such as CLEO, Belle, BABAR and LHCb [1–5] as well as for lattice QCD calculations [6, 7]. Baryons made of light and heavy flavor quarks are an interesting challenge because one has to deal with quark masses at various mass scales. Rapid development has been made in the heavy baryon sector and many new excited states of these heavy flavour baryons are also being investigated [8]. The doubly and triply heavy flavor baryons are determined with different approaches [9–12]. In working towards an understanding of QCD at different energy scales, a rich dynamical study on heavy flavor baryons with their properties is necessary [13]. The mass spectra, width, lifetime, decays and form factors have been often reported by numerous experimental groups but the spin and parity identification of some states are still missing. The future experiments at J-PARC, PANDA [14] and LHCb are expected to give further information on charmed baryons soon.

Various phenomenological models have been used to study heavy baryons by different approaches such as the non-relativistic Isgur-Karl model [15], relativized potential quark model [16], relativistic quark model [17], variational approach [18], relativistic flux tube (RFT) model [19], Hamiltonian model [20], heavy quark symmetry [21], the Fadeev approach [22], the algebraic approach [23], the Goldstone boson exchange model [24], the interacting quark-diquark model [25], the Feynman-Hellmann theorem [26], the hypercentral model [27], the combined expansion in $1/m_Q$ and $1/N_c$ [28], the chiral quark model [29], QCD sum rules [30], a soliton model [31] and many more. Also, many lattice QCD studies have been done for heavy flavor baryons [6, 7, 32–36]. In all the models they first reproduce the baryon spectra and later describe various baryonic properties. A recent review article explained different quark models along with the hypercentral constituent quark model and its application to baryonic properties [37].

The hypercentral constituent quark model (hCQM) with color Coulomb plus power potential has been used for systematic calculations of ground state masses of various baryons [38–41]. The hCQM scheme amounts to an average two-body potential for the three quark system over the hyper angle. After successful calculation of ground state masses, we are using this model to calculate the masses of singly charmed baryons up to higher radial and orbital excitations. In this paper, we use the hyper color Coulomb plus linear potential, as we have used in case of spectroscopy of mesons [42]. We study four singly charmed baryons Λ_c^+(udc), Σ_c^0(ddc), Ξ_c^0(dsc) and Ω_c^0(ssc) and compute their excited states (radial and orbital) masses. As we know, in $SU(3)$ multiplicity Σ_c^0

Received 6 August 2016
∗ Supported in part (A. K. Rai) by DST, India (SERB Fast Track Scheme SR/FTP/PS-152/2012)
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and Ω_c^0 are symmetric sextets(6_s), whereas Λ_c^+ and Ξ_c^0 are anti-symmetric triplets($\bar{3}_A$) [43]. The available experimental observed states of these baryons are listed in Table 1.

Table 1. The possible states for experimentally known (J^P) states.

name	mass	name	mass	name	mass	name	mass
$\Lambda_c(2286)^+$	2286.46±.014	$\Sigma_c(2455)^0$	2453.74±0.16	$\Xi_c(2468)^0$	2470.88	$\Omega_c(2695)^0$	2695.2 ± 1.7
$\Lambda_c(2595)^+$	2592.25±.028	$\Sigma_c(2520)^0$	2518.8±.06	$\Xi_c(2645)^0$	2645.9±0.5	$\Omega_c(2770)^0$	2765.9 ± 2.0
$\Lambda_c(2625)^+$	2628.11±0.19	$\Sigma_c(2800)^0$	2806±8±10	$\Xi_c(2790)^0$	2791.8±3.3		
$\Lambda_c(2880)^+$	2881.53±.035			$\Xi_c(2815)^0$	2819.6±1.2		
$\Lambda_c(2940)^+$	$2939.3^{+1.4}_{-1.5}$			$\Xi_c(3080)^0$	3079.9±1.4		
$\Lambda_c(2765)^+$	2766.6±2.4			$\Xi_c(2980)^0$	2968.0±2.6		

We also construct the Regge trajectories for these baryons in the (n, M^2) plane, where one can test their linearity, parallelism and equidistance. This study gives information about the hadron dynamics and is also important for hadron production and high energy scattering [17]. It is very important to reproduce the baryon spectrum in the model, likewise the study of other properties is also important. Hence, we calculate the magnetic moment and transition magnetic moment using effective quark masses. We also determine the radiative decay widths.

This paper is organized as follows. A description of the hypercentral constituent quark model (hCQM) is given in Section 2. A systematic mass spectroscopy calculation has been performed in this model for four singly charmed baryons. We analyze and discuss our results in Section 3. We plot the Regge trajectories and explain these for these baryons in Section 4. The magnetic moments and radiative decay widths are determined for these baryons in Section 5. Finally, we draw conclusions in Section 6.

2 The model

Baryons are made of three constituent quarks. The relevant degrees of freedom for their motion are related by the Jacobi coordinates ($\vec{\rho}$ and $\vec{\lambda}$) are given by Ref. [44] as

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}), \tag{1a}$$

$$\vec{\lambda} = \frac{m_1\vec{r_1} + m_2\vec{r_2} - (m_1+m_2)\vec{r_3}}{\sqrt{m_1^2 + m_2^2 + (m_1+m_2)^2}}. \tag{1b}$$

Here m_i and $\vec{r_i} (i = 1,2,3)$ denote the mass and coordinates of the i-th constituent quark. In this paper, the confining three-body potential is chosen within a string-like picture, where the quarks are connected by gluonic strings and the potential strings increases linearly with a collective radius r_{3q} as mentioned in Ref. [37]. Accordingly the effective two body interactions can be written as

$$\sum_{i<j} V(r_{ij}) = V(x) + \cdots \ . \tag{2}$$

The hyper radius x is a collective coordinate in the hypercentral approximation and therefore the hypercentral potential contains the three-body effects. A more detailed description is given in Refs. [37, 45]. The Hamiltonian of a three body baryonic system in the hCQM is then expressed as

$$H = \frac{P_x^2}{2m} + V(x), \tag{3}$$

where $m = \dfrac{2m_\rho m_\lambda}{m_\rho + m_\lambda}$ is the reduced mass and x is the six-dimensional radial hyper central coordinate of the three body system. The respective reduced masses are given by

$$m_\rho = \frac{2m_1m_2}{m_1+m_2}, \tag{4a}$$

$$m_\lambda = \frac{2m_3(m_1^2+m_2^2+m_1m_2)}{(m_1+m_2)(m_1+m_2+m_3)}. \tag{4b}$$

We consider the masses of light quarks (u,d) as unequal. The constituent quark mass parameters used in our calculations are:

- m_u=0.338, m_d=0.350, m_s=0.500 and m_c=1.275. (all in GeV)

The angles of the hyper spherical coordinates are given by $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$. We define the hyper radius x and hyper angle ξ in terms of the absolute values ρ and λ of the Jacobi coordinates [46–48],

$$x = \sqrt{\rho^2 + \lambda^2} \text{ and } \xi = \arctan\left(\frac{\rho}{\lambda}\right), \tag{5}$$

In the center of mass frame ($R_{\rm c.m.} = 0$), the kinetic energy operator can be written as

$$-\frac{\hbar^2}{2m}(\Delta_\rho + \Delta_\lambda) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} + \frac{L^2(\Omega)}{x^2}\right), \tag{6}$$

where $L^2(\Omega)=L^2(\Omega_\rho,\Omega_\lambda,\xi)$ is the quadratic Casimir operator of the six-dimensional rotational group $O(6)$ and its eigenfunctions are the hyperspherical harmonics $Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi)$ satisfying the eigenvalue relation $L^2 Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi)=-\gamma(\gamma+4)Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi)$. Here, γ is the grand angular momentum quantum number. The hyperradial Schrodinger equation corresponding to the Eq. (3) Hamiltonian can be written as

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\frac{5}{x}\frac{\mathrm{d}}{\mathrm{d}x}-\frac{\gamma(\gamma+4)}{x^2}\right]\Psi_\gamma(x)=-2m[E-V(x)]\Psi_\gamma(x), \tag{7}$$

where Ψ_γ(x) is the hypercentral wave function and γ is the grand angular quantum number. We consider a reduced hypercentral radial function, $\phi_\gamma(x)=x^{\frac{5}{2}}\Psi_\gamma(x)$. Thus, the six-dimensional hyperradial Schrodinger equation reduces to

$$\left[\frac{-1}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\frac{\frac{15}{4}+\gamma(\gamma+4)}{2mx^2}+V(x)\right]\phi_\gamma(x)=E\phi_\gamma(x). \tag{8}$$

We now consider the hypercentral potential $V(x)$ as the color Coulomb plus linear potential with spin interaction

$$V(x)=V^0(x)+V_{\mathrm{SD}}(x), \tag{9}$$

where $V^0(x)$ is given by

$$V^{(0)}(x)=\frac{\tau}{x}+\beta x+V_0. \tag{10}$$

Here, the hyper-Coulomb strength $\tau=-\frac{2}{3}\alpha_\mathrm{s}$ where α_s corresponds to the strong running coupling constant; $\frac{2}{3}$ is the color factor for baryons, and β(=0.14) corresponds to the string tension for baryons. V_0 is a constant value used to obtain the ground state masses ($J^P=\frac{1}{2}^+$ and $\frac{3}{2}^+$)[42]. The values for Λ_c^+, Σ_c^0, Ξ_c^0 and Ω_c^0 are taken as $-0.575, -0.465, -0.468$ and -0.411, respectively.
The strong running coupling constant α_s is given by,

$$\alpha_\mathrm{s}=\frac{\alpha_\mathrm{s}(\mu_0)}{1+\frac{33-2n_\mathrm{f}}{12\pi}\alpha_\mathrm{s}(\mu_0)\ln\left(\frac{m_1+m_2+m_3}{\mu_0}\right)}. \tag{11}$$

We get α_s=0.6 with μ_0=1 GeV and 4 flavors. If we compare Eq. (8) with the usual three-dimensional radial Schrodinger equation, the resemblance between angular momentum and hyper angular momentum is given by [23], $l(l+1)\rightarrow\frac{15}{4}+\gamma(\gamma+4)$. The spin-dependent part of Eqn. (9), $V_{\mathrm{SD}}(x)$, contains three types of interaction terms, including the spin-spin term $V_{\mathrm{SS}}(x)$, the spin-orbit term $V_{\gamma\mathrm{S}}(x)$ and a tensor term $V_\mathrm{T}(x)$ given by [49],

$$\begin{aligned}V_{\mathrm{SD}}(x)=&V_{\mathrm{SS}}(x)(\vec{S_\rho}.\vec{S_\lambda})+V_{\gamma\mathrm{S}}(x)(\vec{\gamma}\cdot\vec{S})\\&+V_T(x)\left[S^2-\frac{3(\vec{S}\cdot\vec{x})(\vec{S}\cdot\vec{x})}{x^2}\right].\end{aligned} \tag{12}$$

The spin-orbit and the tensor term describe the fine structure of the states, while the spin-spin term gives the spin singlet triplet splittings. The coefficients of these spin-dependent terms of Eqn. (10) can be written in terms of the vector, $V_\mathrm{V}(x)=\frac{\tau}{x}$, and scalar, $V_\mathrm{S}(x)=\beta x$, parts of the static potential as

$$V_{\gamma\mathrm{S}}(x)=\frac{1}{2m_\rho m_\lambda x}\left(3\frac{\mathrm{d}V_\mathrm{V}}{\mathrm{d}x}-\frac{\mathrm{d}V_\mathrm{S}}{\mathrm{d}x}\right), \tag{13}$$

$$V_\mathrm{T}(x)=\frac{1}{6m_\rho m_\lambda}\left(3\frac{\mathrm{d}^2V_\mathrm{V}}{\mathrm{d}x^2}-\frac{1}{x}\frac{\mathrm{d}V_\mathrm{V}}{\mathrm{d}x}\right), \tag{14}$$

$$V_{\mathrm{SS}}(x)=\frac{1}{3m_\rho m_\lambda}\nabla^2 V_\mathrm{V}. \tag{15}$$

Instead of the six-dimensional delta function which appears in the spin-spin interaction term of Eq. (15), we use a smear function similar to the one given by [39, 41]

$$V_{\mathrm{SS}}(x)=\frac{-A}{6m_\rho m_\lambda}\frac{\mathrm{e}^{-x/x_0}}{xx_0^2}, \tag{16}$$

where x_0 is the hyperfine parameter of the model. We take $A=A_0/\left(n+\gamma+\frac{3}{2}\right)^2$, where A_0 is an arbitrary constant. The baryon spin average mass in this hypercentral model is given by $M_\mathrm{B}=\sum_{i=1}m_i+BE$. We numerically solve the six-dimensional Schrodinger equation using Mathematica notebook [50]. We have followed the ${}^{(2S+1)}\gamma_J$ notations for spectra of baryons.

3 Mass spectra: discussion

We have calculated the masses of the radial and orbital excited heavy baryons Λ_c^+, Σ_c^0, Ξ_c^0 and Ω_c^0. Our masses of ground states are compared with different lattice QCD models [7, 32–36, 51] in Table 2. Refs. [52, 53] have also performed ground state calculations. The obtained masses of radial excited states charm baryons with $J=\frac{1}{2}$, $J=\frac{3}{2}$ and $J=\frac{5}{2}$ (positive parity) and the masses of negative parity states baryons with $J=\frac{1}{2}$, $J=\frac{3}{2}$, $J=\frac{5}{2}$ and $J=\frac{7}{2}$ are listed in Tables 3–6.

Until now, only Ref. [17] has focused on the mass spectra of the radial as well as orbital excited heavy baryons, looking at the $2S$-$5S$ and $1P$-$5P$, $1D$-$2D$ and $1F$ states using the relativistic quark potential model in the quark-diquark picture. Ref. [19] has calculated higher excited states of Λ_c^+ (up to the $3S$, $3P$, $1D$, $1F$ and $1G$ states) and Ξ_c (up to the $4S$ and $3P$, $1D$, $1F$ and $1G$ states) within the relativistic flux tube (RFT) model. A non-relativistic quark model with harmonic oscillator potential also showed excited mass spectra of Λ_c, Σ_c and Ω_c up to the $3S$, $3P$ and $3D$ states [20]. The excited mass specta in the HQS limit is calculated by [21] for Λ_c ($2S$-$4S$, $1P$, $1D$ and $1F$), Σ_c and Ξ_c ($2S$-$3S$, $1P$ and $1D$). Refs. [7, 18, 22] calculated the $1S$ and $1P$ states whereas [30] calculated only the $1P$ state for all these baryons. All these models are compared with our obtained results in their respective tables. Several S-wave, P-wave and D-wave single charm baryon masses are given in Table 1 with their known experimental masses. We anticipated heavy baryons having low lying states with J^P, $\frac{1}{2}^+$ and $\frac{3}{2}^+$ and the higher states with J^P, $\frac{5}{2}^+$, $\frac{7}{2}^+$, $\frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{7}{2}^-$ and $\frac{9}{2}^-$ in accordance to $S=\frac{1}{2}$ and $\frac{3}{2}$. The mass spectra of the Λ_c^+, Σ_c^0, Ξ_c^0 and Ω_c^0 baryons are given in subsections 3.1–3.4.

Table 2. Ground state masses of different lattice QCD results.

particle	J^P	our	Ref.[32]	Ref.[33]	Ref.[34]	Ref.[36]	Ref.[35]	[51]
Λ_c^+	$\frac{1}{2}^+$	2.286	2.254(48)(31)	2.333(112)(10)	2.272(26)(33)	2.291		-
Σ_c^0	$\frac{1}{2}^+$	2.452	2.474(48)(31)	2.467(39)(11)	2.445(32)	2.481	2.490	
Σ_c^0	$\frac{3}{2}^+$	2.518	2.551(43)(25)	2.538(70)(11)	2.513(38)	2.559	2.538	
Ξ_c^0	$\frac{1}{2}^+$	2.471	2.433(35)(30)	2.455(11)(20)	2.469(28)	-	2.473	
Ξ_c^0	$\frac{3}{2}^+$	2.647	2.648(70)(11)	2.674(26)(12)	2.628(33)	2.655	2.554	
Ω_c^0	$\frac{1}{2}^+$	2.695	2.679(37)(20)	2.673(05)(12)	2.629(22)	2.681	2.678	2.783(13)
Ω_c^0	$\frac{3}{2}^+$	2.767	2.738(05)(12)	2.755(37)(24)	2.709(26)	2.764	2.752	2.837(18)

Table 3. Mass spectra of Λ_c^+ baryon.

$n^{2S+1}L_J$	our	Exp.[8]	Ref.[17]	Ref.[19]	Ref.[20]	Ref.[18]	Ref.[21]	Ref.[22]	Ref.[30]	lattice[7]
$1^2S_{\frac{1}{2}}$	2.287	2.286±0.0014	2.286	2.286	2.285	2.268	2.268	2.285		2.280(41)
$2^2S_{\frac{1}{2}}$	2.758	2.766±0.024	2.769	2.766	2.857	2.791	2.791	2.785		
$3^2S_{\frac{1}{2}}$	3.134		3.130	3.112	3.123		2.983			
$4^2S_{\frac{1}{2}}$	3.477		3.430	3.397			3.154			
$5^2S_{\frac{1}{2}}$	3.787		3.715							
$(1^2P_{1/2})$	2.694	2.592±0.0028	2.589	2.591	2.628	2.625	2.625	2.627	2.600	2.578(289)
$(1^2P_{3/2})$	2.640	2.628±0.0019	2.627	2.629	2.630	2.816	2.830	2.880	2.650	
$(2^2P_{1/2})$	3.062	$2.939^{+1.4}_{-1.5}$	2.983	2.989	2.890	2.636				
$(2^2P_{3/2})$	3.015		3.005	3.000	2.917	2.830				
$(3^2P_{1/2})$	3.397		3.303	3.296	2.933	2.872				
$(3^2P_{3/2})$	3.354		3.222	3.301	2.956					
$(4^2P_{1/2})$	3.705		3.588							
$(4^2P_{3/2})$	3.668		3.606							
$(5^2P_{1/2})$	3.997		3.852							
$(5^2P_{3/2})$	3.962		3.869							
$(1^2D_{3/2})$	2.924		2.874	2.857			3.12			
$(1^2D_{5/2})$	2.854	2.881±0.0035	2.880	2.879	2.922	2.887	3.125	2.888	2.882	
$(2^2D_{3/2})$	3.263		3.189	3.188			3.194			
$(2^2D_{5/2})$	3.204		3.209	3.198	3.202		3.194			
$(1^2F_{5/2})$	3.130		3.097	3.075			3.092			
$(1^2F_{7/2})$	3.052		3.078	3.092			3.128			

3.1 Λ_c^+ baryon

Λ_c^+ was the first singly charmed baryon to be discovered at BNL [54] in 1975. The experimental known and unknown states are tabulated in Table 1. The experimental known ground state is $\Lambda_c(2286)^+$. The first orbital excited states are $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$ with quantum numbers $\Lambda_c^+\left(\frac{1}{2}^-\right)$ and $\Lambda_c^{*+}\left(\frac{3}{2}^-\right)$ respectively. $\Lambda_c(2880)^+$ with $J^P=\frac{5}{2}^+$ is also known. Two more states, namely $\Lambda_c(2765)^+$ and $\Lambda_c(2940)^+$, were observed by the CLEO collaboration [2] and Babar collaboration [4] but their J^P values are still unknown. We have calculated the mass spectra of the Λ_c^+ baryon for the $1S$-$5S$, $1P$-$5P$, $1D$-$2D$ and $1F$ states (see Table 3).

Our ground state is close to the lattice QCD results (see Table 2) and other theoretical predictions. The radial excited states $2S$-$5S$ show (≈4–78 MeV) difference from Ref. [17]. Comparisons with other models are also given. We obtained 2.758 GeV in the $2S$ state, which is close to the results from Refs. [8, 17, 19]. Thus, we assigned $J^P=\frac{1}{2}^+$ to $\Lambda_c(2765)^+$. The first orbital $\Lambda_c^+\left(\frac{1}{2}^-\right)$ state value is 102 MeV less than our result, but Refs. [18, 20–22] are closer while the $\Lambda_c^{*+}\left(\frac{3}{2}^-\right)$ state difference value is only 12 MeV. Our $2P$ state value is higher then other predictions, though we would like to assign $\Lambda_c^{*+}\left(\frac{3}{2}^-\right)$ as a $2P$ state with 123 MeV difference. Ebert et al. [17], Chen et al. [19] and Yamaguchi et al. [21] have calculated D $\left(\frac{3}{2}^+,\frac{5}{2}^+\right)$ and F $\left(\frac{5}{2}^-,\frac{7}{2}^-\right)$ states and our values are higher than theirs. Our $1D$ state $\left(\frac{5}{2}^+\right)$ is also closer to the experimental value (27 MeV difference).

3.2 Σ_c^0 baryon

PDG(2014) has listed two ground states $\Sigma_c(2455)^0$ and $\Sigma_c(2520)^0$ with their J^P values $\frac{1}{2}^+$ and $\frac{3}{2}^+$. $\Sigma_c(2800)$ is also found but the quantum number is undefined. We have calculated the mass spectroscopy of the Σ_c^+ baryon for the $1S$-$5S$, $1P$-$4P$, $1D$-$2D$ and $1F$ states (see Table 4).

The ground states are close to the lattice QCD results (see Table 2) as well as experimental results. The radial excited states $2S$-$5S$ show ≈ 10–39 MeV difference for Σ_c and ≈ 19–33 MeV difference for Σ_c^* compared to Ref. [17]. These results are also close to those of Refs. [20, 21]. The J^P value of the $\Sigma_c(2800)$ state is unknown experimentally. We compare our $1P$ state with J^P $\left(\frac{1}{2}^-,\frac{3}{2}^-\text{ and }\frac{5}{2}^-\right)$ and it shows 3, 51 and 96 MeV difference respectively. These states are also calculated by Refs. [7, 17, 18, 20, 21, 30] and are reasonably close. Ref. [18, 21] shows only 35 MeV difference and [17] shows 7 MeV difference with our $1D$ $\left(\frac{5}{2}^+\right)$ state. Ref. [55] has also calculated $J^P\left(\frac{1}{2}^-\text{ and }\frac{3}{2}^-\right)$ and obtained 2.74±0.20 GeV and Ref. [56] has also calculated $m(\Sigma_c)=2.84\pm0.11$ GeV for $J^P=\frac{3}{2}^-$.

3.3 Ξ_c^0 baryon

The Ξ_c baryon was announced by the Belle [3] and BABAR collaborations [4]. $\Xi_c(2470)$ and $\Xi_c^*(2645)$ are the two lowest-lying ground states. The first orbital excited states $\Xi_c(2790)$ with $J^P=\frac{1}{2}^-$ and $\Xi_c^*(2815)$ with $J^P=\frac{3}{2}^-$ are well established experimentally [8]. The 1D state $\left(\frac{5}{2}^+\right)$ $\Xi_c^0(3080)$ is also experimentally known. $\Xi_c(2930)$ has only been observed by BABAR [4] and $\Xi_c(2980)^0$ has been found by the Belle collaboration [5]. Although these baryons are well established experimentally, their quantum numbers are still unknown. We have calculated the mass spectroscopy of the Ξ_c^0 baryon for the $1S$-$5S$, $1P$-$4P$, $1D$-$2D$ and $1F$ states (see Table 5).

The ground state Ξ_c^0 is in accordance with different phenomenological models and lattice QCD results (Table 2). The $\Xi_c^{*0}\left(\frac{3}{2}^+\right)$ results are perfectly matched with Refs. [8, 22, 32]. Our $2S$ state mass is 31 MeV lower than $\Xi_c(2980)^0$. Other theoretical results [17, 19, 21] are also close to the experimental result. So, it could be a $2S\left(\frac{1}{2}^+\right)$ state. Our obtained $1P$ state shows a higher value than Refs. [7, 8, 17, 19–22, 30] while the other $1P$ state with $J^P\left(\frac{3}{2}^-\right)$ is closer to the experimental result and all the above models. We compare our $\Xi_c^0\left(\frac{1}{2}^-\right)$ with one more $\Xi_c(2930)$ state. We predict this unknown state as $1P$, as the difference of the values is just 32 MeV. The $1D$ state mass is 21 MeV lower than the experimental value. The rest of the states ($2D$ and $1F$) with $S=\frac{1}{2}$ are also reasonably close to Refs. [17, 19].

3.4 Ω_c^0 baryon

The WA62 collaboration announced the first ground state of Ω_c [57]. This baryon is the least known experimentally. Only two ground states $\Omega_c(2695)$ and $\Omega_c(2770)$ have been observed yet, though we have calculated the mass spectrum of the Ω_c^0 baryon for the $1S$-$5S$, $1P$-$4P$, $1D$-$2D$ and $1F$ states (see Table 6) like other baryons.

Table 4. Mass spectra of Σ_c^0 baryon.

$n^{2S+1}L_J$	our	Exp. [8]	Ref.[17]	Ref.[18]	Ref.[20]	Ref.[21]	Ref.[22]	[30]	lattice [7]
$1^2S_{\frac{1}{2}}$	2.452	2.453±0.0016	2.443	2.455	2.460	2.455	2.435		2.434(46)
$2^2S_{\frac{1}{2}}$	2.891		2.901	2.958	3.029	2.958	2.904		
$3^2S_{\frac{1}{2}}$	3.261		3.271		3.271	3.115			
$4^2S_{\frac{1}{2}}$	3.593		3.581						
$5^2S_{\frac{1}{2}}$	3.900		3.861						
$1^4S_{\frac{3}{2}}$	2.518	2.518±0.0006	2.519	2.519	2.523	2.519	2.502		3.713(16)
$2^4S_{\frac{3}{2}}$	2.917		2.936	2.995	3.065	2.995	2.944		
$3^4S_{\frac{3}{2}}$	3.274		3.293		3.094	3.116			
$4^4S_{\frac{3}{2}}$	3.601		3.598						
$5^4S_{\frac{3}{2}}$	3.906		3.873						
$(1^2P_{1/2})$	2.809	2.806±0.0018	2.799	2.748	2.802	2.848	2.772	2.730	3.785(16)
$(1^2P_{3/2})$	2.755		2.798	2.763	2.807	2.763	2.772	2.800	2.740(99)
$(1^4P_{1/2})$	2.835		2.713	2.768					
$(1^4P_{3/2})$	2.782		2.773	2.776					
$(1^4P_{5/2})$	2.710		2.789	2.790	2.839	2.790		2.890	
$(2^2P_{1/2})$	3.174		3.172		2.826		2.893		
$(2^2P_{3/2})$	3.128		3.172		2.837				
$(2^4P_{1/2})$	3.196		3.125						
$(2^4P_{3/2})$	3.151		3.151						
$(2^4P_{5/2})$	3.090		3.161		3.316				
$(3^2P_{1/2})$	3.505		3.488		2.909				
$(3^2P_{3/2})$	3.465		3.486		2.91				
$(3^4P_{1/2})$	3.525		3.455						
$(3^4P_{3/2})$	3.485		3.469						
$(3^4P_{5/2})$	3.433		3.475		3.521				
$(4^2P_{1/2})$	3.814		3.770						
$(4^2P_{3/2})$	3.777		3.768						
$(4^4P_{1/2})$	3.832		3.743						
$(4^4P_{3/2})$	3.796		3.753						
$(4^4P_{5/2})$	3.747		3.757						
$(1^4D_{1/2})$	3.036		3.041						
$(1^2D_{3/2})$	3.112		3.043			3.095			
$(1^4D_{3/2})$	3.061		3.040						
$(1^2D_{5/2})$	2.993		3.038	3.003	3.099	3.003			
$(1^4D_{5/2})$	2.968		3.023						
$(1^4D_{7/2})$	2.909		3.013			3.015			
$(2^4D_{1/2})$	3.376		3.370						
$(2^2D_{3/2})$	3.398		3.366						
$(2^4D_{3/2})$	3.442		3.364						
$(2^2D_{5/2})$	3.316		3.365		3.114				
$(2^4D_{5/2})$	3.339		3.349						
$(2^4D_{7/2})$	3.265		3.342						
$(1^4F_{3/2})$	3.332		3.288						
$(1^2F_{5/2})$	3.245		3.283						
$(1^4F_{5/2})$	3.268		3.254						
$(1^4F_{7/2})$	3.189		3.253						
$(1^2F_{7/2})$	3.165		3.227						
$(1^4F_{9/2})$	3.094		3.209						

Table 5. Mass spectra of Ξ_c^0 baryon.

$n^{2S+1}L_J$	our	Exp. [8]	Ref.[17]	Ref.[19]	Ref.[21]	Ref.[22]	Ref.[30]	lattice [7]
$1^2S_{\frac{1}{2}}$	2.471	$2.470^{+0.34}_{-0.80}$	2.476	2.467	2.466	2.471		2.442(31)
$2^2S_{\frac{1}{2}}$	2.937	2.968± 0.0026	2.959	2.959	2.924			
$3^2S_{\frac{1}{2}}$	3.303		3.323	3.325	3.183			
$4^2S_{\frac{1}{2}}$	3.626		3.642	3.629				
$5^2S_{\frac{1}{2}}$	3.921		3.909					
$1^4S_{\frac{3}{2}}$	2.647	2.645± 0.0005				2.642		2.608(35)
$2^4S_{\frac{3}{2}}$	3.004							
$3^4S_{\frac{3}{2}}$	3.338							
$4^4S_{\frac{3}{2}}$	3.646							
$5^4S_{\frac{3}{2}}$	3.934							
$(1^2P_{1/2})$	2.877	2.791± 0.033	2.792	2.779	2.773	2.799	2.790	2.761(156)
$(1^2P_{3/2})$	2.834	2.819± 0.012	2.819	2.814	2.783	2.902	2.830	2.891(68)
$(1^4P_{1/2})$	2.899	2.931±0.0008						
$(1^4P_{3/2})$	2.856							
$(1^4P_{5/2})$	2.798							
$(2^2P_{1/2})$	3.222		3.179	3.195				
$(2^2P_{3/2})$	3.189		3.201	3.204				
$(2^4P_{1/2})$	3.239							
$(2^4P_{3/2})$	3.206							
$(2^2P_{5/2})$	3.162							
$(3^2P_{1/2})$	3.544		3.5	3.521				
$(3^2P_{3/2})$	3.512		3.519	3.525				
$(3^4P_{1/2})$	3.561							
$(3^4P_{3/2})$	3.528							
$(3^4P_{5/2})$	3.484							
$(4^2P_{1/2})$	3.837		3.785					
$(4^2P_{3/2})$	3.808		3.804					
$(4^4P_{1/2})$	3.851							
$(4^4P_{3/2})$	3.823							
$(4^4P_{5/2})$	3.784							
$(1^4D_{1/2})$	3.147							
$(1^2D_{3/2})$	3.109		3.059	3.055				
$(1^4D_{3/2})$	3.090							
$(1^2D_{5/2})$	3.058	3.079± 0.0017	3.076	3.076	3.049	3.071		
$(1^4D_{5/2})$	3.039							
$(1^4D_{7/2})$	2.995							
$(2^4D_{1/2})$	3.470							
$(2^2D_{3/2})$	3.417		3.388	3.407				
$(2^4D_{3/2})$	3.434							
$(2^2D_{5/2})$	3.701		3.407	3.416				
$(2^4D_{5/2})$	3.388							
$(2^4D_{7/2})$	3.330							
$(1^4F_{3/2})$	3.360							
$(1^2F_{5/2})$	3.291		3.278	3.286				
$(1^4F_{5/2})$	3.310							
$(1^4F_{7/2})$	3.247							
$(1^2F_{7/2})$	3.228		3.292	3.301				
$(1^4F_{9/2})$	3.172							

Table 6. Mass spectra of Ω_c^0 baryon.

$n^{2S+1}L_J$	our	Exp. [8]	Ref.[17]	Ref.[18]	Ref.[20]	Ref.[30]	Ref.[22]	Ref.[7]	Ref.[21]
$1^2S_{\frac{1}{2}}$	2.695	2.695±0.0017	2.698	2.718	2.731		2.699	2.648(28)	2.718
$2^2S_{\frac{1}{2}}$	3.100		3.088	3.152	3.227		3.159	2.709(32)	3.152
$3^2S_{\frac{1}{2}}$	3.436		3.489		3.292				3.275
$4^2S_{\frac{1}{2}}$	3.737		3.814		3.814				3.299
$5^2S_{\frac{1}{2}}$	4.015		4.102		4.102				
$1^4S_{\frac{3}{2}}$	2.767	2.766±0.0002	2.768	2.776	2.779		2.767		
$2^4S_{\frac{3}{2}}$	3.126		3.123	3.190	3.257		3.202		
$3^4S_{\frac{3}{2}}$	3.450		3.51		3.285				
$4^4S_{\frac{3}{2}}$	3.745		3.83		3.83				
$5^4S_{\frac{3}{2}}$	4.021		4.114		4.114				
$(1^2P_{1/2})$	3.011		3.055	2.977	3.030	3.250	2.980	2.995(46)	3.046
$(1^2P_{3/2})$	2.976		3.054	2.986	3.033	3.260	2.980	3.016(69)	2.986
$(1^4P_{1/2})$	3.028		2.966	2.977			3.035		
$(1^4P_{3/2})$	2.993		3.029	2.959					
$(1^4P_{5/2})$	2.947		3.051	3.014	3.057	3.320			3.014
$(2^2P_{1/2})$	3.345		3.435		3.048		3.125		
$(2^2P_{3/2})$	3.315		3.433		3.056				
$(2^4P_{1/2})$	3.359		3.384						
$(2^4P_{3/2})$	3.330		3.415						
$(2^4P_{5/2})$	3.290		3.427		3.477				
$(3^2P_{1/2})$	3.644		3.754		3.048				
$(3^2P_{3/2})$	3.620		3.752		3.056				
$(3^4P_{1/2})$	3.656		3.717						
$(3^4P_{3/2})$	3.632		3.737						
$(3^4P_{5/2})$	3.601		3.744		3.620				
$(4^2P_{1/2})$	3.926		4.037						
$(4^2P_{3/2})$	3.903		4.036						
$(4^4P_{1/2})$	3.938		4.009						
$(4^4P_{3/2})$	3.915		4.023						
$(4^4P_{5/2})$	3.884		4.028						
$(1^4D_{1/2})$	3.215		3.287						
$(1^2D_{3/2})$	3.231		3.298						
$(1^4D_{3/2})$	3.262		3.282						
$(1^2D_{5/2})$	3.188		3.297	3.196	3.288				3.273
$(1^4D_{5/2})$	3.173		3.286						
$(1^4D_{7/2})$	3.136		3.283						
$(2^4D_{1/2})$	3.524		3.623						
$(2^2D_{3/2})$	3.538		3.627						
$(2^4D_{3/2})$	3.565		3.613						
$(2^2D_{5/2})$	3.502		3.626						
$(2^4D_{5/2})$	3.488		3.614						
$(2^4D_{7/2})$	3.456		3.611						
$(1^4F_{3/2})$	3.457		3.533						
$(1^2F_{5/2})$	3.403		3.522						
$(1^4F_{5/2})$	3.418		3.515						
$(1^4F_{7/2})$	3.369		3.54						
$(1^2F_{7/2})$	3.354		3.498						
$(1^4F_{9/2})$	3.310		3.485						

Our ground states Ω_c^0 and Ω_c^{*0} are in good agreement with other theoretical predictions as well as experimental measurements and lattice QCD results. We mainly compare our results with Ebert et al. [17] and Yoshida et al. [20]. As we move to higher excited states we can observe that the difference of masses compared to other models increases, though our obtained masses are lower than theirs. Our results are ($\approx$ 100 MeV) close to the W. Roberts calculation [18].

4 Regge trajectories

The calculated masses are used to plot Regge trajectories for Λ_c^+, Σ_c^0, Ξ_c^0 and Ω_c^0 in the $M^2 \to n$ plane. The output of the graphs show good agreement with experimental measurements. We use the (n, M^2) Regge trajectories

$$n = \beta M^2 + \beta_0, \tag{17}$$

where β and β_0 are the slope and intercept, respectively, and $n = \boldsymbol{n} - 1$, where $\boldsymbol{n}$ is the principal quantum number. The values of β and β_0 are shown in Table 7 for the different baryons. As described in the previous section, we have calculated the masses of the S, P and D states which are used to construct Regge trajectories in the (n, M^2) plane. The ground and radial excited states S with $J^P = \frac{1}{2}^+$ and the orbital excited state P with $J^P = \frac{1}{2}^-$, D with $J^P = \frac{5}{2}^+$ are plotted. These trajectories for different baryons are presented in Figs. 1–2.

Table 7. Fitted slope (β) and intercept (β_0) of the Regge trajectories

baryon	J^P	state	β	β_0
Λ_c	$\frac{1}{2}^+$	S	0.440±0.0029	−2.327±0.030
Λ_c	$\frac{1}{2}^-$	P	0.459±0.003	−3.314±0.036
Λ_c	$\frac{5}{2}^+$	D	0.472	−3.841
Σ_c	$\frac{1}{2}^+$	S	0.436±0.001	−2.628±0.015
Σ_c	$\frac{1}{2}^-$	P	0.451±0.003	−3.548±0.029
Σ_c	$\frac{5}{2}^+$	D	0.459	−4.230
Ξ_c	$\frac{1}{2}^+$	S	0.4335±0.006	−2.696±0.073
Ξ_c	$\frac{1}{2}^-$	P	0.465±0.002	−3.841±0.028
Ξ_c	$\frac{5}{2}^+$	D	0.497	−4.651
Ω_c	$\frac{1}{2}^+$	S	0.453±0.005	−3.320±0.054
Ω_c	$\frac{1}{2}^-$	P	0.473±0.001	−4.285±0.017
Ω_c	$\frac{5}{2}^+$	D	0.476	−4.790

Λ_c^+ and Ξ_c^0 are presented in Fig. 1 and Σ_c^0 and Ω_c^0 are shown in Fig. 2. Our obtained results up to ($L=2$) are shown with the experimental states. These known charmed baryons are mentioned with their names in all the figures. Straight lines were obtained by linear fitting in both figures. We observe that the square of the calculated masses fits very well to a linear trajectory and is almost parallel and equidistant in the S, P and D states. We can determine the possible quantum numbers and prescribe them to a particular Regge trajectory with the help of our obtained results.

5 Magnetic moments and radiative decays

The magnetic moments of baryons are obtained in terms of the spin, charge and effective mass of the bound quarks as [38–41, 58]

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_{iz} | \phi_{sf} \rangle)$$

where

$$\mu_i = \frac{e_i \sigma_i}{2 m_i^{\mathrm{eff}}}, \tag{18}$$

where e_i is a charge and σ_i is the spin of the respective constituent quark corresponding to the spin flavor wavefunction of the baryonic state. The effective mass for each of the constituting quarks m_i^{eff} can be defined as

$$m_i^{\mathrm{eff}} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i} \right) \tag{19}$$

where $\langle H \rangle$ = E + $\langle V_{\mathrm{spin}} \rangle$.

The electromagnetic radiative decay width can be expressed in terms of the radiative transition magnetic moment (in μ_N) and photon energy (k) as [41, 60]

$$\gamma_r = \frac{k^3}{4\pi} \frac{2}{2J+1} \frac{e^2}{m_p^2} \mu^2_{B \to B'}. \tag{20}$$

Here, m_p is the proton mass. $\mu^2_{B \to B'}$ is the radiative transition magnetic moment (in nuclear magnetons), which is expressed in terms of the magnetic moments of the constituent quarks (μ_i) of the initial and final state of the baryon. The radiative transition magnetic moment is calculated as (in terms of keV) [38, 41]

- $\Sigma_c^{*0} \to \Sigma_c^0$ $\quad \frac{2\sqrt{2}}{3}(\mu_d - \mu_c)$ $\quad -1.037$
- $\Xi_c^{*0} \to \Xi_c^0$ $\quad \frac{\sqrt{2}}{\sqrt{3}}(\mu_d - \mu_s)$ $\quad -0.182$
- $\Omega_c^{*0} \to \Omega_c^0$ $\quad \frac{2\sqrt{2}}{3}(\mu_u - \mu_s)$ $\quad -0.876$
- $\Sigma_c^0 \to \Lambda_c^+$ $\quad \frac{\sqrt{2}}{\sqrt{3}}(\mu_u - \mu_d)$ $\quad 1.844$

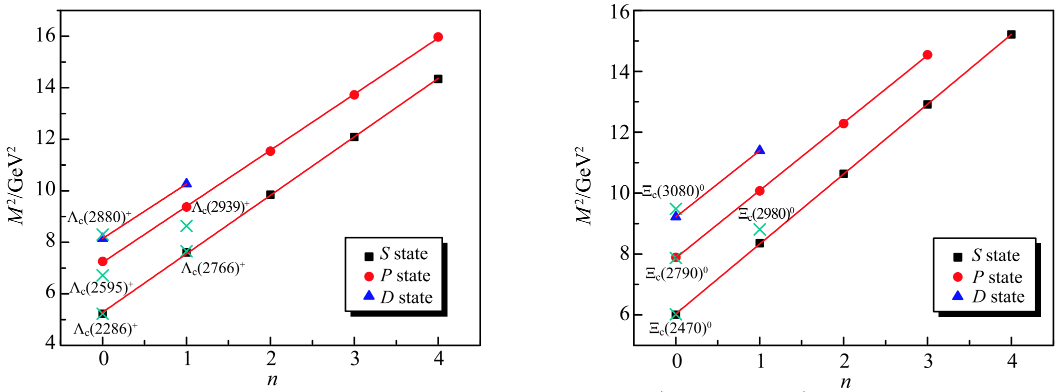

Fig. 1. Regge trajectory of Λ_c^0 (left) and Ξ_c^+ (right) for J^P values $\frac{1}{2}^+$, $\frac{1}{2}^-$ and $\frac{5}{2}^+$ with different states. Available experimental states are given with the particle name.

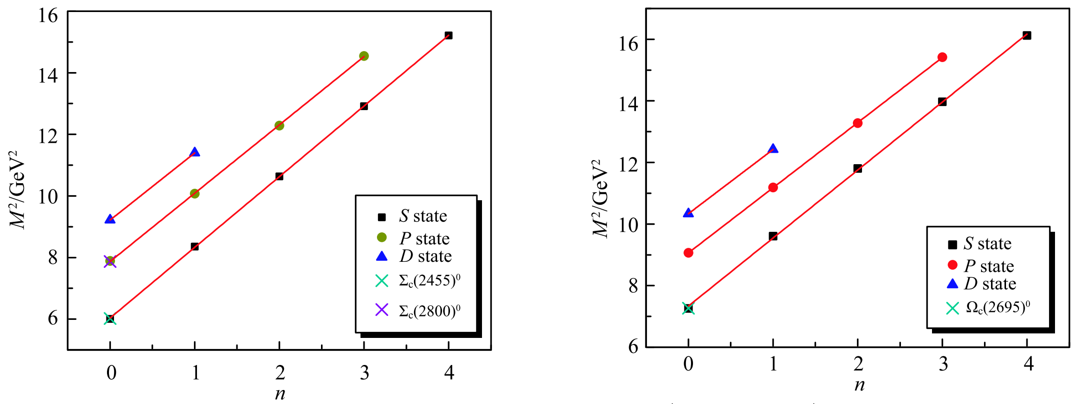

Fig. 2. Regge trajectory of Σ_c^0 (left) and Ω_c^+ (right) for J^P values $\frac{1}{2}^+$, $\frac{1}{2}^-$ and $\frac{5}{2}^+$ with different states. Available experimental states are given with the particle name.

Table 8. Magnetic moment (in nuclear magnetons) of J^P $\frac{1}{2}^+$ and $\frac{3}{2}^+$ singly charmed baryons.

baryons	function	our	Ref.[61]	Ref.[38]	Ref.[62]	Ref.[63]
Λ_c^0	μ_c	0.422	0.411	0.385	0.39	
Σ_c^0	$\frac{4}{3}\mu_d-\frac{1}{3}\mu_c$	−1.091	−1.043	−1.015	−1.60	
Σ_b^{*0}	$2\mu_d+\mu_c$	−1.017	−0.958	−0.850	−1.99	−1.18
Ξ_c^0	$\frac{2}{3}\mu_d+\frac{2}{3}\mu_s-\frac{1}{3}\mu_c$	−1.011	−0.914			
Ξ_c^{*0}	$\mu_d+\mu_s+\mu_b$	−0.825	−0.746	−0.690	−1.49	−1.020
Ω_c^-	$\frac{4}{3}\mu_s-\frac{1}{3}\mu_c$	−0.842	−0.774	−0.960	−0.900	
Ω_c^{*-}	$2\ \mu_s+\ \mu_c$	−0.625	−0.547	−0.867	−0.860	−0.840

Using these radiative transition magnetic moments for particular baryons, we calculated the radiative decay width and the results are tabulated in Table 9.

6 Conclusion

The mass spectra of the Λ_c, Σ_c, Ξ_c and Ω_c baryons have been calculated in the hypercentral constituent quark model (hCQM) with Coulomb plus linear potential. The mass spectra of the Λ_c, Σ_c, Ξ_c and Ω_c baryons are close to known experimental observations and other theoretical predictions as shown in Tables 3–6. We have listed various experimentally known singly charmed baryonic states in Table 10 and compared them with our calculated results. We have also assigned J^P values to unknown experimental states. We can say that there are not many theoretical calculations which provide spectra starting from S to F states. So, we have compared all our results to Ebert's calculations [17] and these match up to a certain energy scale. This study will help experimentalists as well as theoreticians to understand the dynamics of singly charmed baryons.

Table 9. Radiative decay widths (in keV).

decay	our	Ref.[61]	Ref.[64]	Ref.[65]
$\Sigma_c^{*0} \to \Sigma_c^0$	1.553	1.080	2.670	0.08 ± 0.03
$\Xi_c^{*0} \to \Xi_c^0$	0.906	0.908	-	0.66 ± 0.32
$\Omega_c^{*0} \to \Omega_c^0$	1.441	1.070	0.850	-
$\Sigma_c^{*0} \to \Lambda_c^+$	213.3	126	176.7	130±45

Table 10. Comparison between our predicted baryonic states and experimental unknown (J^P) excited states.

names	m/GeV[8]	baryon state
$\Lambda_c^+(2765)$	2766.6±2.4	$(2^2S_{\frac{1}{2}})$
$\Lambda_c^+(2940)$	$2939.3^{+1.4}_{-1.5}$	$(1^2P_{\frac{1}{2}})$,$(1^2P_{\frac{3}{2}})$
$\Sigma_c^0(2800)$	2806±8±10	$(1^2P_{\frac{1}{2}})$,$(1^2P_{\frac{3}{2}})$,$(1^4P_{\frac{5}{2}})$
$\Xi_c(2930)$	2931±3±5	$(1^2P_{\frac{1}{2}})$,$(1^2P_{\frac{3}{2}})$
$\Xi_c^0(2980)$	2968.0±2.6	$(2^2S_{\frac{1}{2}})$,$(2^4S_{\frac{3}{2}})$
$\Xi_c^0(3080)$	3079.9±1.4	$(1^2D_{\frac{3}{2}})$, $(1^4D_{\frac{3}{2}})$

The higher excited orbital and radial states mass calculation allowed us to construct the Regge trajectories in the (n_r, M^2) plane. The experimental masses in Figs. 1–2 are near to the obtained masses, such that we get almost linear, parallel and equidistant lines for each type of baryon. In Fig. 1, the $1S$ and $1D$ states of Λ_c^+ match our results while our $1P$ state is higher than the experimental value. $\Lambda_c(2765)^+$ is perfectly matched with our $2S$ state and $\Lambda_c^+(2940)$ can be assigned as $2P$. The $1S$, $1P$ and $1D$ states of Ξ_c^0 are reasonably close to the obtained results and the $\Xi_c(2990)^0$ value is nearer to the $2S$ state, so we assigned it as the $2S$ state. In Fig. 2, $\Sigma_c(2455)^0$ and $\Sigma_c(2800)^0$ exactly coincide with our $1S$ and $1P$ states. Our results for Σ_c^0 are quite close to the theoretical models and experiments in comparison to other obtained baryon states. The only (experimentally available) ground state value of Ω_c^0 matches our $1S$ state.

The magnetic moments and radiative decay widths were also calculated for ground state baryons with J^P $\frac{1}{2}^+$ and $\frac{3}{2}^+$. The obtained results are close to other theoretical predictions. Thus, our aim to calculate the excited states has been fulfilled. The obtained masses of four singly charmed baryons have also been checked by bound state baryonic properties, Regge trajectories and radiative decay widths. The model succeeds in determining their properties, so in future we would like to extend this scheme to calculate the decay rates of these baryons. We will also use the model to calculate the properties of doubly and triply heavy baryons.

One of the authors, Z. Shah, would like to thank Dr. Nilmani Mathur for his valuable suggestions at CCP 2015 at IIT, Guwahati.

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