General scan in flavor parameter space in models with vector quark doublets and an enhancement in the $B \to X_s\gamma$ process*

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Abstract: In models with vector-like quark doublets, the mass matrices of up and down type quarks are related. Precise diagonalization of the mass matrices has become an obstacle in numerical studies. In this work we first propose a diagonalization method. As its application, in the Standard Model with one vector-like quark doublet we present the quark mass spectrum and Feynman rules for the calculation of $B \to X_s\gamma$. We find that i) under the constraints of the CKM matrix measurements, the mass parameters in the bilinear term are constrained to a small value by the small deviation from unitarity; ii) compared with the fourth generation extension of the Standard Model, there is an enhancement to the $B \to X_s\gamma$ process in the contribution of vector-like quarks, resulting in a non-decoupling effect in such models.

Keywords: FCNC, vector like particle, CKM matrix

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1 Introduction

Though the Standard Model (SM) has been verified to be correct time after time, many new physics models beyond standard model are proposed to solve both experimental and aesthetic problems, such as neutrino masses, the μ anomalous magnetic movement problem, the hierarchy problem, etc. Many new models introduce vector-like particles (VLP) [1] whose right-handed and left-handed components transform in the same way under the weak $SU(2)\times U(1)$ gauge group. The extension is acceptable because the anomalies generated by the VLPs cancel automatically, and vector quarks can be heavy naturally. VLPs also arise in some grand unification theories. For example, in order to explain the little hierarchy problem between the traditional GUT scale and string scale, a testable flipped $SU(5)\times U(1)_X$ model is proposed in Ref. [2] in which TeV-scale VLPs are introduced [3]. Such models can be constructed from free fermionic string constructions at Kac-Moody level one [4, 5] and from the local F-theory model [2, 6].

However, when we do flavor physics with doublet VLPs in these models [7, 8], a problem always appears when we are dealing with the mass spectrum of quarks and leptons. Let us start with the SM, in which all fermion masses come from the Yukawa couplings. After spontaneous gauge symmetry breaking, we get two separate mass matrices M_U, M_D for the up and down type fermions. The mass eigenstates are obtained after the diagonalization

$$Z_U^\dagger M_U U_U = M_U^D, \quad Z_D^\dagger M_D U_D = M_D^D, \tag{1}$$

where $M_U^D = \mathrm{diag.}[m_u, m_c, m_t]$, $M_D^D = \mathrm{diag.}[m_d, m_s, m_b]$. The physical measurable parameters are m_i and the so-called CKM matrix

$$V_{\rm CKM} = U_U^\dagger U_D. \tag{2}$$

Since M_U, M_D come from separate Yukawa couplings, we can always set one of the matrices diagonal, for example M_U, and use the CKM matrix to get the Yukawa couplings

$$Z_D \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} V_{\rm CKM}^\dagger = \begin{pmatrix} Y_{11}^D v & Y_{12}^D v & Y_{13}^D v \\ Y_{21}^D v & Y_{22}^D v & Y_{23}^D v \\ Y_{31}^D v & Y_{32}^D v & Y_{33}^D v \end{pmatrix} \tag{3}$$

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for the calculation in flavor physics. Note that v is the vacuum expectation value (VEV) of the Higgs, and Z_D is a random unitary matrix.

Such a trick cannot be used in the case of the participation of a vector doublet, namely Q with gauge charge $\mathbf{3}$, $\mathbf{2}$, $\mathbf{1/6}$ and $\bar{Q}$ with gauge charge $\bar{\mathbf{3}}$, $\mathbf{2}$, $-\mathbf{1/6}$, resulting in a bilinear term in the Lagrangian

$$M^V Q\cdot\bar{Q}.$$

It is clear that in the model, there are the same input parameters in the matrices $M_U,\ M_D$

$$M_U=\begin{pmatrix} Y^U_{11}v & Y^U_{12}v & Y^U_{13}v & \cdots \\ Y^U_{21}v & Y^U_{22}v & Y^U_{23}v & \cdots \\ Y^U_{31}v & Y^U_{32}v & Y^U_{33}v & \cdots \\ M^V_{41} & M^V_{42} & M^V_{43} & \cdots \end{pmatrix},$$

$$M_D=\begin{pmatrix} Y^D_{11}v & Y^D_{12}v & Y^D_{13}v & \cdots \\ Y^D_{21}v & Y^D_{22}v & Y^D_{23}v & \cdots \\ Y^D_{31}v & Y^D_{32}v & Y^D_{33}v & \cdots \\ -M^V_{41} & -M^V_{42} & -M^V_{43} & \cdots \end{pmatrix}. \quad (4)$$

The mass matrices for up and down type quarks are related to each other. Therefore, we cannot set one of the matrices diagonal and the CKM matrix cannot be obtained easily. The shooting method is always used to treat such an obstacle. Random M_U and M_D are generated to meet the requirements after diagonlization: the mass of the eigenstate and the measurements of elements of the CKM matrix. However this is too time-consuming, and a precise solution for diagonalization is almost unobtainable. Although this is just a numerical problem, when one treats the VLP contributions to the flavor physics seriously, diagonalization of quark matrices will be the first and an important step.

In this paper, we will first propose a general method to solve the obstacle in models with vector-like quark doublets. As its application, we will study the rare B decay $\mathrm{B}\to\mathrm{X_s}\gamma$ in the SM with one vector-like quark doublet. The paper is organized as follows. We show the details of the trick in Section 2. The simple application to the $\mathrm{B}\to\mathrm{X_s}\gamma$ process, including the quark mass spectrum, Feynman rules and Wilson coefficients, as well as the numerical analysis for calculation of $\mathrm{B}\to\mathrm{X_s}\gamma$, is shown in Section 3. A summary is given in Section 4.

2 Diagonalization trick for vector quark doublets

Firstly, we address the problem of how to deal with the diagonalization of an $N\times N$ matrix M_U and M_D:

$$Z^\dagger_U M_U U_U = M^D_U,\ Z^\dagger_D M_D U_D = M^D_D \quad (5)$$

in which M^D_U, M^D_D are the diagonal mass matrices for up and down type quarks, respectively. Note that N should be greater than 3 and the first three elements in the matices should be the three generations of quark multiplets in the SM. Other elements with $N>3$ are the new multiplets introduced in new physics beyond the SM. Then we have

$$M_U=\begin{pmatrix} Y^U_{11}v & Y^U_{12}v & Y^U_{13}v & \cdots & M_{U1N} \\ Y^U_{21}v & Y^U_{22}v & Y^U_{23}v & \cdots & M_{U2N} \\ Y^U_{31}v & Y^U_{32}v & Y^U_{33}v & \cdots & M_{U3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ M^V_{N1} & M^V_{N2} & M^V_{N3} & \cdots & M_{UNN} \end{pmatrix},$$

$$M_D=\begin{pmatrix} Y^D_{11}v & Y^D_{12}v & Y^D_{13}v & \cdots & M_{D1N} \\ Y^D_{21}v & Y^D_{22}v & Y^D_{23}v & \cdots & M_{D2N} \\ Y^D_{31}v & Y^D_{32}v & Y^D_{33}v & \cdots & M_{D3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -M^V_{N1} & -M^V_{N2} & -M^V_{N3} & \cdots & M_{DNN} \end{pmatrix}. \quad (6)$$

The last line of the two matrices has the same parameters except for the last elements.

Considering that there are some of the same parameters in M_U and M_D, we find that a very simple way is to add the two matrices in Eq. (6)

$$M_U+M_D=\left(Z_U M^D_U U_{\mathrm{CKMN}}+Z_D M^D_D\right)U^\dagger_D. \quad (7)$$

The left side of the equation is

$$M_U+M_D=\begin{pmatrix} Y^U_{11}v+Y^D_{11}v & Y^U_{12}v+Y^D_{12}v & Y^U_{13}v+Y^D_{13}v & \cdots & M_{U1N}+M_{D1N} \\ Y^U_{21}v+Y^D_{21}v & Y^U_{22}v+Y^D_{22}v & Y^U_{23}v+Y^D_{23}v & \cdots & M_{U2N}+M_{D2N} \\ Y^U_{31}v+Y^D_{31}v & Y^U_{32}v+Y^D_{32}v & Y^U_{33}v+Y^D_{33}v & \cdots & M_{U3N}+M_{D3N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & M_{UNN}+M_{DNN} \end{pmatrix}. \quad (8)$$

Obviously, the mass inputs from bilinear terms vanish. We can denote the matrix in this form as

$$M_U+M_D=M_{UD}=\begin{pmatrix} \mathbf{M_A} & \mathbf{M_B} \\ \mathbf{M_0} & M_C \end{pmatrix}, \quad (9)$$

in which $\mathbf{M_A}$, $\mathbf{M_B}$, $\mathbf{M_0}$ are $(N-1)\times(N-1)$, $(N-1)\times 1$ and $1\times(N-1)$ matrices correspondingly.

To prepare for the diagonalization, we choose the diagonal mass matrix elements of quarks $(m_\mathrm{u}, m_\mathrm{c}, m_\mathrm{t}, \cdots$

m_X), $(m_d, m_s, m_b, \cdots, m_Y)$ and a matrix U_{CKMN}, which are determined partly by experimental measurements as input parameters

$$U_{CKMN} = U_U^\dagger U_D = \begin{pmatrix} (U_{CKM})_{3\times3} & \cdots \\ \cdots & U_{NN} \end{pmatrix}$$
$$= \begin{pmatrix} \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} & \cdots \\ \cdots & U_{NN} \end{pmatrix}. \quad (10)$$

Note that the above Z_U, Z_D, U_U, U_D are unitary matrices, but $(U_{CKM})_{3\times3}$ is not an ordinary CKM matrix V_{CKM} which is non-unitary in this case. Detailed dicussion will be shown in the following section.

What we need to do next is to generate a unitary matrix U_D. In a similar way we denote U_D as

$$U_D = \begin{pmatrix} \mathbf{U_{DA}} & \mathbf{U_{DB}} \\ \mathbf{U_{D0}} & U_{DNN} \end{pmatrix}. \quad (11)$$

Multiplying both sides of Eq. (7) by the matrix U_D, we can get

$$M_{UD}U_D = \begin{pmatrix} \mathbf{M_A}\mathbf{U_{DA}} + \mathbf{M_B}\mathbf{U_{D0}} & \mathbf{M_A}\mathbf{U_{DB}} + \mathbf{M_B}U_{DNN} \\ M_C\mathbf{U_{D0}} & M_C U_{DNN} \end{pmatrix}$$
$$= \left(Z_U M_U^D U_{CKMN} + Z_D M_D^D\right). \quad (12)$$

From the above equation, we can get the last line of U_D simply by inputting M_U^D, M_D^D, U_{CKMN} and random Z_U, Z_D:

$$\left(Z_U M_U^D U_{CKMN} + Z_D M_D^D\right)_{\text{last line}}$$
$$= \begin{pmatrix} M_C\mathbf{U_{D0}} & M_C U_{DNN} \end{pmatrix}$$
$$= M_C \mathbf{U_D}_N, \quad (13)$$

where

$$\mathbf{U_D}_N = \begin{pmatrix} U_{DN1} & U_{DN2} & \cdots & U_{DNN} \end{pmatrix} \quad (14)$$

is a unit vector in N dimensions.

Next we use the unit vector to generate the total U_D. Since $\mathbf{M_A}$ and $\mathbf{M_B}$ are random matrices, U_D can be random too. The unit vector $\mathbf{U_D}_{N-1}$ of U_D can be determined as

$$\mathbf{U_D}_{N-1} = \begin{pmatrix} -\frac{U_{DN2}^*}{\sqrt{|U_{DN1}|^2+|U_{DN2}|^2}} & \frac{U_{DN1}^*}{\sqrt{|U_{DN1}|^2+|U_{DN2}|^2}} & 0 & \cdots & 0 \end{pmatrix}. \quad (15)$$

It is clear that the vector is orthogonal to $\mathbf{U_D}_N$ and normalized to 1. Then we use the first three elements of $\mathbf{U_D}_N$ and $\mathbf{U_D}_{N-1}$ to generate $\mathbf{U_D}_{N-2}$. We normalize the algebraic complements of the first line of the 3×3 matrix. Step by step, we can finally get ($\mathbf{U_D}_1$, $\mathbf{U_D}_2$, $\cdots$, $\mathbf{U_D}_{N-1}$) and form a special U_D^S

$$U_D^S = \begin{pmatrix} \mathbf{U_D}_1 \\ \cdots \\ \mathbf{U_D}_{N-2} \\ \mathbf{U_D}_{N-1} \\ \mathbf{U_D}_N \end{pmatrix}$$
$$= \begin{pmatrix} U_{D11} & U_{D12} & U_{D13} & \cdots & U_{D1N} \\ \cdots & \cdots & \cdots & \cdots & 0 \\ U_{D(N-2)1} & U_{D(N-2)2} & U_{D(N-2)3} & \cdots & 0 \\ U_{D(N-1)1} & U_{D(N-1)2} & 0 & \cdots & 0 \\ U_{DN1} & U_{DN2} & U_{DN3} & \cdots & U_{DNN} \end{pmatrix}. \quad (16)$$

From the above steps, we see that ($\mathbf{U_D}_1$, $\mathbf{U_D}_2$, $\cdots$, $\mathbf{U_D}_{N-1}$) can be rotated into any other orthogonal $N-1$ vectors to construct a random matrix $\mathbf{M_A}$ and $\mathbf{M_B}$, and only $\mathbf{U_D}_N$ must be kept unchanged. Therefore, a general unitary matrix can be realized by multiplying by a unitary $N\times N$ matrix U_R,

$$U_D = U_R U_D^S = \begin{pmatrix} \mathbf{U_R}_{N-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{pmatrix} U_D^S \quad (17)$$

in which $\mathbf{U_R}_{N-1}$ is a $(N-1)\times(N-1)$ unitary matrix. We finish the work by

$$U_U^\dagger = U_{CKMN} U_D^\dagger \quad (18)$$
$$M_U = Z_U M_U^D U_U^\dagger \quad (19)$$
$$M_D = Z_D M_D^D U_D^\dagger \quad (20)$$

At this stage, we would like to summarize our method here

- Step 1: Choose $(m_u, m_c, m_t, \cdots, m_X, m_d, m_s, m_b, \cdots, m_Y)$ and U_{CKMN} and generate random unitary matrices Z_U and Z_D as the inputs for the model;

- Step 2: Determine the last line of matrix $Z_U M_U^D U_{CKMN} + Z_D M_D^D$ as

$$M_C \begin{pmatrix} U_{DN1} & U_{DN2} & \cdots & U_{DNN} \end{pmatrix} \quad (21)$$

and normalize it into a unit vector $\mathbf{U_D}_N$.

- Step 3: Use the unit vector $\mathbf{U_{D}}_N$ to generate other $N{-}1$ unitary vectors ($\mathbf{U_{D}}_1$, $\mathbf{U_{D}}_2$, $\cdots$, $\mathbf{U_{D}}_{N-1}$), and form a special U_D^S

$$U_D^S=\begin{pmatrix} \mathbf{U_{D}}_1 & \cdots & \mathbf{U_{D}}_{N-2} & \mathbf{U_{D}}_{N-1} & \mathbf{U_{D}}_N \end{pmatrix}^{\mathrm{T}}. \quad (22)$$

- Step 4: Generate a $N-1$ unitary matrix $\mathbf{U_{R}}_{N-1}$ to form a unitary matrix U_R which is

$$U_R=\begin{pmatrix} \mathbf{U_{R}}_{N-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{pmatrix}, \quad (23)$$

then, a general U_D is obtained by

$$U_D=U_RU_D^S. \quad (24)$$

- Step 5: Use these equations

$$\begin{aligned} U_U^\dagger &= U_{\mathrm{CKMN}}U_D^\dagger, \\ M_U &= Z_U M_U^D U_U^\dagger, \\ M_D &= Z_D M_D^D U_D^\dagger, \end{aligned} \quad (25)$$

to get the inputs for the flavor physics.

We see that by this trick we can skip the inputs of the bilinear mass terms M_{Ni}^V. In physical analysis, the masses of eigenstates $m_{\mathrm{X,\ Y}}$ in the VLP models are input freely. Z_U and Z_D can be generated randomly, U_U and U_D can also be scanned the most generally if we vary U_R randomly. Thus the method can do the most general scan in the parameter space of mass matrices in models with VLPs for numerical studies, which will be shown in the following section.

3 $\mathrm{B}\to\mathrm{X_s}\gamma$ process in extension of the SM with one vector-like quark doublet

3.1 Standard Model with vector-like quarks

As an application of the method, in this section we study the VLP contribution to $\mathrm{B}\to\mathrm{X_s}\gamma$ in a very simple VLP extension of SM, as a demonstration. In Table 1, we list the gauge symmetry of the matter multiplets, in which the first two columns show the quarks in the SM and the last two columns show the VLPs with the anti-gauge symmetry. Note that we ignore partners of the last two columns whose gauge symmetry is exactly the same as the first two columns of the SM. As discussed in the introduction, these VLPs can be heavy naturally. Since the gauge symmetry of Higgs $H=(h^+,\ h^0)^{\mathrm{T}}$ is $(\mathbf{1},\ \mathbf{2},\ \mathbf{1/2})$, the Lagrangian for two quarks of the model is written as:

$$\begin{aligned} \mathcal{L}=&Y_{\mathrm{d}}\bar{Q}Hd_R+Y_{\mathrm{u}}\bar{Q}\cdot\bar{H}u_R+Y_{V\mathrm{u}}\bar{V_Q}H\bar{V}_{\mathrm{u}L}+Y_{V\mathrm{d}}\bar{V_Q}\cdot\bar{H}\bar{V}_{\mathrm{d}L} \\ &+M_QV_{\mathrm{q}}\cdot Q+M_{\mathrm{u}}\bar{V}_{\mathrm{u}L}u_R+M_{\mathrm{d}}\bar{V}_{\mathrm{d}L}d_R+\mathrm{h.c.}, \end{aligned} \quad (26)$$

in which $A\cdot B=\epsilon^{ij}A_iB_j$. The first line of the Lagrangian is Yukawa terms, and the second line is the bilinear terms. Note that Y_{u}, Y_{d} are 3×3 matrices, and without the bilinear terms, the model will be almost the same as the fourth generation standard model (SM4).

After the electro-weak symmetry breaking, we can get the mass matrices of up and down quarks in the basis of (u, c, t, V_{u}) and (d, s, b, V_{d}):

$$M_U=\begin{pmatrix} Y_{\mathrm{u}}^{11}v & Y_{\mathrm{u}}^{12}v & Y_{\mathrm{u}}^{13}v & M_{\mathrm{u}}^1 \\ Y_{\mathrm{u}}^{21}v & Y_{\mathrm{u}}^{22}v & Y_{\mathrm{u}}^{23}v & M_{\mathrm{u}}^2 \\ Y_{\mathrm{u}}^{31}v & Y_{\mathrm{u}}^{32}v & Y_{\mathrm{u}}^{33}v & M_{\mathrm{u}}^3 \\ -M_Q^1 & -M_Q^2 & -M_Q^3 & Y_{V_{\mathrm{u}}}v \end{pmatrix},$$

$$M_D=\begin{pmatrix} Y_{\mathrm{d}}^{11}v & Y_{\mathrm{d}}^{12}v & Y_{\mathrm{d}}^{13}v & M_{\mathrm{d}}^1 \\ Y_{\mathrm{d}}^{21}v & Y_{\mathrm{d}}^{22}v & Y_{\mathrm{d}}^{23}v & M_{\mathrm{d}}^2 \\ Y_{\mathrm{d}}^{31}v & Y_{\mathrm{d}}^{32}v & Y_{\mathrm{d}}^{33}v & M_{\mathrm{d}}^3 \\ M_Q^1 & M_Q^2 & M_Q^3 & Y_{V_{\mathrm{d}}}v \end{pmatrix}, \quad (27)$$

where v is the VEV for H. The first three elements of the last line of the matrices have the same parameter, making the scan of the parameter space very difficult. These two matrices can be diagonalized by unitary matrices U and Z,

$$\begin{aligned} Z_{\mathrm{u}}^\dagger M_U U_{\mathrm{u}} &= \mathrm{diag.}[m_{\mathrm{u}},m_{\mathrm{c}},m_{\mathrm{t}},m_{\mathrm{X}}], \\ Z_{\mathrm{d}}^\dagger M_D U_{\mathrm{d}} &= \mathrm{diag.}[m_{\mathrm{d}},m_{\mathrm{s}},m_{\mathrm{b}},m_{\mathrm{Y}}]. \end{aligned} \quad (28)$$

The product of the two matrices is denoted as

$$U_{\mathrm{CKM4}}=U_{\mathrm{u}}^\dagger U_{\mathrm{d}}, \quad (29)$$

which is a unitary 4×4 matrix. We stress that the trick we introduced in the above section seems to just give us a numerical tool for quark masses and some quark mixing matrices, but it is important in studying the flavor physics in such models.

Table 1. A simple extension of the standard model with one vector-like quark doublet

	$\mathbf{SU(3)}$, $\mathbf{SU(2)}$, $\mathbf{U(1)}$		$\mathbf{SU(3)}$, $\mathbf{SU(2)}$, $\mathbf{U(1)}$
$Q=\begin{pmatrix} U \\ D \end{pmatrix}_L$	$\mathbf{3},\ \mathbf{2},\ \mathbf{\frac{1}{6}}$	$V_Q=\begin{pmatrix} \bar{V}_{\mathrm{d}} \\ \bar{V}_{\mathrm{u}} \end{pmatrix}_R$	$\bar{\mathbf{3}},\ \mathbf{2},\ -\mathbf{\frac{1}{6}}$
u_R	$\mathbf{3},\ \mathbf{1},\ \mathbf{\frac{2}{3}}$	$\bar{V}_{\mathrm{u}L}$	$\bar{\mathbf{3}},\ \mathbf{1},\ -\mathbf{\frac{2}{3}}$
d_R	$\mathbf{3},\ \mathbf{1},\ -\mathbf{\frac{1}{3}}$	$\bar{V}_{\mathrm{d}L}$	$\bar{\mathbf{3}},\ \mathbf{1},\ \mathbf{\frac{1}{3}}$

For studying VLP contributions to $B\to X_s\gamma$, we now present the Feynman rules for the interaction of $\bar{u}_l d_j \chi^+$, $\chi=W$, G and $\bar{d}_l d_j Z$ in the Feynman gauge which read:

$$\mathrm{i}\frac{g}{\sqrt{2}}\gamma^\mu\left[g_L^\chi(i,j)P_L+g_R^W(i,j)P_R\right],\quad (\chi=W,Z),\tag{30}$$

$$\mathrm{i}\frac{g}{\sqrt{2}m_W}\left[g_L^\chi(i,j)P_L+g_R^\chi(i,j)P_R\right]\ (\chi=G)\tag{31}$$

where

$$g_L^W(i,j)=\sum_{m=1}^{3}U_{\mathrm{u}}^{*mi}U_{\mathrm{d}}^{m,j},\quad g_R^W(i,j)=Z_{\mathrm{u}}^{*4i}Z_{\mathrm{d}}^{4j},\tag{32}$$

$$g_L^G(i,j)=\sum_{k,m=1}^{3}Y_{\mathrm{u}}^{km}vZ_{\mathrm{u}}^{*ki}U_{\mathrm{d}}^{mj}+Y_{V\mathrm{d}}vZ_{\mathrm{u}}^{*4i}U_{\mathrm{d}}^{4j},\tag{33}$$

$$g_R^G(i,j)=-\sum_{k,m=1}^{3}Y_{\mathrm{d}}^{*mk}vZ_{\mathrm{d}}^{*kj}U_{\mathrm{u}}^{mi}-Y_{V\mathrm{u}}^{*}vZ_{\mathrm{d}}^{*4j}U_{\mathrm{d}}^{4i}.\tag{34}$$

$$g_L^Z(i,j)=-\frac{1}{\sqrt{2}\cos\theta_W}\left[\left(1-\frac{2}{3}\sin^2\theta_W\right)\delta^{ij}-U_{\mathrm{d}}^{*4i}U_{\mathrm{d}}^{4j}\right],\tag{35}$$

$$g_R^Z(i,j)=-\frac{1}{\sqrt{2}\cos\theta_W}\left[-\frac{2}{3}\sin^2\theta_W\delta^{ij}+Z_{\mathrm{d}}^{*4i}Z_{\mathrm{d}}^{4j}\right].\tag{36}$$

Note that $U(1)_{EM}$ interaction is not changed by the VLPs, thus the vertices of photons and quarks are still the same as those in the SM. From the above mass matrices and Feynman rules, we can see that the model has two points to be explored:

1) The CKM matrix is obtained from the $W^+\bar{u}_i d_j$ vertex in Eq. (32)

$$V_{\mathrm{CKM4}}^{ij}=\sum_{m=1}^{3}U_{\mathrm{u}}^{*mi}U_{\mathrm{d}}^{mj}=U_{\mathrm{CKM4}}^{ij}-U_{\mathrm{u}}^{*4i}U_{\mathrm{d}}^{4j},\tag{37}$$

which is non-unitary for indices i,j ranging from 1 to 4, but the summation of index m is from 1 to 3. V_{CKM4}^{ij} is also a 4×4 matrix of which the upper left elements $(i,j\neq4)$ are a physically measurable value of the CKM matrix V as in the SM. This is the key difference between VLP models and the SM4. Nevertheless, the loop-level flavor change neutral current (FCNC) will be changed by the Yukawa interactions, then the prediction of process $B\to X_s\gamma$ may be changed significantly.

2) The last terms in Eqs.(32)–(36), which we call the "tail terms", violate the gauge universality of fermions and cause tree-level FCNC processes induced by the processes such as $b\to sl^+l^-$, so the constraints on the parameter space need to be explored.

3.2 Enhancement in b → s transition

In this subsection we focus on VLP contributions to the rare B decay $B\to X_s\gamma$. The starting point for rare B decays is the determination of the low-energy effective Hamiltonian obtained by integrating out the heavy degrees of freedom in the theory. For $b\to s$ transition, this can be written as

$$\mathcal{H}_{\mathrm{eff}}=-\frac{G_F}{\sqrt{2}}V_{\mathrm{tb}}V_{\mathrm{ts}}^{*}\sum_{i=1}^{10}[C_i(\mu)O_i(\mu)+C_i^{'}(\mu)O_i^{'}(\mu)]\ ,\tag{38}$$

where the effective operators O_i are same as those in the SM defined in Ref. [9]. The chirality-flipped operators O_i' are obtained from O_i by the replacement $\gamma_5\to-\gamma_5$ in quark current [7]. We calculate the Wilson coefficient C_7 at matching scale m_W. The leading order Feynman diagrams are shown in Fig. 1 and C_7 reads

$$\begin{aligned}&C_7(m_{\mathrm{W}})\\&=\frac{1}{V_{\mathrm{tb}}V_{\mathrm{ts}}^{*}}\sum_{i=1}^{4}\Bigg[g_L^{\mathrm{W}*}(i,2)g_L^{\mathrm{W}}(i,3)A(x_i)\\&\quad+\frac{g_L^{G*}(i,2)g_L^{G}(i,3)}{m_{\mathrm{u}_i}^2}x_iB(x_i)\\&\quad+\frac{g_L^{G*}(i,2)g_R^{G}(i,3)}{m_{\mathrm{u}_i}m_{\mathrm{b}}}x_iC(x_i)+\frac{g_L^{\mathrm{W}*}(i,2)g_R^{G}(i,3)}{m_{\mathrm{b}}}D(x_i)\\&\quad+\frac{m_{\mathrm{u}_i}}{m_{\mathrm{b}}}g_L^{\mathrm{W}*}(i,2)g_R^{\mathrm{W}}(i,3)E(x_i)+\frac{g_L^{G*}(i,2)g_R^{\mathrm{W}}(i,3)}{m_{\mathrm{b}}}D(x_i)\Bigg]\end{aligned}\tag{39}$$

where $x_i=m_{\mathrm{u}_i}^2/m_{\mathrm{W}}^2$ and the loop functions $A(x)$, $B(x)$, $C(x)$, $D(x)$, $E(x)$ are listed in the Appendix A. The first two lines are similar contributions to those in the SM, while the last lines are from the tail terms. Note that the contribution of the right-hand diagram in the second line of Fig. 1 is zero in the SM. The terms with $1/m_{\mathrm{b}}$ in the above equation are extracted to compose the operator $\mathcal{O}_7$. There are two differences in the calculation of $B\to X_s\gamma$ processes compared with the SM. One is the tail terms of the gauge or Yukawa interactions, the other is the new type of Yukawa interactions listed in Eqs. (33, 34), which cannot be written into the simple form in the SM such as

$$g_L^{G,\mathrm{SM}}(i,2)=m_{\mathrm{u}_i}V_{is},\ g_R^{G,\mathrm{SM}}(i,3)=-m_{\mathrm{b}}V_{i\mathrm{b}}.\tag{40}$$

In the model with three generations of quarks, the CKM matrix unitarity is already used in the calculations of the loop-level FCNC-induced rare B decays. For consistency, in numerical analysis the constraints on the CKM matrix elements are not from processes occurring at loop level, such as rare B decays, but from the tree-level processes shown in Table 2 [10, 11]. Since there are no tree-level measurements of V_{td}, V_{ts} now, we use the above inputs and the unitarity to get a 3×3 unitary matrix at first. The method is that we scan $(V_{\mathrm{ud}},V_{\mathrm{us}},V_{\mathrm{ub}})$ randomly (keeping $|V_{\mathrm{ud}}|^2+|V_{\mathrm{us}}|^2+|V_{\mathrm{ub}}|^2=1$) in the range listed in Table 2, then we define two parameters α, β and solve them by the equations

$$V_{\rm ud}^*(V_{\rm cd}+\alpha)+V_{\rm us}^*(V_{\rm cs}+\beta)+V_{\rm ub}^*V_{\rm cb}=0,\\ |V_{\rm cd}+\alpha|^2+|V_{\rm cs}+\beta|^2+|V_{\rm cb}|^2=1. \quad (41)$$

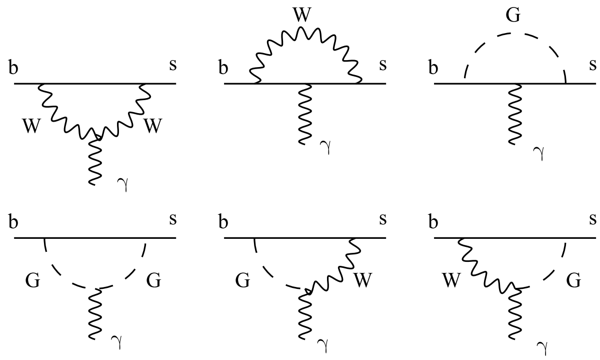

Fig. 1. Leading order Feynman diagram of the $B\rightarrow X_s\gamma$ process.

Table 2. The CKM matrix elements constrained by the tree-level B decays.

	absolute value	direct measurement from
$V_{\rm ud}$	0.97425 ± 0.00022	nuclear beta decay
$V_{\rm us}$	0.2252 ± 0.0009	semi-leptonic K-decay
$V_{\rm ub}$	0.00415 ± 0.00049	semi-leptonic B-decay
$V_{\rm cd}$	0.230 ± 0.011	semi-leptonic D-decay
$V_{\rm cs}$	1.006 ± 0.023	(semi-)leptonic D-decay
$V_{\rm cb}$	0.0409 ± 0.0011	semi-leptonic B-decay
$V_{\rm tb}$	0.89 ± 0.07	(single) top-production

$(V_{\rm td},V_{\rm ts},V_{\rm tb})$ are obtained by the unitarity relation with $(V_{\rm ud},V_{\rm us},V_{\rm ub})$ and $(V_{\rm cd},V_{\rm cs},V_{\rm cb})$. After that we multiply the 3×3 unitary matrix with three matrices

$$\begin{pmatrix} \mathbf{1} & \cdots & \cdots & \cdots \\ \cdots & \cos\theta_{4i} & \cdots & \sin\theta_{4i} \\ \cdots & \cdots & \mathbf{1} & \cdots \\ \cdots & -\sin\theta_{4i} & \cdots & \cos\theta_{4i} \end{pmatrix} \quad (42)$$

in which $i=1,\ 2,\ 3$ and $\max(|\theta_{41}|,\ |\theta_{42}|,\ |\theta_{43}|)<0.01\pi$, to generate a 4×4 unitary matrix $U_{\rm CKM4}$. $V_{\rm CKM4}$ are obtained by Eq. (37). All the corresponding elements should satisfy the experiment bound list in Table 2 and $V_{\rm td}$, $V_{\rm ts}$ ($|V_{\rm ts}|\simeq0.04$, which is consistent with the fitting results in Ref. [10]) can be obtained too. With these inputs in hand, the first task is to check the scale of the mass parameters of the model, such as M_Q, $m_{\rm X},m_{\rm Y}$. From the $Z\bar{b}b$ vertexes in Eq. (35) and Eq. (36), we can see that in order to keep gauge universality of quarks, the tail terms in the Feynman rules must be much smaller than the SM-like terms, namely $|Z_{\rm u,d}^{4i}|^2_{i=1,2,3},\ |U_{\rm u,d}^{4i}|^2_{i=1,2,3}\ll\sin^2\theta_W$. Thus in the numerical studies we require

$$|Z_{\rm u,d}^{4i}|^2_{i=1,2,3},\ |U_{\rm u,d}^{4i}|^2_{i=1,2,3}<10^{-4}. \quad (43)$$

Note that though these elements are greater than λ^3 (parameter in the Wolfenstein parameterization [12]), they are much smaller than the product of $V_{\rm CKM3}^\dagger V_{\rm CKM3}$ (almost equals $\mathbf{1}$), thus the requirements are suitable for indicating the contraints from the deviation from unitarity.

Since the scanning in the parameter space is free, we set $m_{\rm X}=1172$ GeV (mass of top quark plus 1000 GeV) and scan $m_{\rm Y}$ in the range of $(4.2,1004)$ GeV (mass of bottom quark plus 1000 GeV), and $Z_{\rm u,d}$, $U_{\rm u,d}$ randomly (ignoring the CP phases). M_V is defined by

$$M_V=\max(|M_Q^1|,\ |M_Q^2|,\ |M_Q^3|). \quad (44)$$

The result for M_V versus $M_{\rm Y}$ is shown in Fig. 2, which checks the mass input of the vector doublet. We can see that M_V increases as $m_{\rm Y}$ goes up. However M_V is much smaller than $m_{\rm X}$ and $m_{\rm Y}$. Small mixings lead to the parameter M_Q, which determines the mixing between SM quarks and vector-like quarks, also being suppressed. This is in agreement with the deviation from unitarity being suppressed by the ratio $m/m_{\rm X,Y}$ where m denotes generically the standard quark masses, which is a typical result of VLP models. [13–17]

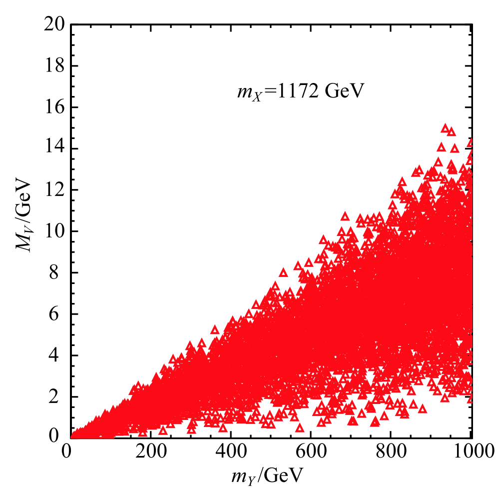

Fig. 2. M_V versus $M_{\rm Y}$ under constraints $|Z_{\rm u,d}^{4i}|^2_{i=1,2,3},\ |U_{\rm u,d}^{4i}|^2_{i=1,2,3}<10^{-4}$.

The second task is to check the VLP contribution to $B\rightarrow X_s\gamma$. We find the Wilson coefficient of the FCNC operator O_7 is not so suppressed as the mixing. The new contributions, from the terms of the last line in Eq. (39), are suppressed by the mixing, whereas the terms of first line are almost the same as the SM. The enhancement comes mainly from the $g_R^G(4,3)/m_{\rm b}$ terms which come from the Goldstone loop in the $b\rightarrow s$ transition. (The right-hand diagram in the first line and diagrams in the second line of Fig. 1) In order to show the enhancement clearly, we define two factors

$$K_1=\frac{g_R^G(4,3)g_L^G(4,2)^*}{m_\mathrm{X}m_\mathrm{b}V_\mathrm{tb}V_\mathrm{ts}^*}=\frac{g_R^{GV\mathrm{b}}g_L^{GV\mathrm{s}*}}{m_\mathrm{X}m_\mathrm{b}V_\mathrm{tb}V_\mathrm{ts}^*}, \tag{45}$$

$$K_2=\frac{U^{43}U^{42*}}{V_\mathrm{tb}V_\mathrm{ts}^*}=\frac{U^{V\mathrm{b}}U^{V\mathrm{s}*}}{V_\mathrm{tb}V_\mathrm{ts}^*}, \tag{46}$$

in which K_2 denotes the deviation from the unitarity of a 3×3 CKM matrix, while K_1 shows the enhancement of the contribution from vector-like particles. K_1 is in fact obtained from the coefficient of the first term in the second line of the analytical expression of $C_7(m_\mathrm{W})$ in Eq. (39) when $i=4$. It will be changed into exactly K_2 in case of the SM4. Note that other terms with $g_R^G(4,3)/m_\mathrm{b}$ can give enhancement too. We chose factor K_1 for a typical demonstration since it seems that it will be suppressed by m_X. Results are shown in Fig. 3 in which the left panel shows K_1 and K_2 versus M_V while the right panel shows $|C_7(m_\mathrm{W})|$ versus K_1. From the left panel, we can see that though K_2 increases as M_V increases, it is still much smaller than $V_\mathrm{tb}V_\mathrm{ts}^*$, implying that deviation from unitarity is negligible. However, the factor K_1 can be enhanced up to order $\mathcal{O}(1)$ by the increase of M_V. From the right panel, we see that K_1 enhances C_7 up to a value much larger that the result of the SM. The reason for the enhancement mainly comes from the new type of Yukawa couplings. Combining Eqs. (28, 33, 34), one can get a similar form to that of the SM4

$$\frac{g_L^G(4,2)}{m_\mathrm{X}}=U_\mathrm{CKM4}^{42}+\frac{1}{m_\mathrm{X}}\left[\sum_{m=1}^{3}\left(M_Q^m U_\mathrm{d}^{m2}Z_\mathrm{u}^{*44}-M_\mathrm{u}^m U_\mathrm{d}^{42}Z_\mathrm{u}^{*m4}\right)+(Y_{V\mathrm{d}}-Y_{V\mathrm{u}})U_\mathrm{d}^{42}Z_\mathrm{u}^{*44}v\right], \tag{47}$$

$$\frac{g_R^G(4,3)}{m_\mathrm{b}}=-U_\mathrm{CKM4}^{*43}-\frac{1}{m_\mathrm{b}}\left[\sum_{m=1}^{3}\left(M_Q^{*m}U_\mathrm{u}^{*m4}Z_\mathrm{d}^{43}+M_\mathrm{d}^{*m}U_\mathrm{u}^{*44}Z_\mathrm{d}^{m3}\right)+(Y_{V\mathrm{d}}^*-Y_{V\mathrm{u}}^*)U_\mathrm{u}^{*44}Z_\mathrm{d}^{43}v\right]. \tag{48}$$

Since $m_\mathrm{X}\simeq Y_{V\mathrm{u}}v$, Z_u^{44}, $U_\mathrm{u}^{44}\simeq1$, one can easily obtain that

$$\frac{g_L^G(4,2)}{m_\mathrm{X}}\sim V_\mathrm{CKM4}^{42}. \tag{49}$$

The suppression of Z_d^{43} (order of $m/m_\mathrm{X,Y}$) in Eq. (48) is enhanced by terms with factors such as $\frac{Y_{V\mathrm{u}}v}{m_\mathrm{b}}$, etc., resulting in

$$\frac{g_R^G(4,3)}{m_\mathrm{b}}\gg V_\mathrm{CKM4}^{43}. \tag{50}$$

Thus the term $V_{4\mathrm{b}}V_{4\mathrm{s}}^*$ satisfying the unitary constraint

$$V_\mathrm{ub}V_\mathrm{us}^*+V_\mathrm{cb}V_\mathrm{cs}^*+V_\mathrm{tb}V_\mathrm{ts}^*+V_{4\mathrm{b}}V_{4\mathrm{s}}^*=0 \tag{51}$$

is enhanced greatly by heavy VLPs, then the factor leads the enhancement to C_7. This is different from those in the SM4 in which the contribution from the fourth generation can be neglected.

In the numerical scan, we vary $Z_\mathrm{u,d}$ and $U_\mathrm{u,d}$ randomly, keeping the constraints of $|V_\mathrm{u,d}^{4i}|^2_{i=1,2,3}$, $|U_\mathrm{u,d}^{4i}|^2_{i=1,2,3}$, scan m_X and m_Y in the range of (1,2000)GeV. Apart from the CKM limits, we use the $\mathrm{B}\to\mathrm{X_s}\gamma$ process to constrain parameter space. The branching ratio of $\mathrm{B}\to\mathrm{X_s}\gamma$ is normalized by the process $\mathrm{B}\to\mathrm{X_c}\mathrm{e}\bar{\nu}_\mathrm{e}$:

$$\mathrm{Br}(\mathrm{B}\to\mathrm{X_s}\gamma)=\mathrm{Br}^\mathrm{ex}(\mathrm{B}\to\mathrm{X_c}\mathrm{e}\bar{\nu}_\mathrm{e})\frac{|V_\mathrm{ts}^*V_\mathrm{tb}|^2}{|V_\mathrm{cb}|^2}\frac{6\alpha}{\pi f(z)}\cdot[|C_7^\mathrm{eff}(\mu_\mathrm{b})|^2+|C_7^{\prime,\mathrm{eff}}(\mu_\mathrm{b})|^2]. \tag{52}$$

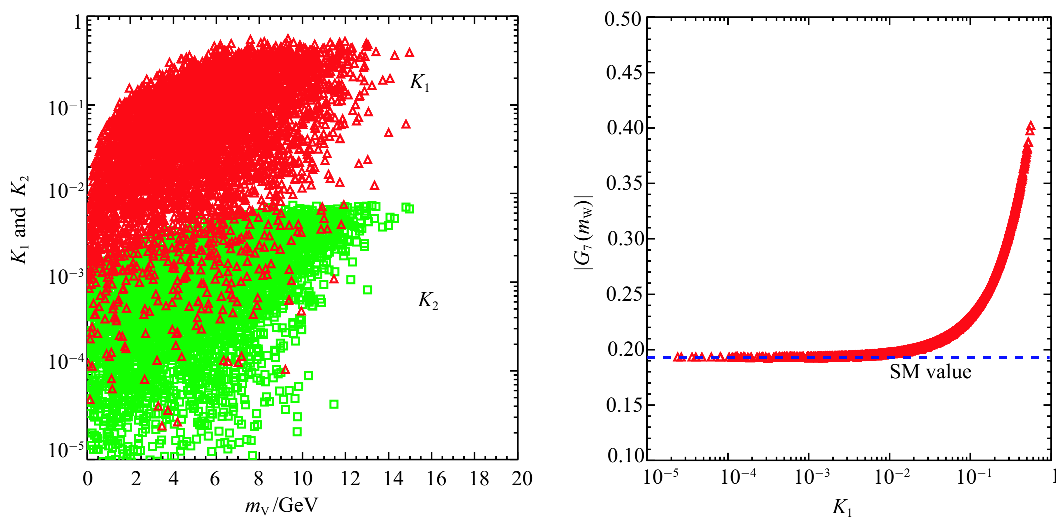

Fig. 3. (color online)K_1, (red △) K_2 (green □) versus M_V and enhancement of $|C_7(m_\mathrm{W})|$ case of $|Z_\mathrm{u,d}^{4i}|^2_{i=1,2,3}$, $|U_\mathrm{u,d}^{4i}|^2_{i=1,2,3}<10^{-4}$.

Here $z=\frac{m_c}{m_b}$, and $f(z)=1-8z^2+8z^6-z^8-24z^4\ln z$ is the phase-space factor in the semi-leptonic B decay. The method of running of the operators from m_W scale to μ_b scale can be found in Ref. [7]. We use the following bounds on the calculation [10]

$$Br^{\rm ex}(b\to ce\overline{\nu}_c)=(10.72\pm0.13)\times10^{-2}, \quad (53)$$

$$Br^{\rm ex}(B\to X_s\gamma)=(3.55\pm0.24\pm0.09)\times10^{-4}. \quad (54)$$

The numerical results show that $C_7^{\prime,\rm eff}(\mu_b)$ is much smaller than $C_7^{\rm eff}(\mu_b)$, so we do not present the formula of $C_7^{\prime,\rm eff}(m_W)$ here.

The branching ratio as a function of m_V is shown in Fig. 4, from which we can see that $Br(B\to X_s\gamma)$ can be enhanced much more than the experiment bound. Then the measurements of FCNC processes can give a stringent constraint on the vector-like quark model, especially when the masses of vector quarks are much greater than the electro-weak scale. A few remarks should be made:

1) There is one point of view on the unitarity of the CKM matrix where the 3×3 ordinary quark mixing matrix is regarded as nearly unitary, and deviation from unitarity is suppressed by heavy particles in the new physics beyond the SM. In other word, one admits that the extended CKM matrix elements exist, but they approach to zero while mass scale of the new physics approaches to infinity. All the new physical effects should decouple from the flavor sector and what should be checked is if 3×3 unitarity is consistent in all kinds of flavor processes.

2) Another point of view is that, as in the SM case, the 3×3 ordinary quark mixing matrix elements are only extracted by experiments in the measurements of tree and loop level processes. The unitarity should be checked, and experimental measurements on the elements of the matrix can be used as the constraints to new physics beyond the SM. In the numerical analysis, the elements of the CKM matrix are regarded as inputs. Thus what should be done is to scan the parameter space generally under these constraints, and no prejudice should be imposed. Then the enhancement effect in $B\to X_s\gamma$ will be more clear.

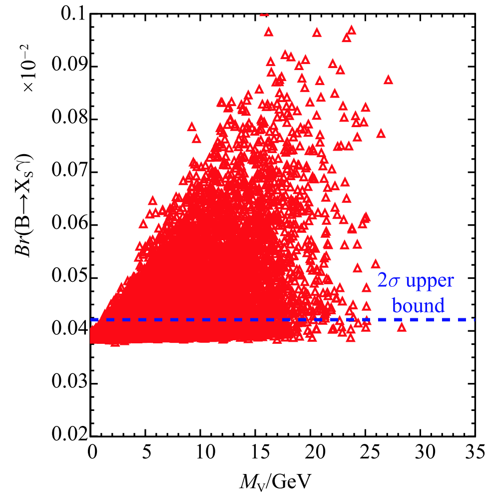

Fig. 4. $B\to X_s\gamma$ prediction in random scan.

A large area of parameter space is excluded by the measured branching fraction of $B\to X_s\gamma$ as shown in Fig. 4. The enhancement effect of the VLPs can be seen in Fig. 5, in which the left panel shows the enhancement factor K_1 versus m_X while the right panel shows K_2 versus m_X. From the right panel we can see that deviation from unitarity is very small and almost unrelated to m_X since we are doing a general scan of $Z_{u,d}$ and $U_{u,d}$. However as we see from the left panel, as m_X increases, $Br(B\to X_s\gamma)$ measurement will constrain the enhancement factor and then constrain the input parameters of

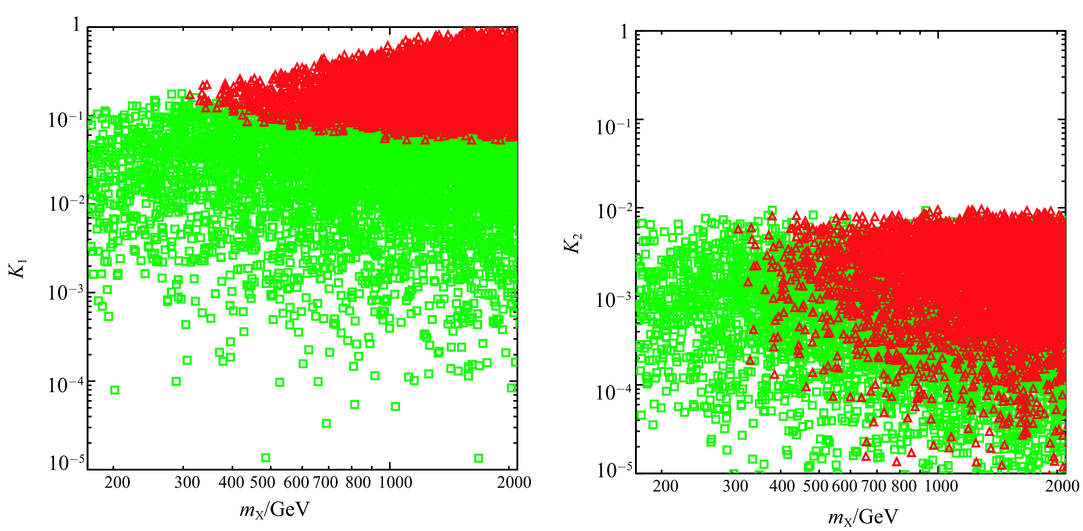

Fig. 5. (color online) Enhancement factor and deviation from unitarity versus m_X, where red △ are excluded by the bound of $B\to X_s\gamma$ measurements, in which the green □ are the surviving points .

m_X. Overall, the enhancement can be summarized as when the mass of the vector-like particle increases, it will increase the mass parameter m_V and thus give an enhancement factor under very small deviation from unitarity. This should be a special point when we study the vector-like quark models.

4 Summary

In models with vector doublets, there exist bilinear terms in the Lagrangian, making general scans of the Yukawa coupling very difficult. In this paper, we show a trick to deal with the scan. Our scan method is exact and efficient. We use this trick to study a very simple extension of the SM with vector-like quarks. We studied one of the most important rare B decay processes, $B \to X_s\gamma$, in which we found that even when the deviations from unitarity of the quark mixing matrix are small, the enhancement to rare B decays from VLPs are still significant. The enhanced effect is an important feature in the vector-like particle model. In this work we just show the scan method, the key point of the enhancement and how stringent constraints are on the parameter space from $B \to X_s\gamma$ measurements. Other models should be examined, including extension of the SM with VLPs, two Higgs doublets models [18] and supersymmetry models [19]. Such an effect should be checked in all kinds of rare decays such as inclusive process $b \to sl^+l^-$ and exclusive processes $B_s \to \mu^+\mu^-$, $B_s \to l^+l^-\gamma$ and $B\bar{B}$ mixing etc. Detailed studies on the parameter space, including other rare B decays and new models, will appear in our future work.

Appendix A

- The loop functions for calculating the Wilson coefficients at the matching scale are the following:

$$A(x) = \frac{55-170x+127x^2}{36(1-x)^3} + \frac{4x-17x^2+15x^3}{6(1-x)^4}\ln x,$$

$$B(x) = \frac{-7+5x+8x^2}{36(1-x)^3} + \frac{-2x+3x^2}{6(1-x)^4}\ln x,$$

$$C(x) = \frac{3-5x}{6(1-x)^2} + \frac{2-3x}{3(1-x)^3}\ln x,$$

$$D(x) = \frac{3x-1}{4(1-x)^2} + \frac{x^2}{2(1-x)^3}\ln x,$$

$$E(x) = \frac{-17+19x}{6(1-x)^2} + \frac{-8x+9x^2}{3(1-x)^3}\ln x.$$

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