Effect of Wigner energy on the symmetry energy coefficient in nuclei *

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Abstract: The nuclear symmetry energy coefficient (including the coefficient $a_{\rm sym}^{(4)}$ of the I^4 term) of finite nuclei is extracted by using the differences of available experimental binding energies of isobaric nuclei. It is found that the extracted symmetry energy coefficient $a_{\rm sym}^{*}(A,I)$ decreases with increasing isospin asymmetry I, which is mainly caused by Wigner correction, since $e_{\rm sym}^{*}$ is the summation of the traditional symmetry energy $e_{\rm sym}$ and the Wigner energy $e_{\rm W}$. We obtain the optimal values $J=30.25\pm0.10$ MeV, $a_{\rm ss}=56.18\pm1.25$ MeV, $a_{\rm sym}^{(4)}=8.33\pm1.21$ MeV and the Wigner parameter $x=2.38\pm0.12$ through a polynomial fit to 2240 measured binding energies for nuclei with $20\leqslant A\leqslant 261$ with an rms deviation of 23.42 keV. We also find that the volume symmetry coefficient $J\simeq 30$ MeV is insensitive to the value x, whereas the surface symmetry coefficient $a_{\rm ss}$ and the coefficient $a_{\rm sym}^{(4)}$ are very sensitive to the value of x in the range $1\leqslant x\leqslant 4$. The contribution of the $a_{\rm sym}^{(4)}$ term increases rapidly with increasing isospin asymmetry I. For very neutron-rich nuclei, the contribution of the $a_{\rm sym}^{(4)}$ term will play an important role.

Keywords: symmetry energy, Wigner energy, binding energy, isobaric nuclei

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1 Introduction

It is evident that the symmetry energy coefficient plays an extremely important role, not only in nuclear physics, where it is involved in processes such as the dynamics of heavy-ion collisions induced by radioactive beams, the proper description of the nuclear binding energies along the periodic table, and the structure of exotic nuclei near the nuclear drip lines [1–12], but also in astrophysics, including the dynamical evolution of the core collapse of a massive star and the associated explosive nucleosynthesis [13–19]. In the global fitting of nuclear masses in the framework of the liquid drop mass formula, the symmetry energy per particle is usually written as $e_{\rm sym}=a_{\rm sym}I^2$, where $I=(N-Z)/A$ is the isospin asymmetry and the symmetry energy coefficient $a_{\rm sym}$ enters as a mass-dependent phenomenological parameter [20–25]. In fact, $a_{\rm sym}$ is also a function of the isospin asymmetry I, which is usually written as $a_{\rm sym}(A,I)=J-a_{\rm ss}/A^{1/3}+a_{\rm sym}^{(4)}I^2$ by neglecting the higher order term [26, 27]. But how does $a_{\rm sym}$ change with increasing isospin asymmetry I for given mass number A? It is mainly determined by the high-order I^4 term coefficient $a_{\rm sym}^{(4)}$ of the symmetry energy. However, the coefficient $a_{\rm sym}^{(4)}$ is difficult to determine. It is necessary to investigate the symmetry energy coefficient of finite nuclei.

In Ref. [28], Min Liu et al. obtained the mass dependence of $a_{\rm sym}(A)$ through performing a two-parameter parabolic fitting to the energy per particle after removing the Coulomb energy $e_{\rm n}(A,I)=e(A,I)-e_{\rm c}(A,I)$ for a series of nuclei with the same mass number A. The extracted $a_{\rm sym}$ is only dependent on mass number A. In this work, with similar approach to Ref. [28], we consider the mass and isospin dependence of $a_{\rm sym}$, and at the same time include the higher-order (I^4) term of the symmetry energy. It is found that the Wigner energy $E_{\rm W}$ should be considered in the extraction of the nuclear symmetry energy coefficient. However, the Wigner energy was not included in our previous paper [29]. The nature of the symmetry and Wigner energy are intertwined in the nuclear mass formula and one term cannot be reliably determined without knowledge of the other [30]. This leads to considerable uncertainty in the value for the symmetry energy, especially the coefficient $a_{\rm sym}^{(4)}$ of the I^4 term in the symmetry energy coefficient expression.

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The paper is organized as follows. In Section 2, the symmetry energy and the Wigner energy are presented, and the summation of both are extracted by using the experimental binding energies differences between isobaric nuclei. In Section 3, the method of extracting the symmetry energy coefficient is described and the corresponding coefficients are obtained through polynomial fitting. The effect of the Wigner energy on the symmetry energy coefficient is studied in Section 4. Finally, a summary is given in Section 5.

2 Symmetry energy and Wigner energy

Nuclear mass is one of the most precisely measured quantities in nuclear physics. It can provide information about the symmetry energy coefficient through the liquid drop mass systematics. In the semi-empirical Bethe-Weizsäcker mass formula [31, 32], the energy per particle $e(A,I)$ of a nucleus can be expressed as a function of the mass number A and the isospin asymmetry I,

$$e(A,I)=a_{\rm v}+a_{\rm s}A^{-1/3}+e_{\rm c}(A,I)+a_{\rm sym}I^2+\delta, \tag{1}$$

with

$$\delta=\pm a_{\rm p}A^{-3/2} \text{ or } 0, \tag{2}$$

where the "+" is for even-even nuclides, the "–" is for odd-odd nuclides, and for odd-A nuclides (i.e. even-odd and odd-even) $\delta=0$. The $a_{\rm v}$, $a_{\rm s}$, $a_{\rm sym}$ and $a_{\rm p}$ are the volume, surface, symmetry and pairing energy coefficients, respectively. The Coulomb energy per particle is $e_{\rm c}(A,I)=E_{\rm c}/A$, where the Coulomb energy of a nucleus $E_{\rm c}=0.71\dfrac{Z^2}{A^{1/3}}(1-0.76Z^{-2/3})$ and $Z=\dfrac{A}{2}(1-I)$ are usually used [33, 34].

Let us assume the binding energy per particle $e(A,I)=e_{\rm n}(A,I)+e_{\rm c}(A,I)$, $e_{\rm n}(A,I)$ and $e_{\rm c}(A,I)$ denote the nuclear energy part and the Coulomb energy part per particle, respectively. Subtracting the Coulomb energy term from the binding energy, one obtains the nuclear energy part per particle,

$$\begin{aligned}e_{\rm n}(A,I)&=e(A,I)-e_{\rm c}(A,I)\\&=e_0(A)+e_{\rm sym}(A,I)\\&=e_0(A)+a_{\rm sym}(A,I)I^2,\end{aligned} \tag{3}$$

where $e_0(A)=a_{\rm v}+a_{\rm s}A^{-1/3}+\delta$, including the volume, surface and pairing energy terms, is only dependent on nuclear mass number A. $e_{\rm sym}(A,I)$ is the symmetry energy per particle of a nucleus. If we take the difference in the nuclear energy part per particle $e_{\rm n}(A,I)$ between two isobaric nuclei with same odd-even parity, the $e_0(A)$ term is canceled and the difference of the symmetry energy per particle can be written as

$$\begin{aligned}\Delta e_{\rm sym}&=e_{\rm n}(A,I)-e_{\rm n}(A,I_1)\\&=a_{\rm sym}(A,I)I^2-a_{\rm sym}(A,I_1)I_1^2.\end{aligned} \tag{4}$$

Here $e_{\rm n}(A,I_1)$ is the nuclear energy part per particle of a reference nucleus (A,I_1), and the symmetric nucleus $(I_1=0)$ is selected as the reference nucleus if its experimental binding energy exists for even-even nuclei. For any other case, the nucleus with the minimum value of $I_1=I_{\rm min}>0$ is selected as the reference nucleus among each series of isobaric nuclei. $e_{\rm n}(A,I)$ indicates any other value of isobaric nuclei for given mass number A.

If the experimental binding energy of a symmetric nucleus (I_1=0) is known, we obtain

$$e_{\rm sym}(A,I)=e_{\rm n}(A,I)-e_{\rm n}(A,0)=a_{\rm sym}(A,I)I^2, \tag{5}$$

or

$$a_{\rm sym}(A,I)=\frac{e_{\rm sym}(A,I)}{I^2}=\frac{e_{\rm n}(A,I)-e_{\rm n}(A,0)}{I^2}, \tag{6}$$

where only even-even nuclei are taken into account in our calculations to consider the pairing effects for the even mass number nuclei.

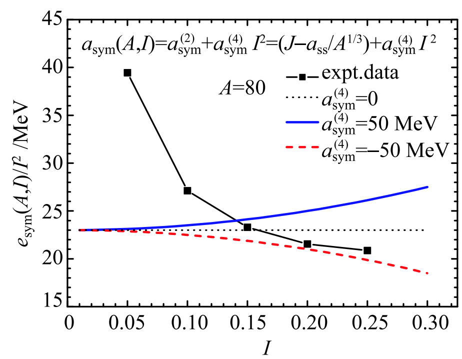

Fig. 1. Experimental symmetry energy coefficients as a function of I extracted from Eq. (6) for all even-even nuclei with mass number A=80 (solid squares). The dotted line ($a^{(4)}_{\rm sym}$=0), the solid line ($a^{(4)}_{\rm sym}$=50 MeV) and the dashed line ($a^{(4)}_{\rm sym}$=–50 MeV) are the results using the expression of symmetry energy coefficient of Eq. (7).

On the other hand, according to the liquid drop model, the symmetry energy coefficient of a finite nucleus is usually written as

$$\begin{aligned}a_{\rm sym}(A,I)&=a^{(2)}_{\rm sym}+a^{(4)}_{\rm sym}I^2+o(I^4)\\&\simeq J-a_{\rm ss}A^{-1/3}+a^{(4)}_{\rm sym}I^2,\end{aligned} \tag{7}$$

by using the Leptodermous expansion in terms of powers of $A^{-1/3}$. $J\approx 28-34$ MeV denotes the symmetry energy of nuclear matter at normal density. $a_{\rm ss}$ is the coefficient of the surface symmetry term. $a^{(4)}_{\rm sym}$ is the coefficient of the I^4 term in the expression of symmetry energy.

Figure 1 shows the experimental symmetry energy coefficients as a function of isospin asymmetry I extracted from Eq. (6) for all even-even nuclei with mass number A=80 (solid squares), where $e_{\rm n}(A,I)=e(A,I)-e_{\rm c}(A,I)$, the experimental binding energy per particle $e(A,I)$ is taken from the mass table AME2012 [35], and $e_{\rm c}(A,I)=0.71\frac{Z^2}{A^{4/3}}(1-0.76Z^{-2/3})$. The dotted line ($a_{\rm sym}^{(4)}$=0), the solid line ($a_{\rm sym}^{(4)}$=50 MeV) and the dashed line ($a_{\rm sym}^{(4)}=-50$ MeV) are the results using the expression of symmetry energy coefficient Eq. (7) with $a_{\rm sym}^{(2)}$=23 MeV. From Fig. 1, one can see that only using the expression of symmetry energy coefficient of Eq. (7), the extracted experimental symmetry energy coefficient cannot be reproduced whether it is positive, zero or negative for $a_{\rm sym}^{(4)}$.

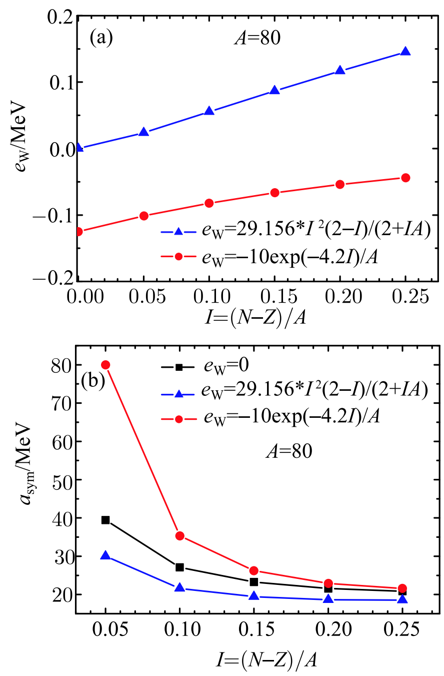

Fig. 2. (a) Two forms of Wigner energy as a function of I, and (b) the extracted symmetry energy coefficients $a_{\rm sym}$ by using the two forms of Wigner energy applied to all even-even nuclei with A=80. The solid squares denote the result of excluded Wigner energy.

The effect of the Wigner energy is responsible for the decrease of $e_{\rm sym}(A,I)/I^2$ with increasing isospin asymmetry I at a given mass number A. To reproduce the experimental data better, one should include the Wigner energy term in Eq. (5). Let us rewrite the expression of Eq. (5) as $e^*_{\rm sym}(A,I)=e_{\rm n}(A,I)-e_{\rm n}(A,0)$, where $e^*_{\rm sym}(A,I)$ is defined as the summation of the traditional symmetry energy $e_{\rm sym}(A,I)$ and the Wigner energy $e_{\rm W}(A,I)$. However, the different Wigner energy expression and parameters will directly affect the extraction of symmetry energy coefficients. Figure 2 (a) presents two forms for Wigner energy as a function of isospin asymmetry I and applied to all even-even nuclei with mass number A=80 in the mass table AME2012. One is $e_{\rm W}=29.156I^2[(2-|I|)/(2+|I|A)]$ (solid triangles), which is proposed in Ref. [33], the other is $e_{\rm W}=-10\exp(-4.2|I|)/A$ [37] (solid circles), which is usually used in the literature. For convenience we denote the former by "form (1)" and the latter by "form (2)". From Fig. 2 (a), the value of $e_{\rm W}$ is positive for form (1) and negative for form (2). While the value $e^*_{\rm sym}(A,I)$ is the summation of the traditional symmetry energy and the Wigner energy, the negative Wigner energy of form (2) will lead to a larger traditional symmetry energy and thus larger symmetry energy coefficient than that with form (1). Figure 2 (b) presents the extracted symmetry-energy coefficients $a_{\rm sym}$ using the two Wigner energy forms for all even-even nuclei with A=80. An obvious discrepancy can be observed by using the two forms for Wigner energy. The solid triangles and solid circles denote the results with form (1) and form (2), respectively. The value of the extracted symmetry-energy coefficients $a_{\rm sym}$ is larger with form (2) than that with form (1), especially for the range of I close to zero, and the discrepancy decreases with increasing isospin asymmetry I. It is therefore necessary to determine the Wigner energy of nuclei for a better description of the symmetry energy coefficient.

3 Theoretical framework

In the semi-empirical mass formula, the Wigner energy is usually decomposed into two parts [38, 39]

$$E_{\rm W}(N,Z)=-W(A)|N-Z|-d(A)\delta_{N,Z}\pi_{\rm np}, \tag{8}$$

where $W(A)$ and $d(A)$ are smooth functions of the nuclear mass number A. The first term on the right-hand side of Eq. (8) contributes to all $N\neq Z$ nuclei. The quantity $\pi_{\rm np}$ equals 1 for odd-odd nuclei and vanishes otherwise, and therefore the second term $d(A)$ is nonzero only for $N=Z$ odd-odd nuclei. The Wigner effect mainly stems from the first term in Eq. (8). By combining the first term in Eq. (8), the traditional symmetry energy term $(N-Z)^2/A$ is replaced by the $T(T+x)$ term [40–43]. So odd-odd symmetric nuclei are not considered in the following calculation. $T=|T_z|=\frac{|N-Z|}{2}$ is the isospin value of the nuclear ground state, and $I=(N-Z)/A$ is the isospin asymmetry of a nucleus. Then one has the relation,

$$T=\frac{|I|A}{2}. \tag{9}$$

The symmetry energy term including the Wigner energy can be expressed as

$$E^*_{\rm sym}(A,T)=\frac{4a_{\rm sym}}{A}T(T+x)=\frac{4a_{\rm sym}}{A}T^2+\frac{4a_{\rm sym}}{A}Tx. \tag{10}$$

Inserting Eq. (9) into Eq. (10), we can obtain the symmetry energy per particle expression as a function of mass number A and isospin asymmetry I,

$$e^*_{\rm sym}(A,I)=\frac{E^*_{\rm sym}(A,I)}{A}=a_{\rm sym}I^2+\frac{2a_{\rm sym}x|I|}{A}, \quad (11)$$

where $e^*_{\rm sym}(A,I)=e_{\rm sym}(A,I)+e_{\rm W}$ and $a_{\rm sym}$ is the symmetry energy coefficient expressed as a function of mass number A and isospin asymmetry I. $2a_{\rm sym}x$ denotes the Wigner energy coefficient, the value of x is not well determined from nuclear masses, $x=1$ is associated with neutron-proton exchange interactions in SU(2) symmetry, while $x=4$ corresponds to the full supermultiplet symmetry SU(4) [44]. Further discussion on the Wigner energy can be found in Ref. [45–48]. Here x as a parameter is introduced, named the Wigner energy parameter. The x values have a crucial effect on the symmetry energy coefficient, since the symmetry energy is the summation of the traditional symmetry energy and the Wigner energy. Different x values denote different Wigner energies. Inserting Eq. (11) into Eq. (3) and replacing $e_{\rm sym}(A,I)$ by $e^*_{\rm sym}(A,I)$, the nuclear energy part per particle Eq. (3) becomes

$$\begin{aligned} e_{\rm n}(A,I)&=e_0(A)+e^*_{\rm sym}(A,I)\\ &=e_0(A)+a_{\rm sym}(A,I)\left(1+\frac{2x}{|I|A}\right)I^2, \end{aligned} \quad (12)$$

Inserting Eq. (7) into Eq. (12), and taking the difference of $e_{\rm n}(A,I)$ between two isobaric nuclei with the same odd-even parity, Eq. (4) becomes

$$\begin{aligned} \Delta e^{*(i)}_{\rm sym}&=e_{\rm n}(A,I)-e_{\rm n}(A,I_i)\\ &=a^{(2)}_{\rm sym}(I^2-I_i^2)+a^{(4)}_{\rm sym}(I^4-I_i^4)\\ &\quad+\frac{2a^{(2)}_{\rm sym}x}{A}(|I|-|I_i|)\\ &\quad+\frac{2a^{(4)}_{\rm sym}x}{A}(|I|^3-|I_i|^3), \end{aligned} \quad (13)$$

where i=1, 2, 3, ..., n, $a^{(2)}_{\rm sym}=J-a_{\rm ss}A^{-1/3}$. The dependence of the reference nuclei (A,I_1), (A,I_2), ... , and (A,I_n) can be cancelled through the summation, and the average value $\overline{\Delta e^*_{\rm sym}}$ of the difference of symmetry energy can be expressed as

$$\begin{aligned} \overline{\Delta e^*_{\rm sym}}&=\frac{1}{n}(\Delta e^{*(1)}_{\rm sym}+\Delta e^{*(2)}_{\rm sym}+...+\Delta e^{*(n)}_{\rm sym})\\ &=e_{\rm n}(A,I)-\frac{1}{n}\sum_{i=1}^{n}e_{\rm n}(A,I_i)\\ &=a^{(2)}_{\rm sym}\left(I^2-\frac{1}{n}\sum_{i=1}^{n}I_i^2\right)+a^{(4)}_{\rm sym}\left(I^4-\frac{1}{n}\sum_{i=1}^{n}I_i^4\right)\\ &\quad+\frac{2a^{(2)}_{\rm sym}x}{A}\left(|I|-\frac{1}{n}\sum_{i=1}^{n}|I_i|\right)\\ &\quad+\frac{2a^{(4)}_{\rm sym}x}{A}\left(|I|^3-\frac{1}{n}\sum_{i=1}^{n}|I_i|^3\right), \end{aligned} \quad (14)$$

When neglecting the microscopic shell corrections of nuclei, the result of Eq. (14) $\overline{\Delta e^*_{\rm sym}}=e_{\rm n}(A,I)-\frac{1}{n}\sum_{i=1}^{n}e_{\rm n}(A,I_i)$ is obtained by the measured binding energy per nucleon of each series of isobaric nuclei compiled in AME2012. By using the expression of the right-hand side in Eq. (14) and fitting $\overline{\Delta e^*_{\rm sym}}$ from more than 2200 measured nuclear binding energies, we obtain the optimal values $J=30.25\pm0.10$ MeV, $a_{\rm ss}=56.18\pm1.25$ MeV, $a^{(4)}_{\rm sym}=8.33\pm1.21$ MeV and $x=2.38\pm0.12$ with an rms deviation of 23.42 keV.

4 Results and discussion

In Fig. 3(a), we show the extracted symmetry energy coefficients of nuclei as a function of nuclear mass number. The solid squares denote the extracted symmetry energy coefficients from the measured nuclear masses by using $\frac{\Delta e^{*(1)}_{\rm sym}}{I^2-I_1^2}$ in Eq. (13). The open circles denote the fitting results by Eq. (14) with the optimum parameter values. The experimental value of $\frac{\Delta e^{*(1)}_{\rm sym}}{I^2-I_1^2}$ obtained in our approach by Eq. (13) shows some oscillations and fluctuations, which is probably caused by the shell effect and other nuclear structure effects. In Fig. 3 (b), we show the same data as in Fig. 3(a), but as a function of isospin asymmetry I. From Fig. 1 and Fig. 3(b), we find that the extracted symmetry energy coefficients depend on the corresponding isospin asymmetry of nuclei, which decreases with increasing isospin asymmetry I for the same mass number A, the largest values located in the range of nearly symmetric nuclei. However, the Wigner energy parameter x value influences every parameter in Eq. (14). Figure 4 shows the coefficients J, $a_{\rm ss}$, $a^{(4)}_{\rm sym}$ (in MeV) and σ deviation (in keV) as a function of Wigner energy parameter x. From Fig. 4, the coefficients J (solid squares), $a_{\rm ss}$ (solid circles) and $a^{(4)}_{\rm sym}$ (solid triangles) first increase then decrease with increasing x values in the range from 0 to 12. The rms deviation σ (downward triangles) first decreases then increases with increasing x values. The minimum value of $\sigma=23.42$ keV corresponds to the set optimal parameters values. One may thus expect the coefficient x to lie somewhere between 1 and 4. The volume symmetry coefficient $J\simeq30$ MeV is insensitive to the value x in the range $1\leqslant x\leqslant4$. The surface symmetry coefficient $a_{\rm ss}$ is sensitive to the value x in the range $1\leqslant x\leqslant4$, and its value changes from 38.72 MeV to 65.85 MeV. The coefficient $a^{(4)}_{\rm sym}$ is more sensitive to the value x in the range

$1 \leqslant x \leqslant 4$, changing from -6.98 MeV to 16.56 MeV. So we conclude from the figure that $a_{\rm sym}^{(4)}$ is not well determined from nuclear masses since x is ill-determined. For example, if we change x somewhat from 1.5 to 1.6, the value of $a_{\rm sym}^{(4)}$ changes from negative to positive. So the sign (positive or negative) of $a_{\rm sym}^{(4)}$ is very sensitive to the value of x.

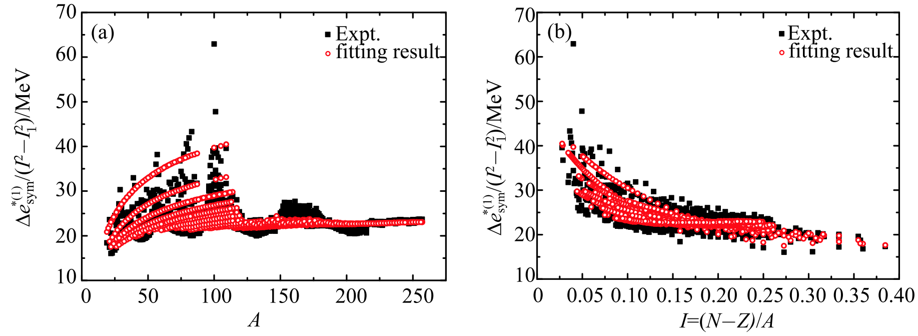

Fig. 3. Symmetry energy coefficients of nuclei as a function of (a) nuclear mass number A and (b) of isospin asymmetry I. The solid squares and open circles denote the experimental data $\dfrac{\Delta e_{\rm sym}^{*(1)}}{I^2-I_1^2}$ and the fitting results by Eq. (14) with the optimum parameter values $J=30.25$ MeV, $a_{\rm ss}=56.18$ MeV, $a_{\rm sym}^{(4)}=8.33$ MeV and $x=2.38$.

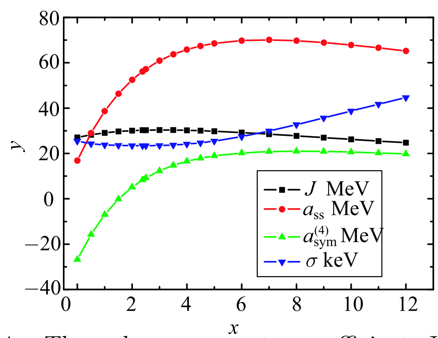

Fig. 4. The volume symmetry coefficient J, surface symmetry coefficient $a_{\rm ss}$, the coefficient $a_{\rm sym}^{(4)}$ of I^4 term (in MeV) and σ deviation (in keV) as a function of Wigner energy parameter x.

The contributions of symmetry energy and Wigner energy were also studied. As an example, the contribution per term is shown in Fig. 5, where the asymmetric nucleus $I_1 = 0.07$ is selected as the reference nucleus, since it is the minimum value of known nuclei in the mass table AME2012 for $A = 168$. From Fig. 5 (a) one can see that the values of all three terms increase with increasing isospin asymmetry I. When $I < 0.39$, the value of the $a_{\rm sym}^{(4)}$ term $a_{\rm sym}^{(4)}(I^4-I_1^4)$ is less than that of the Wigner term $\dfrac{2x}{A}[a_{\rm sym}^{(2)}(|I|-|I_1|)+a_{\rm sym}^{(4)}(|I|^3-|I_1|^3)]$ in Eq. (13), and when $I \geqslant 0.39$ the value of the $a_{\rm sym}^{(4)}$ term is larger than that of the Wigner term. Figure 5 (b) shows the contribution ratio of each term. The ratio is calculated by the ratio of each term value to the sum of the three term values. From Fig. 5 (b) we can see the changing details per term with increasing isospin asymmetry I. The average contribution ratios of the three terms are 87.92%, 8.27% and 3.81% for the $a_{\rm sym}^{(2)}$ term $a_{\rm sym}^{(2)}(I^2-I_1^2)$, Wigner term and $a_{\rm sym}^{(4)}$ term respectively in the range $I=0.07-0.5$. With the increasing of isospin asymmetry I, the $a_{\rm sym}^{(2)}$ term is the major contributor, which first increases and reaches a maximum at $I=0.27$, and then decreases with increasing isospin asymmetry I. The Wigner term decreases and the $a_{\rm sym}^{(4)}$ term increases with increasing isospin asymmetry I. The contribution ratio of $a_{\rm sym}^{(4)}$ term is less than that of the Wigner term in

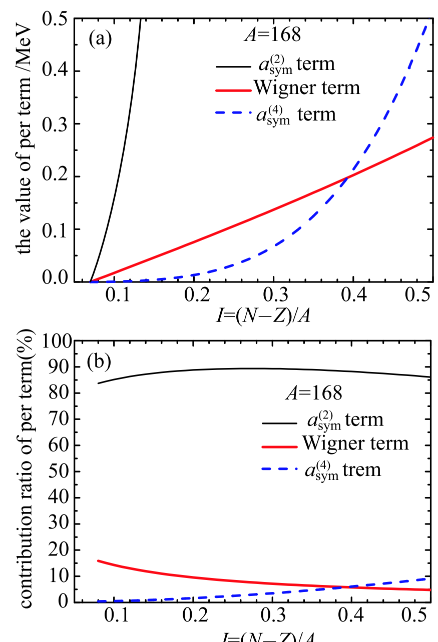

Fig. 5. (a) The values of the $a_{\rm sym}^{(2)}$ term (thin curve), Wigner term (thick curve) and $a_{\rm sym}^{(4)}$ term (dashed curve) in Eq. (13) as a function of I, and (b) the contribution ratio per term for $A=168$.

the range of $I=0.07-0.39$ and larger than the Wigner term when $I\geqslant 0.39$.

5 Summary

In summary, we have proposed a method to extract the symmetry energy coefficient (including the coefficient $a_{\rm sym}^{(4)}$ of I^4 term) from the differences of available experimental binding energies of isobaric nuclei. The advantage of this approach is that one can efficiently remove the volume, surface and pairing energies in the process. It is found that the extracted experimental symmetry energy $e_{\rm sym}^{*}(A,I)$ should be the summation of the traditional symmetry energy $e_{\rm sym}(A,I)$ and the Wigner energy $e_{\rm W}(A,I)$. $a_{\rm sym}^{*}(A,I)$ decreases with increasing isospin asymmetry I, which is mainly caused by the Wigner energy effect. Through the polynomial fit to the result of $\overline{\Delta e_{\rm sym}^{*}}$ by the right-hand side expression of Eq. (14), we have obtained the optimum parameters values $J=30.25\pm0.10$ MeV, $a_{\rm ss}=56.18\pm1.25$ MeV, $a_{\rm sym}^{(4)}=8.33\pm1.21$ MeV and the Wigner parameter $x=2.38\pm0.12$. We also find that the volume symmetry coefficient $J\simeq 30$ MeV is insensitive to the value x, while the surface symmetry coefficient $a_{\rm ss}$ and the coefficient $a_{\rm sym}^{(4)}$ are very sensitive to the value x in the range $1\leqslant x\leqslant 4$, especially for $a_{\rm sym}^{(4)}$, whose value can change from negative to positive due to the change of x value in the range 1 to 4. The contribution of the Wigner energy term decreases and the contribution of the $a_{\rm sym}^{(4)}$ term increases with increasing isospin asymmetry I. For very neutron-rich nuclei, the $a_{\rm sym}^{(4)}$ term will play an important role since its contribution is larger than that of the Wigner energy term.

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