A comparison of direct and indirect determinations of the masses of the Higgs Boson and the top quark

J. H. Field[1)]

Département de Physique Nucléaire et Corpusculaire, Université de Genève 24, quai Ernest-Ansermet CH-1211 Genève 4

Abstract: The indirect estimation of the Higgs Boson mass from electroweak radiative corrections within the Standard Model is compared with the directly measured value obtained by the ATLAS and CMS Collaborations at the CERN LHC collider. Treating the direct measurement of $m_{\rm H}$ as input, the Standard Model indirect estimation of the top-quark mass is also obtained and compared with its directly measured value. A model-independent analysis finds an indirect value of $m_{\rm H}$ of $\simeq$70 GeV, below the directly measured value of 125.7±0.4 GeV and an indirect value: $m_{\rm t}=177.3\pm1.0$ GeV, above the directly measured value: 173.21±0.87 GeV. A goodness-of-fit test to the Standard Model using all Z-pole observables and $m_{\rm W}$ has a χ^2 probability of $\simeq$2%. The reason why probability values about a factor of ten larger than this, and indirect estimates of $m_{\rm H}$ about 30 GeV higher, have been obtained in recent global fits to the same data is recalled.

Keywords: standard electroweak model, LEP, SLC and LHC data, Z-decays, Higgs boson and top quark masses

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1 Introduction

The final results of the analysis of Z-peak data from LEP and SLC were obtained and published in 2006 [1]. Since then, much improved measurements have been made of important parameters appearing in the equations giving indirect predictions, within the Standard Electroweak Model (SM), of the mass of the Higgs boson. In particular the uncertainties in the quantity $\Delta\alpha^{(5)}_{\rm had}(m_{\rm Z})$, appearing in the formula for the electromagnetic coupling constant, the mass, $m_{\rm t}$, of the top quark, and the mass, $m_{\rm W}$, of the W boson have been reduced by factors 3.5, 4.9 and 2.3 respectively. More importantly, a candidate Higgs boson with a precisely measured mass of 125.7(4)[2)] GeV [2] has been discovered [3, 4], at the CERN LHC, by the ATLAS and CMS collaborations.

As discussed in detail in Ref. [1], precise measurements of decay parameters of the Z boson enable the mass, $m_{\rm H}$, of the Higgs boson to be predicted, assuming the correctness of the SM. The subject of the present article is a quantitative statistical comparison of the indirect value of $m_{\rm H}$ obtained in this way with the directly measured value, quoted above, determined at the LHC, as well as a similar comparison for $m_{\rm t}$.

Within the SM, information about the value of $m_{\rm H}$ is contained in the measured values of the effective vector ($\overline{v}_f$) and axial vector ($\overline{a}_f$) coupling constants that describe the decay of the Z boson into fermion (f) anti-fermion ($\bar{f}$) pairs, $f=\ell,\ \nu,\ q$, where ℓ, ν, q denote the charged leptons, neutrinos and quarks of the three known fermion generations of the SM. Further precise information on $m_{\rm H}$ is provided by the measured value of $m_{\rm W}$.

Following previous work by the present author [5–10] it will be found convenient to consider certain combinations of effective coupling constants that are directly related to measured physical 'pseudo observables' [1] (referred to in the following simply as 'observables') chosen in such a way as to minimise uncertainly correlations. Two such observables are [5]:

$$A_f \equiv \frac{2(\sqrt{1-4\mu_f})\overline{r}_f}{1-4\mu_f+(1+2\mu_f)(\overline{r}_f)^2}, \tag{1}$$

where

$$\overline{r}_f \equiv \overline{v}_f/\overline{a}_f,$$

and

$$\overline{s}_f \equiv (\overline{a}_f)^2(1-6\mu_f)+(\overline{v}_f)^2. \tag{2}$$

The parameter $\mu_f=(\overline{m}_f(m_{\rm Z})/m_{\rm Z})^2$, where $\overline{m}_f(Q)$ is the running fermion mass at the scale Q, can be set to zero for $f=\ell,q\neq{\rm b}$ to sufficient accuracy, while for b quarks $(\overline{m}_b(m_{\rm Z})/m_{\rm Z})^2=1.0\times10^{-3}$ [11]. The observable A_f is directly related to charge asymmetry and polarisation

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1) E-mail: john.field@cern.ch

2) Uncertainties on the last significant figures of measured quantities are given enclosed in brackets, so that 100.957±0.063 is written, more compactly, as: 100.957(63).

measurements, $\overline{s}_f$ to decay widths into $f\bar{f}$ pairs. The observables A_f and $\overline{s}_f$ are extracted from experimental data in a rigorously model-independent way i.e. without assuming the validity of the SM predictions for any of the effective coupling constants. This will be contrasted with the tacit model-dependent assumptions that are made in the published global fits to the precision electroweak data.

Statistical methodology has three distinct functions in data analysis [10]: (1) To check the internal consistency of different measurements of the same physical quantity[1)]. (2) To compare suitably-averaged measured physical quantities with theoretical predictions. (3) In the case of satisfactory internal consistency of the data, to adjust the values of unknown parameters of the theoretical model to best describe the data, and to quantify the goodness-of-fit of the resulting description. In the global electroweak fits performed prior to the discovery of a candidate Higgs boson the functions (1)–(3) were mixed-up in a somewhat arbitrary fashion [10]. With the advent of a precise direct measurement of $m_{\rm H}$ all parameters relevant for a test of the Higgs sector of the SM are known experimentally, so only the functions (1) and (2) remain. The analysis presented in the present letter is concerned only with these.

2 Comparison of observables with SM predictions

Since all relevant SM parameters have now been directly measured, simple comparisons (without any parameter fitting) of measured observables can be used to investigate the goodness-of-fit, for different fermion flavours, of the SM predictions. The measurements of A_f and $\overline{s}_f$ for leptons (assuming charged-lepton universality), c-quarks and b-quarks obtained in the model-independent analysis of Ref. [10] as well as the current measurement of $m_{\rm W}$, are presented in Table 1 together with the corresponding SM predictions [10, 12, 13] for: $m_{\rm Z}=91.1876$ GeV, $m_{\rm H}=125.7$ Gev, $m_{\rm t}=173.21$ GeV, $\alpha_s(m_{\rm Z})=0.118$ [2] and $\Delta^{(5)}_{\rm had}(m_{\rm Z})=0.02757$ [14]. Also shown in Table 1 are the 'pulls' (the differences between measurements and predictions divided by experimental uncertainties) and the quantities $S(m_{\rm H})$ and $S(m_{\rm t})$ for each observable. The latter are measures of the sensitivity of the observable to the values of $m_{\rm H}$ or $m_{\rm t}$ [10]. $S(m_{\rm H})$ is defined as the difference between the SM predictions for $m_{\rm H}=200$ GeV and 100 GeV divided by the experimental uncertainity, while $S(m_{\rm t})$ is the difference between the SM predicions for $m_{\rm t}=164$ GeV and 184 GeV divided by the experimental uncertainity. The values of $S(m_{\rm H})$ and $S(m_{\rm t})$ in Table 1 show that practically all sensitivity to both $m_{\rm H}$ and $m_{\rm t}$ resides in the observables A_ℓ, $\overline{s}_\ell$ and $m_{\rm W}$. Indeed since the greatest sensitivity to both masses resides in A_ℓ and $m_{\rm W}$, only these observables are considered in the following indirect determinations of $m_{\rm H}$ and $m_{\rm t}$. In view of the relations defining the square of the effective leptonic weak mixing angle:

$$(\overline{s}^{\ell}_{\rm W})^2\equiv\sin^2\Theta^{\rm lept}_{\rm eff}=\frac{1-\overline{r}_\ell}{4}=\frac{[1-1/A_\ell+\sqrt{(1/A_\ell)^2-1}]}{4}, \quad (3)$$

it can be seen that the observables $\overline{r}_\ell$, A_ℓ and $(\overline{s}^{\ell}_{\rm W})^2$ are mappings of each other and so equally sensitive to the values of $m_{\rm H}$ and $m_{\rm t}$. It will be found convenient in the following, following Ref. [1], to employ the observable $(\overline{s}^{\ell}_{\rm W})^2$.

The sensitivities of A_ℓ or $(\overline{s}^{\ell}_{\rm W})^2$ and $m_{\rm W}$, to the values of $m_{\rm H}$ and $m_{\rm t}$ arise from Feynman diagrams containing loop insertions that contribute to the Z and W boson self-energies [15]. For $m_{\rm H}$ sensitivity, these loops contain trilinear ZZH, WWH vertices or quadrilinear ZZHH, WWHH vertices; for $m_{\rm t}$ sensitivity, $\rm Zt\bar{t}$ and $\rm Wt\bar{b}$ vertices. Recent LHC measurements [16] have set direct limits on the coupling of the Higgs boson to gauge bosons but these are currently too weak to give any appreciable constraint in global electroweak analyses.

Before deriving the indirect predictions for $m_{\rm H}$ and $m_{\rm t}$ from the measured values of $(\overline{s}^{\ell}_{\rm W})^2$ and $m_{\rm W}$ it is interesting to use the data in Table 1 to calculate the statistical goodness-of-fit of various combinations of observables to the SM predictions. This is done by calculation of the Pearson χ^2 statistic for each combination, taking into account any important correlations between the uncertainties of different observables. In fact, only the correlation between the uncertainties of A_ℓ, $A_{\rm c}$ and $A_{\rm b}$ need to be taken into account, in the case that $A_{\rm c}$ and $A_{\rm b}$ are determined from forward-backward charge asymmetries at LEP: $A_{\rm Q}({\rm LEP})=4A^{(0,{\rm Q})}_{FB}/(3A_\ell)$, Q=c,b. The correlation coefficients are [5]:

$$\rho_{\ell{\rm c}}=-0.29, \qquad \rho_{\ell{\rm b}}=-0.52.$$

The polarisation asymmetries of heavy quarks at SLC give the statistically independent measurements: $A_{\rm c}$(SLC) and $A_{\rm b}$(SLC) albeit with larger uncertainties than the LEP ones [10]. In Table 2 the entries in each row are: the combination of observables considered, the Pearson χ^2 statistic, the number of degrees of freedom (d.o.f.) and the corresponding χ^2-probability (CL). The latter is the probability that the value of χ^2 will be equal to, or greater than, the observed value in the case that the theory correctly describes the data, and the quoted uncertainies are interpreted as standard deviations of Gaussian distributions i.e. it is a Confidence Level for agreement of theory and data under these assumptions.

1) This is done, for example in great detail, taking into account uncertainty correlations, in Ref. [1].

The level of agreement between the individual model-independent observables listed in Table 1, and the SM predictions is now summarised. Only A_ℓ, $\overline{s}_\nu$ and A_b show deviations greater than two standard deviations. The situation regarding $\overline{s}_\nu$ and A_b is essentially unchanged since that discussed almost a decade ago in Refs. [1, 10]. However due mainly to new, much improved, measurements of m_t, the agreement for A_ℓ is significantly worse; the previous CL of 0.21 is reduced to 0.039. A similar effect is found when the combination of the two most Higgs-mass-sensitive observables, A_ℓ and m_W, is considered. Previously (see for example Table 9 and Fig. 8 of Ref. [10]) the agreement with the SM for m_H=120 GeV was good (CL = 0.30) to be compared with CL = 0.033 now found in the seventh row of Table 2.

Table 1. Measured values of electroweak observables extracted by the model-independent analysis of Ref. [10], and the measured value of m_W [2] compared with SM predictions for: m_Z=91.1876 GeV m_H=125.7 GeV, m_t=173.21 GeV, $\alpha_s(m_\mathrm{Z})$= 0.118 [2] and $\Delta^{(5)}_\mathrm{had}(m_\mathrm{Z}) = 0.02757$ [14]. Pull≡ [O(expt)-O(SM)]/σ(expt). See text for the definition of S(m_H).

observable	Expt.	SM	Pull	S(m_H)	S(m_t)
A_ℓ	0.1501(16)	0.1468	2.06	-1.74	3.1
$\overline{s}_\ell$	0.25268(26)	0.25264	0.15	-0.7	2.4
$\overline{s}_\nu$	0.5014(15)	0.5050	-2.4	-0.16	0.77
A_c	0.653(20)	0.668	-0.71	-0.045	0.10
$\overline{s}_c$	0.2897(50)	0.2884	0.26	-0.067	0.18
A_b	0.902(13)	0.9347	-2.51	-0.017	0.012
$\overline{s}_b$	0.3663(13)	0.3648	1.20	-0.27	-0.012
m_W/GeV	80.385(15)	80.361	1.6	-2.76	7.9

There has been extensive discussion of the possibly anomalous value of A_b in the literature [1, 2, 5, 7, 17, 18], much less so that of $\overline{s}_\nu$. The NuTeV neutrino scattering experiment [24, 25] is potentially sensitive to the coupling, in the space-like domain, of the Z-boson to neutrino-pairs. The conventional interpretation of the experiment [24, 26] assumes the SM value for the $\mathrm{Z} \to \nu\overline{\nu}$ coupling and extracts an indirect estimate of $\sin^2\theta_\mathrm{W}^\mathrm{on-shell} \equiv 1-(m_\mathrm{W}/m_\mathrm{Z})^2$ that, for $m_\mathrm{t} = 173.21(87)$ GeV and $m_\mathrm{H} = 125.7$ GeV, is 0.22773(164), to be compared with the value calculated directly from m_W and m_Z which is: 0.22290(103) — a 2.5 σ discrepancy. Alternatively, in a model-independent analysis [10, 25], the value of $\sin^2\theta_\mathrm{W}^\mathrm{on-shell}$ as determined by m_W and m_Z can be used to extract from the NuTeV measurement the parameter ρ_0 that gives $\overline{s}_\nu$ as: $\overline{s}_\nu = \rho_0^2\overline{s}_\nu(\mathrm{SM})$ [10]. In this way the value $\overline{s}_\nu(\mathrm{NuTeV}) = 0.4992(21)$ is obtained, that is quite consistent with the value of the observable $\overline{s}_\nu(\mathrm{LEP}) = 0.5014(15)$ shown in Table 1. The weighted average of the LEP and NuTeV measurements is: $\overline{s}_\nu(\mathrm{WA}) = 0.5006(12)$ which differs from $\overline{s}_\nu(\mathrm{SM}) = 0.5050$ by 3.7σ. This is the largest deviation from the SM prediction so far seen for any single precision electroweak observable. Replacing $\overline{s}_\nu$(LEP) in the data in the last row of Table 2 by $\overline{s}_\nu$(WA) increases χ^2 to 28.2, corresponding to CL = 0.0017. Possible theoretical ramifications of an anomalously low value of $\overline{s}_\nu$ have been considered in Refs. [27, 28]. For a discussion of possible QCD-related uncertainties in the interpretation of the NuTeV experiment see Ref. [2] and references therein.

Table 2. Levels of agreement of different combinations of model-independent observables from Table 1 with SM predictions.

observables	χ^2	d.o.f.	CL
A_ℓ	4.24	1	0.039
A_c	0.45	1	0.50
A_b	7.56	1	0.0060
$\overline{s}_\nu$	5.76	1	0.016
m_W	2.56	1	0.11
A_ℓ, $\overline{s}_\nu$	10.0	2	0.0067
A_ℓ, m_W	6.80	2	0.033
A_ℓ, m_W, $\overline{s}_\nu$	12.56	3	0.0057
A_ℓ, A_c(LEP), A_b(LEP)	10.26	3	0.016
A_ℓ, A_c(LEP), A_b(LEP) $\overline{s}_\nu$	16.0	4	0.003
A_ℓ, A_c(LEP), A_b(LEP) m_W	12.8	4	0.012
A_ℓ, A_c(LEP), A_b(LEP) m_W, $\overline{s}_\nu$	18.6	5	0.0023
A_ℓ, A_c(LEP), A_b(LEP) m_W, $\overline{s}_\nu$, A_c(SLC), A_b(SLC) $\overline{s}_\ell$, $\overline{s}_c$, $\overline{s}_b$	20.5	10	0.025

The general question of the goodness-of-fit of the SM to the ensemble of, or subsets of, previously-averaged precision electroweak data, as illustrated by the results presented in Table 2 above, has been little discussed in published global analyses [1, 2, 20, 21]. The level of agreement of the data with the SM is conventionally assessed by the χ^2-probability of a fit to the totality of the electroweak data, which typically includes several different measurements of the same electroweak observable, or different observables such as A_LR, $(\overline{s}^\ell_\mathrm{W})^2$, or $A_{FB}^{(0,\ell)}$ that have equivalent sensitivities to the values of m_H and m_t. The global fit to 18 data in Ref. [1] gave a χ^2 of 18.3 for 13 d.o.f. corresponding to CL =0.15. A similar more recent fit [20], including also the directly measured value of m_H gave χ^2/d.o.f. = 17.8/14, CL = 0.22. The recent fit of Ref. [2] to about 40 experimental data, including also the directly measured Higgs boson mass, claimed CL = 0.30. In contrast, the SM comparison, including all m_H-sensitive observables, in the last row of Table 2 has CL = 0.025 —an order of magnitude lower. The reason for such discrepancies was explained in Ref. [10] and will be briefly recalled below. The fit of Ref. [21] to essentially the same set of observables as Ref. [20] obtained closely similar fit results, but no confidence level for the overall data/theory agreement was quoted.

3 Indirect estimation of the Higgs mass

Numerical approximation formulas, at two-loop level, giving the SM predictions for $(\overline{s}_{\mathrm{W}}^{\ell})^2$ [12] and m_{W} [13] have been published by Awramik et al. The input parameters for the calculations are:

$$m_{\mathrm{Z}},\ \ m_{\mathrm{t}},\ \ m_{\mathrm{H}},\ \ \Delta\alpha_{\mathrm{had}}^{(5)}(m_{\mathrm{Z}}),\ \ \alpha_s(m_{\mathrm{Z}}).$$

These predictions, as a function of m_{H}, are shown as shaded $\pm 1\sigma$ bands, in comparison with the experimental measurements, in Fig. 1 for $(\overline{s}_{\mathrm{W}}^{\ell})^2$ and Fig. 2 for m_{W}. These bands include the effect of parametric uncertainties due to limited experimental knowledge of the input parameters as well as estimated theoretical uncertainties (related to the effect of missing higher order quantum corrections) of 4.9×10^{-5} for $(\overline{s}_{\mathrm{W}}^{\ell})^2$ [12] and 4 MeV for m_{W} [13]. Also shown are the 95% CL direct lower limit on m_{H} of 114.4 GeV from LEP [19] and the direct LHC measurement: $m_{\mathrm{H}} = 125.7(4)$ GeV [2]. The two determinations of $(\overline{s}_{\mathrm{W}}^{\ell})^2$ in Fig. 1 labelled 'EWWG2006 average'(EWWG) and 'LEP and SLC Model Independent' (MI) correspond to the data analyses of Ref. [1] and [10] respectively. For the former analysis it was assumed that all data —in particular the couplings of the Z boson to b-quarks— are correctly described by the SM; in the latter the model-independent observables presented in Table 1 were extracted from the data before any comparison with SM predictions; i.e. without making any assumption whatever as to the agreement, or not, of the measured value of any observable with the SM prediction of it. As pointed out in Refs. [6, 8, 17, 18] and also discussed in Ref. [1], the apparent deviation of A_{b} from the SM prediction gives a value of A_ℓ (or, equivalently, $(\overline{s}_{\mathrm{W}}^{\ell})^2$) derived from the precise LEP measurement of $A_{FB}^{(0,\mathrm{b})}$ that is markedly different from the value presented in Table 1 above, that is derived from the SLC A_{LR} measurement and forward-backward asymmetries and polarisation measurements for leptonic final states at LEP. Instead of the essentially perfect agreement seen in Fig. 1 between the direct and the indirect determination of m_{H} provided by $(\overline{s}_{\mathrm{W}}^{\ell})^2_{\mathrm{EWWG}}$ a much lower indirect value of $m_{\mathrm{H}} \simeq$ 60 GeV is obtained using $(\overline{s}_{\mathrm{W}}^{\ell})^2_{\mathrm{MI}}$. This is the explanation of the low CL value of 0.039 found in the comparison of the measured value of A_ℓ with the SM prediction seen in Table 2. Inspection of Fig. 2 shows that the measured value of m_{W} yields an indirect determination of m_{H} of about 80 GeV, again lower than the direct LHC measurement. As shown in Table 2, the χ^2-probability for consistency of direct and indirect determinations of m_{W} is 0.11, while that of the combination of A_ℓ and m_{W} is 0.033.

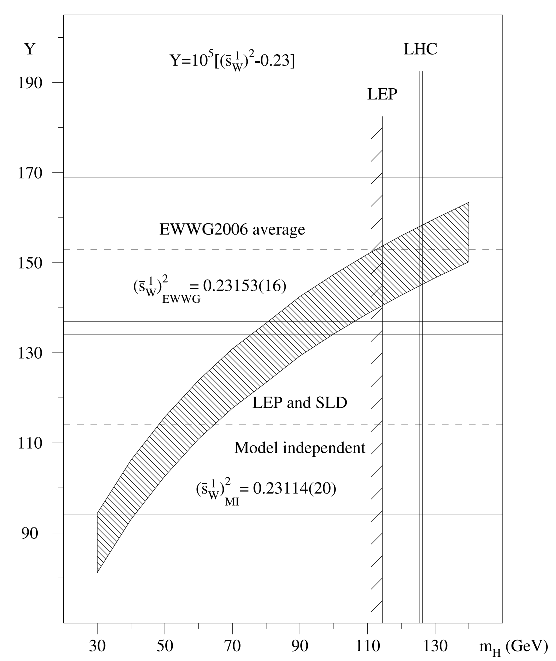

Fig. 1. Comparison of measured values of the observables $(\overline{s}_{\mathrm{W}}^{\ell})^2$ with the Standard Model predictions as a function of m_{H} as given in Ref. [12]. the shaded band corresponds to the $\pm 1\sigma$ parametric and theoretical uncertainties in the prediction. The LEP lower limit on, and the LHC measurement of, m_{H} are also shown.

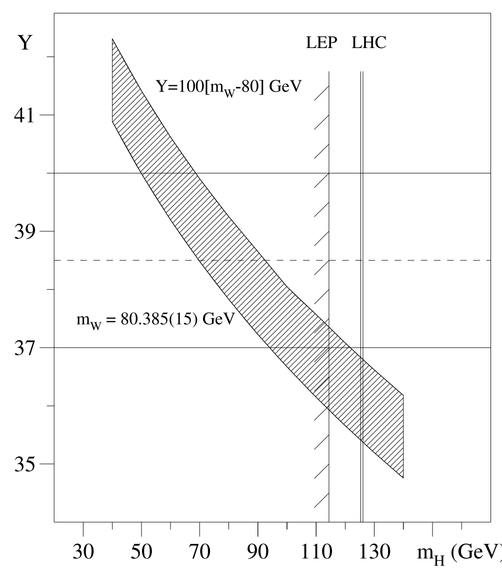

Fig. 2. Comparison of the measured value of m_{W} with the Standard Model prediction as a function of m_{H} as given in Ref. [13]. the shaded band corresponds to the $\pm 1\sigma$ parametric theoretical uncertainties in the prediction. The LEP 95% CL lower limit on, and the LHC measurement of, m_{H} are also shown.

Recent global electroweak fits have given determinations of $(\overline{s}^{\ell}_{\rm W})^2$ including also the directly measured value of $m_{\rm H}$ in the analysis. For example Ref. [20] finds $(\overline{s}^{\ell}_{\rm W})^2 = 0.23149(7)$ which is very close to, but two times more precise than, the value of Ref. [1]: $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG} = 0.23153(16)$ obtained before the candidate Higgs boson discovery. The bias in the value of $(\overline{s}^{\ell}_{\rm W})^2$ resulting from the inclusion of $A^{(0,{\rm b})}_{FB}$ in the fitted data, as discussed below in the present article, is also apparent in the more recent estimation. As will be seen below, the indirect prediction of $m_{\rm H}$ based on $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$ agrees almost perfectly with the directly measured value, so no significant change is expected in the global fit value of $(\overline{s}^{\ell}_{\rm W})^2$ on inclusion of the directly measured value of $m_{\rm H}$.

The values of $m_{\rm H}$ derived from the analytical formulas of Refs. [12, 13] with:

$$(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}{=}0.23153(16),\quad (\overline{s}^{\ell}_{\rm W})^2_{\rm MI}{=}0.23114(20),$$

$$m_{\rm W}{=}80.385(15)\ {\rm GeV}$$

$$m_{\rm Z}{=}91.1876(21)\ {\rm GeV},\quad m_{\rm t}{=}173.21(87)\ {\rm GeV}$$

$$\Delta\alpha{=}0.05907(10),\quad \alpha_{\rm s}(m_{\rm Z}){=}0.118(2)$$

are presented, together with the corresponding uncertainties, in Table 3. The upper and lower uncertainties quoted correspond to $+\sigma$ and $-\sigma$ variations for each source listed in the top row. The uncertainty associated with the value of $m_{\rm Z}$ is negligible and not shown. The 'parametric' uncertainies in the measured values of $m_{\rm t}$, $\Delta\alpha_{\rm had}$ and $\alpha_{\rm s}$, as well as the theory uncertainties, are much smaller than those due to the measurements of $(\overline{s}^{\ell}_{\rm W})^2$ and $m_{\rm W}$.

Table 3. Indirect SM estimates of the Higgs Boson mass given by experimental measurements of $(\overline{s}^{\ell}_{\rm W})^2$ and $m_{\rm W}$ according to the predictions of Refs. [12, 13]. One standard deviation uncertainties on $m_{\rm H}$ due to experimental, parametric and theoretical uncertainties are shown.

observable	$m_{\rm t}$/GeV	uncertainity source				
		observable	$m_{\rm t}$	$\Delta\alpha_{\rm h}$	$\alpha_{\rm s}(m_{\rm Z})$	theory
$(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$	129	51	8	-7	-1	16
		-36	7	11	1	-11
$(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$	56	30	3	-4.5	-0.5	7.4
		-21	-3.5	4.0	0.4	-5.1
$m_{\rm W}$	80	30	10	-3	-1	8.0
		-20	-8	4.0	2	-5.3

Taking weighted averages (including correlations of parametric uncertainities) of values of $m_{\rm H}$ derived from either $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$ or $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$ and $m_{\rm W}$ give:

$$m_{\rm H}[(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG},m_{\rm W}]_{\rm WA}{=}99.6{\pm}26.1\ {\rm GeV},$$

$$m_{\rm H}[(\overline{s}^{\ell}_{\rm W})^2_{\rm MI},m_{\rm W}]_{\rm WA}{=}72.5{+}19.2{-}19.0\ {\rm GeV}.$$

The uncertainties shown reflect correctly the quadratically added $\pm1\sigma$ deviations of input parameters , but are not expected, unlike the latter, to approximately correspond to the σ parameters of Gaussian distributions. This is due to the logarithmic dependences on $m_{\rm H}$ of $(\overline{s}^{\ell}_{\rm W})^2$ and $m_{\rm W}$. For example, treating the uncertainities on $m_{\rm H}[(\overline{s}^{\ell}_{\rm W})^2_{\rm MI},m_{\rm W}]_{\rm WA}$, quoted above, as Gaussian distributed, gives a discrepancy of 53(18.1) GeV with the direct LHC measurement, corresponding to CL = 0.0034, as compared with the estimation (assuming instead Gaussian uncertainties in $(\overline{s}^{\ell}_{\rm W})^2$ and $m_{\rm W}$ and logarithmic dependence on $m_{\rm H}$) given by the combination of A_ℓ and $m_{\rm W}$ in Table 2 of CL = 0.033. The analyses of [20] and [21] which, like the EWWG analysis above, assume that, in the fits, all observables are correctly described by the SM, found similar indirect estimates of $m_{\rm H}$: $m_{\rm H}{=}94{+}25{-}22$ GeV [20], and 99.9±26.64 GeV [21].

Inspection of the first two rows of Table 3 shows that the values of $m_{\rm H}$ obtained from $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$ and $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$ differ by more than a factor of two. It is clearly important to know which, if any, of the two values is favoured statistically. The quantity $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$ is derived, according to Eq. (3), from the weighted average value of A_ℓ as presented in the first row of Table 1. This was derived from all the purely leptonic LEP and SLC charge asymmetry and polarisation measurements, on the assumption of charged lepton universality, as given in Eq. (7.10) of Ref. [1]. The consistency confidence level of the average with the individual measurements is 56%. The corresponding value of $m_{\rm H}$ is calculated on the hypothesis that the actual value of A_ℓ is correctly estimated by the weighted average, which then has a conditional probability of 56%. The quantity $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$ is taken from Fig. 7.6 of Ref. [1]. It is derived with the same conditional probability for data consistency of $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$ and the assumption, in addition, that all values of $A_{\rm had}$ (h = u,d,s,c,b) are given by the SM predictions that (see the last two columns of Table 1) have very weak sensitivity to the values of $m_{\rm H}$

and m_t. This implies that in a global fit, where the latter are varied, all values of $A_{\rm had}$ are assigned essentially the fixed SM values as shown, for example, for A_b and A_c, in Table 1. The weighted average value of $(\overline{s}^{\ell}_{\rm W})^2_{\rm had}$ derived from the hadronic observables $A^{(0,b)}_{FB}$, $A^{(0,c)}_{FB}$ and $A^{(0,Q)}_{FB}$ as shown in Fig. 7.6 of Ref. [1], with the above assumption concerning $A_{\rm had}$, is 0.23222(27). The consistency χ^2 of the weighted average value $(\overline{s}^{\ell}_{\rm W})^2_{\rm had}$ derived from the three hadronic observables is 6.2×10^{-3} for d.o.f. = 2, giving CL=0.9969. This suggests that experimental errors may have been over-estimated, but that the contribution of the consistency χ^2 of $(\overline{s}^{\ell}_{\rm W})^2_{\rm had}$ to the overall consistency χ^2 of $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$ may be neglected. The deviation of $(\overline{s}^{\ell}_{\rm W})^2_{\rm had}$ from the value of $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$ derived from leptonic observables is then:

$$(\overline{s}^{\ell}_{\rm W})^2_{\rm had}-(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}=-10.9(3.3)\times10^{-5}.$$

This shows a 3.3σ deviation with a one-sided CL = 0.001.

The overall consistency confidence level of $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$: CL(EWWG) is then given by combining the independent confidence levels associated with $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$: CL(MI)$\equiv \alpha_1 = 0.56$ and the agreement of $(\overline{s}^{\ell}_{\rm W})^2_{\rm had}$ with $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$: α_2=0.001 [29]:

$$\mathrm{CL(EWWG)}=\alpha_1\alpha_2(1-\ln\alpha_1\alpha_2)=4.8\times10^{-3}.$$

This gives CL(MI)/CL(EWWG) = 118 so that the value of $m_{\rm H}$ derived from $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$ is strongly favoured statistically due to the much better internal consistency of the data contributing to the weighted average. In addition, the MI value of $(\overline{s}^{\ell}_{\rm W})^2$, although having a slightly larger statistical uncertainty[1)] than $(\overline{s}^{\ell}_{\rm W})^2_{\rm EWWG}$, is free from any possible biases (experimental or theoretical) related to the values of $A^{(0,b)}_{FB}$ or A_b.

4 Overall confidence levels of global fits

Concerning the overall goodness-of-fit of the combined LEP, SLC and LHC precision electroweak data to the predictions of the SM, Ref. [2] contains the statement:

'The agreement is generally very good. Despite the few discrepancies discussed in the following the fit describes the data well, with a χ^2/d.o.f. = 48.3/44. The probability of a larger χ^2 is 30%.'

This seems to imply that the 'agreement' of the SM predictions with the data is also very good. However, the χ^2 of the comparison with the SM of the data in the last row of Table 2, including essentially all Higgs-mass sensitive data (including the LHC direct mass measurement) had a CL of only 2.5% —more than an order of magnitude smaller than that claimed for the global fit of Ref. [2].

The reason for this large difference between the SM confidence levels shown in Table 2 based on previously-averaged observables and those quoted for global fits [1, 2, 20] to partially-averaged data was explained in Ref. [10]. The total χ^2 of the global fit can be split into two independant contributions, one from the comparison of the averaged observables with the SM predictions, the other from the comparison of different measurements of each observable with the weighted average of the observable, as well as employing different observables that have equivalent sensitivities to $m_{\rm H}$ and m_t. As shown in detail in Ref. [10] when the 'data consistency' contribution is subtracted from the global χ^2 and the number of degrees of freedom appropriately reduced, the 'SM averaged-data' comparison gives a CL compatible with those presented in Table 2. The confidence level of global fits is also improved by including well-measured quantities in the global fit rather than treating them as fixed input parameters. This increases the number of degrees of freedom at the cost of a negligibly small increase in the χ^2 of the fit. These effects are illustrated by some numbers given in Table 24 of Ref. [10] for the EWWG global fit to 2003 data. Separating the contributions to the global χ^2 of '$m_{\rm H}$-sensitive', 'other' and 'measured' observables gives effective confidence levels for these subsets of 0.0071, 0.78 and 0.998 respectively, whereas the overall χ^2, dominated by the 'data averaging' contribution, has a confidence level of 0.15.

Although the '$A^{(0,b)}_{FB}$ anomaly' is typically discussed in the context of global fits, and the huge difference in the indirect determination of $m_{\rm H}$ given by inclusion or exclusion of the $A^{(0,b)}_{FB}$ datum is clear in plots and tables shown (for example Figs. 8.4, 8.5 and 8.15 of Ref. [1], Table 10.7 of Ref. [2] and Fig. 2 of Ref. [30]), the texts contains no discussion of the effect. In particular the model-dependent assumption that all values of $A_{\rm had}$ are correctly given by the SM predictions (corresponding, as shown above, to a statistically-unlikely premise) is not pointed out, nor that an unbiased estimation of $m_{\rm H}$,(i.e. one that does not depend on any theoretical of experimental considerations concerning $A^{(0,b)}_{FB}$ or the other hadronic forward/backward asymmetries), is given, at the cost of a slightly larger statistical uncertainty, by exclusion of the hadronic asymmetries from the fitted data.

5 Indirect estimation of the top quark mass

The numerical formulas of Awramik et al. giving the SM predictions for $(\overline{s}^{\ell}_{\rm W})^2$ [12] and $m_{\rm W}$ [13] may be used, in conjunction with the direct LHC $m_{\rm H}$ measure-

1) The inclusion of the hadronic observables in the determination of $(\overline{s}^{\ell}_{\rm W})^2$, in the case that all observables are in agreement with SM predictions, gives only at 20% reduction in the uncertainity of $(\overline{s}^{\ell}_{\rm W})^2$.

ment to provide indirect determinations of the mass of the top quark. Predicted values of m_t as a function of m_H as determined by the measured values of $(\overline{s}^{\ell}_W)^2$ and m_W are shown as $\pm1\sigma$ bands in Fig. 3. The values of m_t as determined by different observables and the direct LHC m_H measurement are presented, in a similar format to Table 3, in Table 4. Combining the $(\overline{s}^{\ell}_W)^2$ and m_W determinations it is found that:

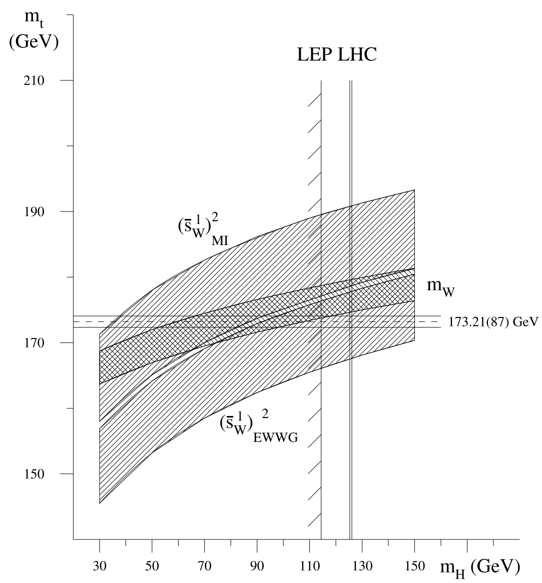

Fig. 3. Comparison of the measured value of m_t with the Standard Model predictions as a function of m_H, given in Refs. [12, 13], as determined by measured values of $(\overline{s}^{\ell}_W)^2$ and m_W. The shaded bands correspond to the $\pm1\sigma$ uncertainties in the predictions. The LEP lower limit on, and the LHC measurement of, m_H are also shown.

$$m_t[(\overline{s}^{\ell}_W)^2_{EWWG}, m_W]_{WA} = 176.2(2.3)\ \text{GeV},$$
$$m_t[(\overline{s}^{\ell}_W)^2_{MI}, m_W]_{WA} = 177.3(1.0)\ \text{GeV}.$$

The weighted average values of m_t derived from $(\overline{s}^{\ell}_W)^2_{EWWG}$ or $(\overline{s}^{\ell}_W)^2_{MI}$ differ by 1.22σ, 3.1σ, respectively, from the direct measurement: m_t=173.21(87) GeV. This is a statistically equivalent way of presenting the larger discrepancy between direct and indirect determinations of m_H found in the model independent analysis, that is also clear by inspection of Fig. 3. The smaller value of m_t obtained using $(\overline{s}^{\ell}_W)^2_{EWWG}$ is also a direct consequence of assuming the SM value of A_{had} in determining the value of this observable.

The weighted average value of m_t, derived from $(\overline{s}^{\ell}_W)^2_{EWWG}$ and m_W, quoted above, is close to the value: $175.8+2.7-2.4$ GeV in a recent global fit [30], that includes the directly measured value of m_H, as well as that given by the similar fit of Ref. [21]: 176.6±2.5. The value of $(\overline{s}^{\ell}_W)^2$ obtained in the fit of [30] is: 0.23150(10) which is very close to $(\overline{s}^{\ell}_W)^2_{EWWG}$.

The larger value of m_t preferred, in the model-independent analysis, in order to accomodate, in the SM, the direct LEP lower limit on m_H was previously pointed out in Ref. [22].

It may also be noted that the model-independent determination of m_t (assumed to be a measurement of m_t(pole)) differs by 2.3σ from the result of the recent QCD analysis of top-quark pair production at the LHC [23] that finds: m_t(pole)=171.2(2.4) GeV.

6 Summary and conclusions

The evolution of the data/theory comparison for the Higgs-mass-sensitive observables A_ℓ and m_W since the 2006 publication [1] of the definitive Z-pole precision electroweak data, including the impact of the direct m_H measurement, is now summarised.

All three quantites: $\Delta\alpha^{(5)}_{had}(m_Z)$, m_W and m_t controlling the indirect determination of m_H are now known with much higher precision than in 2006. While the value of $\Delta\alpha^{(5)}_{had}(m_Z)$ remains essentially unchanged, the values of m_W and m_t have shifted down, respectively, by 41 MeV and 4790 MeV (-2.7σ and -5.5σ, in terms of the current experimental uncertainties). As can be seen from Figs. 2 and 3 these mass shifts favour, respectively, higher and lower values of m_H, but the m_t variation is dominant, favouring a significantly lower indirect

Table 4. Indirect SM estimates of the top quark mass mass given by experimental measurements of $(\overline{s}^{\ell}_W)^2$ and m_W according to the predictions of Refs. [12, 13]. One standard deviation uncertainties on m_t due to experimental, parametric and theoretical uncertainties are shown.

observable	m_t/GeV	uncertainity source				
		observable	m_H	$\Delta\alpha_h$	$\alpha_s(m_Z)$	theory
		5.0.	0.04	1.0	0.2	1.5
$(\overline{s}^{\ell}_W)^2_{EWWG}$	172.8					
		-5.2	-0.04	-1.0	-0.2	-1.6
		6.0	0.05	1.1	0.2	1.5
$(\overline{s}^{\ell}_W)^2_{MI}$	184.9					
		-6.0	-0.05	-1.1	-0.2	-1.5
		0.9	0.02	0.35	0.1	0.24
m_W	177.1					
		-2.6	-0.02	-0.35	-0.1	-0.69

determination of $m_{\rm H}$. This explains why the confidence level of the SM comparison with A_ℓ and $m_{\rm W}$ (see the seventh row of Table 2) is now only 0.033 as compared to the value 0.30 for $m_{\rm H}$=120 GeV given [10] by the 2006 values of $m_{\rm W}$ and $m_{\rm t}$.

The indirect value of $m_{\rm H}$ obtained from A_ℓ and $m_{\rm W}$ (equivalently $(\overline{s}^{\ell}_{\rm W})^2_{\rm MI}$ and $m_{\rm W}$) makes no assumption concerning the SM prediction of the effective coupling constants $\overline{v}_f$ and $\overline{a}_f$. This is no longer the case if the value of $m_{\rm H}$ is determined in a global fit to all data, including, in particular, the observable $A_{FB}^{(0,{\rm b})}$ with a small quoted uncertainty. Since $A_{FB}^{(0,{\rm b})}=3A_\ell A_{\rm b}/4$, and (see Table 1) $A_{\rm b}$ is 100 times less sensitive to the value of $m_{\rm H}$ than is A_ℓ, $A_{\rm b}$ takes essentially a fixed SM value in the fit and the value of A_ℓ determined by the measured value of $A_{FB}^{(0,{\rm b})}$ in the fit differs by three standard deviations from the value of the same parameter as determined by A_{LR} and purely leptonic LEP charge asymmetry and polarisation measurements. The weighted average of these two determinations of A_ℓ corresponds to the observable $(\overline{s}^{\ell}_{\rm W})^2_{{\rm E}WWG}$ discussed above.

Three possible explanations [7, 9, 10, 31, 32] for the anomalous value observed for $A_{FB}^{(0,{\rm b})}$ (and the corresponding values of A_ℓ or $(\overline{s}^{\ell}_{\rm W})^2$) are:

1) An unknown systematic error in the experimental value of $A_{FB}^{(0,{\rm b})}$.

2) 'New Physics' [33–38] (presumably at tree-level) in the effective coupling constants $\overline{v}_f$ and $\overline{a}_f$.

3) A statistical fluctuation in the measured value of $A_{FB}^{(0,{\rm b})}$.

In cases a) and b) the $A_{FB}^{(0,{\rm b})}$ observable can yield no reliable information concerning the value of $m_{\rm H}$ and so must be excluded from any analysis that aims to determine it. In the global fits, the interpretation c) is implicit. The probability of a fluctuation of the size observed is, under the assumption of Gaussian uncertainties, $\simeq$0.001. In comparision the consistency confidence level of the different data contributing to the A_ℓ value quoted in Table 1 is 0.56.

In conclusion, the statistically favoured indirect value of $m_{\rm H}$ is that determined by A_ℓ (or $(\overline{s}^{\ell}_{\rm W})^2_{MI}$) and $m_{\rm W}$:

$$m_{\rm H}(\text{indirect})=72.5+19.2-19.0\ \text{GeV}.$$

The probability that this value is consistent with the LHC direct measurement of 125.7(4) GeV is 0.033.

The statistically-favoured indirect estimation of $m_{\rm t}$ determined by A_ℓ and $m_{\rm W}$ and the directly measured value of $m_{\rm H}$ is:

$$m_{\rm t}(\text{indirect})=177.30(10)\ \text{GeV}.$$

This differs from the directly measured value: 173.21(87) GeV by 3.1σ and by 2.3σ from the top quark pole mass: 171.20(240) GeV given by a QCD analysis of top-quark pair production at the LHC [23].

As explained in detail in Ref. [10] the confidence levels of 0.2−0.3 obtained for global electroweak fits to the SM [1, 2, 20, 30] are quantitatively explained by the dilution of the hypothesis-testing power of the analysis by using unaveraged or equivalent observables and by treating well-measured parameters as fit variables.

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