

# Isospin splitting of nucleon effective mass and symmetry energy in isotopic nuclear reactions<sup>\*</sup>

Ya-Fei Guo(郭亚飞)<sup>1,2</sup> Peng-Hui Chen(陈鹏辉)<sup>2,4</sup> Fei Niu(牛菲)<sup>2,3</sup> Hong-Fei Zhang(张鸿飞)<sup>1</sup>  
Gen-Ming Jin(靳根明)<sup>2</sup> Zhao-Qing Feng(冯兆庆)<sup>2;1)</sup>

<sup>1</sup>School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China

<sup>2</sup>Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China

<sup>3</sup>Institute of Particle and Nuclear Physics, Henan Normal University, Xinxiang 453007, China

<sup>4</sup>University of Chinese Academy of Sciences, Beijing 100190, China

**Abstract:** Within an isospin and momentum dependent transport model, the dynamics of isospin particles (nucleons and light clusters) in Fermi-energy heavy-ion collisions are investigated for constraining the isospin splitting of nucleon effective mass and the symmetry energy at subsaturation densities. The impacts of the isoscalar and isovector parts of the momentum dependent interaction on the emissions of isospin particles are explored, i.e., the mass splittings of  $m_n^* = m_p^*$  and  $m_n^* > m_p^*$  ( $m_n^* < m_p^*$ ). The single and double neutron to proton ratios of free nucleons and light particles are thoroughly investigated in the isotopic nuclear reactions of  $^{112}\text{Sn} + ^{112}\text{Sn}$  and  $^{124}\text{Sn} + ^{124}\text{Sn}$  at incident energies of 50 and 120 MeV/nucleon, respectively. It is found that both the effective mass splitting and symmetry energy impact the kinetic energy spectra of the single ratios, in particular at the high energy tail (larger than 20 MeV). The isospin splitting of nucleon effective mass slightly impacts the double ratio spectra at the energy of 50 MeV/nucleon. A soft symmetry energy with stiffness coefficient of  $\gamma_s = 0.5$  is constrained from the experimental data with the Fermi-energy heavy-ion collisions.

**Keywords:** isotopic reactions, isospin and momentum dependent transport model, symmetry energy, effective mass splitting

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## 1 Introduction

The mass of a nucleon in nuclear matter is different from the in-vacuum case due to its interaction with the surrounding nucleons [1, 2]. In neutron-rich nuclear matter, there exists a splitting of neutron and proton effective masses (the nonrelativistic mass or Landau mass). The strength increases with the isospin asymmetry and the nucleon density. There are wide differences in the predictions of the isospin splitting of nucleon effective mass based on nuclear many-body theories. For example, calculations using Landau-Fermi-liquid theory [3], the nonrelativistic Brueckner-Hartree-Fock theory [4] and Dirac-Brueckner-Hartree-Fock (DBHF) [5] present a neutron-proton mass splitting of  $m_n^* > m_p^*$ . Calculations using relativistic mean-field theory [6], however, give a different result. The Skyrme-Hartree-Fock approach predicts both splittings of  $m_n^* > m_p^*$  and  $m_n^* < m_p^*$  exist with different Skyrme parameters [7]. Recently, the optical

potential has been investigated within the chiral effective field theory and concluded the mass splitting of  $m_n^* > m_p^*$  [8]. The nucleon Landau mass splitting in neutron-rich matter results from the momentum-dependence in the nucleon-nucleon interaction, which directly affects the isospin transport in heavy-ion collisions and consequently the extraction of the density dependence of the symmetry energy. Some observables have been proposed for extracting the isospin splitting of nucleon effective mass, i.e., the neutron/proton ratio at high momenta or kinetic energies, elliptic flows in the mid-rapidity domain, elliptic flow difference between neutrons and protons at the high momentum tail, etc [9, 10].

There are a lot of studies on the constraints of the density dependence of the symmetry energy from heavy-ion collisions, which has important applications in nuclear physics [7, 11–13] and also in astrophysics [14, 15]. It is possible to use heavy-ion reactions with neutron-rich beams to explore the density-dependent symmetry

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1) E-mail: fengzfq@impcas.ac.cn (corresponding author)

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energy and the effective mass splitting of neutrons and protons in nuclear matter. In this work, we investigate the effective mass splitting of neutrons and protons and the density-dependent symmetry energy at sub-saturation densities in heavy-ion collisions within the Lanzhou quantum molecular dynamics (LQMD) transport model. The momentum dependence of the symmetry potential is included in the model, which results in the splitting of the nucleon effective mass in a nuclear medium [16]. We concentrate on the isospin dynamics of fast nucleons and light clusters.

## 2 Model description

In the LQMD model, the temporal evolutions of the baryons (nucleons and resonances) and mesons in the reaction system under the self-consistently generated mean-field are governed by Hamilton's equations of motion. Based on the Skyrme interactions, we constructed an isospin-, density-, and momentum-dependent potentials originating from the Hamiltonian, which consists of the relativistic energy, the effective interaction and the momentum-dependent potentials. The effective interaction potential is composed of the Coulomb potential and the local interactions.

The local interaction potential is derived from the energy-density functional in the form  $U_{\text{loc}} = \int V_{\text{loc}}(\rho(\mathbf{r}))d\mathbf{r}$ . The functional reads

$$V_{\text{loc}}(\rho) = \frac{\alpha}{2} \frac{\rho^2}{\rho_0} + \frac{\beta}{1+\gamma} \frac{\rho^{1+\gamma}}{\rho_0^\gamma} + E_{\text{sym}}^{\text{loc}}(\rho)\rho\delta^2 + \frac{g_{\text{sur}}}{2\rho_0} (\nabla\rho)^2 + \frac{g_{\text{sur}}^{\text{iso}}}{2\rho_0} [\nabla(\rho_n - \rho_p)]^2, \quad (1)$$

where  $\rho_n$ ,  $\rho_p$  and  $\rho = \rho_n + \rho_p$  are the neutron, proton and total densities, respectively, and  $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$  is the isospin asymmetry. The parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $g_{\text{sur}}$   $g_{\text{sur}}^{\text{iso}}$  and  $\rho_0$  are taken to be  $-215.7$  MeV,  $142.4$  MeV,  $1.322$ ,  $23$  MeV  $\text{fm}^2$ ,  $-2.7$  MeV  $\text{fm}^2$  and  $0.16$   $\text{fm}^{-3}$ , respectively. A compression modulus of  $K=230$  MeV for isospin symmetric nuclear matter is obtained with these parameters. The local part  $E_{\text{sym}}^{\text{loc}}(\rho) = \frac{1}{2}C_{\text{sym}}(\rho/\rho_0)^{\gamma_s}$  with  $\gamma_s=0.5$ , 1 and 2 leads to the soft, linear and hard symmetry energies, respectively. The parameter  $C_{\text{sym}}$  is taken as the values of  $52.5$  MeV and  $23.5$  MeV for the effective mass splittings of  $m_n^* > m_p^*$  and  $m_n^* < m_p^*$ , respectively, which leads to the symmetry energy of  $31.5$  MeV at saturation density.

The nucleon effective mass in nuclear medium is contributed from the momentum-dependent interaction in the LQMD model. A Skyrme-type momentum-dependent potential is used in the LQMD model [17]

$$U_{\text{mom}} = \frac{1}{2\rho_0} \sum_{i,j,j \neq i} \sum_{\tau,\tau'} C_{\tau,\tau'} \delta_{\tau,\tau_i} \delta_{\tau',\tau_j} \int \int \int dp dp' d\mathbf{r}$$

$$\times f_i(\mathbf{r}, \mathbf{p}, t) [\ln(\epsilon(\mathbf{p} - \mathbf{p}')^2 + 1)]^2 f_j(\mathbf{r}, \mathbf{p}', t). \quad (2)$$

Here  $C_{\tau,\tau} = C_{\text{mom}}(1+x)$ ,  $C_{\tau,\tau'} = C_{\text{mom}}(1-x)$  and the isospin symbols  $\tau(\tau')$  represent the proton or neutron. The parameters  $C_{\text{mom}}$  and  $\epsilon$  were determined by fitting the real part of the optical potential as a function of incident energy from the proton-nucleus elastic scattering data. The effective (Landau) mass in nuclear matter is calculated through the potential as  $m_\tau^* = m_\tau / \left(1 + \frac{m_\tau}{|\mathbf{p}|} \left| \frac{dU_\tau}{d\mathbf{p}} \right| \right)$  with the in-vacuum mass  $m_\tau$  at Fermi momentum of  $\mathbf{p} = \mathbf{p}_F$ . Therefore, the nucleon effective mass only depends on the momentum-dependent interactions. In the calculation, we take the values of  $1.76$  MeV,  $500$   $\text{c}^2/\text{GeV}^2$  for the  $C_{\text{mom}}$  and  $\epsilon$ , respectively, which result in the effective mass  $m^*/m=0.75$  in nuclear medium at saturation density for symmetric nuclear matter. The parameter  $x$  changes as the strength of the mass splitting, and the values of  $-0.65$  and  $0.65$  are respective to the cases of  $m_n^* > m_p^*$  and  $m_n^* < m_p^*$ , respectively.

## 3 Results and discussion

The dynamics of isospin particles (nucleons and light clusters) produced in heavy-ion collisions is influenced by the isospin dependent interactions in the mean-field potentials, which could be used as observables for extracting the density dependence of symmetry energy. Fermi-energy heavy-ion collisions have a longer isospin relaxation time, which lead to the pronounced isospin effect in comparison to high-density probes such as  $\pi^-/\pi^+$ ,  $K^0/K^+$ ,  $\Sigma^-/\Sigma^+$ , etc [18, 19]. It has been found that the momentum-dependent potential plays an important role in fast nucleon emissions in heavy-ion collisions [10, 17]. The pre-equilibrium neutron and proton transverse emission in isotopic reaction systems at an incident energy of  $50$  MeV/A was measured at the National Superconducting Cyclotron Laboratory (Michigan State University, East Lansing, MI, USA) [13]. Recently, the data were updated with lower statistical errors [20]. Shown in Fig. 1 is the kinetic energy spectra of free nucleons and gas-phase nucleons (nucleons, hydrogen and helium isotopes) in collisions of  $^{124}\text{Sn}+^{124}\text{Sn}$  at a beam energy of  $50$  MeV/nucleon with the effective mass splittings of  $m_n^* > m_p^*$  and  $m_n^* < m_p^*$ , respectively. The particles emitted from an impact parameter domain of  $0-3$  fm and perpendicular to the beam direction with a polar angle cut of  $70^\circ < \theta < 110^\circ$  ( $\cos\theta = p_z/\sqrt{p_x^2 + p_y^2 + p_z^2}$ ) are analyzed. The reaction system evolves up to the time of  $300$  fm/c. After that, the neutron to proton ratio reaches a constant value. We count the gas-phase proton and neutron yields by multiplying the proton and neutron numbers on the fragments, respectively. It is interesting to find that the neutron spectra are not sensitive to the isospin splitting of nucleon effective mass. However, the proton yields depend on the splitting, in particular at high kinetic ener-

gies. This is because the momentum-dependent interaction of  $m_n^* > m_p^*$  has a negative contribution to symmetry energy and the effect is more pronounced with increasing nucleon momentum [17]. The negative symmetry energy reduces neutron emission, but is favorable to energetic proton production. The case of  $m_n^* < m_p^*$  gives

exactly the opposite contribution. The influence of the isospin splitting of effective mass on fast nucleon emission in heavy-ion collisions has also been investigated with the stochastic mean-field (SMF) model [21] and the improved quantum molecular dynamics (ImQMD) model [22].

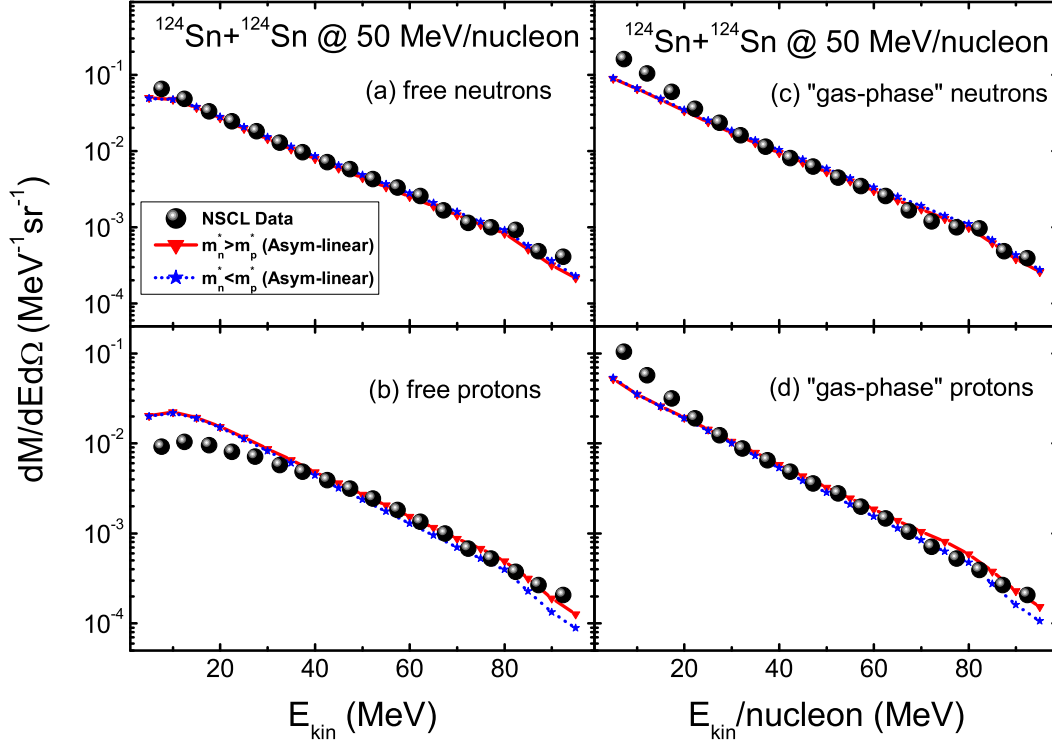


Fig. 1. (color online) Kinetic energy spectra of free nucleons [panels (a) and (b)] and gas-phase nucleons (neutrons, hydrogen and helium isotopes) [panels (c) and (d)] in  $^{124}\text{Sn}+^{124}\text{Sn}$  reactions at an incident energy of 50 MeV/nucleon in the impact parameter domain of 0-3 fm. The experimental data from NSCL [20] are shown for comparison.

Both the isospin splitting of nucleon effective mass and symmetry energy impact the isospin dynamics in heavy-ion collisions, but the specific structure of isospin observable is different with the two quantities. Shown in Figs. 2 and 3 is the neutron to proton (n/p) ratios from the free nucleons and gas-phase fragments in collisions of  $^{112}\text{Sn}+^{112}\text{Sn}$  and  $^{124}\text{Sn}+^{124}\text{Sn}$  at incident energies of 50 MeV/nucleon and 120 MeV/nucleon, respectively. The same constraint condition as in Fig. 1 is used to analyze the transverse emission particles. It is found that the isospin splitting appears at kinetic energies above 20 MeV/nucleon. The case of  $m_n^* < m_p^*$  has a larger n/p ratio for both incident energies. The effect is more pronounced for the neutron-rich system. The bump structure around the energy of 10 MeV/nucleon comes from

the competition of free nucleons and light fragments to the n/p ratios. The light fragments with  $Z \leq 2$  are mainly produced at low kinetic energy and have smaller n/p ratios in comparison to the free nucleons [12]. At kinetic energies above 30 MeV/nucleon, the yields of protons and neutrons are mainly contributed from the free nucleons. The impact of the stiffness of symmetry energy on the kinetic energy spectra is investigated in collisions of  $^{112}\text{Sn}+^{112}\text{Sn}$  and  $^{124}\text{Sn}+^{124}\text{Sn}$  at the incident energy of 50 MeV/nucleon as shown in Fig. 4. The difference between the hard ( $\gamma_s=2$ ) and soft ( $\gamma_s=0.5$ ) symmetry energies is obvious over the whole energy range. A flat structure in the  $^{112}\text{Sn}+^{112}\text{Sn}$  reaction appears with the soft case. The n/p ratios increase with kinetic energy for the neutron-rich system.

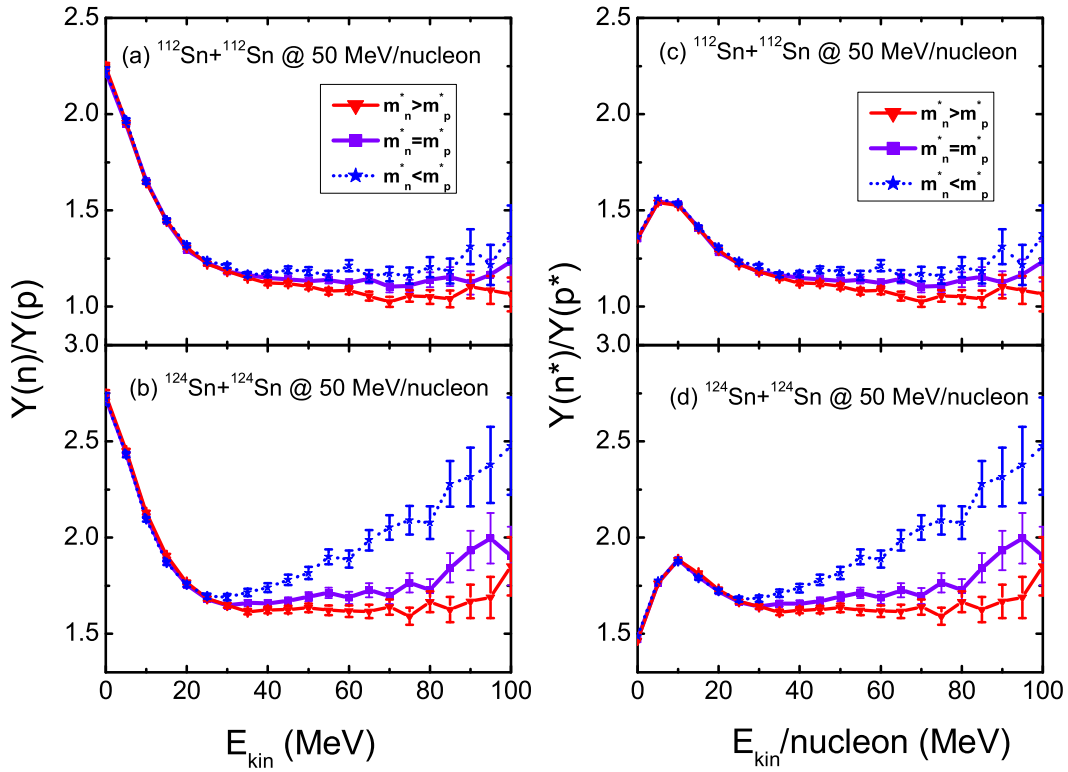


Fig. 2. (color online) Kinetic-energy spectra of neutron to proton ratios from the yields of free nucleons [panels (a) and (b)] and gas-phase fragments (nucleons, hydrogen and helium isotopes) [panels (c) and (d)] in  $^{112}\text{Sn}+^{112}\text{Sn}$  and  $^{124}\text{Sn}+^{124}\text{Sn}$  reactions at 50 MeV/nucleon with effective mass splittings of  $m_n^* > m_p^*$ ,  $m_n^* = m_p^*$  and  $m_n^* < m_p^*$ , respectively.

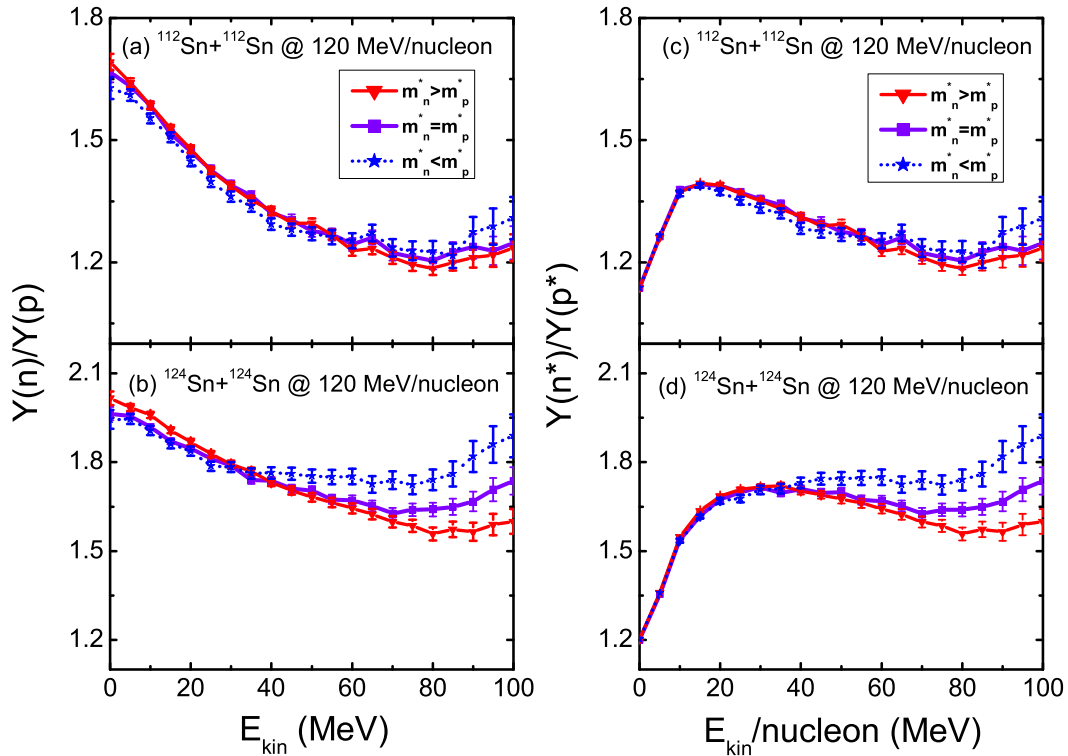


Fig. 3. (color online) The same as Fig. 2, but for the energy of 120 MeV/nucleon.

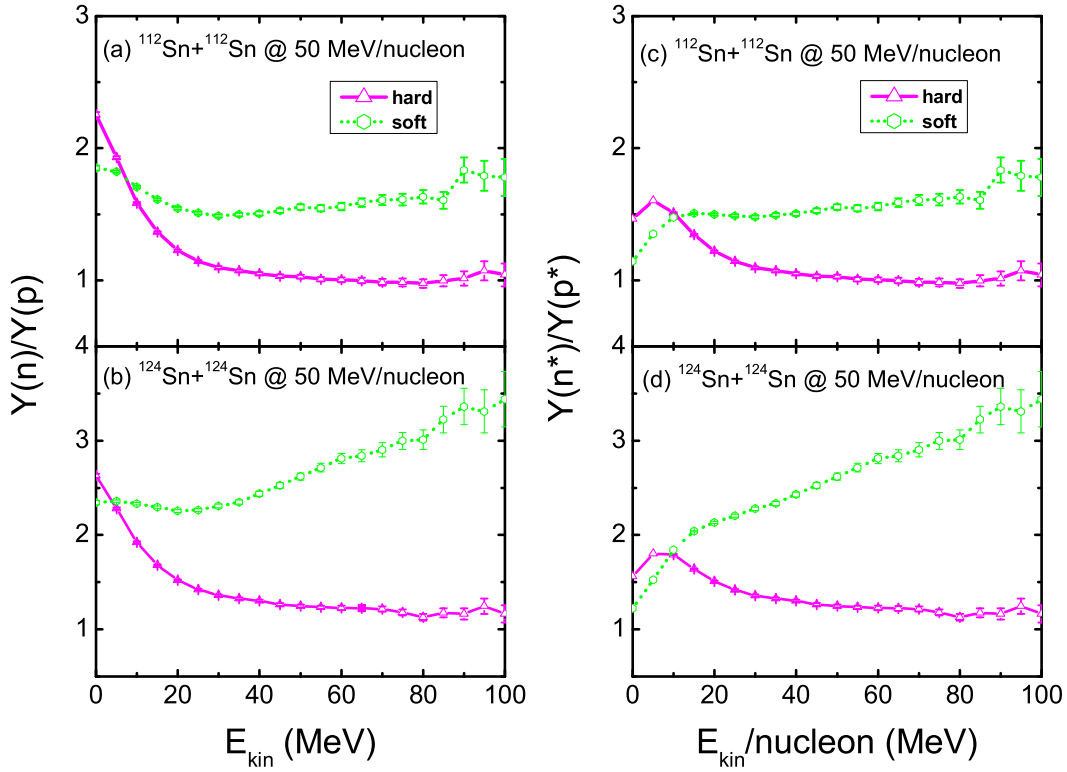


Fig. 4. (color online) The  $n/p$  ratios of free nucleons [panels (a) and (b)] and gas-phase fragments [panels (c) and (d)] as a function of kinetic energy with hard and soft symmetry energies, respectively.

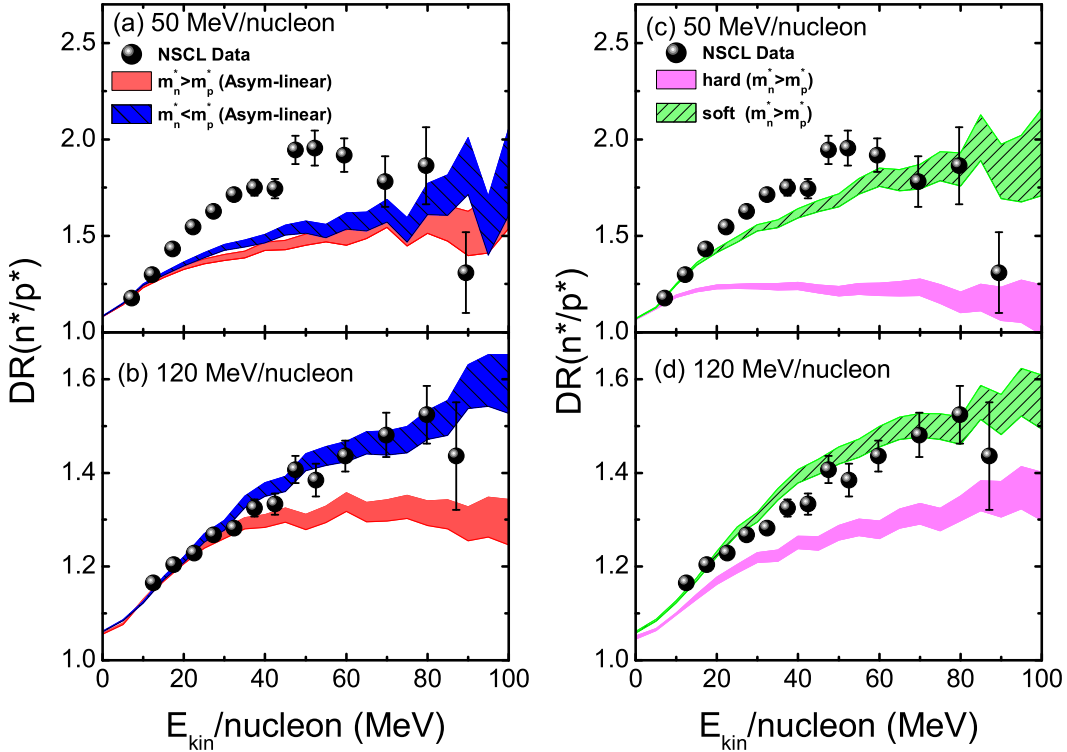


Fig. 5. (color online) The double neutron to proton ratios of gas-phase particles in collisions of  $^{124}\text{Sn}+^{124}\text{Sn}$  over  $^{112}\text{Sn}+^{112}\text{Sn}$  at the incident energy of 50 MeV/nucleon (upper panels) and 120 MeV/nucleon (lower panels) with different isospin splitting of effective mass and different stiffness of symmetry energy. The available data were measured at NSCL [20].

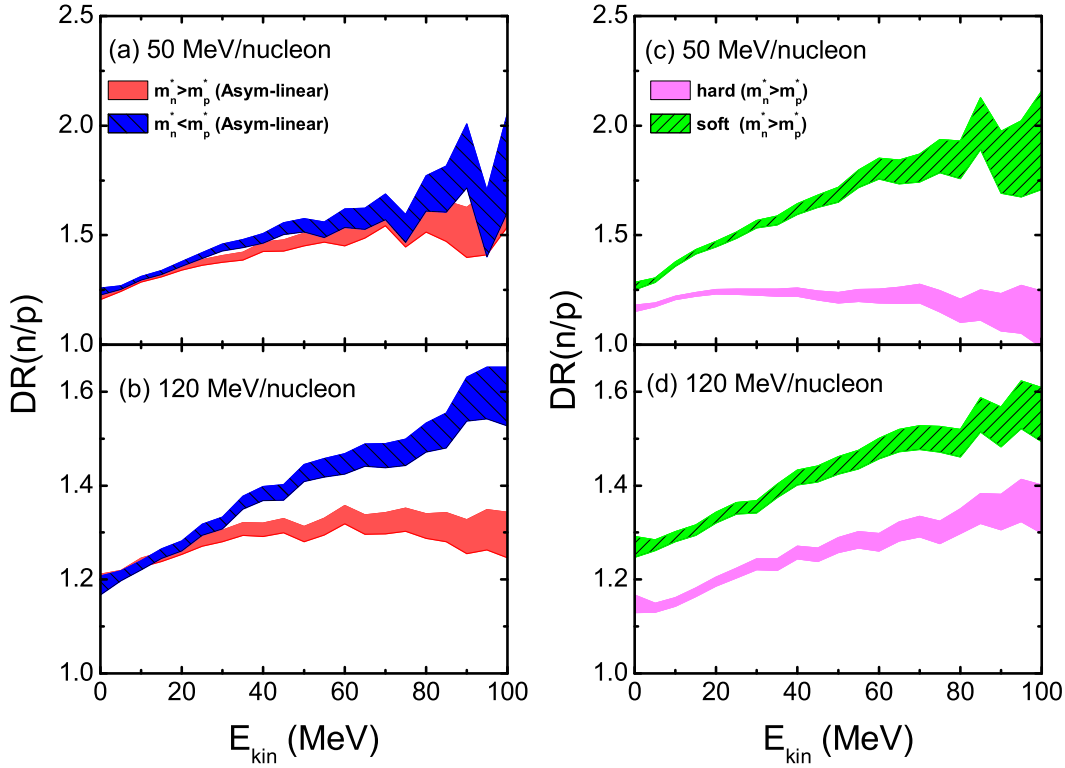


Fig. 6. (color online) The same as Fig. 5, but for the double ratio spectra of free nucleons.

The spectra of the isospin ratios are influenced by both the symmetry energy and the isospin splitting of nucleon effective mass, in particular at high kinetic energies, which confuse the extraction of the density dependence of symmetry energy. On the other hand, the  $n/p$  ratios are also influenced by the Coulomb potential and the detector efficiencies of protons and neutrons in experiments. To eliminate the uncertainties, the double ratios of two isotopic systems would be nice probes for constraining the isospin splitting of nucleon effective mass and the symmetry energy beyond the saturation density from the experimental data. Shown in Fig. 5 are the double ratio spectra in collisions of  $^{124}\text{Sn}+^{124}\text{Sn}$  over  $^{112}\text{Sn}+^{112}\text{Sn}$ . The influence of the isospin splitting with a linear symmetry energy (left panels) and the stiffness of symmetry energy with the mass splitting of  $m_n^* > m_p^*$  (right panels) on the spectra is compared with new data from the NSCL [20]. The effective mass splitting of neutrons and protons in nuclear medium is pronounced at kinetic energies above 30 MeV/nucleon and the splitting of  $m_n^* < m_p^*$  is nicely consistent with the available data at the beam energy of 120 MeV/nucleon. However, the difference of both splittings on the spectra is very small at the beam energy of 50 MeV/nucleon. The symmetry energy effect is obvious and appears at the kinetic energy of 10 MeV/nucleon. A soft symmetry energy ( $\gamma_s=0.5$ ) is constrained and independent of the isospin

splitting of nucleon effective mass. Furthermore, the difference of the soft and hard cases is more pronounced at the beam energy of 50 MeV/nucleon. The double ratio distributions from the free nucleons are further investigated as shown in Fig. 6. A more pronounced effect from the stiffness of symmetry energy is observed, in particular at kinetic energies above 30 MeV. The gas-phase particles at high kinetic energies are mainly contributed from the free nucleons. The effective mass effect is smaller at the incident energy of 50 MeV/nucleon. However, the spectra are pronounced with the different stiffness of symmetry energy. This is because the mass splitting of  $m_n^* < m_p^*$  contributes the repulsive interaction for neutrons and the attractive force for protons in neutron-rich matter. The strength is pronounced with increasing nuclear density [17]. Therefore, the case of  $m_n^* < m_p^*$  leads to larger  $n/p$  ratios, in particular at the incident energy of 120 MeV/nucleon. The contribution of the momentum-dependent interaction to the double ratio spectra is smaller at the lower incident energy, i.e., 50 MeV/nucleon. The conclusions are slightly different from the calculations of the ImQMD model [22] because of the different momentum-dependent interactions. On the other hand, calculations from the SMF transport model have also concluded that the  $n/p$  ratios in Fermi-energy heavy-ion collisions depend weakly on the momentum-dependent interactions [21].

## 4 Conclusions

Within the LQMD transport model we have investigated the isospin dynamics in heavy-ion collisions. The mass splitting of neutrons and protons in a nuclear medium and the symmetry energy impact the kinetic energy spectra of isospin particles. The isospin splitting of nucleon effective mass is more pronounced at the beam energy of 120 MeV/nucleon. However, the difference between soft and hard symmetry energies is obvious at the lower energy (50 MeV/nucleon). The soft symmetry energy with  $\gamma_s=0.5$  is constrained from the

double ratio spectra at the energy of 50 MeV/nucleon. The single n/p ratio decreases with the kinetic energy and then goes up for the case of  $m_n^* < m_p^*$ , in particular for the neutron-rich system. A flat structure of the n/p ratio from the free nucleons appears with the soft symmetry energy until the kinetic energy of 30 MeV, and the ratios in the gas-phase go up monotonically with increasing kinetic energy in the  $^{124}\text{Sn}+^{124}\text{Sn}$  reaction at incident energies of 50 MeV/nucleon.

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