Charmed baryon decay asymmetry in e⁺e⁻ annihilation *

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Abstract: We propose to measure the decay asymmetry parameters in the hadronic weak decays of singly charmed baryons, such as $\Lambda_c^+ \to \Lambda\pi^+, \Sigma^0\pi^+, p\bar{K}_0, \Xi_c^0 \to \Xi^-\pi^+$ and $\Omega_c^0 \to \Omega^-\pi^+$. The joint angular formulae for these processes are presented, and are used to extract the asymmetry parameters in e⁺e⁻ annihilation data. Base on the current Λ_c^+ data set collected at BESIII, we estimate the experimental sensitivities to measure the parameters $\alpha_{\Lambda\pi^+}$ for $\Lambda_c^+ \to \Lambda\pi^+$, $\alpha_{\Sigma^+\pi^0}$ for $\Lambda_c^+ \to \Sigma^+\pi^0$ and $\alpha_{\Sigma^0\pi^+}$ for $\Lambda_c^+ \to \Sigma^0\pi^+$.

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1   Introduction

As hyperon decays, the hadronic weak decay of singly charmed baryons, $\mathcal{B}_c$, is expected to violate parity conservation. In the quark model, $\mathcal{B}_c$ consists of a charm quark (c) and two other light quarks (u, d and s). The ground state $\mathcal{B}_c$ has spin $\frac{1}{2}$, and decays via the weak interaction, dominantly producing final states involving a particle with strangeness. For example, the two body decay, $\Lambda_c^+ \to \Lambda\pi^+$, goes via a W-interaction, $c \to W^+ + s$, where P-parity is not conserved. S- and P-waves are allowed between the Λ and π^+ particles. The parity violation is manifested by the polarization of charmed baryons, which is characterized by the angular distribution of Λ in the Λ_c^+ rest frame, taking the form of

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta_\Lambda} \propto 1 + \alpha_{\Lambda\pi}\cos\theta_\Lambda,$$

where $\alpha_{\Lambda_c}^+$ is the decay asymmetry parameter.

Some decay asymmetry parameters in $\mathcal{B}_c$ decays, e.g., $\alpha_{\Lambda\pi}$ for $\Lambda_c^+ \to \Lambda\pi^+$ and $\alpha_{\Sigma^+\pi^0}$ for $\Lambda_c^+ \to \Sigma^+\pi^0$, have been measured in the FOCUS [1], CLEO [2–4] and AR-GUS [5] experiments, as listed in Table 1. Measurements of $\alpha_{\Lambda\pi^+}$ are consistent with each other either in the e⁺e⁻ annihilation or photoproduction experiments within one standard deviation. However, their precisions are poor with relative uncertainties larger than 15%. Average values of these parameters [6] are $\alpha_{\Lambda\pi^+} = -0.91 \pm 0.15$ and $\alpha_{\Sigma^0\pi^+} = -0.45 \pm 0.31 \pm 0.06$. For $\Xi_c^0 \to \Xi^-\pi^+$ decays, only one measurement from e⁺e⁻ annihilation was performed, and $\alpha_{\Xi^-\pi^+}$ was measured with relative uncertainty of 41% as listed in Table 1.

Table 1.   Experimental measurements of decay asymmetry parameters $\alpha_{\Lambda\pi^+}$ for $\Lambda_c^+ \to \Lambda\pi^+$, $\alpha_{\Sigma^+\pi^0}$ for $\Lambda_c^+ \to \Sigma^+\pi^0$ and $\alpha_{\Xi^-\pi^+}$ for $\Xi_c^0 \to \Xi^-\pi^+$. The theoretical predictions are also listed.

experiment	data sets	$\alpha_{\Lambda\pi^+}$	$\alpha_{\Sigma^+\pi^0}$	$\alpha_{\Xi^-\pi^+}$
FOCUS[1]	$\gamma A \sim 80$ GeV	$-0.78\pm0.16\pm0.19$		
CLEO [2]	e⁺e⁻ $\sim \Upsilon(4S)$	$-0.94^{-0.21+0.12}_{-0.06-0.06}$	$-0.45\pm0.31\pm0.06$	
ARGUS[5]	e⁺e⁻ ~ 10.4 GeV	-0.96 ± 0.42		
CLEO [3]	e⁺e⁻ ~10.6 GeV	$-1.1^{+0.4}_{-0.1}$		
CLEO [4]	e⁺e⁻ $\sim \Upsilon(4S)$			$-0.56\pm0.39^{+0.10}_{-0.09}$
theoretical predictions		$-0.70[7], -0.67[8]$ $-0.95[9], -0.95[10]$ $-0.99[11], -0.99[12]$	$0.71[7], 0.92[8]$ $0.78[9], 0.43[10]$ $0.39[11], -0.31[12]$	$-0.38[7], -0.99[8]$ $-0.38[9], -0.84[10]$ $-0.79[11], -0.97[12]$

These asymmetry parameters have been predicted by many model calculations, as shown in Table 1. The calculated decay asymmetry $\alpha_{\Lambda\pi^+}$ ranges from -0.67 to -0.99, which is consistent with the current world average -0.91 ± 0.15 within 2 standard deviations. The agreement between theory and experiment implies the V-A structure of the decay process $\Lambda_c^+ \to \Lambda\pi^+$. The decay asymmetry $\alpha_{\Xi^-\pi^+}$ ranges from -0.38 to -0.99, which is compatible with the CLEO measurement [4]. However, most model calculations predict the decay asymmetry $\alpha_{\Sigma^+\pi^0}$ to have a positive value, while the measurement from CLEO [2] gives a negative result of $-0.45\pm0.31\pm0.06$. Additional measurements are important to test the sign of this parameter.

Furthermore, improved measurements with better precisions are desirable to constrain the different model calculations. This is helpful to shed light on the decay mechanism and to test the CP symmetry in charmed baryon sector [1]. Based on a data sample with integrated luminosity of 567 pb^{-1} at $\sqrt{s}=4.6$ GeV accumulated at BESIII recently [13], large statistics of Λ_c^+ events are available through the process $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$. In this paper, we present a proposal to study the decay asymmetry parameters at e^+e^- experiments in the τ-charm energy region, and estimate the sensitivities of measuring $\alpha_{\Lambda\pi^+}$, $\alpha_{\Sigma^0\pi^+}$ and $\alpha_{\Xi^-\pi^+}$ parameters, based on the current BESIII data set.

2   Charmed baryon decay asymmetry

In the process of $e^+e^- \to \gamma^* \to \mathcal{B}_c\bar{\mathcal{B}}_c$, the charmed baryon $\mathcal{B}_c$ pairs are produced from the electromagnetic process. Hence, they are unpolarized, if we ignore the Z-boson contribution, since the Z mass is far from the τ-charm energy region. A direct measurement on the Λ_c^+ polarization was performed at $\Upsilon(4S)/(5S)$ energy points [3], and the results were consistent with the expectation of Λ_c^+ unpolarized production. Based on the unpolarized charmed baryons in e^+e^- annihilation experiment, we will present formulae to measure the Λ_c^+ decay asymmetry parameters in two-body hadronic decays, i.e. $\Lambda_c^+ \to \Lambda\pi^+, \Sigma^0\pi^+, p\bar{K}_0$. In addition, we will extend the discussions to other singly charmed baryon decays, such as $\Xi_c \to \Xi^-\pi^+$ and $\Omega_c^0 \to \Omega^-\pi^+$.

2.1 $\Lambda_c^+ \to \Lambda\pi^+$ and $\Sigma^0\pi^+$

Parity violations in the weak decays of charmed baryons, such as $\Lambda_c^+ \to \Lambda\pi^+$ and $\Sigma^0\pi^+$, give rise to the polarization of the produced hyperons in the final states. For the unpolarized Λ_c^+, the decay asymmetry parameters, $\alpha_{\Lambda\pi^+}$ and $\alpha_{\Sigma^0\pi^+}$, cannot be observed in the hyperon angular distributions, but they would be related to the hyperon polarizations. This implies that one needs to investigate the hyperon decays to probe the hyperon polar-

ization, and then study the Λ_c^+ asymmetry parameters. Experimentally, these hyperon states are reconstructed with the decays $\Lambda \to p\pi^-$, and $\Sigma^0 \to \gamma\Lambda, \Lambda \to p\pi^-$. We choose the nonleptonic decay $\Lambda \to p\pi^-$ as the polarization analyzer.

The amplitude in the weak decay $\Lambda \to p\pi^-$ is conventionally constructed with S- and P-waves, and related to the Λ asymmetry parameter as $\alpha_- = \dfrac{2\mathrm{Re}(S^* \cdot P)}{|S|^2 + |P|^2}$. Instead, we construct the amplitude under the helicity basis. The angular distribution for the decays of Λ into $p\pi^-$ is defined by

$$f(\theta_p, \phi_p) = \sum_{\lambda_p=\pm 1/2} \rho_{M,M'} D_{M,\lambda_p}^{\frac{1}{2}}(\phi_p, \theta_p, 0)$$
$$\times D_{M',\lambda_p}^{\frac{1}{2}*}(\phi_p, \theta_p, 0)|H_{\lambda_p}|^2, \qquad (1)$$

where ρ is the spin density matrix for Λ, and (θ_p, ϕ_p) is the solid angle of the proton in the Λ helicity system, $M(\lambda_p)$ is the helicity of $\Lambda(p)$, and H_{λ_p} is the helicity amplitude, which is related to the asymmetry parameter by $|H_{1/2}|^2 = (1+\alpha_-)/2$ and $|H_{-1/2}|^2 = (1-\alpha_-)/2$, where α_- is the decay asymmetry parameter for $\Lambda \to p\pi^-$. After integrating over the azimuthal angle ϕ_p, Eq. (1) is reduced to

$$f(\theta_p) = \mathrm{Tr}\rho R,$$
$$\text{with } R = \begin{pmatrix} 1+\alpha_- \cos\theta_p & 0 \\ 0 & 1-\alpha_- \cos\theta_p \end{pmatrix}. \qquad (2)$$

The spin density matrix, ρ, measures the Λ polarization, and it is determined from the Λ production process. For the decay $\Lambda_c^+ \to \Lambda\pi^+$, the elements of the ρ matrix are defined by

$$\rho_{\lambda_1,\lambda_2} = \sum_{\lambda=\pm 1/2} D_{\lambda,\lambda_1}^{\frac{1}{2}}(\phi_\Lambda, \theta_\Lambda, 0) D_{\lambda,\lambda_2}^{\frac{1}{2}*}(\phi_\Lambda, \theta_\Lambda, 0) A_{\lambda_1} A_{\lambda_2}^*, \qquad (3)$$

where the sum over the Λ_c^+ helicity value, $\lambda = \pm 1/2$, takes the same probability due to unpolarization of Λ_c^+, $(\theta_\Lambda, \phi_\Lambda)$ is the solid angle with reference to the Λ_c^+ helicity system, and A_λ is the helicity amplitude of this decay, which is related to the asymmetry parameter by $|A_{+\frac{1}{2}}|^2 = (1+\alpha_{\Lambda\pi^+})/2$ and $|A_{-\frac{1}{2}}|^2 = (1-\alpha_{\Lambda\pi^+})/2$. After integrating over ϕ_Λ in Eq. (3), one has

$$\rho = \begin{pmatrix} 1+\alpha_{\Lambda\pi^+} & 0 \\ 0 & 1-\alpha_{\Lambda\pi^+} \end{pmatrix}. \qquad (4)$$

Then the joint angular distribution for $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-$, $\Lambda_c^+ \to \Lambda\pi^+$, $\Lambda \to p\pi^-$ is calculated to be [3]

$$\frac{dN}{d\cos\theta_p} \propto 1 + \alpha_{\Lambda\pi^+}\alpha_- \cos\theta_p, \qquad (5)$$

where α_- is the decay asymmetry parameter for $\Lambda \to p\pi^-$.

It is natural to apply the above formulae to other singly charmed baryon decays like $(\Lambda_c,\ \Xi_c,\ \Omega_c)\left(\frac{1}{2}^+\right) \to Y\left(\frac{1}{2}^+\right)P(0^-),\ Y \to \mathcal{B}\left(\frac{1}{2}^+\right)P(0^-)$, where P denotes a pseudoscalar particle, and Y is a hyperon particle, decaying to a baryon ($\mathcal{B}$) and a pseudoscalar particle. The decay asymmetry parameters, $\alpha_{\Lambda\pi} + \alpha_-$ in Eq. (5), are replaced with those in the $\Lambda_c,\ \Xi_c,\ \Omega_c$ and subsequent hyperon decays. For example, for the decay $\Lambda_c^+ \to \Sigma^+\pi^0,\ \Sigma^+ \to p\pi^0$, its angular distribution has a similar form to [2]

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta_p} \propto 1 + \alpha_{\Sigma^+\pi^0}\alpha_{p\pi^0}\cos\theta_p, \tag{6}$$

where $\alpha_{p\pi^0}$ is the decay asymmetry parameter for $\Sigma^+ \to p\pi^0$.

For the case of Λ production from $\Lambda_c^+(\lambda) \to \Sigma^0(\lambda_2)\pi^+,\ \Sigma^0 \to \Lambda(\lambda_3)\gamma(\lambda_4)$, where (λ_i) represents the helicity value for the corresponding particle. The elements of the spin density matrix are defined by

$$\rho_{\lambda_3,\lambda_3'} = \sum_{\lambda=\pm 1/2} D_{\lambda,\lambda_2}^{\frac{1}{2}}(\phi_{\Sigma^0},\theta_{\Sigma^0},0)D_{\lambda,\lambda_2}^{\frac{1}{2}*}(\phi_{\Sigma^0},\theta_{\Sigma^0},0)$$

$$\times D_{\lambda_2,\lambda_3-\lambda_4}^{\frac{1}{2}}(\phi_\Lambda,\theta_\Lambda,0)D_{\lambda_2',\lambda_3'-\lambda_4}^{\frac{1}{2}*}(\phi_\Lambda,\theta_\Lambda,0)$$

$$\times A_{\lambda_2}A_{\lambda_2'}^*B_{\lambda_3,\lambda_4}B_{\lambda_3',\lambda_4}^*, \tag{7}$$

where $(\theta_{\Sigma^0},\phi_{\Sigma^0})$ is the Σ^0 solid angles in the Λ_c^+ helicity system, whose z-axis is taken along the direction of Σ^0 flight, and A_λ is the helicity amplitude of the decay $\Lambda_c^+ \to \Sigma^0\pi^+$, which is related to the asymmetry parameter by $|A_{+\frac{1}{2}}|^2 = (1+\alpha_{\Sigma^0\pi^+})/2$ and $|A_{-\frac{1}{2}}|^2 = (1-\alpha_{\Sigma^0\pi^+})/2$. $(\theta_\Lambda,\phi_\Lambda)$ is the Λ solid angles in the Σ^0 helicity system, following the convention that the z-axis is taken along the direction of Λ flight, and B_{λ_3,λ_4} is the helicity amplitude of the decay $\Sigma^0 \to \Lambda\gamma$. The parity conservation in this radiative decay implies that $B_{\frac{1}{2},1} = B_{-\frac{1}{2},-1}$. After integrating over angles ϕ_Σ^0 and ϕ_Λ, one has

$$\rho = \begin{pmatrix} 1-\alpha_{\Lambda_c^+}\cos\theta_\Lambda & 0 \\ 0 & 1+\alpha_{\Lambda_c^+}\cos\theta_\Lambda \end{pmatrix}. \tag{8}$$

Therefore, the joint angular distribution for the sequential decay $e^+e^- \to \Lambda_c^+\bar{\Lambda}_c^-,\ \Lambda_c^+ \to \Sigma^0\pi^+,\Sigma^0 \to \Lambda\gamma,\Lambda \to p\pi^-$ reads [2]

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta_\Lambda\mathrm{d}\cos\theta_p} \propto 1 - \alpha_{\Sigma^0\pi^+}\alpha_-\cos\theta_\Lambda\cos\theta_p. \tag{9}$$

2.2 $\Xi_c^0 \to \Xi^-\pi^+,\ \Xi^- \to \Lambda\pi^-$

For the case of Λ production from $\Xi_c^0(\lambda) \to \Xi^-(\lambda_2)\pi^+,\ \Xi^- \to \Lambda(\lambda_3)\pi^-$, the elements of the spin density matrix are defined by

$$\rho_{\lambda_3,\lambda_3'} = \sum_{\lambda=\pm 1/2} D_{\lambda,\lambda_2}^{\frac{1}{2}}(\phi_{\Xi^-},\theta_{\Xi^-},0)D_{\lambda,\lambda_2'}^{\frac{1}{2}*}(\phi_{\Xi^-},\theta_{\Xi^-},0)$$

$$\times D_{\lambda_2,\lambda_3}^{\frac{1}{2}}(\phi_\Lambda,\theta_\Lambda,0)D_{\lambda_2',\lambda_3'}^{\frac{1}{2}*}(\phi_\Lambda,\theta_\Lambda,0)$$

$$\times A_{\lambda_2}A_{\lambda_2'}^*B_{\lambda_3}B_{\lambda_3'}^*, \tag{10}$$

where $(\theta_{\Xi^-},\phi_{\Xi^-})$ is the Ξ^- solid angles in the Ξ_c^0 helicity system, whose z-axis is taken along the direction of Ξ^- flight, and A_λ is the helicity amplitude of the decay $\Xi_c^0 \to \Xi^-\pi^+$, which is related to the asymmetry parameter by $|A_{+\frac{1}{2}}|^2 = (1+\alpha_{\Xi^-\pi^+})/2$ and $|A_{-\frac{1}{2}}|^2 = (1-\alpha_{\Xi^-\pi^+})/2$. $(\theta_\Lambda,\phi_\Lambda)$ is the Λ solid angles in the Ξ^- helicity system, where the z-axis is taken along the direction of Λ flight, and B_{λ_3} is the helicity amplitude of the decay $\Xi^- \to \Lambda\pi^-$, which is related to the Ξ^- asymmetry parameter by $|B_{+\frac{1}{2}}|^2 = (1 + \alpha_{\Xi^-\pi^+})/2$ and $|A_{-\frac{1}{2}}|^2 = (1 - \alpha_{\Xi^-\pi^+})/2$. Then one has

$$\rho =$$
$$\begin{pmatrix} (1+\alpha_{\Lambda\pi^-})(1+\alpha_{\Xi^-\pi^+}\cos\theta_\Lambda) & 0 \\ 0 & (1-\alpha_{\Lambda\pi^-})(1-\alpha_{\Xi^-\pi^+}\cos\theta_\Lambda) \end{pmatrix}. \tag{11}$$

The joint angular distribution for the sequential decay $e^+e^- \to \Xi_c^0\bar{\Xi}_c^0,\Xi_c^0 \to \Xi^-\pi^+,\Xi^- \to \Lambda\pi^-,\Lambda \to p\pi^-$ reads [2]

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta_\Lambda\mathrm{d}\cos\theta_p} \propto 1 + \alpha_{\Xi^-\pi^+}\alpha_{\Lambda\pi^-}\cos\theta_\Lambda$$

$$+ \alpha_-\alpha_{\Xi^-\pi^+}\cos\theta_\Lambda\cos\theta_p$$

$$+ \alpha_-\alpha_{\Lambda\pi^-}\cos\theta_p. \tag{12}$$

2.3 $\Omega_c^0 \to \Omega^-\pi^+,\ \Omega^- \to \Lambda K^-$

Let us consider the decay $\Omega_c^0\left(\frac{1}{2}^+\right) \to \Omega^-\left(\frac{3}{2}^+\right)\pi^+$ and $\Omega^-\left(\frac{3}{2}^+\right) \to \Lambda\left(\frac{1}{2}^+\right)K^-(0^-)$. The element of spin density matrix for the Λ is defined by

$$\rho_{\lambda_3,\lambda_3'} = \sum_{\lambda,\lambda_2,\lambda_2'} D_{\lambda,\lambda_2}^{\frac{1}{2}}(\phi_{\Omega^-},\theta_{\Omega^-},0)D_{\lambda,\lambda_2'}^{\frac{1}{2}*}(\phi_{\Omega^-},\theta_{\Omega^-},0)$$

$$\times D_{\lambda_2,\lambda_3}^{\frac{3}{2}}(\phi_\Lambda,\theta_\Lambda,0)D_{\lambda_2',\lambda_3'}^{\frac{3}{2}*}(\phi_\Lambda,\theta_\Lambda,0)$$

$$\times A_{\lambda_2}A_{\lambda_2'}^*B_{\lambda_3}B_{\lambda_3'}^*, \tag{13}$$

where $(\theta_{\Omega^-},\phi_{\Omega^-})$ is the Ω^- solid angles in the Ω_c^0 helicity system, whose z-axis is taken along the direction of Ω^- flight, and A_λ is the helicity amplitude of the decay

$\Omega_c^0 \to \Omega^- \pi^+$, which is related to the asymmetry parameter by $|A_{+\frac{1}{2}}|^2 = (1+\alpha_{\Omega^-\pi^+})/2$ and $|A_{-\frac{1}{2}}|^2 = (1-\alpha_{\Omega^-\pi^+})/2$. $(\theta_\Lambda, \phi_\Lambda)$ is the Λ solid angles in the Ω^- helicity system under the same convention in Eq. (10) and B_{λ_3} is the helicity amplitude of the decay $\Omega^- \to \Lambda K^-$, which is related to the Ω^- asymmetry parameter by $|B_{+\frac{1}{2}}|^2 = (1+\alpha_{\Lambda K^-})/2$ and $|A_{-\frac{1}{2}}|^2 = (1-\alpha_{\Lambda K^-})/2$. The joint angular distribution for the sequential decay $e^+ e^- \to \Omega_c^0 \bar{\Omega}_c^0, \Omega_c^0 \to \Omega^- \pi^+, \Omega^- \to \Lambda K^-, \Lambda \to p\pi^-$ reads [2]

$$\frac{dN}{d\cos\theta_{\Omega^-} d\cos\theta_p} \propto (9\cos^2\theta_{\Omega^-} - 5)(\alpha_- \cos\theta_p + \alpha_{\Lambda K^-})$$

$$\times \alpha_{\Omega^-\pi^+} \cos\theta_{\Omega^-} + (3\cos^2\theta_{\Omega^-} + 1)$$
$$\times (1 + \alpha_- \alpha_{\Lambda K^-} \cos\theta_p). \tag{14}$$

If the angle θ_{Ω^-} is integrated out, and we look for the angular distribution of the proton, one has

$$\frac{dN}{d\cos\theta_p} \propto 1 + \alpha_- \alpha_{\Lambda\pi^-} \cos\theta_p. \tag{15}$$

2.4 $e^+ e^- \to \gamma^* \to \Lambda_c^+ \bar{\Lambda}_c^-$ with $\Lambda_c^- \to \bar{p}K^0$

We consider another situation, where the strangeness in the final state goes to the meson $\bar{K}^0$, e.g., $\Lambda_c^+ \to p\bar{K}^0$. In this process, the proton is produced to be polarized from the Λ_c^+ weak decay, but its polarization cannot be analyzed directly, due to having no subsequent decays. In this case, the decay asymmetry parameters could be measured by the correlation of Λ_c^+ and $\bar{\Lambda}_c^-$ spins in the process $e^+ e^- \to \gamma^* \to \Lambda_c^+ \bar{\Lambda}_c^-$ with $\Lambda_c^+ \to p\bar{K}^0$ and $\bar{\Lambda}_c^- \to \bar{p}K^0$. The joint angular distribution is similar to that of process $e^+ e^- \to J/\psi \to \Lambda\bar{\Lambda} \to (p\pi^-)(\bar{p}\pi^+)$ [14]:

$$\frac{dN}{d\cos\theta \ d\cos\theta_1 d\phi_1 d\cos\bar{\theta}_1 d\bar{\phi}_1}$$

$$\propto 4|A_{1/2,1/2}|^2 \sin^2\theta [1 + \alpha_{p\bar{K}^0} \alpha_{\bar{p}K^0} (\cos\theta_1 \cos\bar{\theta}_1$$

$$+ \sin\theta_1 \sin\bar{\theta}_1 \cos(\phi_1 + \bar{\phi}_1))]$$

$$- 2|A_{1/2,-1/2}|^2 (1 + \cos^2\theta)(\alpha_{p\bar{K}^0} \alpha_{\bar{p}K^0} \cos\theta_1 \cos\bar{\theta}_1 - 1), \tag{16}$$

where $A_{1/2,\pm 1/2}$ is the helicity amplitude of $\gamma^* \to \Lambda_c^+ \bar{\Lambda}_c^-$, θ is the polar angle for Λ_c^+ in the $e^+ e^-$ center-of-mass system, and $\theta_1 (\bar{\theta}_1)$ and $\phi_1 (\bar{\phi}_1)$ are the solid angle of $p(\bar{p})$ in the $\Lambda_c^+ (\bar{\Lambda}_c^-)$ helicity system. $\alpha_{p\bar{K}^0} (\alpha_{\bar{p}K^0})$ is the decay asymmetry parameter for $\Lambda_c^+ \to p\bar{K}^0 (\bar{\Lambda}_c^- \to \bar{p}K^0)$.

2.5   Statistical sensitivity

With the measured asymmetry parameters of hyperon decays, we can extract the parameters $\alpha_{\Lambda\pi^+}$, $\alpha_{\Sigma^0\pi^+}$ for Λ_c^+ decays, $\alpha_{\Xi^-\pi^+}$ for Ξ_c^0 decays, and

parameter product $\alpha_{p\bar{K}^0} \alpha_{\bar{p}K^0}$ by fitting the formulae of joint angular distributions to data. For a given data set with N events observed, a likelihood function is defined by

$$\mathcal{L} = \prod_{i=1}^{N} \tilde{f}(\theta_i, \alpha), \tag{17}$$

where $\tilde{f}$ is a normalized function of angular distribution with helicity angle θ_i and parameter α. The maximum likelihood method is used to estimate the parameter, and its statistical sensitivity is defined by relative statistical uncertainty as

$$\delta(\alpha) = \frac{\sqrt{V(\alpha)}}{|\alpha|}, \tag{18}$$

where $V(\alpha)$ denotes the variance of the parameter α. The variance can be determined by

$$V^{-1}(\alpha) = N \int \frac{1}{\tilde{f}(\cos\theta_i, \alpha)} \left[\frac{\partial \tilde{f}(\cos\theta_i, \alpha)}{\partial \alpha} \right]^2 \prod_i d\cos\theta_i, \tag{19}$$

where N denotes the observed signal yield.

The parameters, $\alpha_{\Lambda\pi^+}$ and $\alpha_{\Sigma^+\pi^0}$, are estimated with Eq. (5). With the input of the accurate results of α_- [6], their sensitivities are obtained from Eq. (18):

$$\frac{2.13}{\sqrt{N}} \leqslant \delta(\alpha_{\Lambda\pi^+}) \leqslant \frac{3.27}{\sqrt{N}},$$

$$\frac{1.81}{\sqrt{N}} \leqslant \delta(\alpha_{\Sigma^+\pi^0}) \leqslant \frac{13.53}{\sqrt{N}}. \tag{20}$$

Here we fix the parameters at $\alpha_{\Lambda\pi^+} = -0.91 \pm 0.15$ [6], and $\alpha_{\Sigma^+\pi^0} = -0.45 \pm 0.32$ [6]. The parameter $\alpha_{\Sigma^0\pi^+}$ is estimated with Eq. (9), and its sensitivity is calculated by Eq. (18)

$$\frac{4.30}{\sqrt{N}} \leqslant \delta(\alpha_{\Sigma^0\pi^+}) \leqslant \frac{15.01}{\sqrt{N}}, \tag{21}$$

where the low sensitivity is estimated with the low limit of $\alpha_{\Sigma^0\pi^+} = -1$, and the high sensitivity is estimated with a theoretical prediction -0.31 [12]. The parameter $\alpha_{\Xi^-\pi^+}$ is estimated with Eq. (12), and its sensitivity is calculated by Eq. (18)

$$\delta(\alpha_{\Xi^-\pi^+}) = \frac{5.38}{\sqrt{N}}. \tag{22}$$

Here all hyperon decay parameters are fixed at PDG values [6], i.e., $\alpha_{\Sigma^+\pi^-} = -0.56$, $\alpha_{\Xi^-\pi^-} = -0.458$.

The parameter $\alpha_{\Omega^-\pi^+}$ is estimated with Eq. (14), and its sensitivity is calculated to be

$$\delta(\alpha_{\Omega^-\pi^+}) = \frac{214.90}{\sqrt{N}}, \tag{23}$$

if we take $\alpha_{\Omega^-\pi^+} = 0.17$ from a theoretical calculation [16], and $\alpha_- = 0.642$, $\alpha_{\Lambda K^-} = 0.018$ from the PDG [6].

The parameter product $\alpha_{p\bar{K}^0}\alpha_{\bar{p}K^0}$ is estimated with Eq. (16), and according to Eq. (18), the statistical sensitivity reads

$$\delta(\alpha_{p\bar{K}^0}\alpha_{\bar{p}K^0}) \approx \frac{1}{|\alpha_{p\bar{K}^0}\alpha_{\bar{p}K^0}|}$$

$$\times\sqrt{\frac{9(d-2)^2(d+1)\sqrt{d^2-4}}{N\left\{48d^2\mathrm{i}\tanh^{-1}\left(\sqrt{\frac{2-d}{2+d}}\right)+(d+2)[(d-9)d+2]\sqrt{d^2-4}\right\}}},\tag{24}$$

where $d = \dfrac{2(1+\beta)}{1-\beta}$, and β is the angular distribution parameter for Λ_c^+ in the e^+e^- rest frame, which takes the form $\dfrac{\mathrm{d}N}{\mathrm{d}\cos\theta} \propto 1 + \beta\cos^2\theta$ with $\beta = \dfrac{|A_{1/2,-1/2}|^2 - 2|A_{1/2,1/2}|^2}{|A_{1/2,-1/2}|^2 + 2|A_{1/2,1/2}|^2}$. If we take $|\alpha_{p\bar{K}^0}\alpha_{\bar{p}K^0}| = 1$ from a theoretical prediction [15], and assume $\beta = -0.3$, we have

$$\delta(\alpha_{p\bar{K}^0}\alpha_{\bar{p}K^0}) = \frac{8.54}{\sqrt{N}}.\tag{25}$$

To be more straightforward for scaling, we estimate these statistical sensitivities based on a sample of integrated luminosity $\mathcal{L} = 1$ fb^{-1}, and the number of observed events is calculated with

$$N = \mathcal{L}\sigma\mathcal{B}\epsilon,$$

where σ, $\mathcal{B}$ and ϵ denote the cross section, combined branching fraction and detection efficiency, respectively. The results, in Table 2, indicate that one needs a huge data sample to improve the sensitivities for measuring $\alpha_{\Xi^-\pi^+}$, $\alpha_{\Omega^-\pi^+}$ and $\alpha_{p\bar{K}^0}\alpha_{\bar{p}K^0}$.

If we assume that the Λ_c^+, Ξ_c^0 and Ω_c^0 decays conserve the CP transformation, the charged conjugate events can be combined to measure the asymmetry parameters, so that the sensitivities in Table 2 are improved by a factor of $1/\sqrt{2}$. Based on the 567 pb^{-1} data sample taken at 4.6 GeV collected at BESIII, an analysis [13] has shown that there are 707 ± 27 signal events for $\Lambda_c^+(\bar{\Lambda}_c^-) \to \Lambda\pi^+(\bar{\Lambda}\pi^-)$, and 309 ± 24 events for $\Lambda_c^+(\bar{\Lambda}_c^-) \to \Sigma^+\pi^0(\bar{\Sigma}^-\pi^0)$, and a precision of $(8.0\text{–}12.3)\%$ for $\alpha_{\Lambda\pi^+}$, and $(10.3\sim77.0)\%$ for $\alpha_{\Sigma^+\pi^0}$ will be achieved. This data sample will provide a first measurement of the asymmetry parameter, $\alpha_{\Sigma^0\pi^+}$, with a precision of $(18.8\text{–}65.6)\%$ determined from observed 522 ± 27 events of $\Lambda_c^+(\bar{\Lambda}_c^-) \to \Sigma^0\pi^+(\bar{\Sigma}^0\pi^-)$ [13]. To resolve the sign issue in the decay $\Lambda_c^+ \to \Sigma^+\pi^0$, the statistical uncertainty of $\alpha_{\Sigma^+\pi^0}$ is required to be at least less than the systematic uncertainty of 13% [2], and a data sample is required with a integrated luminosity large than 0.5 fb^{-1}. To study the decays of heavier charmed baryons Ξ_c^0 and Ω_c^0 at BESIII, one needs to collect data going beyond 4.6 GeV.

Table 2.   Estimation of statistical sensitivities $\delta(\alpha)$ with a sample of integrated luminosity of 1 fb^{-1}, taken at the energy point $\sqrt{s}$.

decay	$\Lambda_c^+ \to \Lambda\pi^+$	$\Lambda_c^+ \to \Sigma^+\pi^0$	$\Lambda_c^+ \to \Sigma^0\pi^+$	$\Xi_c^0 \to \Xi^-\pi^+$	$\Omega_c^0 \to \Omega^-\pi^+$	$\Lambda_c^+\bar{\Lambda}_c^- \to p\bar{K}^0\bar{p}K^0$
parameter	$\alpha_{\Lambda\pi^+}$	$\alpha_{\Sigma^+\pi^0}$	$\alpha_{\Sigma^0\pi^+}$	$\alpha_{\Xi^-\pi^+}$	$\alpha_{\Omega^-\pi^+}$	$\alpha_{p\bar{K}^0}\alpha_{\bar{p}K^0}$
$\sqrt{s}$	4.6 GeV	4.6 GeV	4.6 GeV	10.5 GeV	10.6 GeV	4.6 GeV
$\sigma\cdot\mathcal{B}$	1.88 pb [13]	2.80 pb[13]	1.92 pb [13]	0.77 pb[17]	11.3 fb[18]	0.05 pb[13]
ϵ	42.2%[13]	23.8%[13]	29.9%[13]	10.8%[17]	15%[18]	31.2%[13]
N	793	666	574	83	2	16
$\delta(\alpha)$	(7.6–11.6)%	(7.0–52.4)%	(17.9–62.0)%	59%	152.0%	213.5%

3   Summary

To summarize, we have presented several approaches to measure the decay asymmetry parameters in the two-body hadronic weak decay of singly charmed baryons, such as $\Lambda_c^+ \to \Lambda\pi^+$, $\Sigma^0\pi^+$, $\Sigma^0\pi^+$, $p\bar{K}^0$, $\Xi_c^0 \to \Xi^-\pi^+$ and $\Omega_c^0 \to \Omega^-\pi^+$. With the reasonable assumption that the charmed baryons produced from e^+e^- annihilations are unpolarized, one can probe the asymmetry parameter by measuring the polarization of the subsequent hyperons via the decay $\Lambda \to p\pi^-$. We present formulae of joint angular distributions for these processes, which will be used to fit to data to extract the asymmetry parameters. The sensitivities of the proposed measurements are estimated. Based on the 567 pb^{-1} data collected at BESIII, a rough precision of 10% for $\alpha_{\Lambda\pi^+}$ should be achieved, and the sensitivity for measuring $\alpha_{\Sigma^0\pi^+}$ for $\Lambda_c^+ \to \Sigma^0\pi^+$, is estimated to be $(19\text{–}66)\%$. A huge data sample is crucial to improve the precision of these parameters in the future and even to study the CP violation of the decay asymmetries of charmed baryons and their anti-baryons.

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