Investigation of fission fragments average spin based on four dimensional Langevin dynamical model

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Abstract: We applied the four dimensional Langevin dynamical model to investigate the average spin of fission fragments. Elongation, neck thickness, asymmetry parameter, and the orientation degree of freedom (K coordinate) are the four dimensions of the dynamical model. We assume that the collective modes depend on the emission angle of the fragments, then different parameters related to the average spin of fission fragments are calculated dynamically. The angle dependence of average spin of fission fragments is investigated by calculating the spin at angles 90° and 165°. Also, the obtained results based on the transition state model at scission point are presented. One can obtain better agreement between the results of the dynamical model and experimental data in comparison with the results of the transition state model.

Keywords: Langevin equations, average spin, fission fragments

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1 Introduction

The study of the spin distribution in fission fragments provides important information on the mechanism of spin generation and the excitation of collective degrees of freedom in fission processes. The total spin of the final fragments in the fission process is largely determined by the excitation of various angular momentum bearing modes, such as wriggling, bending, twisting, and tilting [1–3]. In heavy ion induced fission reactions, where the compound nucleus is populated with an initial spin distribution, a part of the initial angular momentum also gets transferred as the spin of the fission fragments.

An alternative way of determining compound nucleus spin distribution is to measure the fission fragment spins by gamma ray multiplicity measurements. In the case of a fission exit channel, the compound nucleus spin gets fractionated into fragment spin due to rigid rotation of the nascent fragments at scission and the relative orbital angular momentum between them. Also an additional spin is induced in the fragments due to statistical excitation of the spin-bearing collective modes in the fissioning nucleus [4–6].

A proper description of the fusion and fission reactions requires an understanding of the total potential energy as a function of the collective coordinates describing these processes. The macroscopic coordinates generally used in describing the fission process are the deformation parameters characterizing the elongation, neck and mass asymmetry degrees of freedom. The liquid drop model and its later variants such as droplet model, rotating liquid drop model and rotating finite range model have been quite successful in calculating the potential energy of a nucleus undergoing fission. Various shape parameterizations have been adopted to describe the family of shapes that the nucleus spans as it deforms from the ground state to the saddle and scission configurations [7]. Based on these studies the fission barrier parameters such as the barrier height and saddle point moments of inertia as a function of Z^2/A and angular momentum, have been calculated [8–10]. It is seen that for heavy actinide nuclei ($Z^2/A=35$), the fission barrier is in the range of 5 to 6 MeV for zero angular momentum and goes to zero when the angular momentum increases in the range of 30–40 $\hbar$. This has an important bearing on the study of heavy ion induced fusion-fission reactions, since the compound nuclei formed in heavy ion reactions at above-barrier energies have very large angular momenta.

In the last two decades the stochastic model [11–17] has been engaged to successfully reproduce a great number of experimental data on prescission particle multiplicities and evaporation residue cross sections for a lot of compound systems over a wide range of excitation energy, angular momentum, and fissility. The Langevin equations in one, two, three and four dimensions have been applied to calculate the angular distribution of fis-

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sion fragments, mass distribution, kinetic energy, fission probability, and fission cross section. In this paper, for the first time, we study the angular dependence of average spin of fission fragments in fusion fission reactions using four dimensional Langevin equations. The paper is organized as follows. In Section 2 we explain the Langevin equations and theoretical calculations of average spin of fission fragments. The obtained results are given in Section 3. Finally, a summary and conclusion of our results are presented in Section 4.

2 Model

2.1 Statistical model

The standard empirical relation between the measured gamma ray multiplicity, M_γ, and the total fragment spin, $S_{\rm T}$, has the form [18–20]

$$S_{\rm T}=n(M_\gamma-2M_{\rm st})+0.5\nu_{\rm tot}, \tag{1}$$

where $2M_{\rm st}$ is the number of statistical gamma rays escaping from two complementary fragments and carrying, on average, zero angular momentum, and $\nu_{\rm tot}$ is the total number of fission neutrons, each of which carries away an angular momentum of 0.5 $\hbar$ [18]. The coefficient n takes values ranging between 1.5 and 2.0. If $n=2.0$ [18], all gamma rays are assumed to be quadrupole for transition downward along the yrast band, but, if $n=1.7$ [19], an admixture featuring possible dipole transitions is taken into account.

According to the standard transition state model, the angular distribution of fission fragments are determined by K. The mean square of the K, $\langle K^2\rangle$ is given by [21]

$$\langle K^2\rangle=\frac{\sum_{K=-I,M}^{I}K^2W_{M=0,K}^{I}(\theta)\exp(-K^2/2K_0^2)}{\sum_{K=-I}^{I}W_{M=0,K}^{I}(\theta)\exp(-K^2/2K_0^2)}. \tag{2}$$

After substitution for the angular yield, $W_{M=0,K}^{I}(\theta)$, the above expression can be written in an analytical form as

$$\langle K^2\rangle=0.5(I+0.5)^2\sin^2(\theta)\left[1-\frac{I_1(\beta^2)}{I_0(\beta^2)}\right], \tag{3}$$

where $\beta=\alpha\sin(\theta)/\sqrt{2}$, $\alpha=(I+0.5)/(2K_0^2)^{1/2}$. I_0 and I_1 represent the zeroth and first order modified Bessel functions. The Gaussian distribution of K is centered at zero with the mean equal to zero and having a variance equal to K_0^2 as

$$K_0^2=\frac{J_{\rm eff}T}{\hbar^2}, \tag{4}$$

where $J_{\rm eff}$ and T are the effective moment of inertia and the temperature of the compound nucleus, respectively.

In the transition state model the fragment average spin is evaluated by using the following relation [22, 23]

$$\langle S_{\rm T}\rangle=\sqrt{f^2\langle I^2\rangle+(1-f^2)\langle K^2\rangle+S_{\rm coll}^2}. \tag{5}$$

In this relation $S_{\rm coll}$ is average spin related to collective modes, $\langle I^2\rangle$ is the average square of compound nucleus spin that can be calculated using coupled channel analysis, and f is a parameter that indicates how compound nuclear spin is divided along the fission and perpendicular-to-fission axes,

$$f=\frac{J_1+J_2}{J_1+J_2+\mu R^2}, \tag{6}$$

where J_1 and J_2 are perpendicular moments of inertia of fission fragments and μ is reduced mass. R is the distance between the centers of two fission fragments. We can calculate $S_{\rm coll}$ by the following relation:

$$S_{\rm coll}=kA^{5/6}T^{1/2}, \tag{7}$$

where k is a proportionality constant. Also, the temperature of the nucleus at the scission point can be calculated as

$$T=\sqrt{\frac{8E^*}{A}}, \tag{8}$$

where E^* and A are the excitation energy of the nucleus at the scission point and the mass number of the compound nucleus, respectively.

2.2 Dynamical model

In our dynamical calculations we use a well-known (c,h,α) parametrization. In cylindrical coordinates the surface of the nucleus is given by [24]

$$\rho_{\rm s}^2(z)=(c^2-z^2)(A_{\rm s}/c^2+B_{\rm sh}z^2/c^2+\alpha z/c), \tag{9}$$

where $\rho_{\rm s}$ and z are radial and parallel coordinates relative to the symmetry axis, respectively. c denotes the elongation parameter. $A_{\rm s}$ and $B_{\rm sh}$ are defined in Ref. [24]. The neck thickness parameter can be defined by

$$h=-1.047c^3+4.297c^2-6.309c+4.073. \tag{10}$$

Also, the asymmetry parameter is defined as

$$\alpha=0.11937\alpha_{\rm as}^2+0.24720\alpha_{\rm as}, \tag{11}$$

where

$$\alpha_{\rm as}=\frac{A_1-A_2}{A}. \tag{12}$$

In this relation A_1, A_2 are the mass numbers of two fission fragments and A is the mass number of the compound nucleus. In the stochastic approach, evaluation of the collective coordinates is considered as motion of Brownian particles which interact stochastically with a large number of internal degrees of freedom, constituting the surrounding heat bath [25, 26]. Based on this approach, the Langevin equations can be used as

$$\frac{\mathrm{d}q_i}{\mathrm{d}t}=\frac{p_j}{m_{ij}},$$
$$\frac{\mathrm{d}p_i}{\mathrm{d}t}=-\frac{p_jp_k}{2}\frac{\partial}{\partial q_i}\left(\frac{1}{m_{jk}}\right)-\frac{\partial F}{\partial q_i}-\gamma_{ij}\frac{p_j}{m_{jk}}+g_{ij}\xi_j, \quad (13)$$

where q_i are the vectors of collective coordinates, and p_i are their conjugate momenta. m_{ij} are the inertia tensors, which are evaluated using the Werner-Wheeler formula [27]

$$m_{ij}=\pi\rho_{\mathrm{m}}\int_{z_{\min}}^{z_{\max}}\rho_{\mathrm{s}}^2(z)(A_iA_j+\rho_{\mathrm{s}}^2(z)A_i'A_j'/8)\mathrm{d}z, \quad (14)$$

where ρ_{m} is the mass density of the nucleus. $z_{\min}$ and $z_{\max}$ are the positions of the left and right ends of the nuclear shape. Also, A_i is calculated as

$$A_i=-\frac{1}{\rho_{\mathrm{s}}^2(z)}\frac{\partial}{\partial q_i}\int_{-c}^{z}\rho_{\mathrm{s}}^2(z')\mathrm{d}z'. \quad (15)$$

The quantity A' is the first derivative of A with respect to z and F represents the free energy of the system. $g_{ij}\xi_j$ is a random force. g_{ij} is the random force amplitude and ξ_j is Gaussian white noise satisfying the relations

$$\langle\xi_j(t)\rangle=0, \quad (16)$$
$$\langle\xi_i(t)\xi_j(t')\rangle=2\delta_{ij}\delta(t-t'). \quad (17)$$

The temperature of the "heat bath" is defined as follows [28]:

$$T=\sqrt{\frac{E_{\mathrm{int}}}{a_{\mathrm{v}}A+a_{\mathrm{s}}A^{2/3}B_{\mathrm{s}}}}, \quad (18)$$

where the values of the parameters $a_{\mathrm{v}}=0.073$ MeV^{-1} and $a_{\mathrm{s}}=0.095$ MeV^{-1} are taken from Ref. [29]. The intrinsic energy of the system, (E_{int}) is calculated at every step of the Langevin trajectory by the relation

$$E_{\mathrm{int}}=E_{\mathrm{cm}}+Q-V(q)-\frac{p_ip_j}{2m_{ij}}, \quad (19)$$

where E_{cm}, Q, and $V(q)$ are the energy of the system in center of mass frame, the reaction Q value and potential energy, respectively.

The potential energy of the system contains the surface energy, the Coulomb energy and the rotational energy, which are described below [30]:

$$V(q,I,K)=E_{\mathrm{s}}^0(Z,A)(B_{\mathrm{s}}(q)-1)+E_{\mathrm{c}}^0(Z,A)(B_{\mathrm{c}}(q)-1)+\frac{[I(I+1)-K^2]\hbar^2}{2J_{\perp}(q)}+\frac{K^2\hbar^2}{2J_{\parallel}(q)}, \quad (20)$$

where Z is the charge of the fissioning nucleus, and E_{s}^0 and E_{c}^0 are the surface and the Coulomb energy respectively of a spherical nucleus within the liquid drop model with a sharp edge. Also, $B_{\mathrm{s}}(q)$and $B_{\mathrm{c}}(q)$ are the dimensionless surface energies and Coulomb energies respectivelyof the spherical system, and are given by

$$B_{\mathrm{s}}(q)=\pi\int_{-c}^{c}\sqrt{4\rho_{\mathrm{s}}^2(z)+\left(\frac{\partial\rho_{\mathrm{s}}^2(z)}{\partial z}\right)^2}\mathrm{d}z, \quad (21)$$

and

$$B_{\mathrm{c}}(q)=\frac{15}{4\pi}\int_{-c}^{c}\mathrm{d}z\int_{-c}^{c}\mathrm{d}\acute{z}\int_{0}^{\pi}\mathrm{d}\varphi\frac{\rho_{\mathrm{s}}^2(z)\rho_{\mathrm{s}}^2(\acute{z})\sin^2\varphi}{z-\acute{z}+f}, \quad (22)$$

where $f=\sqrt{(z-\acute{z})^2+\rho_{\mathrm{s}}^2(z)+\rho_{\mathrm{s}}^2(\acute{z})-2\rho_{\mathrm{s}}^2(z)\rho_{\mathrm{s}}^2(\acute{z})\cos\varphi}$. The moments of inertia ($J_{\perp}$ and $J_{\parallel}$) are obtained as [31, 32]

$$J_{\perp(\parallel)}(q)=J_{\perp(\parallel)}^{(\mathrm{sharp})}(q)+4Ma_M^2, \quad (23)$$

where $a_M=0.704$ fm is the parameter of the nuclear surface diffuseness. M is the mass of the compound nucleus. $J_{\perp(\parallel)}^{(\mathrm{sharp})}$ stands for moments of inertia that are calculated on the basis of liquid drop model with a sharp edge. In the $\{c,h,\alpha\}$ parametrization, they are given by

$$J_{\perp}^{(\mathrm{sharp})}=\frac{2}{5}MR_0^2\Big[\frac{c^2}{2}\Big(1+c^{-3}+\frac{4}{35}\Big(2h+\frac{c-1}{2}\Big)\times\Big(2c^3+\frac{4}{15}\Big(2h+\frac{c-1}{2}\Big)c^3-1\Big)-\alpha^2c^3\Big(\frac{c^3}{5}-\frac{1}{7}\Big)\Big)\Big], \quad (24)$$

and

$$J_{\parallel}^{(\mathrm{sharp})}=\frac{2}{5}MR_0^2\Big[c^2\Big(c^{-3}+\frac{4}{35}\Big(2h+\frac{c-1}{2}\Big)\times\Big(-1+\frac{4}{15}\Big(2h+\frac{c-1}{2}\Big)c^3\Big)+\frac{1}{7}\alpha^2c^3\Big)\Big], \quad (25)$$

with $R_0=1.2249A^{1/3}$. Based on the Fermi gas model, the free energy is given by

$$F(q,I,K,T)=V(q,I,K)-(a_{\mathrm{v}}A+a_{\mathrm{s}}A^{2/3}B_{\mathrm{s}})T^2. \quad (26)$$

Using one-body dissipation we can calculate wall friction by [26]

$$\gamma_{ij}^{\mathrm{wall}}(c<c_{\mathrm{win}})=\frac{\pi\rho_{\mathrm{m}}}{2}\overline{v}\int_{z_{\min}}^{z_{\max}}\left(\frac{\partial\rho_{\mathrm{s}}^2}{\partial q_i}\right)\left(\frac{\partial\rho_{\mathrm{s}}^2}{\partial q_j}\right)\times\left(\rho_{\mathrm{s}}^2+\left(\frac{1}{2}\frac{\partial\rho_{\mathrm{s}}^2}{\partial z}\right)^2\right)^{-1/2}\mathrm{d}z, \quad (27)$$

and for an elongation greater than the point which a neck is formed in the nuclear system($c\geqslant c_{\mathrm{win}}$), the corresponding friction tensors can be written as

$$\begin{aligned}\gamma_{ij}^{\mathrm{wall}}(c\geqslant c_{\mathrm{win}})=&\frac{\pi\rho_m}{2}\bar{\upsilon}\Big\{\int_{z_{\min}}^{z_{\mathrm{neck}}}\Big(\frac{\partial\rho_{\mathrm{s}}^2}{\partial q_i}+\frac{\partial\rho_{\mathrm{s}}^2}{\partial z}\frac{\partial D_1}{\partial q_i}\Big)\\&\times\Big(\frac{\partial\rho_{\mathrm{s}}^2}{\partial q_j}+\frac{\partial\rho_{\mathrm{s}}^2}{\partial z}\frac{\partial D_1}{\partial q_j}\Big)\\&\times\Big(\rho_{\mathrm{s}}^2+\Big(\frac{1}{2}\frac{\partial\rho_{\mathrm{s}}^2}{\partial z}\Big)^2\Big)^{-1/2}\mathrm{d}z\\&+\int_{z_{\mathrm{neck}}}^{z_{\max}}\Big(\frac{\partial\rho_{\mathrm{s}}^2}{\partial q_i}+\frac{\partial\rho_{\mathrm{s}}^2}{\partial z}\frac{\partial D_2}{\partial q_i}\Big)\\&\times\Big(\frac{\partial\rho_{\mathrm{s}}^2}{\partial q_j}+\frac{\partial\rho_{\mathrm{s}}^2}{\partial z}\frac{\partial D_2}{\partial q_j}\Big)\\&\times\Big(\rho_{\mathrm{s}}^2+\Big(\frac{1}{2}\frac{\partial\rho_{\mathrm{s}}^2}{\partial z}\Big)^2\Big)^{-1/2}\mathrm{d}z\Big\},\end{aligned}\tag{28}$$

$$\gamma_{ij}^{\mathrm{win}}(c\geqslant c_{\mathrm{win}})=\frac{\pi\rho_{\mathrm{m}}}{2}\bar{\upsilon}\Big(\frac{\partial R}{\partial q_i}\frac{\partial R}{\partial q_j}\Big)\Delta\sigma.\tag{29}$$

Here, $\bar{\upsilon}$ is the average nucleon speed inside the nucleus, D_1 and D_2 are the positions of the centers of two parts of the fissioning system relative to the center of mass of the whole system, and z_{neck} is the position of the neck plane that divides the nucleus into two parts. $\Delta\sigma$ is the area of the window between two parts of the system and R is the distance between the centers of mass of future fragments.

The chaoticity factor (μ) gives the average fraction of trajectories which are chaotic when sampling is done uniformly over the surface. In other words, the chaoticity is used to express the degree of irregularity in the dynamics of the system. Each such trajectory is identified as a regular or as a chaotic one by considering the magnitude of its Lyapunov exponent over a long time interval [29]. By introducing chaos into the classical linear response theory for one-body dissipation, the scaled version of wall and window friction, are obtained as

$$\begin{aligned}\gamma_{ij}(c<c_{\mathrm{win}})&=\mu(c)\gamma_{ij}^{\mathrm{wall}}(c<c_{\mathrm{win}}),\\\gamma_{ij}(c\geqslant c_{\mathrm{win}})&=\mu(c)\gamma_{ij}^{\mathrm{wall}}(c\geqslant c_{\mathrm{win}})\\&\quad+\gamma_{ij}^{\mathrm{win}}(c\geqslant c_{\mathrm{win}}),\end{aligned}\tag{30}$$

where the value of μ changes from 0 to 1 as the nucleus evolves from a spherical to a deformed shape. Generally, the friction strength has an important influence on fission processes. In the present work we applied the one-body dissipation using the scaled version of wall and window friction for investigation of the angular dependence of fission fragment average spin.

The evolution of the orientation degree of freedom (K coordinate) is obtained from the solution of the Langevin equations [26, 33]

$$\delta K=-\frac{\gamma_K^2I^2}{2}\frac{\partial V}{\partial K}\delta t+\gamma_K I\xi\sqrt{T\delta t},\tag{31}$$

where ξ is a random number from a normal distribution with unit variance, and γ_K is a parameter which controls the coupling between the orientation degree of freedom and the heat bath. This parameter can be calculated as [26, 33]

$$\gamma_K=\frac{1}{R_{\mathrm{N}}R\sqrt{2\pi^3n_0}}\sqrt{\frac{J_{\parallel}|J_{\mathrm{eff}}|J_R}{J_{\perp}^3}},\tag{32}$$

where R_{N} and n_0 are the neck radius and the bulk flux in standard nuclear matter (0.0263 MeVzsfm^{-4}), respectively. $J_R=MR^2/4$ where M is the compound nucleus mass. Lestone and McCalla proposed the description of evolution of the K collective coordinate using the Langevin equation for overdamped motion [34]. Such a description is more consistent than the application of the Metropolis algorithm [35, 36]. The Langevin equation for the K coordinate allows the modeling of the relaxation process of K states depending on the instantaneous physical properties of the fissioning system, such as temperature and moment of inertia, instead of treating the corresponding relaxation time (τ_K) as a free parameter. However, the main drawback of this approach is the lack of detailed description of the coupling between the orientation degree of freedom and the heat bath. Successful application of the transition state model of angular distributions to a large number of fusion-fission reactions suggests that K can be treated as an overdamped collective coordinate. In other words, the stochastic dynamics of the orientation degree of freedom does not include the inertia parameter and could be described by the overdamped Langevin equation.

The friction coefficient for rotational degree of freedom (γ_K) is defined based on Eq. (32). This equation was obtained by assuming nuclear shapes featuring a well-defined neck. Consequently, we choose γ_K to be a constant equal to 0.077 (MeVzs)$^{(-1/2)}$ [34]. This value is suitable for the explanation of the anisotropy of the angular distribution for the highly excited fissioning compound nuclei with mass about 225–250 [37].

The neutron decay width is calculated using the relation [38]

$$\begin{aligned}\Gamma_{\mathrm{n}}=&\frac{2m_{\mathrm{n}}}{\pi^2\hbar^2\rho_{\mathrm{c}}(E_{\mathrm{int}})}\\&\times\int_0^{E_{\mathrm{int}}-B_{\mathrm{n}}}\rho_R(E_{\mathrm{int}})\varepsilon_{\mathrm{n}}\sigma_{\mathrm{inv}}(\varepsilon_{\mathrm{n}})\mathrm{d}\varepsilon_{\mathrm{n}},\end{aligned}\tag{33}$$

where m_{n} is the reduced mass of the neutron with respect to the residual nucleus and B_{n} shows the binding energy of the compound nucleus. Also, ρ_c is the level density of the compound nuclei and ε_{n} is the average kinetic energy of the emitted neutrons. Here $\sigma_{\mathrm{inv}}(\varepsilon_{\mathrm{n}})=\pi R_{\mathrm{n}}^2$ is the inverse cross section for the reaction $(A-1)+\mathrm{n}\rightarrow A$, and R_{n} is defined as $R_{\mathrm{n}}=1.21[(A-1)^{1/3}+1]+3.4/\sqrt{\varepsilon_{\mathrm{n}}}$.

The width of the gamma emission can be calculated using the following relation [39]

$$\Gamma_{\gamma} \cong \frac{3}{\rho_{\rm c}(E_{\rm int})}\int_0^{E_{\rm int}} {\rm d}\varepsilon \rho_{\rm c}(E_{\rm int}-\varepsilon)f(\varepsilon), \qquad (34)$$

where ε is the energy of the emitted γ quanta. $f(\varepsilon)$ is defined as

$$f(\varepsilon) = \frac{4}{3\pi}\frac{e^2}{\hbar c}\frac{1.75}{m_{\rm n}c^2}\frac{NZ}{A}\frac{\Gamma_{\rm G}\varepsilon^4}{(\Gamma_{\rm G}\varepsilon)^2+(\varepsilon^2-E_{\rm G}^2)^2}, \qquad (35)$$

with $E_{\rm G}=80A^{-1/3}$ and $\Gamma_{\rm G}=5$ MeV [40]. $E_{\rm G}$ and $\Gamma_{\rm G}$ are the position and width of the giant dipole resonance, respectively.

Using Eqs. (33) and (34), we can establish the emission algorithm which decides at each time step, along each of the trajectories, whether a particle is being emitted from the compound nucleus. For this purpose, we define $x=\tau/\tau_{\rm tot}$ where $\tau_{\rm tot}=\hbar/(\Gamma_{\rm n}+\Gamma_{\gamma})$. The probability for emitting a neutron or gamma, for a small enough time step τ, can be written as

$$P(\tau) = 1-{\rm e}^{-\tau/\tau_{\rm tot}} \approx x. \qquad (36)$$

Then we choose a random number r by sampling from a uniformly distributed set between 0 and 1. If we find $r<x$, it will be interpreted as emission of either a neutron or a gamma ray during that interval.

The following relation has been used to calculate the average different quantities

$$\langle \xi \rangle = \frac{\sum_I \sum_{\alpha} \langle \xi \rangle_{I,\alpha}(2I+1)P_I}{\sum_I \sum_{\alpha}(2I+1)P_I}, \qquad (37)$$

where ξ can be the average multiplicity of gamma rays, M_{γ}, and fission fragment spin, $S_{\rm T}$. The probability of the fission barrier penetration, P_I, which depends upon angular momentum, is

$$P_I = \frac{N_I}{N}, \qquad (38)$$

where N and N_I are the total number of trajectories and the number of trajectories which undergo fission, respectively. We made a completely dynamical calculation from ground state configuration to scission point. Each Langevin trajectory can lead to fission if it overcomes the fission barrier and reaches the scission point. The scission criteria can be determined using the following relation [41]

$$c_{\rm sci} = -2.0\alpha^2+0.032\alpha+2.0917. \qquad (39)$$

This cycle of calculations is repeated for typically 50000 Langevin trajectories. For initial values of dynamical parameters we used ($c=1, h=1.01$, and $\alpha=0$). Also, the initial K value was generated using the Monte Carlo method from uniform distribution in the interval $(-I,I)$. Also, the spin I for each Langevin trajectory has been sampled from the spin distribution function [38]

$$\sigma_I = \frac{2\pi}{k^2}\frac{2I+1}{1+\exp[(I-I_{\rm c})/\delta I]}, \qquad (40)$$

where k is the wave number. The critical spin $I_{\rm c}$ scales as

$$\begin{aligned} I_{\rm c} = {} & \sqrt{A_{\rm P}\times A_{\rm T}/A}\times(A_{\rm P}^{1/3}+A_{\rm T}^{1/3}) \\ & \times(0.33+0.205\times\sqrt{E_{\rm cm}-V_{\rm c}}), \end{aligned} \qquad (41)$$

when $0<E_{\rm cm}-V_{\rm c}<120$ MeV; and when $E_{\rm cm}-V_{\rm c}>120$ MeV the term in the last set of brackets is put equal to 2.5. The barrier $V_{\rm c}$ can be calculated by

$$V_{\rm c} = (5/3)\times c_3\times Z_{\rm P}Z_{\rm T}/(A_{\rm P}+A_{\rm T}+1.6), \qquad (42)$$

with $c_3=0.7053$ MeV. Also, $A_{\rm P}$ and $Z_{\rm P}$ are the mass and charge of the projectile nucleus, and $A_{\rm T}$ and $Z_{\rm T}$ are the mass and charge of the target nucleus. The diffuseness δ_I is found to scale as

$$\delta_I = \begin{cases} (A_{\rm P}A_{\rm T})^{3/2}\times 10^{-5}\times[1.5+0.02\times \\ (E_{\rm cm}-V_{\rm c}-10)] \quad E_{\rm cm}>V_{\rm c}+10, \\ (A_{\rm P}A_{\rm T})^{3/2}\times 10^{-5}\times[1.5-0.04\times \\ (E_{\rm cm}-V_{\rm c}-10)] \quad E_{\rm cm}<V_{\rm c}+10. \end{cases} \qquad (43)$$

3 Results

In the present paper the transition state model and four dimensional Langevin approach are applied to study the gamma multiplicity and average spin of fission fragments in the reactions ^{18}O+^{208}Pb, ^{16}O +^{232}Th, and ^{12}C +^{232}Th. The obtained results are shown in Figs. 1 to 5.

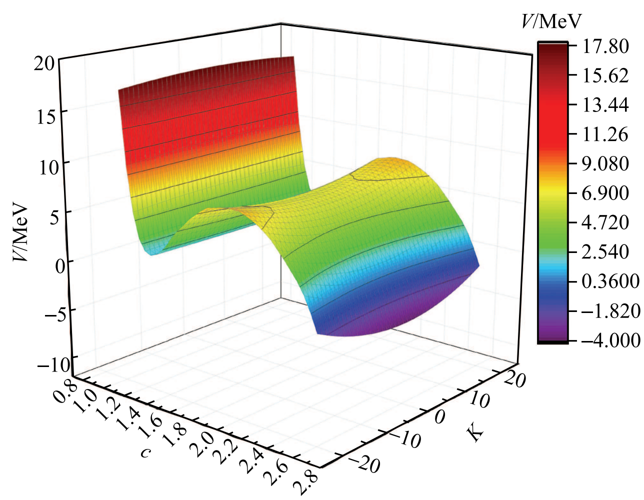

Fig. 1. (color online) Potential energy surface for the compound nucleus ^{226}Th at I=20 $\hbar$.

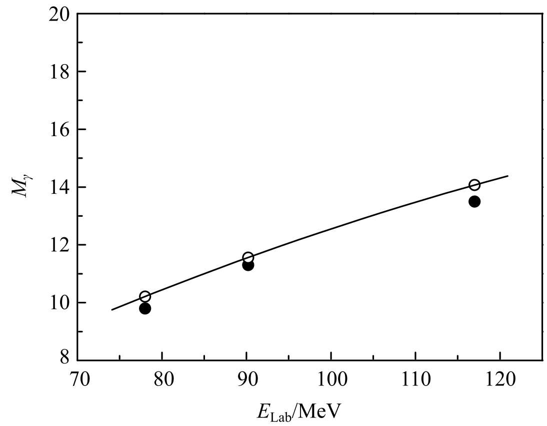

Fig. 2. The average multiplicity of gamma rays as a function of energy for the $^{18}O+^{208}$ Pb reaction. Solid and open circles are experimental data [20] and obtained results based on the four dimensional dynamical model, respectively. The curve is an approximation of theory by a polynomial of second order.

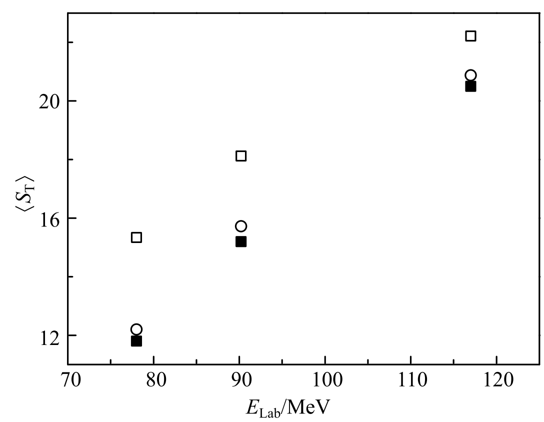

Fig. 3. Average fragment spin as a function of bombarding energy for the ^{18}O $+^{208}Pb$ reaction. Open squares and circles show obtained results based on transition state model and four dimensional Langevin equations, respectively. Solid squares are experimental data [20].

The variation of potential energy as function of elongation parameter and K coordinate is shown in Fig. 1 for the $^{18}O+^{208}Pb$ reaction at $I = 20\ \hbar$. One can see for $K = 0$ the potential energy has the minimum value. In other words, with increasing K the potential energy increases. Also, when the shape of the fissioning system changes from spherical ($c = 1$) to scission point ($c = c_{sci}$), the potential energy initially increases and then decreases. In Fig. 2 we show the gamma multiplicity for the $^{18}O+^{208}Pb$ reaction based on the four dimensional Langevin equations. With increasing energy of projectile, the gamma ray multiplicity increases. One can see good agreement between theoretical calculations and experimental data. Generally, based on these theoretical considerations, the proportionality of gamma multiplicity versus energy is consistent with the available experimental data. Also, we show the variation of average spin of fission fragments with energy for this reaction in Fig. 3. Results of the dynamical model and statistical model are compared with experimental data. It can be seen in Fig. 3 that the dynamical calculation of the average spin of fission fragments is in better agreement with the experimental data.

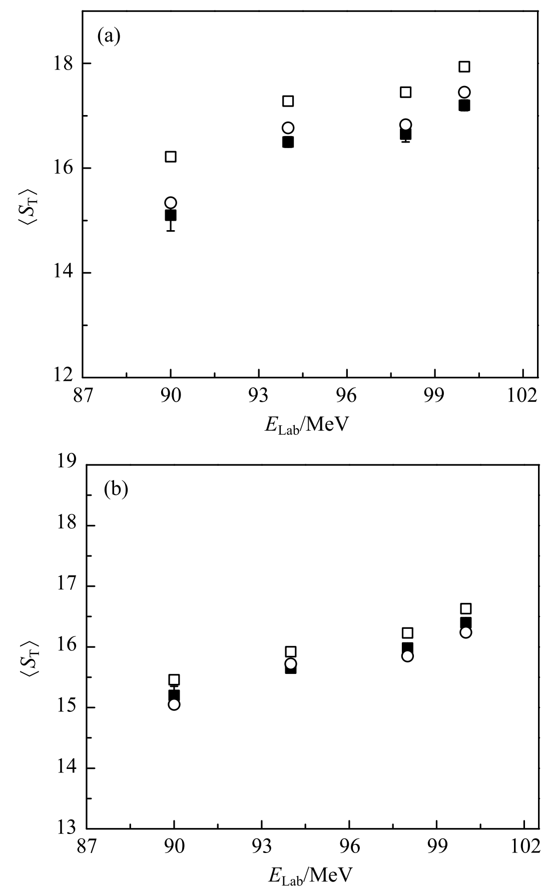

Fig. 4. Average fragment spin as a function of bombarding energy for the ^{16}O $+^{232}Th$ reaction at 90° (a) and 165° (b), respectively. Open squares and circles show obtained results based on transition state model and four dimensional Langevin equations, respectively. Solid squares are experimental data [3].

For better comparison between the results of the dynamical model and experimental data and also investigating the angular dependence of average total spin of the fission fragments, we calculated the average fragment spin as a function of bombarding energy for ^{16}O $+^{232}Th$ and ^{12}C $+^{232}Th$ reactions, respectively, in Figs. 4 and 5 for angles 90° and 165°. The fragments emitted along 90° to the beam result from all possible K states, ranging from $K = -I$ to $K = I$ and hence bear spin components from $K=-I$ to $K=I$. The spins induced in the fragments are, thus, expected to be dependent on their direction of emission. The spin will be maximum for fragments emitted along the 90° direction to the beam. It should be noted that this of course is not the only mode of spin incentive in fission fragments. The spins of fragments emitted at 90° to the beam are higher than those emitted along the beam direction. We can conclude from Figs. 4 and 5 that fragment spin at 90° is higher than for 165° .

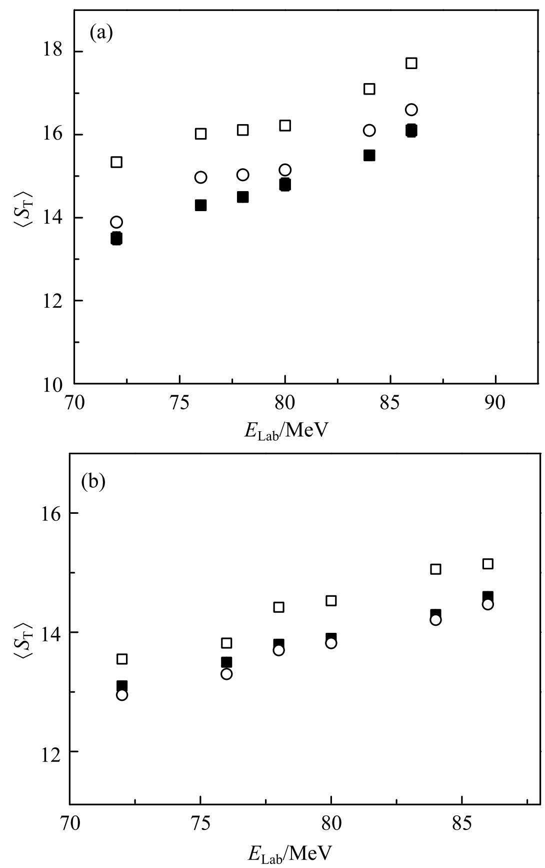

Fig. 5. Average fragment spin as a function of bombarding energy for the $^{12}C + ^{232}Th$ reaction at 90° (a) and 165° (b), respectively. Open squares and circles show the obtained results based on the transition state model and four dimensional Langevin equations, respectively. Solid squares are experimental data [3].

The value of f is related to the shape of the fission fragments and distance between their centers of mass at the scission point. Consequently we can expect that the value of f is equal for 90° and 165° emission of fragments. This is supported largely by the fact that the mass and kinetic energy distributions of fragments do not differ with their angle of emission. In Ref. [3] it is shown that $S_{\rm coll}$ at 90° and 165° are different. In other words, the excitation of the K mode inhibits the excitation of other modes by a suppression factor which varies as $\exp(-2\Delta E_{\rm rot}/T)$, where $\Delta E_{\rm rot}$ is the energy of the tilting mode. Here this factor is calculated by averaging over I and K for the 90° emission. Rotational energy is defined as $E_{\rm rot}=\dfrac{I(I+1)\hbar^2}{2J_\perp(q)}+\dfrac{K^2\hbar^2}{2J_{\rm eff}(q)}$. In Fig. 6 we have shown the ratio of $S_{\rm coll}(\theta)/S_{\rm coll}(180°)$ as a function of projectile energy. With increasing energy of the projectile this ratio decreases. Furthermore, by considering the angular dependence for $S_{\rm coll}$ and dynamically calculating this parameter, we obtain a lower value for $S_{\rm coll}$ in comparison with the statistical model.

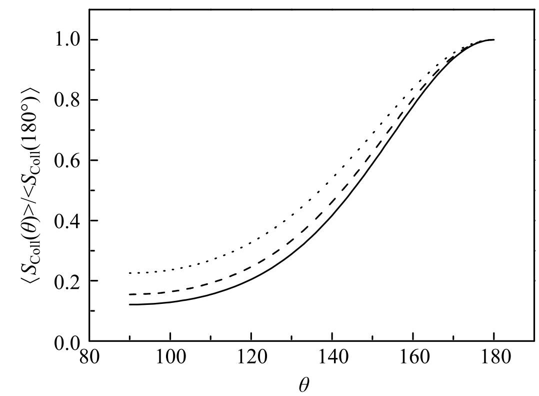

Fig. 6. Ratio of the collective spins for θ and 180° emission angles in the $^{12}C + ^{232}Th$ reaction. Dotted, dashed and solid curves are results for $E_{\rm Lab}$=80, 85, 90 MeV.

Two models are applied to calculate the average spin of fission fragments. One is the transition state model at scission point and the other is the four dimensional dynamical model. The results obtained based on the transition state model at scission point are higher than the results from the four dimensional Langevin approach. It is clear from these figures that the dynamical model can reproduce the experimental data whereas there is a large difference between the predictions of the transition state model and the experimental data.

The angular dependence of average fragment spin can be investigated by comparing $\langle S_{\rm T}\rangle$ at angles of 90° and 165°. The average fragment spin at 90° is higher than at 165°. Based on Eq. (5), the average fragment spin is related to angle via K^2 and $S_{\rm coll}$. In the dynamical model we calculated different parameters of Eq. (5) in Langevin trajectories and then obtained the average of these quantities using Eq. (37). For each trajectory we calculated emitted neutrons and gamma rays from fissioning systems using Monte Carlo simulation. Each neutron and gamma ray carries away 1 $\hbar$ of angular momentum. After each emission the intrinsic excitation energy of residual mass and spin of the compound nucleus is recalculated due to the energy removed by one particle emission. It should be mentioned that the average multiplicity of gamma rays and average fragment spin are sensitive to the uncertainty of the friction strength used for shape degrees of freedom and K degree of freedom.

In Ref. [3], the average total spins of fission fragments were measured. Also, the authors investigated the angular dependence of fragment spin within the statistical model. In Ref. [20], the authors calculated the average total spins of fission fragments based on an empirical relation. Also, they investigated the angular distribution of fission fragments using the three dimensional Langevin equations. In the present work, we studied fission of a compound nucleus as a dynamical process and investigated the fragment spin using the four dimensional Langevin equations. The fourth dimension in the calculations is the orientation degree of freedom, which is calculated based on the overdamped Langevin equation. This dynamical interpretation of average fragments spin in the present calculation is new in comparison with that reported in previous works.

4 Summary and conclusion

In this paper, the fusion fission reactions were studied for typical systems in terms of collective motion through the Langevin equations coupled with a Monte Carlo simulation to allow discrete emission of light particles and gamma rays. Based on the present dynamical Langevin mechanism along with Monte Carlo simulation, we can successfully describe the gamma multiplicity and angular dependence of fission fragment average spin. The obtained results can use in future studies of fusion fission reactions.

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