Is GW151226 really a gravitational wave signal? *

Zhe Chang(常哲)[1,2;1)] Chao-Guang Huang(黄超光)[1,2;2)] Zhi-Chao Zhao(赵志超)[1,2;3)]

[1] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

[2] School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China

Abstract: Recently, the LIGO Scientific Collaboration and Virgo Collaboration published the second observation of a gravitational wave, GW151226 [Phys. Rev. Lett. **116**, 241103 (2016)], from a binary black hole coalescence with initial masses about 14 $M_{\odot}$ and 8 $M_{\odot}$. They claimed that the peak gravitational strain was reached at about 450 Hz, the inverse of which is longer than the average time a photon stays in the Fabry-Perot cavities in the two arms. In this case, the phase-difference of a photon in the two arms due to the propagation of a gravitational wave does not always increase as the photon stays in the cavities. It might even be cancelled to zero in extreme cases. When the propagation effect is taken into account, we find that the claimed signal GW151226 almost disappears.

Keywords: gravitational wave, gravitational wave detector, LIGO

PACS: 04.30.Nk, 04.80.Nn **DOI:** 10.1088/1674-1137/41/2/025001

1 Introduction

The analysis of GW150914 shows that the initial black hole masses are 36 $M_{\odot}$ and 29 $M_{\odot}$ [1], which are heavier than the previous known stellar-mass black holes [2]. In the newly announced black hole merge event, GW151226 [3], the initial black hole masses are about 14 $M_{\odot}$ and 8 $M_{\odot}$, which fall into the known mass range of stellar black holes in previous observations. It seems to make the picture of binary black hole merge and gravitational wave observation more reliable because the signals of GW150914 and GW151226 are extracted from noise by the same methods [4, 5].

However, we notice the response of a detector to a gravitational wave is a function of frequency. When the time a photon moves around in the Fabry-Perot cavities is the same order of magnitude as the period of a gravitational wave, the phase-difference due to the gravitational wave should be an integral along the path. In fact, this propagation effect on Michelson detector response was addressed, for example, in Ref. [6]. Unfortunately, the propagation effect on Fabry-Perot detector response has not been considered properly. In this article, we try to take into account the propagation effect of the gravitational wave and reexamine the LIGO data. We find that when the average time a photon stays in the Fabry-Perot cavities in the two arms is the same order of magnitude as the period of a gravitational wave, the phase-difference of a photon in the two arms due to the gravitational wave may be cancelled. In the case of observation of GW151226, the average time a photon stays in the detector is longer than the period of the gravitational wave at maximum gravitational radiation. When the propagation effect is taken into account, the claimed signal GW151226 almost disappears.

It is well known that a Michelson interferometer is a broadband gravitational wave detector. A Michelson interferometer with a 4 km arm can be used to detect gravitational waves from tens to thousands of hertz. When an interferometer uses Fabry-Perot cavities instead of Michelson interferometer arms, the detection sensitivity is multiplied by a factor of $2N(1-N\epsilon)$, where N is the average number of photons back and forth in the cavity and ϵ is the fractional power lost in one round trip inside the cavity. The above statement seems to imply that the Fabry-Perot cavities do not affect the frequency range of the detector. In fact, there are subtle differences that should be dealt with carefully. In the following, we give a detailed description of the propagation effect of a gravitational wave on detectors. For a 4 km Michelson interferometer, seismic noise and environmental disturbances set the lower frequency limit of a gravitational wave detector. The upper limit comes from two aspects. The first, astrophysical gravitational waves with frequency over 10

Received 13 August 2016

∗ Supported by National Natural Science Foundation of China (11275207, 11375203, 11690022, 11675182) and Strategic Priority Research Program of the Chinese Academy of Sciences “Multi-waveband Gravitational Wave Universe” (XDB23040000)

1) E-mail: changz@ihep.ac.cn

2) E-mail: huangcg@ihep.ac.cn

3) E-mail: zhaozc@ihep.ac.cn

kHz are too weak to detect [7]. The second can be seen clearly from the following discussion.

2 Photons moving in Fabry-Perot cavities with gravitational wave as background

Consider the simplest case, a monochromatic gravitational wave with plus polarization normally incident on the detector. Assume the polarization direction is just consistent with the direction of the arms. The time difference that light travels to the end mirror and back in two arms is

$$\begin{aligned}\Delta t_{\mathrm{M}} &:= \frac{2}{c}\left(\oint(1+\frac{1}{2}h_{11})\mathrm{d}x-\oint(1-\frac{1}{2}h_{11})\mathrm{d}y\right)\\ &= \frac{1}{c}\left(\int_0^L h_{11}\mathrm{d}x-\int_L^0 h_{11}\mathrm{d}x\right.\\ &\quad\left.+\int_0^L h_{11}\mathrm{d}y-\int_L^0 h_{11}\mathrm{d}y\right)\\ &= \frac{4L}{c}\frac{\sin(2\pi fL/c)}{2\pi fL/c}h_{11}(t+L/c)\,|_{z=0},\end{aligned} \tag{1}$$

where $z=0$ denotes the plane of the detector, f is the frequency of the gravitational wave, L is the length of the detector arm, c is the speed of light, and $h_{11}(t-L/c)$ is the gravitational wave at the time $t-L/c$. Thus, the laser field acquires a phase shift as it travels to the end mirror and back:

$$\begin{aligned}\Delta\Phi_{\mathrm{M}} &= 2\pi\nu\Delta t_{\mathrm{M}}\\ &= \frac{2L}{\lambda/2\pi}\frac{\sin(2\pi fL/c)}{2\pi fL/c}h_{11}(t-L/c)|_{z=0}.\end{aligned} \tag{2}$$

In the case of $fL/c=1/2$ (which means $f=c/(2L)=$ 37500 Hz), the signal is just cancelled. That is to say, the detection frequency of the Michelson interferometer for a gravitational wave shall not exceed $c/(4L)=18750$ Hz . A similar result was found in Ref. [6]. When the frequency of the gravitational wave is 1 kHz or less, the time difference and phase shift can be written as

$$\Delta t_{\mathrm{M}}=\frac{2L}{c}h_{11}(t+L/c)_{z=0}, \tag{3}$$

$$\Delta\Phi_{\mathrm{M}}=2\pi\nu\Delta t_{\mathrm{M}}=\frac{2L}{\lambda/2\pi}h_{11}(t+L/c)|_{z=0}. \tag{4}$$

To increase the response of the detector to gravitational waves, Fabry-Perot resonant cavities are used to make the photons go back and forth many times. We denote the average number of round trips of photons in the arm as N. When N is small, we can still use Eqs. (3) and (4). They are good approximations. However, if N is large enough, we should use the general expressions (1) and (2). For a 1 kHz gravitational wave, this approximation no longer holds if $N>40$. For LIGO, $N=140$, so we should reconsider the approximation.

The above analysis seems to be contrary to the fact that a Fabry-Perot cavity can multiply the detection sensitivity. Taking into account the introduction of sidebands, Schnupp asymmetry, and PDH locking technique, the response of a Michelson interferometer is

$$V_s=P_0RJ_0(\beta)J_1(\beta)kLh\sin^2\frac{2\pi\Delta L}{\lambda_\Omega}, \tag{5}$$

where P_0 is the power of the incident laser, R is the response of the photodiode, J_0 and J_1 are the zeroth and first order Bessel functions, β is the phase modulation parameter, k is the wave number of the carrier wave, ΔL is the length difference of the two arms, and λ_Ω is the wavelength of the sideband [8]. For the Fabry-Perot cavities, the response of gravitational wave becomes:

$$V_s=P_0RJ_0(\beta)J_1(\beta)\frac{2F}{\pi}\left(1-\frac{F}{\pi}\varepsilon\right)\sin\left(2\pi\frac{\Delta L}{\lambda_\Omega}\right)kLh, \tag{6}$$

where $F/\pi=N$. Equations (5) and (6) show that the use of the Fabry-Perot cavities increases response by a factor of $2N$ when the loss is ignored. However, Eq. (6) is derived in the assumption of low frequency limit (i.e. $2NL\ll c/f$). Our analysis of GW151226, based on the light moving in the cavities, is not contrary to the property of Fabry-Perot cavities.

Now, let us consider Eqs. (2) (3) and (4) for the LIGO detectors. For the LIGO detectors, the lengths of the Fabry-Perot cavities are $L\approx 4$ km. On average, a photon travels in cavities 140 round trips [1]. It will move back and forth in the cavities for about 0.0037 s. In that period, a gravitational wave with frequency 268 Hz will have propagated a distance of one wavelength. Therefore, the above propagation effect should be taken into account in the analysis of GW151226 because the frequency of the peak gravitational strain is about 450 Hz (>268 Hz).

3 LIGO-GW151226 gravitational wave signal revisited

The LIGO Scientific Collaboration provides a simple program which can be used to search for gravitational wave signals from the noise data. They also provide a set of strain data for the two detectors and a well matched template. In the program, the various subtleties, whose effects are very small, are ignored on the LIGO website [9]. With the detector parameters, the program and the noise data, the propagation effect of a gravitational wave on the signal GW151226 can be re-examined.

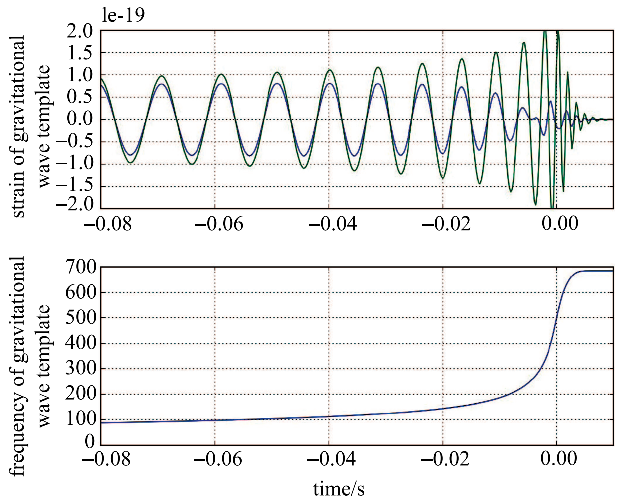

Fig. 1. The green line in the top panel is the response of the detector to the best fit template for GW151226 provided on the LIGO website [9]. When the propagation effect is taken into account, the detector response to the gravitational wave in the template takes the form of the blue line. The bottom panel presents the variation of frequency in time for gravitational wave.

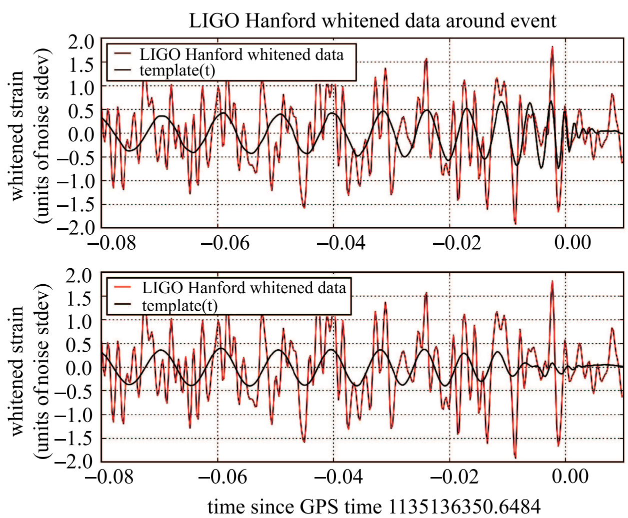

Fig. 2. The upper panel of Fig. 2 is the match of the data from the detector at Hanford and the response to the gravitational wave in the template provided on the LIGO website. The lower panel shows that the data from the detector does not match the response of detector to the gravitational wave in the template if the propagation effect of the gravitational wave is taken into account.

The green line in the top panel of Fig. 1 is the response of the detector to the best fit template for GW151226 provided on the LIGO website [9]. Near the instant the coalescence happens, the signal is strongest. When the propagation effect is taken into account, the detector response to the gravitational wave in the template becomes the blue line. It shows that the response of the detector to the gravitational wave becomes tiny when the real time of a cycle of the gravitational wave is near the time a photon travels in the arms. The bottom panel of Fig. 1 presents the variation of frequency in time for the gravitational wave. The vanishing response happens at about 268 Hz, in the case where the time of a gravitational wave cycle is 0.0037 s. The upper panel of Fig. 2 is the match of the data from the detector at Hanford and the response to the gravitational wave in the template provided on the LIGO website. The lower panel of Fig. 2 shows that the data from the detector does not match the response of the detector to the gravitational wave in the template if the propagation effect of the gravitational wave is taken into account. It should be noted that in Figs. 1 and 2 the templates are obtained by taking into account the propagation effect with varying frequency and amplitude, which is more accurate than what is shown in [10].

4 Conclusion and remarks

As a conclusion, the propagation effect of the gravitational wave is important in matching the signals in observations with templates when the time of a cycle of gravitational wave is of the same order as the time a photon stays in the cavities. By taking into account the propagation effect of the gravitational wave, we find that the LIGO-GW151226 signal almost disappears. For low-frequency gravitational waves the propagation effect is small, so the signal for GW150914 is not affected a lot.

It should be remarked that there is a subtle difference between the effect of a gravitational wave on the light traveling in a detector and the phase variation due to the vibration of mirrors which has been used in the calibration of LIGO's detectors [11], though both the vibration of mirrors and the incidence of a gravitational wave will modify the phase of light traveling in the cavities. The vibrations of the mirrors modify the phase of the light when the photons travel near the vibrating mirrors. The phase shift of light beyond the vibration region will not be affected by the vibrating mirrors. In contrast, a gravitational wave affects the phase of light at every place in the cavities. As the result, the average phase variations due to the vibrating mirrors do not vanish even when the round trip time of a photon in the cavity is the same as the period of the vibration of the end mirrors. But, it will vanish in the gravitational wave background when the round trip time for a photon is the same as the period of a gravitational wave. Therefore, the propagation effect of gravitational waves is not included in the calibration, which is calibrated with the help of the vibrating mirrors.

References

1 B. P. Abbott et al, Phys. Rev. Lett., **116**: 061102 (2016)
2 J. A. Orosz et al, Nature, **449**: 872–875 (2007)
3 B. P. Abbott et al, Phys. Rev. Lett., **116**: 241103 (2016)
4 For example, see: E. D. Black and R. N. Gutenkunst, Am. J. Phys., **71**: 365–378 (2003)
5 B. Allen, W. G. Anderson, P. R. Brady, D. A. Brown, and J. D. E. Creighton, Phys. Rev. D, **85**: 122006 (2012)
6 Rakhmanov, M., J. D. Romano, and John T. Whelan, Class.Quantum Grav., 25.18: 184017 (2008)
7 K. S. Thorne, in *Three hundred years of gravitation*, edited by S. W. Hawking and W. Israel, (Cambridge University Press, Cambridge, 1987), p.330
8 E.D. Black and R.N.Gutenkunst, Am. J. Phys., **69**: 79–87 (2001)
9 https:\\losc.ligo.org\tutorials\
10 Chang Z, Huang CG, Zhao ZC, Sci. China–Phys. Mech. Astro., **59**: 100421 (2016)
11 Abbott, B.P. and LIGO Scientific Collaboration, arXiv:1602.03845, 2016