Some new symmetric relations and prediction of left- and right-handed neutrino masses using Koide's relation *

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Abstract: The masses of the three generations of charged leptons are known to completely satisfy Koide's mass relation, but the question remains of whether such a relation exists for neutrinos. In this paper, by considering the seesaw mechanism as the mechanism generating tiny neutrino masses, we show how neutrinos satisfy Koide's mass relation, on the basis of which we systematically give exact values of both left- and right-handed neutrino masses.

Keywords: Koide's mass relation, neutrino mass, seesaw mechanism.

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1 Introduction

Despite being the most successful model of particle physics, the Standard Model (SM) fails to answer many questions, such as why the parameters of the SM are the way they are, and whether there is any relation among these parameters. Another such question is why Koide's relation for charged leptons is 2/3. Yoshio Koide [1, 2] pointed out that a very simple relationship exists for the pole masses (given in Table 1) of the three generations of charged leptons,

$$(m_{\mathrm{e}}+m_{\mu}+m_{\tau})=\frac{2}{3}(\sqrt{m_{\mathrm{e}}}+\sqrt{m_{\mu}}+\sqrt{m_{\tau}})^2. \tag{1}$$

which is surprisingly precise to a good degree of accuracy. This precision inspired Koide to propose models [3–5] in an attempt to explain the underlying physics. Various attempts have been made to extend this formula to other particles. In Ref. [6] some speculations related to the extension of Eq. (1) to quarks and leptons are given, along with its relations to recent theoretical developments. Different ideas following the implementation of this formula can also be found [7–10]. A geometric interpretation for Koide's relation was given in Ref. [11], in which the square root of the mass of leptons $\sqrt{m_{\mathrm{l}}}$ is used to construct a vector $\overrightarrow{V}$, such that

$$\overrightarrow{V}=\left(\sqrt{m_{\mathrm{e}}},\quad \sqrt{m_{\mu}},\quad \sqrt{m_{\tau}}\right). \tag{2}$$

Then, Koide's formula can be considered equivalent to the angle between the vector $(1,1,1)$ and $\overrightarrow{V}$, which is $\frac{\pi}{4}$. This will be considered in detail in Section 3. The questions that follow from the above interpretation are: why is the vector $(1,1,1)$, and why is the angle $\frac{\pi}{4}$? The aim of this paper is to give a meaning to the geometric interpretation and to extend Koide's formula to neutrinos such that the masses of left-handed and right-handed neutrinos can be predicted.

Table 1. The masses of the leptons.

lepton	mass/MeV
e	$0.510998928\pm0.000000011$
μ	105.6583715 ± 0.0000035
τ	1776.82 ± 0.16

The rest of this paper is structured as follows. In Section 2 we find two analytical formulas to find the masses of neutrinos using the data provided by experiments. Section 3 gives a meaning to the geometrical interpretation given by Foot[11]. In Section 4 we devise a formula to find the value of right-handed neutrino mass terms. Considering a relation between left- and right-handed neutrinos we can solve the analytical formulas for neutrino masses, details of which are given in Section 5. The last section is the summary and conclusion.

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2 Analytical formula for neutrino masses

The neutrino mass term has the form

$$\mathcal{L}=\frac{1}{2}\begin{pmatrix} \bar{\nu}_l & \bar{\nu}_R^c \end{pmatrix}\mathcal{M}\begin{pmatrix} \nu_l^c \\ \nu_R \end{pmatrix}+\text{h.c.} \tag{3}$$

We suppose that the mass matrix $\mathcal{M}_{\text{mass}}$ can be diagonalized as follows:

$$\begin{pmatrix} 0 & 0 & 0 & m_{\mathrm{D}_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{\mathrm{D}_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{\mathrm{D}_3} \\ m_{\mathrm{D}_1} & 0 & 0 & M_1 & 0 & 0 \\ 0 & m_{\mathrm{D}_2} & 0 & 0 & M_2 & 0 \\ 0 & 0 & m_{\mathrm{D}_3} & 0 & 0 & M_3 \end{pmatrix}, \tag{4}$$

where M_1, M_2 and M_3 are the Majorana mass coefficients.

The eigenvalues of the matrix are:

$$\begin{aligned} &\frac{1}{2}M_1\pm\frac{1}{2}\sqrt{M_1^2+4m_{\mathrm{D}_1}^2}, \\ &\frac{1}{2}M_2\pm\frac{1}{2}\sqrt{M_2^2+4m_{\mathrm{D}_2}^2}, \\ &\frac{1}{2}M_3\pm\frac{1}{2}\sqrt{M_3^2+4m_{\mathrm{D}_3}^2}. \end{aligned} \tag{5}$$

When $M_i\gg m_{\mathrm{D}_i} (i=1,2,3)$, the neutrino masses would be

$$\frac{m_{\mathrm{D}_1}^2}{M_1}, \quad \frac{m_{\mathrm{D}_2}^2}{M_2}, \quad \frac{m_{\mathrm{D}_3}^2}{M_3}, \tag{6}$$

which is just the seesaw mechanism. The strict form of $\mathcal{M}_{\text{mass}}$ is given by

$$\begin{pmatrix} 0 & 0 & 0 & m_{\mathrm{D}_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & m_{\mathrm{D}_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{\mathrm{D}_3} \\ m_{\mathrm{D}_1} & 0 & 0 & M_1 & m_1 & m_2 \\ 0 & m_{\mathrm{D}_2} & 0 & m_1 & M_2 & m_3 \\ 0 & 0 & m_{\mathrm{D}_3} & m_2 & m_3 & M_3 \end{pmatrix}. \tag{7}$$

$m_{\mathrm{D}_1}, m_{\mathrm{D}_2}, m_{\mathrm{D}_3}$ are the Dirac masses. The constants $M_1, M_2, M_3, m_1, m_2, m_3$ are unknown so the neutrino masses cannot be calculated directly. The case in which the mass matrix has the most general form involves so many parameters and becomes so complicated that it cannot be solved, so we take a simpler form.

There is no exact data available about the neutrino masses, but the cosmological measurements [12] give a boundary of active neutrino masses

$$\sum_i m_i<0.17\text{eV}. \tag{8}$$

Also, the neutrino mass differences [13] are given by experimental measurements of solar, atmospheric, accelerator and reactor neutrinos.

$$\begin{aligned} |\Delta m_{21}^2| &= (7.53\pm0.18)\cdot10^{-5}\ \text{eV}^2, \\ |\Delta m_{32}^2| &= (2.44\pm0.06)\cdot10^{-3}\ \text{eV}^2. \end{aligned} \tag{9}$$

If we denote neutrino masses as m_{ν_1}, m_{ν_2}, and m_{ν_3}, then with the help of Eq.(9) we can write

$$\begin{aligned} |m_{\nu_1}^2-m_{\nu_2}^2| &= |\Delta m_{21}^2|, \\ |m_{\nu_3}^2-m_{\nu_2}^2| &= |\Delta m_{32}^2|. \end{aligned} \tag{10}$$

Putting the values of $|\Delta m_{21}^2|$ and $|\Delta m_{32}^2|$ in Eq. (10) and considering Mikheyev Smirnov Wolfenstein [14, 15] matter effects on solar neutrinos, we can get the following two sets of analytical formulas,

$$\begin{aligned} m_{\nu_2}^2 &= m_{\nu_1}^2+7.53\times10^{-5}\ \text{eV}^2, \\ m_{\nu_3}^2 &= m_{\nu_2}^2+2.44\times10^{-3}\ \text{eV}^2, \end{aligned} \tag{11}$$

or

$$\begin{aligned} m_{\nu_2}^2 &= m_{\nu_1}^2+7.53\times10^{-5}\ \text{eV}^2, \\ m_{\nu_3}^2 &= m_{\nu_2}^2-2.44\times10^{-3}\ \text{eV}^2. \end{aligned} \tag{12}$$

Following Koide's formula for leptons, we can write a relation for neutrinos as

$$k_{\nu_L}^2=\frac{(m_{\nu_1}+m_{\nu_2}+m_{\nu_3})}{(\sqrt{m_{\nu_1}}+\sqrt{m_{\nu_2}}+\sqrt{m_{\nu_3}})^2}, \tag{13}$$

Using Eq. (11), Eq. (13) can be rewritten as

$$k_{\nu_L}^2=\frac{(m_{\nu_1}+\sqrt{m_{\nu_1}^2+7.53\times10^{-5}\text{eV}^2}+\sqrt{m_{\nu_1}^2+251.53\times10^{-5}\text{eV}^2})}{(\sqrt{m_{\nu_1}}+(m_{\nu_1}^2+7.53\times10^{-5}\text{eV}^2)^{1/4}+(\mathrm{m}_{\nu_1}^2+251.53\times10^{-5}\text{eV}^2)^{1/4})^2} \tag{14}$$

and using Eq. (12), as

$$k_{\nu_L}^2=\frac{(m_{\nu_1}+\sqrt{m_{\nu_1}^2+7.53\times10^{-5}\text{eV}^2}+\sqrt{m_{\nu_1}^2-236.47\times10^{-5}\text{eV}^2})}{(\sqrt{m_{\nu_1}}+(m_{\nu_1}^2+7.53\times10^{-5}\text{eV}^2)^{1/4}+(\mathrm{m}_{\nu_1}^2-236.47\times10^{-5}\text{eV}^2)^{1/4})^2} \tag{15}$$

The above equations can be solved to find the value of m_{ν_1} if we can somehow constrain the value of $k^2_{\nu_L}$.

3 Meaning of Foot's geometrical interpretation

In this section we present Foot's geometrical interpretation, explaining what the vector $\overrightarrow{u}=(1,\quad 1,\quad 1)$ means.

The lepton masses have an equal status in Koide's relation, which indicates the presence of some underlying symmetry. With the help of this symmetry we can give a Koide-like relation for Dirac neutrino mass terms.

The neutrino mass matrix is given by Eq. (7). We consider that there exists a symmetry such that the Dirac mass term of the three flavored neutrinos gives an invariable result for the three generations of neutrinos, which would mean that in the original neutrino mass matrix, the three generation of flavored neutrinos have the same mass coefficient, that is,

$$m_{\mathrm{D}_1}=m_{\mathrm{D}_2}=m_{\mathrm{D}_3}. \tag{16}$$

We can write a vector

$$\overrightarrow{U}=\left(\sqrt{m_{\mathrm{D}_1}},\quad \sqrt{m_{\mathrm{D}_2}},\quad \sqrt{m_{\mathrm{D}_3}}\right), \tag{17}$$

having characteristic

$$\frac{\overrightarrow{U}}{|\overrightarrow{U}|}=\frac{(1,\quad 1,\quad 1)}{|(1,\quad 1,\quad 1)|}, \tag{18}$$

which appears in Eq. (2) of Foot's paper [11]. The lepton masses can form a vector $\overrightarrow{V}=\left(\sqrt{m_{\mathrm{e}}},\quad \sqrt{m_{\mu}},\quad \sqrt{m_{\tau}}\right)$. The angle between $\overrightarrow{U}$ and $\overrightarrow{V}$ is

$$\cos\theta=\frac{\left(\sqrt{m_{\mathrm{e}}},\quad \sqrt{m_{\mu}},\quad \sqrt{m_{\tau}}\right)\left(\sqrt{m_{\mathrm{D}_1}},\quad \sqrt{m_{\mathrm{D}_2}},\quad \sqrt{m_{\mathrm{D}_3}}\right)}{|\left(\sqrt{m_{\mathrm{e}}},\quad \sqrt{m_{\mu}},\quad \sqrt{m_{\tau}}\right)||\left(\sqrt{m_{\mathrm{D}_1}},\quad \sqrt{m_{\mathrm{D}_2}},\quad \sqrt{m_{\mathrm{D}_3}}\right)|} \tag{19}$$

$$\begin{aligned}&=\frac{\left(\sqrt{m_{\mathrm{e}}},\quad \sqrt{m_{\mu}},\quad \sqrt{m_{\tau}}\right)(1,\quad 1,\quad 1)}{|\left(\sqrt{m_{\mathrm{e}}},\quad \sqrt{m_{\mu}},\quad \sqrt{m_{\tau}}\right)||(1,\quad 1,\quad 1)|}\\&=\frac{1}{\sqrt{3}}\frac{\sqrt{m_{\mathrm{e}}}+\sqrt{m_{\mu}}+\sqrt{m_{\tau}}}{\sqrt{m_{\mathrm{e}}+m_{\mu}+m_{\tau}}}.\end{aligned} \tag{20}$$

Using Eq. (1), Eq. (20) gives $\cos\theta=\frac{\sqrt{2}}{2}$, making $\theta=\frac{\pi}{4}$. This relation can be expressed by vectors, as given by Fig. 1.

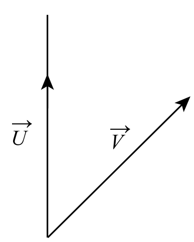

Fig. 1. The vectors $\overrightarrow{U}$ and $\overrightarrow{V}$ form an angle $\frac{\pi}{4}$.

For Dirac neutrino mass terms, using Eq. (20), we can get

$$m_{\mathrm{D}_1}+m_{\mathrm{D}_2}+m_{\mathrm{D}_3}=\frac{1}{3}(\sqrt{m_{\mathrm{D}_1}}+\sqrt{m_{\mathrm{D}_2}}+\sqrt{m_{\mathrm{D}_3}})^2. \tag{21}$$

Because we have

$$\begin{aligned}k_{\mathrm{l}}^2&=\frac{m_{\mathrm{e}}+m_{\mu}+m_{\tau}}{(\sqrt{m_{\mathrm{e}}}+\sqrt{m_{\mu}}+\sqrt{m_{\tau}})^2},\\ k_{\nu}^2&=\frac{m_{\mathrm{D}_1}+m_{\mathrm{D}_2}+m_{\mathrm{D}_3}}{(\sqrt{m_{\mathrm{D}_1}}+\sqrt{m_{\mathrm{D}_2}}+\sqrt{m_{\mathrm{D}_3}})^2},\end{aligned} \tag{22}$$

we can write a new symmetric relation

$$k_{\mathrm{l}}^2+k_{\nu}^2=1, \tag{23}$$

which is similar to

$$\sin^2\alpha+\cos^2\alpha=1. \tag{24}$$

The possible range of the coefficient k_{l}^2 in Koide's relation is (1/3, 1). When the three masses are identical (democratic), we have $k_{\mathrm{l}}^2=1/3$; if the three masses are strongly hierarchical, then $k_{\mathrm{l}}^2=1$. $k_{\mathrm{l}}^2=2/3$ is just the mean value of these two limits. Considering the degeneracy of the Dirac neutrino masses ($k_{\nu}^2=1/3$, Eq. (16)) will lead us to the above-mentioned symmetric relations.

4 Analytical formula for right-handed neutrino masses

We know the matrix given in Eq. (4) has 6 eigenvalues. When $M_i\gg m_{\mathrm{D}_i}(i=1,2,3)$, three would be given by Eqs. (5–6) and the rest of them would be equal to $M_i(i=1,2,3)$. There is another way to find this second set of eigenvalues i.e. by using Eq. (6). Approximately similar relations can be found in Ref. [16] and others.

The Dirac neutrino masses satisfy Eq. (21) and also, as discussed in Section (3), Eq.(16), the Dirac mass term gives an invariable result for the three generations of neutrinos. We can have Dirac neutrino masses proportional to the electroweak scale i.e. $\lambda_{\mathrm{EW}}\approx 246$ GeV. The Dirac masses are forbidden by electroweak gauge symmetry and can appear only after spontaneous symmetry breaking through the Higgs mechanism, as in the case of charged leptons, which implies that Dirac masses are naturally of the order of the vacuum expectation value of the Higgs field in the SM, which is $v=246$ GeV, so $v/\sqrt{2}\approx 174$ GeV. If we consider Dirac neutrino particle masses to be of the order of the electroweak scale, knowing the masses of neutrinos, we can get the masses for Majorana neutrinos, which is to say that using the expression in Eq. (6), right-handed neutrino masses can be written as

$$M_1=\frac{m_{\mathrm{D}_1}^2}{m_{\nu_1}},\quad M_2=\frac{m_{\mathrm{D}_2}^2}{m_{\nu_2}},\quad M_3=\frac{m_{\mathrm{D}_3}^2}{m_{\nu_3}}. \tag{25}$$

which also implies that no left-handed neutrino should have a zero mass.

5 Relation between left- and right-handed neutrino masses

Equation (23) gives a relation between leptons and Dirac neutrinos masses. We assume a similar kind of relation must exist for the right- and left-handed neutrino masses, since left- and right-handed neutrinos take part in the seesaw mechanism and follow Eq.(25). So,

$$k_{\nu_R}^2+k_{\nu_L}^2=1, \tag{26}$$

where $k_{\nu_R}^2$ is

$$k_{\nu_R}^2=\frac{(M_1+M_2+M_3)}{(\sqrt{M_1}+\sqrt{M_2}+\sqrt{M_3})^2}, \tag{27}$$

and $k_{\nu_L}^2$ is given by Eq. (13). Using Eq. (25), Eq. (27) can be re-written as

$$k_{\nu_R}^2=\frac{\left(\frac{m_{\mathrm{D}_1}^2}{m_{\nu_1}}+\frac{m_{\mathrm{D}_2}^2}{m_{\nu_2}}+\frac{m_{\mathrm{D}_3}^2}{m_{\nu_3}}\right)}{\left(\sqrt{\frac{m_{\mathrm{D}_1}^2}{m_{\nu_1}}}+\sqrt{\frac{m_{\mathrm{D}_2}^2}{m_{\nu_2}}}+\sqrt{\frac{m_{\mathrm{D}_3}^2}{m_{\nu_3}}}\right)^2}. \tag{28}$$

For neutrinos there are two possible mass schemes, the normal mass scheme, in which $m3>m2>m1$ and the inverted mass scheme, in which $m2>m1>m3$. Both schemes should be considered to obtain the possible neutrino masses.

5.1 Normal hierarchy

When the masses follow the normal mass hierarchy i.e. $m_{\nu_1}<m_{\nu_2}<m_{\nu_3}$,

$$k_{\nu_R}^2=\frac{\left(\frac{m_{\mathrm{D}_1}^2}{m_{\nu_1}}+\frac{m_{\mathrm{D}_2}^2}{\sqrt{m_{\nu_1}^2+7.53\times10^{-5}\ \mathrm{eV}^2}}+\frac{m_{\mathrm{D}_3}^2}{\sqrt{m_{\nu_1}^2+251.53\times10^{-5}\ \mathrm{eV}^2}}\right)}{\left(\sqrt{\frac{m_{\mathrm{D}_1}^2}{m_{\nu_1}}}+\sqrt{\frac{m_{\mathrm{D}_2}^2}{\sqrt{m_{\nu_1}^2+7.53\times10^{-5}\ \mathrm{eV}^2}}}+\sqrt{\frac{m_{\mathrm{D}_3}^2}{\sqrt{m_{\nu_1}^2+251.53\times10^{-5}\ \mathrm{eV}^2}}}\right)^2}, \tag{29}$$

where we used Eq. (11) to make Eq. (28) dependent only on one unknown parameter, m_{ν_1}. Now we have $k_{\nu_L}^2$ from Eq. (14) and $k_{\nu_R}^2$ from Eq. (29) both dependent on only one parameter, m_{ν_1}. Substituting these equations in Eq. (26) and running an analysis on the value of m_{ν_1}, such that Eq. (26) is completely satisfied, gives the value of m_{ν_1}. Using the obtained value of m_{ν_1}, we can extract the mass values of the remaining left-handed neutrino masses, i.e. m_{ν_2} and m_{ν_3}, by using Eq. (11).

It is interesting to note that when we assume the above relation, in Eq. (26), we can deduce the below-mentioned important relations. The only value which satisfies Eq. (26) up to three decimal points gives left-handed neutrino masses to be:

$$m_{\nu_1}=1.07\times10^{-3}\ \mathrm{eV},$$
$$m_{\nu_2}=8.74\times10^{-3}\ \mathrm{eV},$$
$$m_{\nu_3}=5.02\times10^{-2}\ \mathrm{eV}. \tag{30}$$

The above values of ν_2 and ν_3 neutrino masses are not only in accordance with the previous predictions [9, 17] up to three decimal points, but are also more precise. These masses follow the normal mass hierarchy.

Once we have obtained the left-handed neutrino masses, we can use them in Eq. (25) to obtain the right-handed neutrino masses to be

$$M_1=2.83\times10^{16}\ \mathrm{GeV},$$
$$M_2=3.46\times10^{15}\ \mathrm{GeV},$$
$$M_3=6.04\times10^{14}\ \mathrm{GeV}, \tag{31}$$

which are approximately of order 10^{16} GeV. In the case of the normal mass hierarchy, the plot of Eq. (14), Eq. (29) and Eq. (26) dependent on m_{ν_1} is given in Fig. (2).

5.2 Inverted hierarchy

For the case of the inverted hierarchy of left-handed neutrino masses, i.e. $m_{\nu_3}<m_{\nu_1}<m_{\nu_2}$, we use Eq. (12) to make Eq. (28) dependent on only one unknown parameter m_{ν_1},

$$k_{\nu_R}^2 = \frac{\left(\frac{m_{D_1}^2}{m_{\nu_1}} + \frac{m_{D_2}^2}{\sqrt{m_{\nu_1}^2 + 7.53\times10^{-5}\text{eV}^2}} + \frac{m_{D_3}^2}{\sqrt{m_{\nu_1}^2 - 236.47\times10^{-5}\text{eV}^2}}\right)}{\left(\sqrt{\frac{m_{D_1}^2}{m_{\nu_1}}} + \sqrt{\frac{m_{D_2}^2}{\sqrt{m_{\nu_1}^2 + 7.53\times10^{-5}\text{eV}^2}}} + \sqrt{\frac{m_{D_3}^2}{\sqrt{m_{\nu_1}^2 - 236.47\times10^{-5}\text{eV}^2}}}\right)^2}, \tag{32}$$

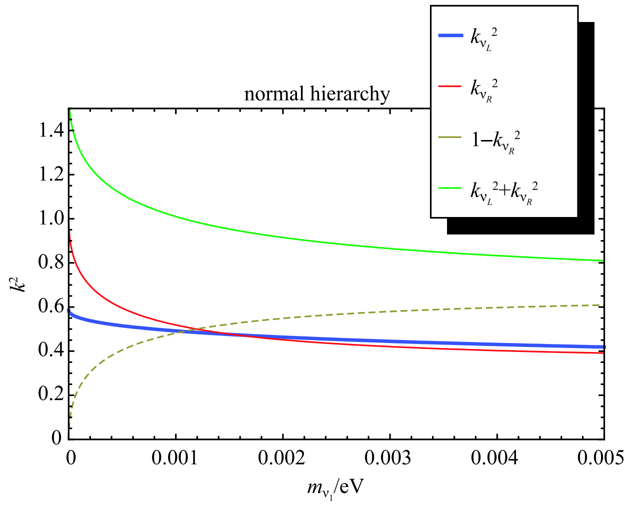

Fig. 2. Relationship with normal hierarchy.

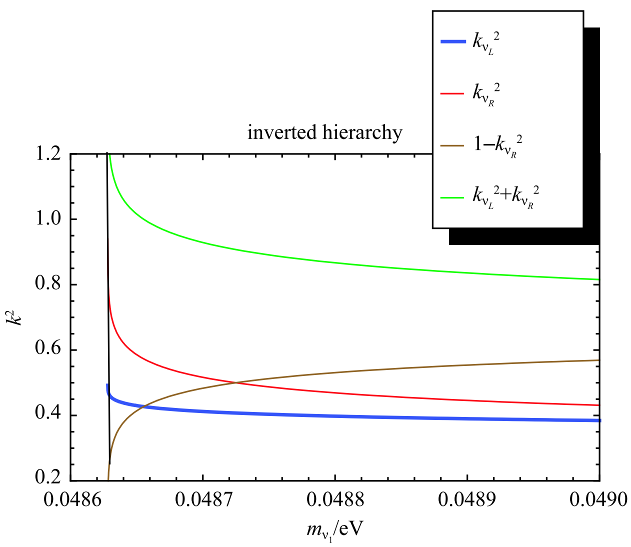

Fig. 3. Relationship with inverted hierarchy.

while $k_{\nu_L}^2$ is given by Eq. (15). Following similar steps and carefully looking for a solution in a very small range with $m_{\nu_1} \geqslant 0.04863$, Eq. (26) is satisfied with

$$m_{\nu_1} = 4.87\times10^{-2}\ \text{eV},$$
$$m_{\nu_2} = 4.94\times10^{-2}\ \text{eV},$$
$$m_{\nu_3} = 1.63\times10^{-3}\ \text{eV}. \tag{33}$$

and in turn the masses of right-handed neutrinos become

$$M_1 = 6.22\times10^{14}\ \text{GeV},$$
$$M_2 = 6.13\times10^{14}\ \text{GeV},$$
$$M_3 = 1.86\times10^{16}\ \text{GeV}. \tag{34}$$

Figure 3 shows the plot of Eq. (15), Eq. (32) and Eq. (26) dependent on m_{ν_1} in the case of the inverted mass hierarchy.

6 Conclusion and summary

In this paper we have shown how neutrino masses can satisfy Koide's relation. We discussed Koide's mass relation and gave the Dirac mass terms a similar symmetry. We consider the Dirac mass terms as invariable and used this to give a meaning to the geometrical interpretation of Koide's formula given in Ref. [11], which in turn leads to a new Koide-like relation for Dirac neutrino mass terms, given by Eq.(21). Koide's relation and this new Koide-like relation for Dirac mass terms, if added together equals one which leads us to Eq. (24). We assume a similar kind of plane must exist for the left- and right-handed neutrinos, because according to the seesaw mechanism the extreme masses of neutrinos are because of the interaction between them, i.e. Eq. (26). This relation can solve the analytical formula for the left-handed neutrinos in Eq.(14), giving masses of the three generations of neutrinos which are precise up to three decimal points with the previously proposed values. Also, we can find the masses of neutrinos following the inverted mass hierarchy. Neutrino oscillations indicates neutrinos are not massless and require all three neutrinos to have different masses.

In our paper we define the Yukawa coupling to be 1 to take m_D to be 174 GeV, but we noticed that it does not affect the mass values given in Eq. (30) for the left-handed neutrino masses, even if changed over a wide range. The masses obtained are dependent on the precision of the experimental data so improved accuracy will give more precise masses. Our model also proposes that the seesaw mechanism is used. Since the masses of the left-handed neutrinos are dependent on the mass of the Dirac neutrino and the right-handed neutrino masses, the Koide-like relation for the neutrino would be

$$k_L{}'^2 = \frac{k_\nu{}^2}{k_{\nu_R}{}^2}, \tag{35}$$

where

$$k^2_{\nu_R} \approx \frac{1}{2}, \tag{36}$$

so we can have,

$$k_L{}'^2 \approx \frac{2}{3}. \tag{37}$$

The above relation indicates that the seesaw mechanism may be the underlying mechanism and thus explains why neutrinos have such extreme masses. This relation explains why neutrinos do not satisfy Koide's relation giving 2/3 as we have for leptons. The reason is that the neutrino masses are the ratio of Dirac and Majorana neutrinos so the ratio of Koide's formula for these would give the same 2/3 as for leptons. This formula is to justify the 2/3 value and not to be used as Koide's formula for neutrinos, which gives a value approximately equal to 1/2, which can be calculated by using the mass values given in Eq. (30).

So far there is not much experimental evidence to explain the mechanism by which neutrinos gain mass. With the present experimental energy range it is not possible to test the speculation about the existence of Majorana neutrinos, but as mentioned above, the values of neutrinos obtained in this paper are in accordance with the masses of neutrinos predicted by several other studies.

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