Spatial and temporal variations of the fine-structure constant in the Finslerian universe*

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Abstract: Recent observations show that the electromagnetic fine-structure constant, α_e, may vary with space and time. In the framework of Finsler spacetime, we propose here an anisotropic cosmological model, in which both spatial and temporal variations of α_e are allowed. Our model naturally leads to the dipole structure of α_e, and predicts that the dipole amplitude increases with time. We fit our model to the most up-to-date measurements of α_e from the quasar absorption lines. It is found that the dipole direction points towards $(l,b)=(330.2^\circ\pm7.3^\circ,-13.0^\circ\pm5.6^\circ)$ in galactic coordinates, and the anisotropic parameter is $b_0=(0.47\pm0.09)\times10^{-5}$, which corresponds to a dipole amplitude $(7.2\pm1.4)\times10^{-8}$ at redshift $z=0.015$. This is consistent with the upper limit of the variation of α_e measured in the Milky Way. We also fit our model to Union2.1 type Ia supernovae, and find that the preferred direction of Union2.1 is consistent with the dipole direction of α_e.

Keywords: Finsler spacetime; fine-structure constant; anisotropy of the Universe

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1 Introduction

The fundamental physical constants play an essential role in modern physics. It is especially important to test the constancy of the fundamental physical constants both in experiments and theories [1]. The stability of the electromagnetic fine-structure constant $\alpha_e=e^2/\hbar c$ has been extensively tested by various experiments. The Oklo phenomenon is the first experimental approach to test the constancy of α_e [2]. Recent analysis of the Oklo phenomenon gives $\Delta\alpha_e/\alpha_e=(3.85\pm5.65)\times10^{-8}$ [3] and $\Delta\alpha_e/\alpha_e=(-0.65\pm1.75)\times10^{-8}$ [4], respectively. Atomic clock data give a strict constraint on the temporal variation of the fine-structure constant, i.e., $\dot{\alpha_e}/\alpha_e=(-1.6\pm2.3)\times10^{-17}\mathrm{yr}^{-1}$ [5], where the dot denotes the derivative with respect to time. The Planck cosmic microwave background data limit the variation of α_e from $z\approx1000$ to the present day to be less than approximately 4×10^{-3} [6]. Recently, observations of quasar absorption spectra show that the fine-structure constant may vary at cosmological scales [7, 8]. Furthermore, in the high-redshift region ($z>1.6$), it has been shown that the variation of α_e is well represented by an angular dipole model pointing in the direction $(l,b)=(330^\circ,-15^\circ)$ in galactic coordinates with $\sim4.2\sigma$ statistical significance, and the dipole amplitude is about $(0.97^{+0.22}_{-0.20})\times10^{-5}$ [7]. Recently, Pinho & Martins [9] carried out a joint analysis of a larger number of old α_e data [8] and ten more recent measurements of α_e [10–17], and found that the dipolar variation of α_e with amplitude $(0.81\pm0.17)\times10^{-5}$ is still a good fit to the combined data set, while the statistical uncertainty is significantly reduced compared with previous results.

Theoretically, several cosmological models have been proposed to explain the spatial variation of α_e, such as models of dark energy (for example, quintessence field or tachyon field) [18]. A more recent analysis of the relation between dark energy models and the variation of α_e can be found in Ref. [19]. Antoniou & Perivolaropoulos [20] found from the Union2 Sne Ia data set that the universe has a preferred direction pointing to $(l,b)=(309^\circ,18^\circ)$. Later on, Mariano & Perivolaropoulos [21] showed that the dipole of α_e is approximately aligned with the corresponding dark energy dipole obtained through the Union2 sample. In a recent paper [22], we proposed an anisotropic cosmological model in Finsler spacetime [23], and showed that both the fine-structure constant and the accelerating cosmic expan-

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sion have a dipole structure. Fitting to the observational data showed that these two dipole directions are almost aligned. However, the dipole amplitude of α_e in this model is constant, and is of the order of $\sim 10^{-5}$. This conflicts with the upper limit of the variation of α_e measured in the Milky Way, i.e., $|\Delta\alpha_e/\alpha_e| < 1.1\times 10^{-7}$ [24]. Webb et al. [25] have shown that the variation of α_e mainly comes from the high redshift region. We may expect that α_e evolves not only with space, but also with time. In this paper, we will construct a new cosmological model based on Finsler spacetime. Unlike the previous model, in our new model both spatial and temporal variations of α_e are allowed.

The rest of the paper is organised as follows. In Section 2, we introduce an anisotropic cosmological model in the framework of Finsler spacetime. In Section 3, we obtain the gravitational field equations and the distance-redshift relation in the Finslerian universe. In Section 4, we use the most up-to-date measurement of α_e and the Sne Ia data to constrain the model parameters. Finally, discussions and conclusions are presented in Section 5.

2 Anisotropic universe in Finsler spacetime

Finsler geometry [23] is a generalization of Riemann geometry and includes the latter as a special case. In fact, Finsler geometry is just Riemann geometry without the quadratic restriction. Finsler geometry is based on the so called Finsler structure F defined on the tangent bundle of a manifold M, with the property $F(x,\lambda y) = \lambda F(x,y)$ for any $\lambda > 0$, where $x\in M$ represents the position and y represents the velocity. The Finslerian metric is given by the second order derivative of F^2 with respect to y [23],

$$g_{\mu\nu} \equiv \frac{\partial}{\partial y^\mu}\frac{\partial}{\partial y^\nu}\left(\frac{1}{2}F^2\right). \tag{1}$$

Throughout this paper, the indices labeled by Greek letters are lowered and raised by $g_{\mu\nu}$ and its inverse matrix $g^{\mu\nu}$, respectively.

In this paper, we propose a Finsler spacetime that describes the anisotropic evolution of the universe. It is of the form

$$F^2 = y^t y^t - a^2(t)F_{Ra}^2, \tag{2}$$

where

$$F_{\rm Ra} = \sqrt{\delta_{ij}y^i y^j} + b(t)y^z \tag{3}$$

is the 3-dimensional Randers space [26], and $a(t)$ is the scale factor of the universe. Hereafter, Greek and Latin indices stand for the 4-dimensional spacetime and 3-dimensional space, respectively. We assume that the Finslerian parameter b is independent of space coordinates, but it is only a function of time. Then, the Killing equations of Randers space $F_{\rm Ra}$ [27, 28] tell us that the translational symmetry of space and the rotational symmetry of the x-y plane are preserved. This means that the symmetry of Finsler spacetime (2) is the same as that of Bianchi type I spacetime [29–31]. However, we will show in the following that the gravitational field equations of Finsler spacetime (2) and Bianchi type I spacetime are different.

The first order variation of Finslerian length gives the geodesic equation which describes the motion of free particles in curved spacetime. The redshift of photon in universe can be deduced from the equation of motion of photon. The geodesic equation in Finsler spacetime is given as

$$\frac{{\rm d}^2x^\mu}{{\rm d}\tau^2} + 2G^\mu = 0, \tag{4}$$

where

$$G^\mu = \frac{1}{4}g^{\mu\nu}\left(\frac{\partial^2F^2}{\partial x^\lambda\partial y^\nu}y^\lambda - \frac{\partial F^2}{\partial x^\nu}\right). \tag{5}$$

It can be easily proven from the geodesic equation (4) that the Finsler structure F is a constant along the geodesic. Substituting Eq. (2) into Eq. (5), through a straightforward calculation we obtain

$$G^t = \frac{1}{2}(a\dot{a}F_{\rm Ra}^2 + a^2\dot{b}y^zF_{\rm Ra}), \tag{6}$$

$$G^i = Hy^iy^t + \frac{1}{2}\dot{b}y^t\left(y^i\frac{y^z}{F_{\rm Ra}} + F_{\rm Ra}\bar{g}^{iz}\right), \tag{7}$$

where $H\equiv\dot{a}/a$ is the Hubble parameter and $\bar{g}^{ij}$ is the Finslerian metric in Randers space F_{Ra}, and the dot represents the derivative with respect to time. In order to get the equation of motion for a photon, one should notice that the null condition of the photon is given by $F=0$ in Finsler spacetime. Substituting the null condition into the geodesic Eq. (4), we obtain the solution

$$\frac{{\rm d}t}{{\rm d}\tau} \propto \frac{1}{a}\exp\left(-b\hat{n}^z\right), \tag{8}$$

where $\hat{n}^z$ denotes the unit vector along the z-axis.

It yields the formula of redshift z

$$1+z = \frac{1}{ca}\exp\left(-b\hat{n}^z\right), \tag{9}$$

where c is the speed of light at any epoch, and we have assumed that the speed of light at present is $c_0 = 1$. In Finsler spacetime (2), the parameters $a(t)$ and $b(t)$ are constant in the local inertial system. We can reparameterize the curve parameter τ such that $a=1$ in the local inertial system. Thus, the speed of light along the radial direction in Finsler spacetime is given by

$$c_r = \frac{1}{1+b\hat{n}^z}. \tag{10}$$

Substituting Eq. (10) into Eq. (9), to first order in b, we obtain

$$1+z = \frac{1}{a}. \tag{11}$$

3 Gravitational field equations and distance-redshift relation

In Finsler geometry, there is a geometrical invariant quantity, i.e., the Ricci scalar. The Ricci scalar is related to the second order variation of Finslerian length. In physics, the second order variation of Finslerian length gives the geodesic deviation equation, which describes the gravitational effect between two particles moving along the geodesics. The analogy between geodesic deviation equations in Finsler spacetime and Riemann spacetime gives the vacuum field equation in Finsler gravity [28, 32], namely, the vanishing of the Ricci scalar. The Ricci scalar in Finsler geometry is given by [23]

$$\mathrm{Ric}\equiv\frac{1}{F^2}\left(2\frac{\partial G^\mu}{\partial x^\mu}-y^\lambda\frac{\partial^2 G^\mu}{\partial x^\lambda\partial y^\mu}+2G^\lambda\frac{\partial^2 G^\mu}{\partial y^\lambda\partial y^\mu}-\frac{\partial G^\mu}{\partial y^\lambda}\frac{\partial G^\lambda}{\partial y^\mu}\right). \tag{12}$$

Substituting Eqs. (6) and (7) into Eq. (12), we obtain

$$\begin{aligned}F^2\mathrm{Ric}=&-3\frac{\ddot a}{a}y^ty^t+(a\ddot a+2\dot a^2)F^2_{Ra}+(6a\dot a\dot b+a^2\ddot b)F_{Ra}y^z\\&-(2\ddot b+4H\dot b)y^ty^tl^z,\end{aligned} \tag{13}$$

where $l^i\equiv y^i/F_{\mathrm{Ra}}$.

In Ref. [22, 28], we have proven that the gravitational field equations in Finsler spacetime are of the form

$$\mathrm{Ric}^\mu_\nu-\frac{1}{2}\delta^\mu_\nu S=8\pi G T^\mu_\nu, \tag{14}$$

where T^μ_ν is the energy-momentum tensor. Here the Ricci tensor is defined as [33]

$$\mathrm{Ric}_{\mu\nu}=\frac{\partial^2\left(\frac{1}{2}F^2\mathrm{Ric}\right)}{\partial y^\mu\partial y^\nu}, \tag{15}$$

and the scalar curvature in Finsler spacetime is given as $S=g^{\mu\nu}\mathrm{Ric}_{\mu\nu}$. Substituting the equation of the Ricci scalar (13) into the field Eq. (14), we obtain

$$8\pi G T^t_t=3H^2+4H\dot b l^z+(\ddot b+2H\dot b)\left(\frac{y^t}{aF_{\mathrm{Ra}}}\right)^2 l^z, \tag{16}$$

$$8\pi G T^t_i=-\frac{y^t}{F_{\mathrm{Ra}}}(\delta^z_i-l^zl_i)(2\ddot b+4H\dot b), \tag{17}$$

$$\begin{aligned}8\pi G T^i_j=&\left[\frac{2\ddot a}{a}+H^2+\left(5H\dot b+\frac{3}{2}\ddot b\right)l^z\right]\delta^i_j\\&-\left(\frac{y^t}{aF_{\mathrm{Ra}}}\right)^2(\ddot b+2H\dot b)(\delta^z_j-2l^zl_j)l^i\\&-\left(3H\dot b+\frac{1}{2}\ddot b\right)(\bar g^{iz}l_j+\delta^z_jl^i-l^zl^il_j),\end{aligned} \tag{18}$$

where $l_i=\bar g_{ij}l^j$. The gravitational field Eqs. (16–18) imply that the energy-momentum tensor in Finsler spacetime involves shear stress and viscosity that would not appear in Bianchi type I spacetime.

The Planck data give severe constraints on the peculiar velocity [34]. Thus, it requires that the energy-momentum tensor in Finsler spacetime (2) does not contain viscosity, i.e., $T^t_i=0$. Therefore, it results from Eq. (17) that

$$2\ddot b+4H\dot b=0. \tag{19}$$

The solution of equation (19) is

$$\dot b=\frac{b_0H_0}{a^2}, \tag{20}$$

where we have set the integral constant to be b_0H_0.

The gravitational field Eqs. (16 − 18) imply that the energy-momentum tensor T^μ_ν is a function of both x and y. However, T^μ_ν in the Finslerian manifold has not been well defined. Therefore, noticing that l^i in gravitational field equations are homogeneous function of degree 0 with respect to the variable y, we can set $l^i=\hat n^i$ such that the geometrical parts of the gravitational field Eqs. (16 − 18) are only functions of x. The energy-momentum tensor T^μ_ν can be treated as the same in general relativity. Then, by making use of Eq. (19), the gravitational field Eqs. (16) and (18) can be reduced to

$$3H^2+4H\dot b\hat n^z=8\pi G\rho, \tag{21}$$

$$2\frac{\ddot a}{a}+H^2+\frac{4}{3}H\dot b\hat n^z=-8\pi G p, \tag{22}$$

where ρ and $p\equiv(p_x+p_y+p_z)/3$ are the energy density and the mean pressure density of the universe, respectively. We find from Eqs. (21) and (22) that

$$\dot\rho+3H(\rho+p)+2\dot b\hat n^z\left(\rho+\frac{1}{3}p\right)=0. \tag{23}$$

In this paper, we assume that the Finslerian universe is made up of a pressureless matter component and an anisotropic dark energy component. The latter has the equation of state $p_\Lambda=-\rho$. Then, combining Eqs. (21) and (23), we obtain

$$\begin{aligned}H^2+\frac{4}{3}H\dot b\hat n^z=H_0^2\,&[\Omega_{m0}a^{-3}\exp(-2b\hat n^z)\\&+\Omega_{\Lambda0}\exp(-4b\hat n^z/3)],\end{aligned} \tag{24}$$

where $\Omega_{\Lambda0}\equiv 8\pi G\rho_{\Lambda0}/(3H_0^2)$ and $\Omega_{m0}\equiv 8\pi G\rho_{m0}/(3H_0^2)$ denote the dimensionless density of dark energy and matter at the present epoch, respectively. We assume that our universe is Riemannian at present epoch, and therefore $\Omega_{\Lambda0}=1-\Omega_{m0}$.

The distance-redshift relation in the Finslerian universe can be derived from the null condition $F=0$ and Eqs. (11) and (24). It is of the form

$$H_0d_L=(1+z)\int_0^z\frac{\mathrm{d}z}{(1+b\cos\theta)f(z)}, \tag{25}$$

where

$$f(z) \equiv \sqrt{\Omega_{m0}(1+z)^3(1-2b\cos\theta)+(1-\Omega_{m0})\left(1-\frac{4b}{3}\cos\theta\right)} \\ -\frac{2}{3}b_0(1+z)^2\cos\theta. \quad (26)$$

In the special case $b_0=0$, Eq. (25) reduces to that of the ΛCDM model.

4 Observational constraints

The variation of the speed of light in the Finslerian universe leads to the variation of the fine-structure constant. Making use of Eqs. (10) and (20), we obtain the variation of the fine-structure constant to first order in b,

$$\frac{\Delta\alpha_e}{\alpha_e} = b\cos\theta, \quad (27)$$

where θ denotes the angle with respect to the z-axis, and

$$b(z) = b_0\int_0^z \frac{(1+z)\mathrm{d}z}{\sqrt{\Omega_{m0}(1+z)^3+1-\Omega_{m0}}} \quad (28)$$

is the dipole amplitude at redshift z. Equations (27) and (28) mean that $\Delta\alpha_e/\alpha_e$ has a dipole distribution at cosmological scale, and $\Delta\alpha_e=0$ at the present epoch. We fit our model to the most complete sample of α_e measured from the quasar absorbtion lines. Our sample contains 303 measurements of $\Delta\alpha_e$ in the redshift range $z\in[0.2223, 4.1798]$, among which 293 measurements are taken from Ref. [8], and the remaining 10 measurements are taken from various studies and are compiled in Ref. [9]. Most of the data are observed by the Ultraviolet and Visual Echelle Spectrograph on the Very Large Telescope in Chile, and the Keck Observatory in Hawaii. The $\Delta\alpha_e/\alpha_e$ values are obtained by comparing the absorption lines from quasars and those from the laboratory using the many-multiplet method [25]. We find the dipole direction pointing towards $(l,b)=(330.2^\circ\pm7.3^\circ, -13.0^\circ\pm 5.6^\circ)$ in galactic coordinates, with the anisotropic parameter $b_0=(0.47\pm0.09)\times10^{-5}$. Here, we have fixed $H_0 = 70.0$ km s^{-1} Mpc^{-1} and $\Omega_{m0}=0.278$, which are derived by fitting to the Union2.1 data set [35] using the standard ΛCDM model. Then, we find from the best-fitting result and Eq. (28) that the dipole amplitude at redshift $z=0.015$ is $b(z=0.015)=(7.2\pm1.4)\times10^{-8}$. Our result is consistent with the upper limit of the variation of α_e measured in the Milky Way, i.e., $|\Delta\alpha_e/\alpha_e|<1.1\times10^{-7}$ [24].

To check the reasonableness of Eq. (28) in describing the temporal variation of α_e, we divide our complete sample into two redshift bins with approximately equal number of data points in each bin. The low-redshift sample contains 153 data points with redshift $z<1.6$, and the high-redshift sample contains 150 data points with redshift $z>1.6$. We assume that the dipole amplitudes are constants in these two redshift bins, and fit the subsamples to the dipole model with constant amplitude, i.e. $\Delta\alpha_e/\alpha_e = A\cos\theta$. For the low-redshift sample, we find $A=(0.44\pm0.24)\times10^{-5}$, and the dipole direction points towards $(l,b)=(354.7^\circ\pm20.0^\circ, -14.2^\circ\pm15.2^\circ)$. For the high-redshift sample, we find $A=(1.29\pm0.26)\times10^{-5}$, and the dipole direction points towards $(l,b)=(321.6^\circ\pm 9.0^\circ, -16.5^\circ\pm6.3^\circ)$. We can see that the dipole directions of both subsamples are consistent with those of the complete sample within 1σ uncertainty, while the high-redshift subsample has larger dipole amplitude than the low-redshift subsample. Using Eq. (28), we can calculate the average dipole amplitude in these two redshift bins. We obtain $\bar{b}(z_{\min}<z<1.6)=(0.48\pm0.09)\times10^{-5}$ and $\bar{b}(1.6<z<z_{\max})=(1.46\pm0.28)\times10^{-5}$, where $z_{\min}=0.2223$ and $z_{\max}=4.1798$ are the minimum and maximum redshifts of the complete sample, respectively. The average dipole amplitudes in these two redshift bins agree very well with the results obtained by fitting to the dipole model with constant amplitude. Therefore, Eq. (28) is quantitatively consistent with the observational data.

It has already been noticed that the dipole of Sne Ia is approximately aligned with that of α_e, although their amplitudes differ by ~2 orders of magnitude [21]. To compare the variation of α_e with the preferred direction of cosmic acceleration, we also fit our model to the Union2.1 data set [35], which is a compilation of 580 Sne Ia in the redshift range $z\in[0.015, 1.414]$. The least-χ^2 fit of Eq. (25) to the Union2.1 data set shows that the preferred direction is at $(l,b)=(312.8^\circ\pm 19.6^\circ, -11.8^\circ\pm11.8^\circ)$, and the anisotropic parameter is $b_0=(-2.57\pm1.15)\times10^{-2}$. We note that the dipole direction of Sne Ia is consistent with that of α_e within 1σ uncertainty. However, the anisotropic parameters differ by more than ~3 orders of magnitude.

5 Discussion and conclusions

In this paper, we proposed an anisotropic cosmological model in Finsler spacetime to account for the spatial and temporal variations of the electromagnetic fine-structure constant α_e. We obtained the gravitational field equations and the distance-redshift relations in the Finslerian universe. In our model, the variation of α_e arises from the variation of the speed of light. The variation of α_e has a dipole structure, and the dipole amplitude increases with time. We fitted our model to a large set of α_e measurements from the quasar absorbtion lines, and found the dipole direction pointing towards $(l,b)=(330.2^\circ\pm7.3^\circ, -13.0^\circ\pm5.6^\circ)$, with the anisotropic parameter $b_0=(0.47\pm0.09)\times10^{-5}$. At the present epoch, the variation of α_e vanishes, which has been tested with high accuracy by various experiments on earth. Our result is also consistent with the upper limit of α_e variation measured in the Milky Way. What's more, the temporal evolution of α_e is also consistent with the present data. A main shortcoming of our model is that the variation of α_e diverges when $z \to \infty$, which is of course unreasonable. Our model is only reasonable below a critical redshift. The furthest data point in our sample has redshift $z \sim 4$, and our model can quantitatively match the current data.

We also fit our model to the Union2.1 Sne Ia data, and found that the dipole direction of Union2.1 is approximately aligned with the dipole of α_x. However, the anisotropic parameter obtained from Union2.1, $b_0 = (-2.57 \pm 1.15) \times 10^{-2}$, is more than 3 orders of magnitude larger than that obtained from α_e. In fact, Mariano & Perivolaropoulos [21] have already noticed that the dipole amplitudes of Sne Ia and α_e differ by about 2 orders of magnitude. None of the existing theoretical models, such as extended topological quintessence [21] and the Finslerian universe [22], including the model proposed in this paper, can reconcile this contradiction. One possible explanation is that the present Sne Ia data are not accurate enough. There are several pieces of evidence for this hypothesis. A recent analysis [36] showed that the dipole directions of two different Sne Ia data sets, Union 2 [37] and JLA [38], are inconsistent. The dipole directions of Union2 derived using two different methods are almost opposite [39]. If the dipoles of Sne Ia and α_e have the same origin, e.g., the universe is Finslerian, we may expect that the dipole amplitudes of these two data sets should also be the same.

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