Analysis of the strong coupling form factors of $\Sigma_b NB$ and $\Sigma_c ND$ in QCD sum rules*

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Abstract: In this article, we study the strong interaction of the vertices $\Sigma_b NB$ and $\Sigma_c ND$ using the three-point QCD sum rules under two different Dirac structures. Considering the contributions of the vacuum condensates up to dimension 5 in the operation product expansion, the form factors of these vertices are calculated. Then, we fit the form factors into analytical functions and extrapolate them into time-like regions, which gives the coupling constants. Our analysis indicates that the coupling constants for these two vertices are $G_{\Sigma_b NB}=0.43\pm0.01$ GeV^{-1} and $G_{\Sigma_c ND}=3.76\pm0.05$ GeV^{-1}.

Keywords: sum rules, coupling constants, baryons

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1 Introduction

To date, many heavy baryons have been observed by the BaBar, Belle and CLEO Collaborations [1–4], including the $\frac{1}{2}^+$ antitriplet states(Λ_c^+, Ξ_c^+,Ξ_c^0), and the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ sextet states (Ω_c, Σ_c, Σ_c') and (Ω_c^*, Σ_c^*, Σ_c^*) [2]. Besides, several S-wave bottom baryon states such as Λ_b, Σ_b, Σ_b^*, Ξ_b and Ω_b have also been observed by the CDF and LHCb Collaborations [5, 6]. The SELEX Collaboration has even reported the observation of a signal for a doubly charmed baryon state Ξ_{cc}^+ [7, 8]. Since then, there has been great interest in studying the properties of these heavy baryons, which contains at least one heavy quark [9–13]. The charm and bottom baryon states which contain one or two heavy quarks are particularly interesting for studying the dynamics of light quarks in the presence of the heavy quark(s), and serve as an excellent ground for testing predictions of the quark models and heavy quark symmetry.

Many studies have looked at the properties of the heavy baryons, such as mass spectrum, and radiative and strong decays, which are very important for us to further understand heavy flavor physics [14–20, 22]. In this regard, the strong coupling constants associated with heavy baryons play an important role in describing the strong interaction among the heavy baryons and other participating hadrons. In addition, the properties of B and D mesons in the nuclear medium are closely related to their interactions with the nucleons [23–25], i.e. D^0+p or $n\to\Lambda_c^+$,Σ_c^+ or Σ_c^0, B^-+p or $n\to\Lambda_b^0$ or Σ_b^-.

From these processes, we can see that it is important to know the values of the related strong coupling constants $G_{\Sigma_b NB}$ and $G_{\Sigma_c ND}$, which are essential to determine the modifications of the masses, decay constants and other parameters of the B and D mesons in the nuclear medium. Up to now, only a few works on the strong coupling constants of the heavy baryons with the nucleon and heavy mesons have been reported [19–21, 26, 27].

The QCD sum rules (QCDSR) are one of the most powerful non-perturbative methods, and are also independent of model parameters. In recent years, numerous articles have reported the precise determination of the strong form factors and coupling constants via QCDSR [28–42, 45–47]. The strong coupling constants of $G_{\Sigma_b NB}$ and $G_{\Sigma_c ND}$ have been analyzed in Ref. [20], which was carried out by considering the $\not{q}\not{p}\gamma_5$ Dirac structure. Here, as a confirmation and verification, we also analyze these two vertices but with the Dirac structures $\not{p}\gamma_5$ and $\not{q}\gamma_5$. Besides, we also consider the $\langle qqG\rangle$ condensate term, which was not considered in Ref. [20].

The outline of this paper is as follows. In Section 2, we study the strong vertices $\Sigma_b NB$ and $\Sigma_c ND$ using the

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three-point QCDSR under two different Dirac structures $p\!\!\!/\gamma_5$ and $q\!\!\!/\gamma_5$. Besides the perturbative contribution, we also consider the contributions of $\langle\overline{q}q\rangle$, $\langle\overline{G}G\rangle$ and $\langle qqG\rangle$ on the OPE side. In Section 3, we present the numerical results and discussions, and our conclusions are given in Section 4.

2 QCD sum rules for Σ_bNB and Σ_cND

We can choose a two-point or three-point correlation function to analyze the strong coupling constants. The vertices Σ_bNB and Σ_cND can be analyzed by choosing light cone QCD sum rules (LCQSR) with a two-point correlator or QCD sum rules with a three-point correlation function. In this work, we choose the latter. The three-point correlation functions of the two vertices Σ_bNB and Σ_cND can be written as:

$$\Pi(p,p',q) = \mathrm{i}^2\int \mathrm{d}^4x\int \mathrm{d}^4y \mathrm{e}^{-\mathrm{i}p.x}\mathrm{e}^{\mathrm{i}p'.y} \times\left\langle 0|\tau(J_N(y)J_{B[D]}(0)\overline{J}_{\Sigma_b[\Sigma_c]}(x))|0\right\rangle, \quad (1)$$

where τ is the time-ordered product and $J_{\Sigma_b[\Sigma_c]}(x)$,$J_N(y)$ and $J_{B[D]}(0)$ are the interpolating currents of the hadrons $\Sigma_b[\Sigma_c]$, N and $B[D]$ respectively. The baryon current, which include several possibilities, is a composite operator with the same quantum numbers as a given baryon. To construct the baryon current, some criteria have to be adopted. For example, the current should include a minimal number of derivatives and maximize the projection onto the considered baryon state [48, 49]. For nucleon N as an example, it can be written as:

$$J_N(x)=\epsilon_{ijk}\left(u^{iT}(x)C\gamma_\mu u^j(x)\right)\gamma_5\gamma^\mu d^k(x)$$

or

$$J_N(x)=\epsilon_{ijk}\left(u^{iT}(x)C\sigma_{\mu\nu} u^j(x)\right)\gamma_5\sigma^{\mu\nu} d^k(x)$$

where C is the charge conjugation operator, and i, j and k are color indices. Other possible currents involve derivatives. Sometimes, we need to also consider linear combinations of the interpolating currents, with the coefficient suitably chosen in order to maximize the overlap. For simplicity, the interpolating currents for baryons $\Sigma_b[\Sigma_c]$, N, and meson B[D], are written as the following form:

$$J_{\Sigma_b[\Sigma_c]}(x) = \epsilon_{ijk}\left(u^{iT}(x)C\gamma_\mu d^j(x)\right)\gamma_5\gamma^\mu b[c]^k(x) \quad (2)$$

$$J_N(y) = \epsilon_{ijk}\left(u^{iT}(y)C\gamma_\mu u^j(y)\right)\gamma_5\gamma^\mu d^k(y) \quad (3)$$

$$J_{B[D]}(0) = \overline{u}(0)\gamma_5 b[c](0) \quad (4)$$

According to the QCD sum rules, the three-point correlation function can be calculated in two different ways. In the first way, the calculation is carried out in hadron degrees of freedom. This is called the phenomenological side. In the second way, called the operator product expansion (OPE) side, it is calculated in quark degrees of freedom. Then, invoking quark-hadron duality, we equate the phenomenological and OPE sides, from which the QCD sum rules for the strong coupling form factors are attained.

2.1 The phenomenological side

We insert a complete set of intermediate hadronic states with the same quantum numbers as the operators $J_{\Sigma_b[\Sigma_c]}(x)$,$J_N(y)$ and $J_{\mathrm{B[D]}}(0)$ into the correlation function Eq. (1) to obtain the phenomenological representations. The positive and negative parity states can also couple to the chosen currents [50]. In our calculations, we mainly take into account the positive parity nucleons. After isolating the ground-state contributions, the correlation function is written as:

$$\Pi^{HAD}(p,p',q)= \frac{\left\langle 0|J_N|N(p')\right\rangle\left\langle 0|J_{B[D]}|B[D](q)\right\rangle\left\langle \Sigma_b[\Sigma_c](p)|\overline{J}_{\Sigma_b[\Sigma_c]}|0\right\rangle}{(p^2-m^2_{\Sigma_b[\Sigma_c]})(p'^2-m^2_N)(q^2-m^2_{B[D]})} \times\left\langle N(p')B[D](q)|\Sigma_b[\Sigma_c](p)\right\rangle+\cdots \quad (5)$$

where $h.r.$ stands for the contributions of higher resonances and continuum states. The matrix elements appearing in the above equation can be parameterized as the following formulas:

$$\langle 0|J_N|N(p')\rangle= \lambda_N u_N(p',s') \quad (6)$$

$$\langle 0|J_{B[D]}|B[D](q)\rangle= \mathrm{i}\frac{m^2_{\mathrm{B[D]}}f_{B[D]}}{m_{\mathrm{u}}+m_{\mathrm{b[c]}}} \quad (7)$$

$$\langle \Sigma_b[\Sigma_c](p)|\overline{J}_{\Sigma_b[\Sigma_c]}|0\rangle= \lambda_{\Sigma_b[\Sigma_c]}\overline{u}_{\Sigma_b[\Sigma_c]}(p,s) \quad (8)$$

$$\langle N(p')B[D](q)|\Sigma_b[\Sigma_c](p)\rangle= G_{\Sigma_bNB[\Sigma_cND]}\overline{u}_N(p',s')i\gamma_5 u_{\Sigma_b[\Sigma_c]}(p,s), \quad (9)$$

where λ_N and $\lambda_{\Sigma_b[\Sigma_b]}$ are residues of the N and $\Sigma_b[\Sigma_b]$ baryons, $f_{B[D]}$ is the leptonic decay constant of the B[D] meson and $G_{\Sigma_bNB[\Sigma_cND]}$ is the strong coupling form factor of the vertices Σ_bNB and Σ_cND. Considering these parameters, Eq. (5) can be written as:

$$\Pi^{HAD}(p,p',q)= \mathrm{i}^2\frac{m^2_{\mathrm{B[D]}}f_{B[D]}}{m_{\mathrm{b[c]}}+m_{\mathrm{u}}}\frac{\lambda_N\lambda_{\Sigma_b[\Sigma_c]}g_{\Sigma_bNB[\Sigma_cND]}}{(p^2-m^2_{\Sigma_b[\Sigma_c]})(p'^2-m^2_{\mathrm{N}})(q^2-m^2_{\mathrm{B[D]}})}$$
$$\times\Big\{(m_{\mathrm{N}}m_{\Sigma_b[\Sigma_c]}-m^2_{\Sigma_b[\Sigma_c]})\gamma_5+(m_{\Sigma_b[\Sigma_c]}-m_{\mathrm{N}})\not{p}\gamma_5+\not{q}\not{p}\gamma_5-m_{\Sigma_b[\Sigma_c]}\not{q}\gamma_5\Big\}+\cdots \tag{10}$$

2.2 The OPE side

Now, we briefly outline the operator product expansion (OPE) for the three-point correlation Eq. (1). First, we contract the quark fields in the correlation with Wich's theorem.

$$\Pi(p,p',q)^{OPE} = \mathrm{i}^2\int\mathrm{d}^4x\int\mathrm{d}^4y\mathrm{e}^{-\mathrm{i}p.x}\mathrm{e}^{\mathrm{i}p'.y}\epsilon_{abc}\epsilon_{ijk}$$
$$\times\Big\{\gamma_5\gamma_\nu S_d^{cj}(y-x)\gamma_\mu CS_u^{biT}(y-x)C\gamma_\nu S_u^{ah}(y)\gamma_5S_{b[c]}^{hk}(-x)\gamma_\mu\gamma_5$$
$$-\gamma_5\gamma_\nu S_d^{cj}(y-x)\gamma_\mu CS_u^{aiT}(y-x)C\gamma_\nu S_u^{bh}(y)\gamma_5S_{b[c]}^{hk}(-x)\gamma_\mu\gamma_5\Big\}. \tag{11}$$

Second, we replace the heavy and light quark propagators with the following full propagators [46, 47, 51],

$$S^{mn}_{b[c]}(x) = \frac{\mathrm{i}}{(2\pi)^4}\int\mathrm{d}^4k\mathrm{e}^{-\mathrm{i}k.x}\Big\{\frac{\delta_{mn}}{\not{k}-m_{\mathrm{b[c]}}}-\frac{g_sG^{\alpha\beta}_{mn}}{4}\frac{\sigma_{\alpha\beta}(\not{k}+m_{\mathrm{b[c]}})+(\not{k}+m_{\mathrm{b[c]}})\sigma_{\alpha\beta}}{(k^2-m^2_{\mathrm{Q}})^2}$$
$$+\frac{\pi^2}{3}\Big\langle\frac{\alpha_sGG}{\pi}\Big\rangle\delta_{mn}m_{\mathrm{b[c]}}\frac{k^2+m_{\mathrm{b[c]}}\not{k}}{(k^2-m^2_{\mathrm{b[c]}})^4}+\cdots\Big\}, \tag{12}$$

$$S^{mn}_{u[d]}(x) = \mathrm{i}\frac{\not{x}}{2\pi^2x^4}\delta_{mn}-\frac{m_{\mathrm{u[d]}}}{4\pi^2x^2}\delta_{mn}-\frac{\langle\overline{q}q\rangle}{12}\Big(1-\mathrm{i}\frac{m_{\mathrm{u[d]}}}{4}\not{x}\Big)-\frac{x^2}{192}m_0^2\langle\overline{q}q\rangle\Big(1-\mathrm{i}\frac{m_{\mathrm{u[d]}}}{6}\not{x}\Big)$$
$$-\frac{\mathrm{i}g_s\lambda^{ij}_AG^A_{\theta\eta}}{32\pi^2x^2}\Big[\not{x}\sigma^{\theta\eta}+\sigma^{\theta\eta}\not{x}\Big]+\cdots \tag{13}$$

where m,n are the color indices, and $\langle q\overline{q}\rangle$ is the $\langle u\overline{u}\rangle$ and $\langle d\overline{d}\rangle$ in Eq. (13). After these above substitutions in Eq. (11), we carry out Fourier transformation in D=4+ϵ dimensions with $\epsilon\rightarrow0$:

$$\frac{1}{[(y-x)^2]^n} = \int\frac{\mathrm{d}^Dt}{(2\pi)^D}\mathrm{e}^{-\mathrm{i}t.(y-x)}\mathrm{i}(-1)^{n+1}2^{D-2n}\pi^{D/2}\times\frac{\Gamma(D/2-n)}{\Gamma(n)}\Big(-\frac{1}{t^2}\Big)^{D/2-n} \tag{14}$$

$$\frac{1}{[y^2]^n} = \int\frac{\mathrm{d}^Dt'}{(2\pi)^D}\mathrm{e}^{-\mathrm{i}t'.y}\mathrm{i}(-1)^{n+1}2^{D-2n}\pi^{D/2}\times\frac{\Gamma(D/2-n)}{\Gamma(n)}\Big(-\frac{1}{t'^2}\Big)^{D/2-n}. \tag{15}$$

Before the preformation of four-x and four-y integrals, the replacements $x_\mu\rightarrow\mathrm{i}\frac{\partial}{\partial p_\mu}$ and $y_\mu\rightarrow-\mathrm{i}\frac{\partial}{\partial p'_\mu}$ are carried out. After these processes, the integrals turn into Dirac delta functions which are used to simplify the four-integrals over k and t'. The following step is to perform the Feynman parametrization, after which the following function is used to carried out the remaining four-integral over t:

$$\int\frac{\mathrm{d}^Dt}{(2\pi)^D}\frac{1}{[t-M^2]^\alpha} = \frac{\mathrm{i}(-1)^\alpha}{(4\pi)^{D/2}}\frac{\Gamma(\alpha-D/2)}{\Gamma(\alpha)}\times\frac{1}{(M^2)^{\alpha-D/2}},$$

where M^2=$m^2_{\mathrm{b[c]}}x+p^2x(x+y-1)+p'^2y(x+y-1)-q^2xy$.

After further simplification, the three-point correlation on the OPE side shows the following Dirac structures:

$$\Pi^{OPE}(p,p',q) = \Pi_1(q^2)\gamma_5+\Pi_2(q^2)\not{p}\gamma_5+\Pi_3(q^2)\not{q}\not{p}\gamma_5+\Pi_4(q^2)\not{q}\gamma_5, \tag{16}$$

where each Π_i denotes contributions coming from the perturbative and nonperturbative parts. In general, we expect that we can choose either Dirac structure Π_i (with i =1,2,3,4) of the correlations $\Pi(p,p',q)$ to study the hadronic coupling constants. In our calculations, p/γ_5 and q/γ_5 are the pertinent Dirac structures.

After taking the imaginary parts of Π_i, we get the spectral densities $\rho_i(s,s',Q^2)$ of the corresponding Dirac structure. Using dispersion relations, each Π_i can be written as:

$$\Pi_i^{\mathrm{OPE}}(Q^2) = \int\mathrm{d}s\int\mathrm{d}s'\frac{\rho_i^{\mathrm{pert}}(s,s',Q^2)+\rho_i^{\mathrm{non-pert}}(s,s',Q^2)}{(s-p^2)(s'-p'^2)}, \tag{17}$$

where $s=p^2$, $s'=p'^2$ and $Q^2=-q^2$. As examples, we give the perturbative and nonperturbative parts of the spectral densities for the two Dirac structures $\not{p}\gamma_5$ and $\not{q}\gamma_5$:

$$\begin{aligned}\rho^{\rm pert}_{\not p\gamma_5}(s,s',Q^2) =& \int_0^1{\rm d}x\int_0^{1-x}{\rm d}y\frac{-1}{32\pi^4(x+y-1)}\Big\{-\Big[2m_{\rm b[c]}(x+y)+m_{\rm d}(3x+3y-1)-m_{\rm u}(x+y-1)\Big]\\&\times\Big[x(m^2_{\rm b[c]}+Q^2y)+sx(x+y-1)+s'y(x+y-1)\Big]+s(x+y)\Big[m_{\rm b[c]}x(2x+2y-1)\\&+3m_{\rm d}(x-1)(x+y-1)-m_{\rm u}(x^2+x(y-4)-2y+3)\Big]+s'(x+y)\Big[m_{\rm b[c]}\Big(x(2y-1)\\&+2(y-1)y\Big)+y\Big(3m_{\rm d}(x+y-1)-m_{\rm u}(x+y-2)\Big)\Big]+9m_{\rm b[c]}m_{\rm d}m_{\rm u}x+9m_{\rm b[c]}m_{\rm d}m_{\rm u}y\\&-6m_{\rm b[c]}m_{\rm d}m_{\rm u}-3m_{\rm b[c]}m^2_{\rm u}x-3m_{\rm b[c]}m^2_{\rm u}y+2m_{\rm b[c]}Q^2x^2y-m_{\rm b[c]}Q^2x^2+2m_{\rm b[c]}Q^2xy^2\\&-m_{\rm b[c]}Q^2xy-6m_{\rm d}m^2_{\rm u}x-6m_{\rm d}m^2_{\rm u}y+6m_{\rm d}m^2_{\rm u}+3m_{\rm d}Q^2x^2y+3m_{\rm d}Q^2xy^2-3m_{\rm d}Q^2xy\\&-3m_{\rm d}Q^2y^2+3m_{\rm d}Q^2y-m_{\rm u}Q^2x^2y-m_{\rm u}Q^2xy^2+2m_{\rm u}Q^2xy+2m_{\rm u}Q^2y^2-3m_{\rm u}Q^2y\Big\}\\&\times\Theta[H_2(s,s',Q^2)],\end{aligned}$$

$$\begin{aligned}\rho^{\rm pert}_{\not q\gamma_5}(s,s',Q^2) =&\int_0^1{\rm d}x\int_0^{1-x}{\rm d}y\frac{1}{32\pi^4(x+y-1)^2}\Big\{(x+y-1)\Big[s\Big(m_{\rm b[c]}xy(2x+2y-1)+3m_{\rm d}(x^2(y-1)\\&+x(y^2-3y+1)-(y-1)y)-m_{\rm u}(x^2(y-3)+x(y^2-7y+3)+(3-2y)y)\Big)\\&+s'y\Big(m_{\rm b[c]}(x(2y-1)+2(y-1)y)+3m_{\rm d}(y-1)(x+y-1)-m_{\rm u}(x(y-3)+y^2-5y+3)\Big)\\&+9m_{\rm b[c]}m_{\rm d}m_{\rm u}y-6m_{\rm b[c]}m_{\rm d}m_{\rm u}-3m_{\rm b[c]}m^2_{\rm u}y+2m_{\rm b[c]}Q^2xy^2-m_{\rm b[c]}Q^2xy-6m_{\rm d}m^2_{\rm u}y+3m_{\rm d}Q^2xy^2\\&-3m_{\rm d}Q^2xy-3m_{\rm d}Q^2y^2+3m_{\rm d}Q^2y-m_{\rm u}Q^2xy^2+3m_{\rm u}Q^2xy+2m_{\rm u}Q^2y^2-3m_{\rm u}Q^2y\Big]\\&-\Big[m_{\rm b[c]}(x(6y-1)+6(y-1)y)+3m_{\rm d}(3y-2)(x+y-1)+m_{\rm u}(-3x(y-2)-3y^2+10y-6)\Big]\\&\times\Big[x(m^2_{\rm b[c]}+Q^2y)+sx(x+y-1)+s'y(x+y-1)\Big]\Big\}\Theta[H_2(s,s',Q^2)],\end{aligned}$$

$$\begin{aligned}\rho^{\rm non-pert}_{\not p\gamma_5}(s,s',Q^2) =&\int_0^1{\rm d}x\int_0^{1-x}{\rm d}y\frac{3(\langle u\overline{u}\rangle-\langle d\overline{d}\rangle)}{12\pi^2}\Big(3x+3y-1\Big)\Theta[H_2(s,s',Q^2)]\\&-\frac{\langle u\overline{u}\rangle}{48\pi^2(m^2_{\rm b[c]}+Q^2)^2}\Big\{6m^3_{\rm b[c]}m_{\rm d}-12m^3_{\rm b[c]}m_{\rm u}-6m^2_{\rm b[c]}m_{\rm d}m_{\rm u}+6m^2_{\rm b[c]}m^2_{\rm u}\\&+s'\Big[2m^2_{\rm b[c]}-m_{\rm b[c]}m_{\rm u}+2Q^2\Big]-2m^2_{\rm b[c]}Q^2+6m_{\rm b[c]}m_{\rm d}Q^2+s\Big[m_{\rm b[c]}(m_{\rm u}-2m_{\rm b[c]})-2Q^2\Big]\\&-11m_{\rm b[c]}m_{\rm u}Q^2-3m_{\rm d}m_{\rm u}Q^2+3m^2_{\rm u}Q^2-2Q^4\Big\}\Theta[H_1[s,s',Q^2]]\\&-\int_0^1{\rm d}x\int_0^{1-x}{\rm d}y\langle\alpha_s\frac{G^2}{\pi}\rangle\frac{x^3(3x+3y-2)m_{\rm b[c]}}{16\pi^2(x+y-1)}\delta[H_2[s,s',Q^2]]\\&+\frac{\langle qqg\rangle}{8\times4\pi^2\times9(m^2_{\rm b[c]}+Q^2)^4}\Big\{18m^6_{\rm b[c]}-18m^5_{\rm b[c]}m_{\rm d}-36m^5_{\rm b[c]}m_{\rm u}-18m^4_{\rm b[c]}m_{\rm d}m_{\rm u}+18m^4_{\rm b[c]}m^2_{\rm u}\\&+39m^4_{\rm b[c]}Q^2-36m^3_{\rm b[c]}m_{\rm d}Q^2-32m^3_{\rm b[c]}m_{\rm u}Q^2+30m^2_{\rm b[c]}Q^4-3s\Big[m^3_{\rm b[c]}(3m_{\rm b[c]}-2m_{\rm u})\\&+4m^2_{\rm b[c]}Q^2+Q^4\Big]+3s'\Big[3m^4_{\rm b[c]}-2m^3_{\rm b[c]}m_{\rm u}+4m^2_{\rm b[c]}Q^2+Q^4\Big]-18m_{\rm b[c]}m_{\rm d}Q^4\\&-2m_{\rm b[c]}m_{\rm u}Q^4+9Q^6\Big\}\Theta[H_1[s,s',Q^2]],\end{aligned}$$

$$\begin{aligned}\rho^{\rm non-pert}_{\not q\gamma_5}(s,s',Q^2) =&\int_0^1 {\rm d}x\int_0^{1-x}{\rm d}y\frac{3}{16\times 12\pi^2}\Big\{(48y-32)\langle d\overline{d}\rangle+(32-48y)\langle u\overline{u}\rangle\Big\}\Theta[H_2(s,s',Q^2)]\\
&-\frac{\langle u\overline{u}\rangle}{48\pi^2(m^2_{\rm b[c]}+Q^2)^2}\Big\{-3m_{\rm b[c]}\Big[2m^2_{\rm b[c]}(m_{\rm d}-2m_{\rm u})+m_{\rm b[c]}m_{\rm u}(m_{\rm u}-5m_{\rm d})+2m_{\rm d}m^2_{\rm u}\Big]\\
&+Q^2\Big[2m^2_{\rm b[c]}-6m_{\rm b[c]}m_{\rm d}+11m_{\rm b[c]}m_{\rm u}+12m_{\rm d}m_{\rm u}\Big]+s\Big[m_{\rm b[c]}(2m_{\rm b[c]}-m_{\rm u})+2Q^2\Big]-2Q^4\Big\}\\
&\times\Theta[H_1[s,s',Q^2]]+\int_0^1{\rm d}x\int_0^{1-x}{\rm d}y\langle\alpha_s\frac{G^2}{\pi}\rangle\frac{x^3(18xy-x+18y^2-18y+2)m_{\rm b[c]}}{96\pi^2(x+y-1)^2}\delta[H_2[s,s',Q^2]]\\
&-\frac{\langle qqg\rangle}{8\times 4\pi^2\times 9(m^2_{\rm b[c]}+Q^2)^4}\Big\{27m^6_{\rm b[c]}-54m^5_{\rm b[c]}m_{\rm d}-54m^5_{\rm b[c]}m_{\rm u}-72m^4_{\rm b[c]}m_{\rm d}m_{\rm u}+18m^4_{\rm b[c]}m^2_{\rm u}\\
&-60m^4_{\rm b[c]}Q^2+36m^3_{\rm b[c]}m_{\rm d}m^2_{\rm u}+s\Big[m^3_{\rm b[c]}(-15m_{\rm b[c]}+36m_{\rm d}+10m_{\rm u})\\
&-4m_{\rm b[c]}Q^2(6m_{\rm b[c]}-9m_{\rm d}-m_{\rm u})-9Q^4\Big]+2s'\Big[m^3_{\rm b[c]}(3m_{\rm b[c]}-72m_{\rm d}-2m_{\rm u})\\
&-2m_{\rm b[c]}Q^2(-3m_{\rm b[c]}+9m_{\rm d}+m_{\rm u})+3Q^4\Big]-72m^3_{\rm b[c]}m_{\rm d}Q^2-64m^3_{\rm b[c]}m_{\rm u}Q^2\\
&-72m^2_{\rm b[c]}m_{\rm d}m_{\rm u}Q^2+45m^2_{\rm b[c]}Q^4-18m_{\rm b[c]}m_{\rm d}Q^4-16m_{\rm b[c]}m_{\rm u}Q^4-18m_{\rm d}m_{\rm u}Q^4\\
&+12Q^6\Big\}\Theta[H_1[s,s',Q^2]].\end{aligned}$$

where δ represents the Delta function, Θ denotes the unit-step function, and $H_1[s,s',Q^2]$, $H_2[s,s',Q^2]$ are defined as:

$$H_1[s,s',Q^2]=s' \tag{18}$$

$$H_2[s,s',Q^2]=x(m^2_{\rm b[c]}+Q^2y)+sx(x+y-1)+s'y(x+y-1). \tag{19}$$

2.3 The strong coupling constant

We perform a double Borel transformation [49] on the phenomenological as well as the OPE side. Then, we equate these two sides, invoking the quark-hadron duality from which the sum rule is obtained. As an example, the form factors for the structure $\not p\gamma_5$ are:

$$\begin{aligned}G^{\not p\gamma_5}_{\Sigma_b NB[\Sigma_c ND]}(Q^2) =& {\rm e}^{\frac{m^2_{\Sigma_b[\Sigma_c]}}{M1^2}}{\rm e}^{\frac{m^2_N}{M2^2}}\frac{(m_{\rm b[c]}+m_{\rm u})(Q^2+m^2_{B[D]})}{m^2_{B[D]}f_{B[D]}\lambda_{\Sigma_b[\Sigma_c]}\lambda_N(m_{\Sigma_b[\Sigma_c]}-m_N)}\\
&\times\Big\{\int_{(m_{\rm b[c]}+m_{\rm u}+m_{\rm d})^2}^{s_0}{\rm d}s\int_{(2m_{\rm u}+m_{\rm d})^2}^{u_0}{\rm d}s'{\rm e}^{-\frac{s}{M1^2}}{\rm e}^{-\frac{s'}{M1^2}}\Big[\rho^{pert}_{\not p\gamma_5}(s,s',Q^2)+\rho^{non-pert}_{\not p\gamma_5}(s,s',Q^2)\Big]\Big\},\end{aligned} \tag{20}$$

where M_1 and M_2 are the Borel parameters, and s_0 and u_0 are two continuum threshold parameters which are introduced to eliminate the *h.r.* terms. These parameters fulfill the following relations: $m_i^2<s_0<m_i'^2$ and $m_o^2<u_0<m_o'^2$, where m_i and m_o are the masses of the incoming and outgoing hadrons respectively and m' is the mass of the first excited state of these hadrons.

3 Results and discussion

This section gives the numerical analysis of the sum rules for the coupling constants. The decay constant parameters used in this work are taken as: f_B =$(248\pm 23_{exp}\pm 25_{Vub})$ MeV [52], f_D =$(205.8\pm 8.5\pm 2.5)$ MeV [53], λ_N^2 =(0.0011 ± 0.0005)GeV^6 [54], λ_{Σ_b} =(0.062 ± 0.018)GeV^3 [55], and λ_{Σ_c} =(0.045 ± 0.015)GeV^3 [55]. We take the masses of the hadrons from Ref. [56], where m_B =(5279.26 ± 0.17) MeV, m_D =(1864.84 ± 0.07) MeV, m_N =(938.272046 ± 0.000021) MeV, m_{Σ_b}=(5811.3 ± 1.9) MeV, m_{Σ_c}=(2452.9 ± 0.4) MeV and for the quarks, m_b =(4.18 ± 0.03) GeV, m_c =(1.275 ± 0.025) GeV, $m_{\rm d}$ =$(4.8^{+0.5}_{-0.3})$ MeV, and $m_{\rm u}$=$(2.3^{+0.7}_{-0.5})$ MeV. The vacuum condensates are taken to be the standard values $\langle\overline{u}u\rangle=\langle\overline{d}d\rangle=-(0.8\pm 0.1)\times(0.24\pm 0.01\text{GeV})^3$ [57], $\langle\overline{s}g_s\sigma Gs\rangle=m_0^2\langle\overline{s}s\rangle$ [57], $m_0^2=(0.8\pm 0.1)\text{GeV}^2$, $\langle g_s^2 GG\rangle=(0.022\pm 0.004)\text{GeV}^4$ [58]. From Eq. (20), we also know that the value of the form factor $G_{\Sigma_b NB[\Sigma_c ND]}$ is a function of the input parameters, including the Borel parameters M_1^2 and M_2^2, the continuum threshold s_0 and u_0, and the momentum Q^2.

The working regions for M_1^2 and M_2^2 are determined by requiring not only that the contributions of the higher states and continuum be effectively suppressed, but also that the contributions of the higher-dimensional operators are small. In other words, we should find a good plateau which will ensure OPE convergence and the stability of our results [49]. The plateau is often called the "Borel window". Considering these factors, the Borel windows are chosen as $7(3)\mathrm{GeV}^2 \leqslant M_1^2 < 14(7)\mathrm{GeV}^2$ and $3(2)\mathrm{GeV}^2 \leqslant M_2^2 < 7(6)\mathrm{GeV}^2$ for the strong vertex $\Sigma_b NB(\Sigma_c ND)$ (see Figs. 1–4). From these figures, the values are rather stable with variations of the Borel parameters, so it is reliable to extract the form factors. In addition, the continuum parameters $s_0=(m_i+\triangle_i)^2$ and $u_0=(m_o+\triangle_o)^2$ are employed to include the pole and to suppress the *h.r.* contributions. The values for $\triangle_i$ and $\triangle_o$ cannot be far from the experimental value of the distance between the pole and the first excited state [49]. In general, these two continuum thresholds s_0 and u_0 are determined by the relations $s_0\sim(m_i+0.5\mathrm{GeV})^2$ and $u_0\sim(m_o+0.5\mathrm{GeV})^2$. According to these considerations, we take $s_0=37.4(7.6)\mathrm{GeV}^2$ and $u_0=1.99(1.99)\mathrm{GeV}^2$ for the strong vertex $\Sigma_b NB(\Sigma_c ND)$.

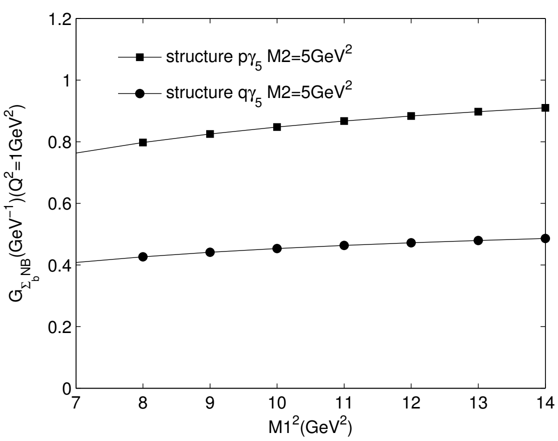

Fig. 1. $G_{\Sigma_b NB}$ as a function of M_1^2 at average values of the continuum thresholds.

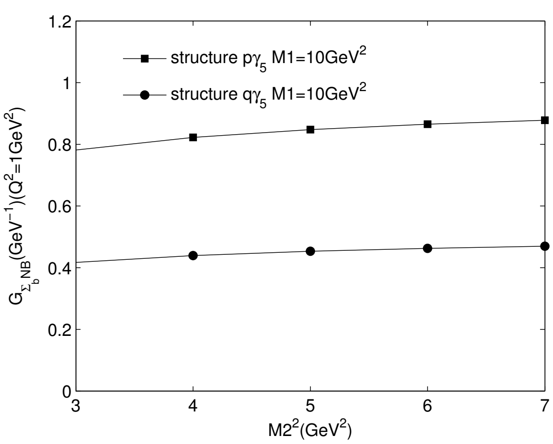

Fig. 2. $G_{\Sigma_b NB}$ as a function of M_2^2 at average values of the continuum thresholds.

However, in order to obtain the coupling constants, it is necessary to extrapolate these results into physical regions ($Q^2<0$), which is realized by fitting the form factors into suitable analytical functions. It is indicated that we should get the same values for the coupling constants for the different Dirac structures p/γ_5 or q/γ_5 when we take $Q^2=-m_{B[D]}^2$. This above procedure can help us minimize the uncertainties in the calculation of the coupling constant, which will be quite clear in the following section. Actually, there is no fixed expression for the fitting function of the form factors in the framework of QCD sum rules. In many cases, it is found that the form factors can be appropriately fitted into a combination of exponential function and power function. After some effort, we observe that the dependence of the form factors on Q^2 can be well fitted into the following two analytical functions for $\Sigma_b NB$ and $\Sigma_c ND$ respectively (see Fig. 5 and Fig. 6):

$$G_{\Sigma_b NB}(Q^2)=C_1 exp^{-\frac{Q^2}{C_2}}+C_3 exp^{-\frac{Q^2}{C_4}} \tag{21}$$

$$G_{\Sigma_c ND}(Q^2)=C_5 exp^{-\frac{Q^2}{C_6}}+C_7 \tag{22}$$

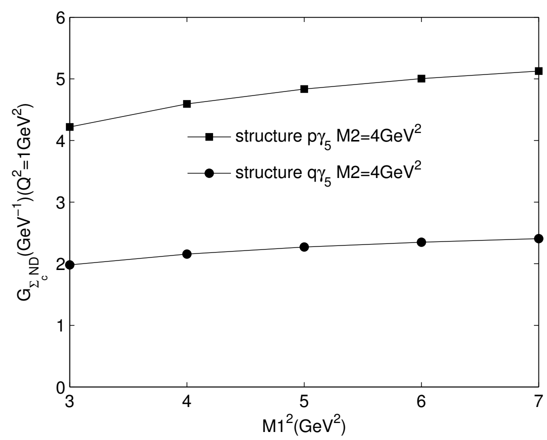

Fig. 3. $G_{\Sigma_c ND}$ as a function of M_1^2 at average values of the continuum thresholds.

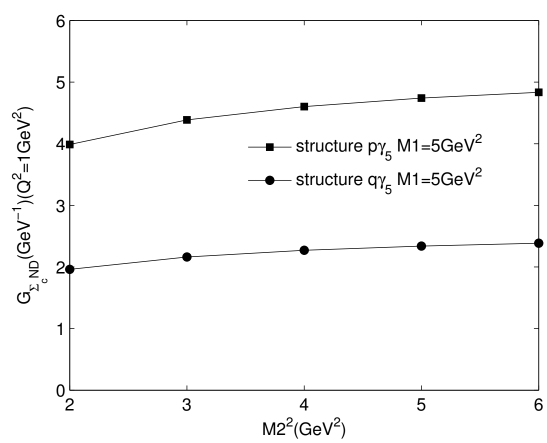

Fig. 4. $G_{\Sigma_c ND}$ as a function of M_2^2 at average values of the continuum thresholds.

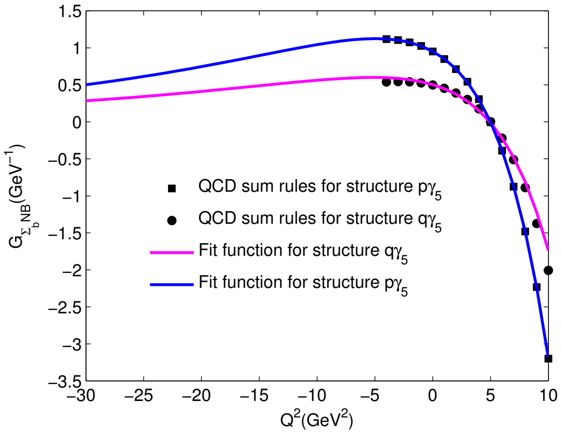

Fig. 5. $G_{\Sigma_b NB}$ as a function of Q^2 at average values of the continuum thresholds and Borel mass parameters.

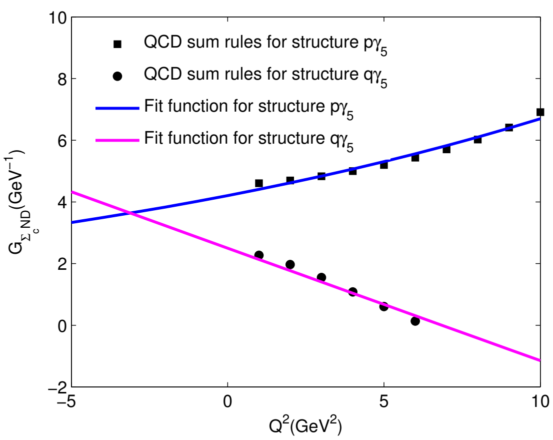

Fig. 6. $G_{\Sigma_c ND}$ as a function of Q^2 at average values of the continuum thresholds and Borel mass parameters.

The values of C_i for different Dirac structures are presented in Table 1 for the two strong vertices $\Sigma_b NB$ and $\Sigma_c ND$. The fit function is used to determine the value of the strong coupling constant at $Q^2\!=\!-m_{B[D]}^2$ for different structures, and the results are also presented in Table 1. The errors in these results mainly arise from the uncertainties of the fitting parameters such as δC_1, δC_2, δC_3 etc. In our calculations, we take the central values of all the input parameters such as the masses of the quark and hadrons, the decay constants, etc. If the uncertainties of these input parameters are considered, one would expect rather large errors at the level of at least 10%.

It is indicated from Fig. 5 and Table 1 that we obtain compatible results for the strong coupling constant $g_{\Sigma_c ND}$ from different Dirac structures when we take $Q^2\!=\!-m_D^2$ in the fitting function Eq. 22. The results of the coupling constants for vertex $\Sigma_c ND$ are 3.58 and 3.94 for the p/γ_5 and q/γ_5 structure respectively. The average value is 3.76±0.05, which is consistent with Azizi's result in Ref. [20]. For the $\Sigma_b NB$ vertex, the results from different Dirac structures are not in good agreement with each other, the values being 0.55 and 0.31 respectively. Comparing these values with Azizi's [20], 12.96, our results are much smaller. The main difference between our calculations and those of Ref. [20] lies in the different choices of the Dirac structure. In Ref. [20], they selected the $\not q\not p\gamma_5$ structure to analyze the strong coupling constants. In our analysis, we choose $\not q\gamma_5$ and $\not p\gamma_5$. Besides, we also consider the contribution from $\langle qqG\rangle$. Our previous work indicated that this condensate contribution should not lead to so much difference in the final results [59]. Thus, this value for the strong coupling constant $\Sigma_b NB$ needs to be further analyzed by other theoretical methods such as the light-cone QCD sum rules (LCQSR) and lattice QCD. At present, we temporarily take the average values of these two different structures as the final results, giving 0.43±0.01 and 3.76±0.05 for $\Sigma_b NB$ and $\Sigma_c ND$ respectively.

4 Conclusion

In this article, we have calculated the form factors of the vertices $\Sigma_b NB$ and $\Sigma_c ND$ in space-like regions by three-point sum rules. We then fit the form factors into analytical functions, extrapolated them into the time-like regions, and obtained the strong coupling constants $G_{\Sigma_b NB}$ and $G_{\Sigma_c ND}$. These calculated results can be used to analyze related experimental results at the LHC as well as heavy ion collision experiments like $\overline{P}ANDA$ at FAIR.

Table 1. Parameters appearing in the fit function of the coupling form factor for $\Sigma_b NB$ and $\Sigma_c ND$.

	structure	C_1/GeV^{-1}	C_2/GeV^2	C_3/GeV^{-1}	C_4/GeV^2	$G_{\Sigma_b NB}$
$\Sigma_b NB$	$\not p\gamma_5$	−0.40±0.05	−4.89±0.02	0.90±0.05	−26.13±4.00	0.55±0.01
	$\not q\gamma_5$	−0.78±0.10	−4.99±0.02	1.74±0.10	−24.22±4.00	0.31±0.01
$\Sigma_c NB$	structure	C_1/GeV^{-1}	C_2/GeV^2	C_3/GeV^{-1}	C_4/GeV^2	$G_{\Sigma_c NB}$
	$\not p\gamma_5$	4.21±0.20	−21.51±3.00	0		3.58±0.02
	$\not q\gamma_5$	663.20±63.50	1728.30±25.60	−660.60±68.70		3.94±0.04

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