Looking for new physics via semi-leptonic and leptonic rare decays of D and D_s *

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Abstract: It is well recognized that looking for new physics at lower energy colliders is a path which is complementary to high energy machines such as the LHC. Using the large volume of data collected by BESIII, we may have a unique opportunity to tackle this. In this paper we calculate the branching ratios of the semi-leptonic processes $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$, and the leptonic processes $D^0 \to e^-e^+$ and $D^0 \to e^-\mu^+$, in the frameworks of the $U(1)'$ model, 2HDM and unparticle model. It is found that both the $U(1)'$ model and 2HDM may influence the semi-leptonic decay rates, but only the $U(1)'$ model offers substantial contributions to the pure leptonic decays, and the resultant branching ratio of $D^0 \to e^-\mu^+$ can be as large as $10^{-7} \sim 10^{-8}$. This might be observed at the future super τ-charm factory.

Keywords: rare decay, D meson, LFV

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1 Introduction

One of tasks of high-intensity but lower-energy colliders is to find traces of new physics beyond the Standard Model (SM) through measuring rare decays with high accuracy, looking for deviations of the measured values from the SM predictions. Generally, it is believed that the new physics scale may exist at several hundreds of GeV to a few TeV, whereas for lower energies the contributions from new physics might be drowned out in the SM background. However, in some rare decays, contributions from the SM are highly suppressed or even forbidden, so new physics beyond the SM (BSM) might emerge and play a leading role. If such processes are observed in high precision experiments, a trace of BSM could be pinned down. Concretely, processes where flavor-changing neutral currents (FCNCs) are involved are the goal of our studies. Even though such results may not determine the kind of new physics, they may offer valuable information about new physics to the high energy colliders such as the LHC. In the SM, FCNCs and lepton flavor violation (LFV) processes can only occur via loop diagrams, so are suppressed. Thus, study of FCNC/LFV transitions would compose a key for BSM searches.

The rare decays of D and B mesons provide a favorable area because they are produced at e^+e^- colliders, where the background is much cleaner than that at hadron colliders. The newest measurements set upper bounds for the branching ratios of $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ as 3.7×10^{-6} and 9.7×10^{-6} respectively [1], and the upper bounds for $D^0 \to e^-e^+$ and $D^0 \to e^-\mu^+$ are 7.9×10^{-8} and 2.6×10^{-7} [1]. Theoretically, those decay processes receive contributions from both short and long distance effects of SM [2]. Especially, for $D_s^+ \to K^+e^-e^+$, its rate is mainly determined by the long distance effect

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and the SM predicted value is 1.8×10^{-6}, which is higher than the short distance contribution (2×10^{-8} [2]) by two orders of magnitude. For the other processes concerned, the contributions from the SM are so small that they can be neglected.

As indicated, at lower energy experiments, one may be able to see traces of new physics, but will not be able to determine what it is. Thus, in collaboration, theorists can offer possible scheme(s) to experimentalists and help them to extract information from the data. This is the main idea of this work.

There are many new physics models (BSM) constructed by numerous theorists. These include the fourth generation [3], the non-universal Z' boson [4–8], the two Higgs doublet model (2HDM) [9–11] and the unparticle model [12–14], etc. In the frameworks of these models, FCNC/LFV processes occur at tree level. Thus, if such rare decays involving FCNC/LFV processes are experimentally observed, one may claim the existence of BSM. Then, comparing the values predicted by different models with the data, one would gain a hint about the type of BSM, which is valuable for high energy colliders.

In Refs. [15, 16], based on several BSM models, the authors derived the formulae and evaluated the decay rates of semi-leptonic and leptonic decays of D mesons while the model parameters were constrained mainly by the data of $D^0-\bar{D}^0$ mixing. Their result was pessimistic, finding that these decay rates cannot provide any trace of the models examined. In this work we choose three new physics models: the $U(1)'$ model, 2HDM type III, and unparticle, but relax the constraint from $D^0-\bar{D}^0$ mixing by supposing there were some unknown reasons to suppress the rate if the present measurements are sufficiently accurate. Instead we consider the constraints obtained by fitting the experimental data for $\tau\to3l$ [1, 8]. Then we calculate the branching ratios of $D_s^+\to K^+e^-e^+$, $D_s^+\to K^+e^-\mu^+$, $D^0\to e^-e^+$ and $D^0\to e^-\mu^+$ in the frameworks of these models. Our numerical results show that only Z', which is from a broken extra $U'(1)$ gauge symmetry, and 2HDM of type III can result in substantial enhancement to the branching ratios of $D_s^+\to K^+e^-e^+$ and $D_s^+\to K^+e^-\mu^+$ up to $10^{-6}\sim10^{-7}$. These results will be tested at the BESIII experiment in future. Indeed, we lay our hope on the huge volume of data collected at BESIII, without which we cannot go any further in our search for new physics.

In this work, we also try to set schemes for analyzing the data on those decays based on the BESIII data and extract information about new physics BSM.

This paper is organized as follows. In Sections 2 and 3, we first briefly review the SM results for the semi-leptonic and pure leptonic rare decays and then derive corresponding contributions induced by new physics models: extra $U(1)'$, 2HDM of type III, and unparticle. In fact, some of these have previously been deduced by other authors and here we only probe their formulation. We add those which were not derived before. We obtain the corresponding Feynman amplitudes and decay widths for $D_s^+\to K^+e^-e^+$, $D_s^+\to K^+e^-\mu^+$, $D^0\to e^-e^+$ and $D^0\to e^-\mu^+$. In Section 4, we present our numerical results along with the constraints on the model parameters obtained by fitting previous experimental data, except for $D^0-\bar{D}^0$ mixing. In Section 5, we set an experimental scheme for analyzing the data which will be collected by the BESIII Collaboration in the near future. In the last section, we present a brief discussion and draw our conclusions.

2 D_s^+ semi-leptonic decay

For the decay processes $D_s^+\to K^+e^-e^+$ and $D_s^+\to K^+e^-\mu^+$, the contributions of SM to these FCNC processes are realized via electroweak penguin diagrams and are suppressed. However, besides the short-distance effects, there exists a long-distance contribution which is larger. Moreover, because of the smallness of the direct SM process, any new physics model whose Hamiltonian includes FCNC interactions may induce semi-leptonic and leptonic decays of D_s^+ and D^0 at tree level. In this section we only explore three possible models: the $U(1)'$ model, 2HDM of type III, and the unparticle model. Since those models have been studied by many authors from various aspects, here we only give a brief review.

2.1 The SM contribution

The authors of Refs. [2, 16, 17] gave the amplitudes for $D_s^+\to K^+e^-e^+$, so here we only list the formulas for readers' convenience. The Feynman amplitude of the decay $D_s^+\to K^+e^-e^+$ in the SM framework is

$$\begin{aligned}\mathcal{M}_{\rm SM} = & \frac{4G_{\rm F}}{\sqrt{2}}[C_7\langle e^+e^-|eA^\delta\bar{l}\gamma_\delta l|\gamma\rangle\frac{1}{q^2}\langle\gamma K^+|O_7|D_s^+\rangle \\ & +C_9\langle e^+e^-K^+|O_9|D_s^+\rangle],\end{aligned} \tag{1}$$

where

$$\begin{aligned}O_7 &= \frac{e}{16\pi^2}m_c(\bar{u}_{\rm L}\sigma^{\alpha\beta}c_{\rm R})F_{\alpha\beta} \\ O_9 &= \frac{e^2}{16\pi^2}(\bar{u}_{\rm L}\gamma^\alpha c_{\rm L})\bar{l}\gamma_\alpha l\end{aligned} \tag{2}$$

and $G_{\rm F}=1.17\times10^{-5}{\rm GeV}^{-2}$ is the Fermi constant. After some simple manipulations, one has

$$\begin{aligned}&\langle K^+(p')|\bar{u}\gamma_\alpha(1\pm\gamma_5)c|D_s^+(p)\rangle \\ &=f_+(q^2)[(p+p')_\alpha-\frac{m_{\rm D_s}^2-m_{\rm K}^2}{q^2}q_\alpha]+f_0(q^2)\frac{m_{\rm D_s}^2-m_{\rm K}^2}{q^2}q_\alpha, \\ &\langle K^+(p')|\bar{u}\sigma^{\alpha\beta}(1\pm\gamma_5)c|D_s^+(p)\rangle \\ &=\frac{f_T(q^2)}{m_{\rm D_s}}[(p+p')^\alpha q^\beta-(p+p')^\beta q^\alpha\pm{\rm i}\epsilon^{\alpha\beta\rho\sigma}(p+p')_\rho q_\sigma],\end{aligned} \tag{3}$$

and $\mathcal{M}_{\rm SM}$ is transformed into

$$\mathcal{M}_{\rm SM} = \frac{4G_{\rm F}}{\sqrt{2}}\frac{e^2m_{\rm c}}{16\pi^2}C_7\frac{\bar{u}(p_2)(\gamma_\beta q_\alpha-\gamma_\alpha q_\beta)v(p_1)}{2q^2}\frac{f_T(q^2)}{m_{\rm D_s}}\left[(p+p')^\alpha q^\beta-(p+p')^\beta q^\alpha+{\rm i}\epsilon^{\alpha\beta\rho\sigma}(p+p')_\rho q_\sigma\right]$$
$$+\frac{e^2}{32\pi^2}C_9\bar{u}(p_2)\gamma^\delta v(p_1)\left\{f_+(q^2)\left[(p+p')_\delta-\frac{m^2_{\rm D_s}-m^2_{\rm K}}{q^2}q_\delta\right]+f_0(q^2)\frac{m^2_{\rm D_s}-m^2_{\rm K}}{q^2}q_\delta\right\}, \quad (4)$$

where $q=p_1+p_2$, and $C_7=4.7\times10^{-3}$[18]. Following Refs. [2, 16], we also consider the resonance processes $\rm D_s^+\to K^+V_i\to K^+e^-e^+$ with $\rm i=\rho,\ \omega,\ \phi$ which are accounted as long distance contributions. The corresponding Feynman diagram is shown in Fig. 1.

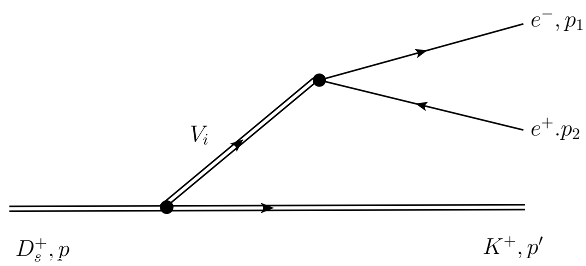

Fig. 1. The Feynman diagram of the process $\rm D_s^+\to K^+e^-e^+$ through SM long distance.

Thus C_9 can be written as

$$C_9 = \left(0.012+\frac{3\pi}{\alpha_{\rm e}^2}\sum_{\rm i=\rho,\omega,\phi}\kappa_{\rm i}\frac{m_{\rm V_i}\Gamma_{\rm V_i\to e^+e^-}}{m^2_{\rm V_i}-q^2-{\rm i}m_{\rm V_i}\Gamma_{\rm V_i}}\right)\times(V_{\rm ud}V_{\rm cd}+V_{\rm us}V_{\rm cs}) \quad (5)$$

with $\kappa_\rho=0.7$, $\kappa_\omega=3.1$ and $\kappa_\phi=3.6$. The second part in the parenthesis corresponds to the long-distance contributions.

Following Refs. [15, 18], the hadronic form factors are written as

$$f_T(q^2)=\frac{f^T_{\rm D_sK}(0)}{(1-q^2/m^2_{\rm D_s})(1-a_Tq^2/m^2_{\rm D_s})}$$
$$f_+(q^2)=\frac{f^+_{\rm D_sK}(0)}{(1-q^2/m^2_{\rm D_s})(1-\alpha_{\rm D_sK}q^2/m^2_{\rm D_s})} \quad (6)$$
$$f_0(q^2)=\frac{f^+_{\rm D_sK}(0)}{1-q^2/(\beta_{\rm D_sK}m^2_{\rm D_s})}$$

where $f^T_{\rm D_sK}(0)=0.46$, $a_T=0.18$, $f^+_{\rm D_sK}(0)=0.75\pm0.08$, $\alpha_{\rm D_sK}=0.30\pm0.03$ and $\beta_{\rm D_sK}=1.3\pm0.07$.

The long-distance contribution is of the order of 10^{-6} [2]. Thus the contribution from the SM may be close to or even larger than that of BSM, so they would interfere among each other. We will discuss this in Section 4.

2.2 Contribution of Z′ in the $U(1)'$ model

The $U(1)'$ model has been proposed and applied by many authors [5, 6, 19, 20]. The corresponding Lagrangian is

$$\mathcal{L}_{\rm Z'}=\sum_{i,j}[\bar{l}_i\gamma^\mu(\omega^{\rm L}_{ij}P_{\rm L}+\omega^{\rm R}_{ij}P_{\rm R})l_jZ'_\mu+\bar{q}_i\gamma^\mu(\varepsilon^{\rm L}_{ij}P_{\rm L}+\varepsilon^{\rm R}_{ij}P_{\rm R})q_jZ'_\mu]+{\rm h.c.} \quad (7)$$

where $P_{\rm L(R)}=\frac{1-(+)\gamma_5}{2}$, and ω_{ij} (ε_{ij}) denotes the chiral couplings between the new gauge boson Z′ and various leptons (quarks). In considering whether it can be applied to solve some phenomenological anomalies, the key point is the intensity of the coupling and the mass of the Z′ gauge boson, which could be fixed by fitting available data.

For the decay processes $\rm D_s^+\to K^+e^-e^+$ and $\rm D_s^+\to K^+e^-\mu^+$, the corresponding Feynman diagrams are shown in Fig. 2.

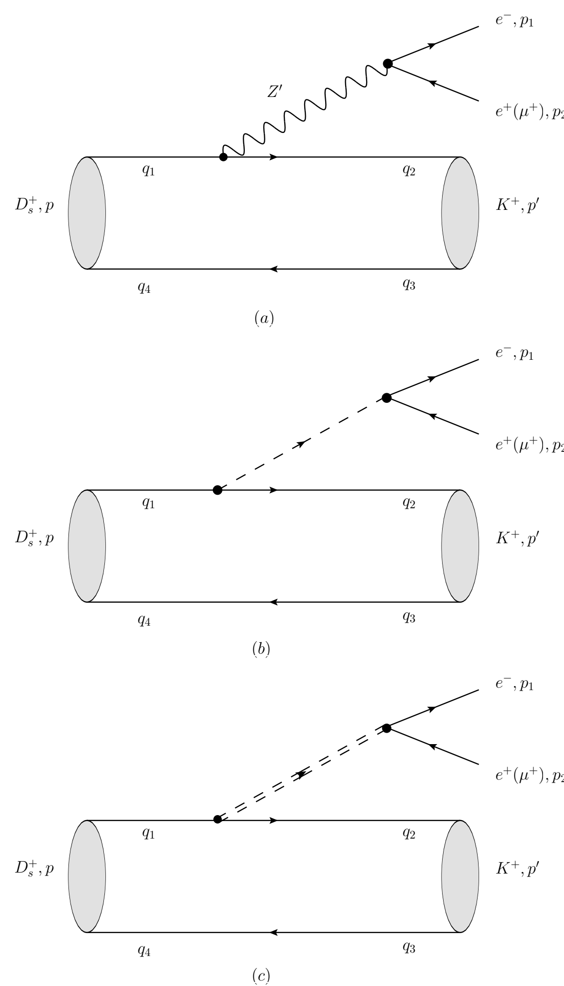

Fig. 2. The Feynman diagrams for $\rm D_s^+\to K^+e^-e^+$ and $\rm D_s^+\to K^+e^-\mu^+$ in (a) the $U(1)'$ model, (b) 2HDM type III and (c) the unparticle model.

The corresponding Feynman amplitude with Z' as the mediating particle was derived by the authors of Refs. [5, 6, 19, 20] as

$$\begin{aligned}&\mathcal{M}_{Z'}(D_s^+\to K^+ l_i\bar{l}_j)\\&=\left\{f_+(q^2)\left[(p+p')_\sigma-\frac{m_{D_s}^2-m_K^2}{q^2}q_\sigma\right]\right.\\&\left.+f_0(q^2)\frac{m_{D_s}^2-m_K^2}{q^2}q_\sigma\right\}\frac{\varepsilon_{cu}^L+\varepsilon_{cu}^R}{g/\sqrt{2}}\frac{1}{q^2-m_{Z'}^2}\\&\times\bar{u}(p_2)(\omega_{ij}^L P_L+\omega_{ij}^R P_R)\gamma^\sigma v(p_1),\end{aligned}\tag{8}$$

where $g^2=4\sqrt{2}m_W^2G_F$, with m_W being the mass of the $W^\pm$ boson, $\omega_{ij}=\omega_{ee}$ for $D_s^+\to K^+e^-e^+$ and $\omega_{ij}=\omega_{e\mu}$ for $D_s^+\to K^+e^-\mu^+$.

The SM contributions (indeed the main contribution originates from the long-distance part) and Z' might be of the same order depending on the model parameters, so we should consider their interference. Here we introduce a free phenomenological phase between the SM and new physics (BSM) which should be determined by fitting data. Then we have

$$\begin{aligned}|\mathcal{M}|^2&=|\mathcal{M}_{SM}+\mathcal{M}_{Z'}e^{i\phi}|^2\\&=|\mathcal{M}_{SM}|^2+|\mathcal{M}_{Z'}|^2+2|\mathrm{Im}(\mathcal{M}_{SM})\mathcal{M}_{Z'}|\sin\phi\\&\quad+2|\mathrm{Re}(\mathcal{M}_{SM})\mathcal{M}_{Z'}|\cos\phi,\end{aligned}\tag{9}$$

and because $\mathrm{Im}(\mathcal{M}_{SM})$ is much greater than $\mathrm{Re}(\mathcal{M}_{SM})$, the formula above can be simplified as

$$|\mathcal{M}|^2\simeq|\mathcal{M}_{SM}|^2+|\mathcal{M}_{Z'}|^2+2|\mathrm{Im}(\mathcal{M}_{SM})\mathcal{M}_{Z'}|\sin\phi.\tag{10}$$

Averaging initial spin and summing over final spin polarizations, the decay width $\Gamma(D_s^+\to K^+e^-e^+)$ is

$$\begin{aligned}\frac{d\Gamma}{dq^2}=&\left\{\frac{G_F^2\alpha_e^2}{1536\pi^5 m_{D_s}^3}\left|C_9f_+(q^2)+2C_7f_T(q^2)\frac{m_c}{m_{D_s}}\right|^2+\frac{(\varepsilon_{cu}^L+\varepsilon_{cu}^R)^2((\omega_{ee}^L)^2+(\omega_{ee}^R)^2)}{192\pi^3g^2m_{Z'}^4m_{D_s}^3}f_+(q^2)^2\right.\\&\left.+\frac{(\varepsilon_{cu}^L+\varepsilon_{cu}^R)(\omega_{ee}^L+\omega_{ee}^R)G_F\alpha_e}{384\pi^4gm_{Z'}^2m_{D_s}^3}f_+(q^2)\left|\mathrm{Im}\left[C_9f_+(q^2)+2C_7f_T(q^2)\frac{m_c}{m_{D_s}}\right]\right|\sin\phi\right\},\lambda^{3/2}(q^2,m_{D_s}^2,m_K^2)\end{aligned}\tag{11}$$

where $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ca$ is the Kallen function. We can obtain the total decay width by integrating over dq^2, as

$$\Gamma=\int_{4m_e^2}^{(m_{D_s}-m_K)^2}\frac{d\Gamma}{dq^2}dq^2\ .\tag{12}$$

2.3 Contribution of heavy neutral Higgs in the two-Higgs-doublet model of type III

In the 2HDM of type III [10, 11, 21], there are two neutral CP-even Higgs bosons, one of which is the SM Higgs boson and the other a heavy Higgs boson. The corresponding Lagrangian for the heavy Higgs boson is

$$\begin{aligned}\mathcal{L}_{Yukawa}=&\sum_{i,j}\left[\bar{l}_i\left(\frac{m_l^i}{v}\cos\alpha\delta_{ij}-\frac{\rho_{ij}^E}{\sqrt{2}}\sin\alpha\right)l_jH\right.\\&\left.+\bar{q}_i\left(\frac{m_q^i}{v}\cos\alpha\delta_{ij}-\frac{\rho_{ij}^U}{\sqrt{2}}\sin\alpha\right)q_jH\right]+\text{h.c.}\end{aligned}\tag{13}$$

where ρ_{ij}^E and ρ_{ij}^U stand for effective coupling constants for leptons and quarks respectively. $\cos\alpha$ is the mixing angle between the light and heavy Higgs bosons. Following Refs. [11, 21], we take $\cos\alpha\to0$ and do not adopt the so-called Cheng-Sher ansatz for ρ_{ij}^f which was discussed in Ref. [9]. Instead, we take a range of ρ_{ij}^f to $0.1\sim0.3$ as suggested in Ref. [21].

The Feynman amplitude corresponding to contributions through exchanging a heavy Higgs boson is

$$\begin{aligned}&\mathcal{M}_{hh}(D_s^+\to K^+l_i\bar{l}_j)\\&=\left\{f_+(q^2)\left[\frac{(p'+p)\cdot p}{m_{D_s}}-\frac{m_{D_s}^2-m_K^2}{q^2}\frac{q\cdot p}{m_{D_s}}\right]\right.\\&\left.+f_0(q^2)\frac{m_{D_s}^2-m_K^2}{q^2}\frac{q\cdot p}{m_{D_s}}\right\}\rho_{cu}^U\frac{1}{q^2-m_{hh}^2}\bar{u}(p_1)v(p_2)\rho_{ij}^E\end{aligned}\tag{14}$$

where $\rho_{ij}=\rho_{ee}$, $\rho_{ij}=\rho_{e\mu}$ stand for $D_s^+\to K^+e^-e^+$ and $D_s^+\to K^+e^-\mu^+$ respectively.

The differential decay width $d\Gamma(D_s^+\to K^+e^-e^+)$ is

$$\begin{aligned}\frac{d\Gamma}{dq^2}=&\left[\frac{G_F^2\alpha_e^2}{1536\pi^5m_{D_s}^3}\left|C_9f_+(q^2))^2+2C_7f_T(q^2)\frac{m_c}{m_{D_s}}\right|^2\lambda(q^2,m_{D_s}^2,m_K^2)\right.\\&\left.+\frac{(\rho_{cu}^U\rho_{ee}^E)^2\{f_0(q^2)(m_{D_s}^2-m_K^2)(m_{D_s}^2-m_K^2+q^2)-f_+(q^2)[m_{D_s}^4+(m_K^2-q^2)^2-2m_{D_s}^2(m_K^2+q^2)]\}^2}{64g^2m_{D_s}^5m_{hh}^4\pi^3q^2}\right]\\&\times\lambda^{1/2}(q^2,m_{D_s}^2,m_K^2).\end{aligned}\tag{15}$$

Then we obtain the total decay width by integrating over $\mathrm{d}q^2$ as done in Eq. (12).

2.4 Contribution from unparticles

The idea of unparticles was proposed by Georgi [12] some years ago. Many authors then followed him to explore relevant phenomenology and study the basic theory. In the unparticle scenario, a flavor changing term exists in the basic Lagrangian, so FCNCs can occur at tree level. One is naturally tempted to conjecture that the unparticle mechanism may contribute to $\mathrm{D_s^+}\to\mathrm{K^+e^-e^+}$ and $\mathrm{D_s^+}\to\mathrm{K^+e^-\mu^+}$. Following Refs. [13, 22, 23], we only consider the interactions between fermions and scalar unparticles. The corresponding effective interaction is:

$$\mathcal{L}=\sum_{f',f}\frac{c_{\mathrm{s}}^{f'f}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\bar{f}'\gamma_\mu(1-\gamma_5)f\partial^\mu\mathcal{O}_{\mathcal{U}}+\text{h.c.} \tag{16}$$

where $c_{\mathrm{s}}^{f'f}$ stands for coupling constants between unparticles and fermions, $\mathcal{O}_{\mathcal{U}}$ is the scalar unparticle field, $d_{\mathcal{U}}$ is a nontrivial scale dimension and $\Lambda_{\mathcal{U}}$ is an energy scale of the order of TeV. The propagator of the scalar unparticle is [14, 23, 24]

$$\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}P\cdot x}\langle 0|T\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle$$

$$=\mathrm{i}\frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}(P^2)^{2-d_{\mathcal{U}}}\mathrm{e}^{-\mathrm{i}(d_{\mathcal{U}}-2)\pi}, \tag{17}$$

where $A_{d_{\mathcal{U}}}$ is

$$A_{d_{\mathcal{U}}}=\frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}. \tag{18}$$

Supposing $\mathrm{D_s^+}\to\mathrm{K^+e^-e^+}$ and $\mathrm{D_s^+}\to\mathrm{K^+e^-\mu^+}$ occur via exchanging a scalar unparticle, the corresponding Feynman amplitude is

$$\begin{aligned}&\mathcal{M}(D_{\mathrm{s}}^+\to K^+l_i\bar{l}_j)\\ &=\left\{2f^+_{\mathrm{D_sK}}(q^2)p'\cdot q+[f^+_{\mathrm{D_sK}}(q^2)+f^-_{\mathrm{D_sK}}(q^2)]q^2\right\}\\ &\quad\times\frac{c_{\mathrm{s}}^{cu}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\frac{1}{(q^2)^{2-d_{\mathcal{U}}}}\mathrm{e}^{-\mathrm{i}(d_{\mathcal{U}}-2)\pi}\bar{u}(p_1)\not{q}(1-\gamma_5)v(p_2)\frac{c_{\mathrm{s}}^{ij}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}.\end{aligned} \tag{19}$$

where $c_{\mathrm{s}}^{ij}=c_{\mathrm{s}}^{ee}$ and $c_{\mathrm{s}}^{ij}=c_{\mathrm{s}}^{e\mu}$ correspond to $\mathrm{D_s^+}\to\mathrm{K^+e^-e^+}$ and $\mathrm{D_s^+}\to\mathrm{K^+e^-\mu^+}$ respectively.

Since numerically the unparticle contribution to $\mathrm{D_s^+}\to\mathrm{K^+e^-e^+}$ and $\mathrm{D_s^+}\to\mathrm{K^+e^-\mu^+}$ is much smaller than that from SM and other models BSM, we list the formula involving unparticles, and for completeness we include the numerical results of the unparticle contribution in the corresponding tables. The differential decay width $\Gamma(\mathrm{D_s^+}\to\mathrm{K^+e^-e^+})$ is

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}&=\frac{1}{256\pi^3m_{\mathrm{D_s}}^3}(c_{\mathrm{s}}^{cu}c_{\mathrm{s}}^{ee})^2\frac{2^{12-4d_{\mathcal{U}}}me^2\pi^{5-4d_{\mathcal{U}}}(2me^2+s_{12})}{s_{12}^{6-2d_{\mathcal{U}}}}\frac{\Gamma^2[1/2+d_{\mathcal{U}}]}{\Lambda_{\mathcal{U}}^{4d_{\mathcal{U}}}\sin^2 d_{\mathcal{U}}\pi}\\ &\quad\times\frac{(f^0_{\mathrm{D_sK}}(q^2)(m^2_{\mathrm{D_s}}-m^2_{\mathrm{K}})(2m^2_{\mathrm{e}}+s_{12})+2f^+_{\mathrm{D_sK}}(q^2)m^2_{\mathrm{e}}(-m^2_{\mathrm{D_s}}+m^2_{\mathrm{K}}+s_{12}))^2}{g^2\Gamma^2[d_{\mathcal{U}}-1]\Gamma^2[2d_{\mathcal{U}}]}\lambda^{1/2}(q^2,m^2_{\mathrm{D_s}},m^2_{\mathrm{K}}).\end{aligned} \tag{20}$$

2.5 Semi-leptonic decay of $\mathrm{D^+}$

Decays of $\mathrm{D^+}\to\pi^+\mathrm{e^-e^+}$ and $\mathrm{D^+}\to\pi^+\mathrm{e^-\mu^+}$ are similar to $\mathrm{D_s^+}\to\mathrm{K^+e^-e^+}$ and $\mathrm{D_s^+}\to\mathrm{K^+e^-\mu^+}$. The only difference is the species of the spectators. Therefore all the formulas of $\mathrm{D_s^+}\to\mathrm{K^+l}_i\bar{\mathrm{l}}_j$ can be transferred to $\mathrm{D^+}\to\mathrm{K^+l}_i\bar{\mathrm{l}}_j$ by an $SU(3)$ symmetry.

3 Rare leptonic decays of $\mathrm{D^0}$

The rare leptonic decays of $\mathrm{D^0}$ refer to $\mathrm{D^0}\to\mathrm{l\bar{l}}$ and $\mathrm{D^0}\to\mathrm{l}_i\bar{\mathrm{l}}_j$ with $i\neq j$, the latter of which is not only a FCNC but also a lepton-flavor violation (LFV) process. In the SM, in $\mathrm{D^0}\to\mathrm{l\bar{l}}$, the charm-quark and $\bar{u}$ annihilate into a virtual photon via an electroweak penguin which suppresses the reaction rate. For the LFV process, not only in the initial part, c and $\bar{u}$ would annihilate into a Z virtual meson which would later turn into a pair of neutrinos, then via a weak scattering the neutrinos would eventually end with two leptons with different flavors. Because neutrinos are very light, this process would be much more suppressed than $\mathrm{D^0}\to\mathrm{l\bar{l}}$. In fact, if there is no new physics BSM, such LFV processes can never be experimentally measured. Therefore, searching for such LFV processes is a trustworthy probe of BSM. Actually, contributions to the leptonic decays (both lepton-flavor conserving and lepton-flavor violating processes) of the SM are too small to be observed [2], so we only consider the contribution from new physics. Since $\mathrm{D^0}$ is a pseudo-scalar meson and the heavy Higgs is scalar boson, the processes $\mathrm{D^0}\to\mathrm{e^-e^+}$ and $\mathrm{D^0}\to\mathrm{e^-\mu^+}$ cannot occur through exchanging a heavy Higgs boson. In the $\mathrm{Z'}$ and unparticle scenarios, $\mathrm{D^0}\to\mathrm{e^-e^+}$ and $\mathrm{D^0}\to\mathrm{e^-\mu^+}$ might be induced to result in sizable rates.

3.1 The $\mathrm{Z'}$ gauge boson from the $U(1)'$ model

For the decay processes $\mathrm{D^0}\to\mathrm{e^-e^+}$ and $\mathrm{D^0}\to\mathrm{e^-\mu^+}$, the corresponding Feynman diagrams are shown in Fig. 3.

The corresponding Feynman amplitude with $\mathrm{Z'}$ as the mediating particle is written as

$$\mathcal{M}(\mathrm{D}^0\to \mathrm{l}_i\bar{\mathrm{l}}_j) = \langle 0|J_A^{\mu}|D(p)\rangle(\varepsilon_{\mathrm{cu}}^{\mathrm{L}}P_{\mathrm{L}}+\varepsilon_{\mathrm{cu}}^{\mathrm{R}}P_{\mathrm{R}}) \times\frac{1}{m_{\mathrm{D}}^2-m_{\mathrm{Z}'}^2}\bar{u}(p_1)(\omega_{ij}^{\mathrm{L}}P_{\mathrm{L}}+\omega_{ij}^{\mathrm{R}}P_{\mathrm{R}})\gamma_\sigma v(p_2), \tag{21}$$

where $\omega_{ij}=\omega_{\mathrm{ee}}$ for $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mathrm{e}^+$ and $\omega_{ij}=\omega_{e\mu}$ for $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$. Following Ref. [25] we have

$$\langle 0|J_A^{\mu}|D(p)\rangle\sim \mathrm{i}p^{\mu}f_{\mathrm{D}}. \tag{22}$$

with f_{D}=200 MeV. The decay width $\Gamma(\mathrm{D}^0\to \mathrm{e}^-\mathrm{e}^+)$ is

$$\Gamma=\frac{(\varepsilon_{\mathrm{cu}}^{\mathrm{L}}-\varepsilon_{\mathrm{cu}}^{\mathrm{R}})^2(\omega_{\mathrm{ee}}^{\mathrm{L}}-\omega_{\mathrm{ee}}^{\mathrm{R}})^2f_{\mathrm{D}}^2m_{\mathrm{e}}^2\sqrt{m_{\mathrm{D}}^2-4m_{\mathrm{e}}^2}}{2\pi(m_{\mathrm{Z}'}^2-m_{\mathrm{D}}^2)^2}. \tag{23}$$

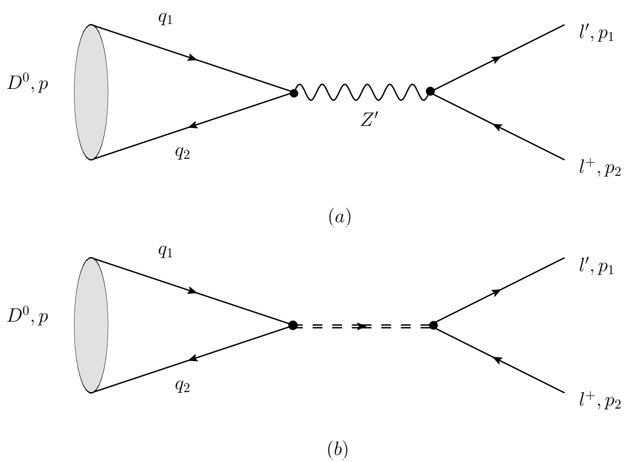

Fig. 3. The Feynman diagrams of processes $\mathrm{D}^0\to \mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}^0\to \mathrm{e}^-\mu^+$ in (a) the $U(1)'$ model and (b) the unparticle model.

3.2 Contribution from unparticles

$\mathrm{D}^0\to \mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}^0\to \mathrm{e}^-\mu^+$ could also be realized via exchanging a scalar unparticle, and the corresponding Feynman amplitude is

$$\mathcal{M} = \mathrm{Tr}[\bar{v}(q_2)\not{p}(1-\gamma_5)u(q_1)]\frac{c_{\mathrm{s}}^{\mathrm{cu}}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}\frac{1}{(m_{\mathrm{D}}^2)^{2-d_{\mathcal{U}}}} \times \mathrm{e}^{-\mathrm{i}(d_{\mathcal{U}}-2)\pi}\bar{u}(p_2)\not{p}(1-\gamma_5)v(p_1)\frac{c_{\mathrm{s}}^{\mathrm{ee}}}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}}, \tag{24}$$

where $c_{\mathrm{s}}^{ij}=c_{\mathrm{s}}^{\mathrm{ee}}$ for $\mathrm{D}^0\to \mathrm{e}^-\mathrm{e}^+$ and $c_{\mathrm{s}}^{ij}=c_{\mathrm{s}}^{\mathrm{e\mu}}$ for $\mathrm{D}^0\to \mathrm{e}^-\mu^+$.

The decay width $\Gamma(\mathrm{D}^0\to \mathrm{e}^-\mathrm{e}^+)$ is

$$\Gamma = \frac{(c_{\mathrm{s}}^{cu}c_{\mathrm{s}}^{\mathrm{ee}})^2f_{\mathrm{D}}^2\sqrt{m_{\mathrm{D}}^2-4m_{\mathrm{e}}^2}me^2 2^{9-4d_{\mathcal{U}}}\pi^{4-4d_{\mathcal{U}}}}{m_{\mathrm{D}}^{4-4d_{\mathcal{U}}}\Lambda_{\mathcal{U}}^{4d_{\mathcal{U}}}} \times\frac{\Gamma^2[1/2+d_{\mathcal{U}}]}{\sin^2 d_{\mathcal{U}}\pi\Gamma^2[d_{\mathcal{U}}-1]\Gamma^2[2d_{\mathcal{U}}]}. \tag{25}$$

4 Numerical results

For $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ where a Z′ boson is exchanged at s-channel, we follow the authors of Ref. [8, 19] and set the ranges of $\varepsilon_{\mathrm{cu}}^{\mathrm{L}}$, $\varepsilon_{\mathrm{cu}}^{\mathrm{R}}$, $\omega_{\mathrm{ee}(\mu)}^{\mathrm{L}}$ and $\omega_{\mathrm{ee}(\mu)}^{\mathrm{R}}$ to $-0.5\sim 0.5$ accordingly.

We plot the branching ratios of $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^+\mathrm{e}^-$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ versus the mixing angle between SM Z and Z′ of $U(1)'$ θ in Fig. 4.

When we calculate the branching ratios of $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ via exchanging a heavy Higgs boson, we just follow Ref. [21] and take the range of ρ_{ij}^f within $0.01\sim 0.3$, rather than adopting the so-called Cheng-Sher ansatz for the couplings ρ_{ij}^f as was done in Ref. [9]. We plot the branching ratios of $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^+\mathrm{e}^-$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ versus the mass of the heavy Higgs boson in Fig. 5.

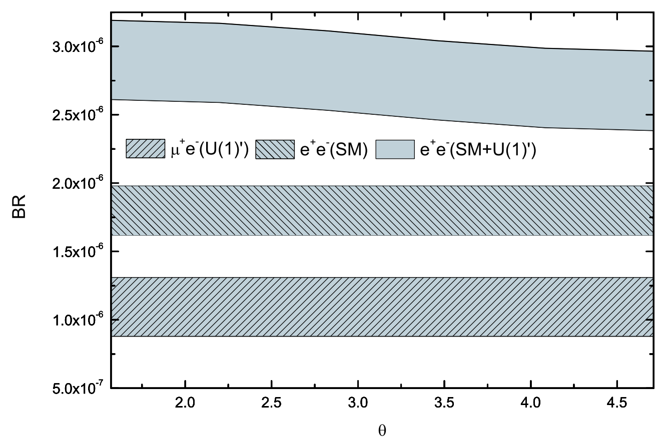

Fig. 4. The branching ratios of $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ versus the mixing angle θ between SM and $U(1)'$ with $\varepsilon_{\mathrm{cu}}^{\mathrm{L}}=\varepsilon_{\mathrm{cu}}^{\mathrm{R}}=\omega_{\mathrm{ee}}^{\mathrm{L}}=\omega_{\mathrm{e\mu}}^{\mathrm{L}}=\omega_{\mathrm{ee}}^{\mathrm{R}}=\omega_{\mathrm{e\mu}}^{\mathrm{R}}=0.2$ and $m_{\mathrm{Z}'}=2000$ GeV (the range of θ varies from $\pi/2$ to $3\pi/2$). The theoretical uncertainty comes from the form factors.

Then, we calculate branching ratios of $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ via exchanging a scalar unparticle. Following Refs. [14, 22–24], we take $\Lambda_{\mathcal{U}}$=1 TeV, $1<d_{\mathcal{U}}<2$ and the range of c_S to be $0.01\sim 0.04$ with the relation

$$c_S^{f'f}=\begin{cases} c_S & f\neq f' \\ \kappa c_S & f=f' \end{cases}, \tag{26}$$

where κ=3 [22]. Then we plot the branching ratio of the decays $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ versus $\Lambda_{\mathcal{U}}$ with different $d_{\mathcal{U}}$ in Fig. 6.

We list the branching ratios of $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mathrm{e}^+$ and $\mathrm{D}_s^+\to \mathrm{K}^+\mathrm{e}^-\mu^+$ predicted by the various new physics models (BSM) in Tables 1 and 2 separately. From those tables we notice that for the $U(1)'$ model [19] and 2HDM of type III [21], the branching ratios can be up to the order of $10^{-6}\sim 10^{-7}$.

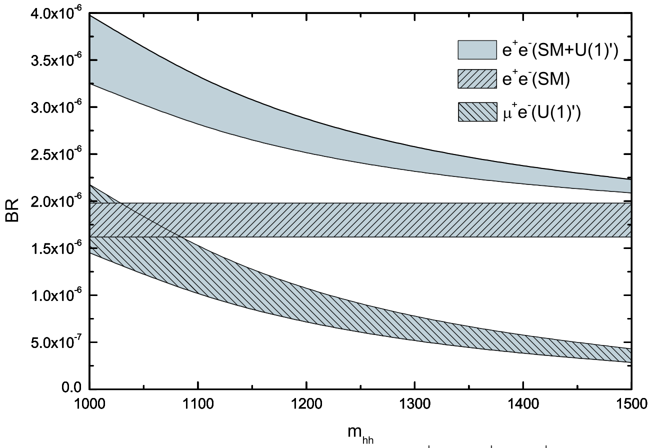

Fig. 5. The branching ratios of $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ versus the mass of the heavy Higgs boson with $\rho_{cu}=\rho_{ee}=\rho_{e\mu}=0.15$. The theoretical uncertainty comes from the form factors.

We also list the branching ratios of the leptonic decays $D^0 \to e^-e^+$ and $D^0 \to e^-\mu^+$ predicted by various models of new physics beyond the SM in Table 3. Since D^0 cannot decay to $l_i\bar{l}_j$ through a scalar particle, only Z' and unparticles could contribute to those leptonic decays.

Our numerical results indicate that as the experimental bounds are taken into account and the corresponding coupling constants in the $U(1)'$ model and 2HDM take their maximum values, the branching ratios of $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ can be up to the order of 10^{-6}, whereas the contribution of scalar unparticles to the branching ratios can only reach the order of $10^{-18}(10^{-15})$.

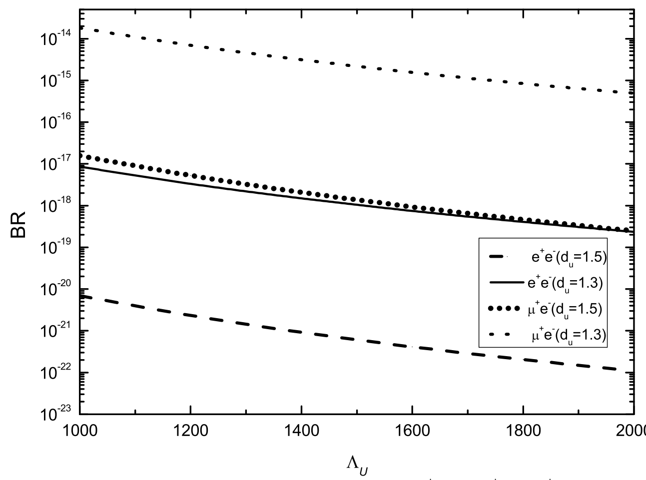

Fig. 6. The branching ratio of $D_s^+ \to K^+e^-e^+$ and $D_s^+ \to K^+e^-\mu^+$ versus the energy scale $\Lambda_{\mathcal{U}}$ with $c_S^{cu}=c_S^{e\mu}=0.04$, $c_S^{ee}=0.12$, $d_{\mathcal{U}}=1.3$ and 1.5.

Table 1. Branching ratios of $D_s^+ \to K^+e^-e^+$ predicted by various models of new physics beyond the SM.

model	mass	couplings constants	BR
$U(1)'$ [19]	2000 GeV	$-0.5\sim0.5$	$10^{-8}\sim10^{-6}$
	1000$\sim$2000 GeV	0.1	$10^{-8}\sim10^{-6}$
2HDM type III[21]	1500 GeV	$0.05\sim0.3$	$10^{-8}\sim10^{-7}$
	1000$\sim$1500 GeV	0.05	10^{-8}
unparticle	2000 GeV	$0.02\sim0.04$	$10^{-22}\sim10^{-21}$
	1000$\sim$2000 GeV	0.02	$10^{-21}\sim10^{-20}$

Table 2. Branching ratios of $D_s^+ \to K^+e^-\mu^+$ predicted by various models of new physics beyond the SM.

model	mass	couplings constants	BR
$U(1)'$ [19]	2000 GeV	$-0.5\sim0.5$	$10^{-8}\sim10^{-6}$
	1000$\sim$2000 GeV	$-0.5\sim0.5$	$10^{-8}\sim10^{-7}$
2HDM type III[21]	1500 GeV	$0.05\sim0.3$	$10^{-8}\sim10^{-7}$
	1000$\sim$1500 GeV	0.05	$10^{-8}\sim10^{-7}$
unparticle	2000 GeV	$0.02\sim0.04$	$10^{-20}\sim10^{-19}$
	1000$\sim$2000 GeV	0.02	$10^{-19}\sim10^{-18}$

5 Searching for semi-leptonic and leptonic decays based on BESIII data

In this section, let us discuss possible constraints and the potential to observe the aforementioned rare semi-leptonic and leptonic decays of D mesons based on the large data samples gathered by BESIII. Unlike hadron colliders, electron-positron colliders have much lower background, which is well understood and helps to reduce contamination from the measurements. Thus, controllable and small systematic uncertainties are expected.

The BESIII experiment has accumulated large data samples at 3.773 and 4.18 GeV, which are just above the production thresholds of $D\bar{D}$ and $D_s^{*+}D_s^-$+c.c.. This provides an excellent opportunity to investigate the decays of these charmed mesons.

At these energies, the charmed mesons are produced in pairs. That is to say, if only one charmed meson is reconstructed in an event, which is defined as a single tag event, there must exist another charmed meson on the recoiling side. With the selected singly tagged events, the rare charm decays concerned can be studied on the recoiling side of the reconstructed charmed meson.

This is called the double-tag technique, and was first employed by the MARK-III Collaboration. It is now widely used in the BESIII experiments. With this method, the two charmed mesons are both tagged in one event. One of the charmed mesons is reconstructed through a well-measured hadronic channel, and the other decays into the signal process of interest. Benefiting from the extremely clean background, the systematic uncertainties in double tag measurements can be reduced to a fully controlled level.

In principle, there are two ways to search for rare or forbidden decays. One is based on the single tag method where one charmed meson is reconstructed for the signal process while no constraint is set for the other. This method can provide higher statistics, but might have more complex and higher background. The other way is the double tag method, which presents a simple and clean background but relatively poorer statistics (see Table 4). Whether or not to employ the double-tag technique to study the relevant processes depends on a balance between reducing background contamination and seeking higher statistics.

Table 3. Branching ratios of $D^0\to e^-e^+$ and $D^0\to e^-\mu^+$ predicted by $U(1)'$ and unparticle models.

decay	$D^0\to e^-e^+$		
model	mass	couplings constants	BR
$U(1)'$	2000 GeV	$-0.5\sim0.5$	$10^{-11}\sim10^{-10}$
	1000∼2000 GeV	0.1	$10^{-13}\sim10^{-12}$
unparticle	2000 GeV	$0.02\sim0.04$	$10^{-15}\sim10^{-14}$
	1000∼2000 GeV	0.02	$10^{-16}\sim10^{-15}$
decay	$D^0\to e^-\mu^+$		
$U(1)'$	2000 GeV	$-0.5\sim0.5$	$10^{-7}\sim10^{-6}$
	1000∼2000 GeV	0.1	$10^{-9}\sim10^{-8}$
unparticle	2000 GeV	$0.02\sim0.04$	$10^{-11}\sim10^{-10}$
	1000∼2000 GeV	0.02	$10^{-11}\sim10^{-10}$

Table 4. Two methods of searching for rare/forbidden D decays.

method	statistics (charged/neutral)	background	sensitivity
single tag method	$1.7\times10^7/2.1\times10^7$	not good	Bkg. vs Stat.
double tag method	$1.6\times10^6/2.8\times10^6$	clean	Bkg. vs Stat.

In the following, we discuss the statistics of the measurements of the rare decays, which may be the main obstacle in searching for new physics in most cases. For the single tag method, the background analysis is severely mode dependent. Thus, to simplify the estimation, we will focus our discussion on the double tag method.

The BESIII experiment has accumulated huge threshold data samples of about 2.95 fb^{-1} and 3.15 fb^{-1} at c.m.s. energies of $\sqrt{s}$ = 3.773 and 4.180 GeV respectively, which are about 3.5 times and 5 times more than the previously accumulated datasets, respectively. According to the published papers of the BESIII experiments, there are more than 1.6×10^6 and 2.8×10^6 singly tagged charged and neutral $D\bar{D}$ mesons, respectively. These modes can be used as the tagging side for the double-tag method. Because of the advantage of the double-tag method in reducing the background and enhancing the confidence level, we suggest adopting the double-tag method for the analysis of the rare decay data, employing the well established modes as the tagging side. Then on the recoiling side, one can look for the expected signal. Omitting some technical details, we know that by adopting this double-tag method, the experimental sensitivity can reach about 10^{-6} at 90% confidence level (CL) if we assume zero-signal and zero-background events. In the next 10 years, 4 to 6 times more charm data can be expected, and we may have a better chance of detecting such rare decays.

Unfortunately, however, according to our predictions this sensitivity is still below the bound of observing the pure leptonic rare decays of D^0 (no matter whether they are lepton-flavor-conserving or lepton-flavor-violating processes). If the size of the BESIII data sample can reach 20 fb^{-1} in the next 10 years, the sensitivity would be at 10^{-7} level, which is almost touching the bottom line of our prediction on the rate of pure leptonic modes.

The analysis is a little more complex at 4.180 GeV even though the method is similar. The sensitivities for the rare semi-leptonic decays of D_s^+ or D_s^{*+} mesons can be expected to reach 10^{-5} at 90% CL. However, this is not enough to test our predictions for the rare D_s^+ semi-leptonic decays.

If the proposed super τ-charm factory (STCF) is launched in the near future, we would be able to collect at least 100 or 1000 times more data, since the designed luminosity of the STCF will be as high as 1×10^{35} $cm^{-2}s^{-1}$, which is 100 times higher than that of BEPCII (the collider at which BESIII is based). Then, the sensitivities of searching for the relevant signals in D or D_s^+ decays should be greatly improved to $10^{-9\sim-10}$ or $10^{-7\sim-8}$ at 90% CL, respectively. With these improved sensitivities, the rates of $D_s^+\to K^+e^+e^-$ and $D_s^+\to K^+\mu^+e^-$ predicted by the $U'(1)$ or 2HDM models become measurable. Then, the more challengeable lepton-flavor-violation modes $D^0\to e^-\mu^+$ predicted by the $U'(1)$ and unparticle models can possibly be tested.

6 Discussion and conclusions

The rare decays of heavy flavored hadrons which are suppressed or even forbidden in the SM can serve as probes for new physics BSM. Experimentally measured "anomalies" which obviously deviate from SM predictions would be candidate signals of BSM, and would at least provide hints of BSM for experiments at high energy colliders, such as the LHC. That is common sense for experimentalists and theorists in high energy physics. However, designing a new experiment which might lead to the discovery of new physics is an art. Following historical experience, besides blind searches in experiments, researchers tend to make measurements according to the predictions made by theorists based on available and reasonable models.

FCNC/LFV processes provide a sensitive test for new physics BSM, which compose a complementary area to high energy collider physics. Processes where the SM contributes substantially definitely do not stand as candidates for seeking new physics BSM, because the new physics contributions would be drowned in the SM background. Researchers are carefully looking for rare processes where SM contributions are very suppressed or even forbidden by some rules. The rare semi-leptonic and leptonic decays of B and D mesons are ideal places because they are caused by FCNC. We are especially interested in lepton-flavor violating decays which cannot be produced in the SM because neutrino masses are too tiny to make any non-negligible contribution.

Recently, most research has focused on B decays. The reason is obvious, with B mesons being at least three times heavier than D mesons, so the processes involving B-mesons are closer to the new physics scale. Moreover, the coupling between the b-quark and top-quark has a large CKM entry. Indeed, there many studies concerning $B\to K^{(*)}l\bar{l}$ [26, 27] and $B^0(B_s)\to l\bar{l}$ have emerged [28, 29]. On another aspect, several authors have studied the case of D mesons, and drawn constraints on the free parameters in the proposed models by fitting available data. The model parameters can be compared with those obtained by fitting the data of B decays. In this work, based on the large data samples from BESIII, we have followed the trend to investigate possibilities of detecting rare semi-leptonic and pure leptonic decays of D mesons, and have paid special attention to analysis of lepton-flavor-violation processes.

In this work, we have calculated the decay rates of $D_s^+\to K^+e^-e^+$, $D_s^+\to K^+e^-\mu^+$, $D^0\to e^-e^+$ and $D^0\to e^-\mu^+$ through exchanging a neutral particle in terms of three BSM new physics models: the extra $U'(1)$ model, 2HDM of type III and the unparticle model. The decay rate of $D_s^+\to K^+e^-e^+$ receives a sizable contribution from SM whose branching ratio is up to orders of 10^{-6}. The branching ratio of the direct decay process via the penguin diagram is small, of the order of 10^{-8}, while the long-distance reaction makes a larger contribution. Our numerical results show that $U(1)'$ and 2HDM of type III can make significant contributions to the process $D_s^+\to K^+e^-e^+$ as long as the model parameters which are obtained by fitting relevant data are adopted, but the unparticle model cannot make any substantial contribution. Recent researchers seem to be more tempted to use the extra $U'(1)$ model and we follow the same trend. But here for fixing the model parameters, we deliberately relax the constraint set by the $D^0-\bar{D}^0$ mixing, as we discussed above. Assuming the new physics contribution is a unique extra one besides the short-distance contribution via the box-diagram, the constraints taken into account would reduce the predicted branching ratio of $D_s^+\to K^+e^-e^+$ by two more orders of magnitude to 10^{-8}, which is much lower than the contribution of the SM long-distance effect. Thus, the new physics contribution would be buried in the SM background. However, as we only consider the constraints on the $U'(1)$ parameter taken by fitting the data of $\tau\to3l$, rather than $D^0-\bar{D}^0$ mixing, the predicted branching ratio can be large, to the order of 10^{-6}, and thus the resultant amplitude might interfere with the SM long-distance contribution.

There is another possibility which is hinted at by the study on the semileptonic decays made above. Namely, the contribution of the long-distance effect to $D_s\to K+l\bar{l}$ is almost two orders of magnitude larger than that of the short-distance effect which is evaluated via the Feynman diagrams. Moreover, the contribution of the Z′, which is the gauge boson of an extra $U'(1)$ model, is of the same order of magnitude as the long-distance effect and they should interfere. In our formulation, we introduce a free relative phase between them. Thus for $D^0\bar{D}^0$ mixing there might also exist a long-distance effect which is larger than the short-distance effect (via the box diagram) by a few orders of magnitude. One may expect it to be of the same order as the contribution of Z′ and moreover, both of them destructively interfere to reduce the D^0–$\bar{D}^0$ value. If this is the case, the D^0–$\bar{D}^0$ data cannot provide constraints on new physics, especially the extra $U'(1)$ model, so that we only consider the constraint from other experiments such as $\tau\to 3l$ where definitely no long-distance effects exists.

In future BESIII experiments, the experimental sensitivity can be up to the order of $10^{-6}\sim10^{-7}$, and thus the data on $D_s^+\to K^+e^-e^+$ might give us some information on new physics.

Our numerical results show that the $U(1)'$ model and 2HDM of type III could give an observable branching ratio of $D_s^+\to K^+e^-\mu^+$ with the BESIII data, as its precision can reach orders of $10^{-6}\sim10^{-7}$.

For the processes $D^0\to e^-e^+$ and $D^0\to e^-\mu^+$, the theoretically predicted ranching ratio of the decay $D^0\to e^-e^+$ is of the order of 10^{-10} since its width is proportional to m_e^2. Such a small value is hard to observe. For the decay $D^0\to e^-\mu^+$, however, its branching ratio can be up to the order of 10^{-7}, which may be observed in a future super τ-charm factory. Moreover, one can expect to observe $D^0\to\mu^-\mu^+$, while unfortunately, $D^0\to\mu^-\tau^+$ is forbidden by the phase space of final states because $m_\mu+m_\tau>m_{D^0}$.

According to the presently available new physics models, $U'(1)$, 2HDM and the unparticle model, the data on D-mesons which will be collected in the next 10 years could marginally detect new physics contributions to $D_s^+\to K^+e^-e^+$, $D_s^+\to K^+e^-\mu^+$+h.c., $D^0\to e^-e^+$ and $D^0\to e^-\mu^+$+h.c. as long as only the constraints set by some experiments are accounted for, but the data of D^0–$\bar{D}^0$ mixing are relaxed. If the recent data on D^0–$\bar{D}^0$ mixing is rigorous and there are no other possible BSM scenarios available, and the evaluation of the SM including the long-distance effects on the mixing is accurate, then BSM contributions to the semileptonic decays of D and D_s, and flavor-conserving and violating leptonic decays of D_0, cannot be observed by any foreseen experiments. But as discussed above, it might be possible to theoretically relax the constraints from the D^0–$\bar{D}^0$ mixing, so we expect experimentalists to continue to search for BSM signals in those channels. In other words, if the present data and only the theoretical prediction via the box diagram on D^0–$\bar{D}^0$ mixing are under consideration, neither BESIII nor the planned high luminosity τ-charm factory will not be able to "see" those rare decays as predicted by these models. However, the situation might be different as indicated above. Even though the argument made above is not sufficient to convince experimentalists, there is by no means any reason to forbid them to search for these rare decays in the charm energy regions based on the huge data sample collected by BESIII and that which will be collected by the future τ- charm factory.

Moreover, the same argument can also be applied to discuss the BSM contributions to the rare decays $B\to Kl\bar{l}$ and $B\to l\bar{l}$. To phenomenologically estimate the coupling of Z′ with quarks or leptons and the mass of the Z′, $D^0-\bar{D}^0$ mixing also provides a possible constraint to determine the model parameters. For $U'(1)$, for example, the coupling is universal for all flavors, so that one should apply the constraint gained from D-physics to study B-related processes. Thus the constraint due to small D^0–$\bar{D}^0$ mixing would be extended to the B-region unless other contributions exist.

Blind experimental searches are not affected by the available theoretical predictions because the present BSM models are only possible ones conjectured by theorists, while nature might give an alternative scenario. If such a new observation were made, we would be stunned and delighted, and would explore new models BSM to explain the phenomena. Thus, theoretical particle physics would make new progress.

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