Holographic superconductor on a novel insulator[*]

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Abstract: We construct a holographic superconductor model, based on a gravity theory, which exhibits novel metal-insulator transitions. We investigate the condition for the condensation of the scalar field over the parameter space, and then focus on the superconductivity over the insulating phase with a hard gap, which is supposed to be Mott-like. It turns out that the formation of the hard gap in the insulating phase benefits the superconductivity. This phenomenon is analogous to the fact that the pseudogap phase can promote the pre-pairing of electrons in high T_c cuprates. We expect that this work can shed light on understanding the mechanism of high T_c superconductivity from the holographic side.

Keywords: gauge/gravity duality, holographic gravity, holographic superconductor

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1   Introduction

Mott physics, a mechanism for metal-insulator transitions due to electron-electron interactions, plays a crucial role in understanding strongly correlated phenomena in a many-body system (see Refs. [1–4] for reviews). Typical examples include transition metal oxides like NiO and CoO, as well as the superconducting cuprates, fullerene compounds like C_{60} and C_{70}, and organic conductors [2, 3]. Most Mott materials exhibit a hard gap rather than a power law behavior of frequency dependence as for the soft gap, which can be observed in the spectral function of single particles as well as in the optical conductivity. The spectral weight transfer and the formation of the hard gap are viewed as two key features of Mott physics [4]. It is widely believed that the strong electron-electron correlation is responsible for these characteristics of Mott materials. This means that conventional methods developed using perturbation techniques are, unfortunately, ineffective. AdS/CFT correspondence, as a powerful tool which in the large N limit maps a strongly coupled quantum field theory to a weakly coupled gravitational theory, may provide a different viewpoint on these strongly correlated systems in condensed matter physics [5–8]. Several novel localization mechanisms have been proposed in recent years using this approach, by constructing different lattice structures as deformations of the bulk geometry [9–19]. In this way, Peierls insulators [9], polaron-localization insulators [10], and Mott-like insulators [11–13] as well as other novel insulating phases, have been implemented and many exciting properties have been observed [14–20], some of which resemble those found in strongly correlated electronic systems. AdS/CFT duality has provided an intuitive and geometric scenario for understanding the phenomena in strongly correlated electronic systems.

It is still challenging to understand the mechanism of high temperature superconductivity. In contrast to conventional superconductors, for which the normal state is metallic and the superconductivity emerges due to the

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formation of Cooper pairs by means of electron-phonon interactions, it is found that high temperature copper-oxide superconductors are strongly correlated electronic systems, which can be generated by doping a Mott insulator [1]. The mechanism of pairing electrons to form superconductivity in high T_c cuprates is still not clear, but it has been found that before entering the superconducting phase, some interesting phases with pseudogap and competing orders are involved. It is very desirable to explore the mechanism of high T_c superconductivity using a holographic approach. Until now, for most holographic models of superconductors in the literature, the normal state is either a perfect conductor [21–24], a metal [25] or an ordinary insulator [26]. Thus, to mimic high T_c superconductivity by holography it is essential to construct a holographic model dual to a Mott insulator at first. As the first step we have successfully proposed a two-gauge formalism in gravity theory and built a Mott-like insulator in Ref. [12], which is characterized by the emergence of a hard gap in the optical conductivity. This appealing progress stimulates us to further explore the superconductivity based on this Mott-like insulating phase in a holographic scenario, which is the main purpose of this work.

Our paper is organized as follows. In Section 2, we present the holographic setup for the construction of a superconductor in two-gauge formalism with a Q-lattice structure. Section 3 contains four subsections where the phase structure, the relation between the gap in the normal state and the critical temperature, and solutions to scalar condensation and optical conductivity in the superconducting phase are studied in detail. The conclusion and discussion are presented in Section 4.

2   Holographic setup

A holographic model which exhibits a novel metal-insulator transition with a Q-lattice structure has been investigated in Ref. [12]. To construct a superconductor based on this model, we introduce an additional charged scalar field with U(1) gauge symmetry into the system such that the action becomes

$$S = S_1 + S_2, \tag{1}$$

where S_1 and S_2 are, respectively,

$$S_1 = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{-g} \Big[R + 6 - |\nabla \Phi|^2 - m^2|\Phi|^2$$
$$- \frac{1}{4}F^2 - \frac{Z(\Phi)}{4}G^2 \Big], \tag{2}$$

$$S_2 = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{-g} \Big(-|D_\mu \Psi|^2 - M^2|\Psi|^2 \Big). \tag{3}$$

Notice that we have set the AdS radius $L = 1$. The action S_1 is proposed in Ref. [12], which contains back-

ground solutions dual to a Mott-like insulating phase. Φ is a neutral complex scalar field with mass m which is responsible for the breaking of translational symmetry in spatial directions, thus dubbed the Q-lattices [16]. $F = dA$, $G = dB$ are the curvatures of two U(1) gauge fields A and B, respectively. The B field is treated as the Maxwell field and we concentrate on its transport properties in this paper. $Z(\Phi) = (1 - \beta|\Phi|^2)^2$ with β being positive, which describes the interaction between the Q-lattice Φ and the Maxwell field B in bulk geometry. This action with two gauge fields plays a crucial role in obtaining an insulating phase with a hard gap. When the hard gap is present, the coupling term $Z \to 0$ on the horizon, which means that the effect of the term $\frac{Z(\Phi)}{4}G^2$ is not strong enough to deform the IR fixed point from AdS_2 geometry, which is dual to a metallic phase, to a new one dual to an insulating phase. Therefore the second gauge field term $F^2/4$ is introduced to obtain an insulating phase with a hard gap. The action S_2 supports a superconducting black brane whenever the U(1) gauge symmetry associated with B is spontaneously broken [21]. Ψ is the charged complex scalar field with mass M, which can be written as $\Psi = \psi e^{i\theta}$ with ψ being a real scalar field and θ a Stückelberg field. $D_\mu = \partial_\mu - ieB_\mu$ is the covariant derivative where e is the charge associated with the Maxwell field B. For convenience, we choose the gauge $\theta = 0$ and therefore S_2 can be rewritten as

$$S_2 = \frac{1}{2\kappa^2} \int \mathrm{d}^4x \sqrt{-g} \Big[-(\nabla_\mu \psi)^2 - (M^2 + e^2 B_\mu B^\mu)\psi^2 \Big]. \tag{4}$$

The equations of motion can be derived from the actions (2) and (4) as

$$R_{\mu\nu} - \Big(3 + \frac{1}{2}R\Big)g_{\mu\nu} + \frac{1}{2}(T^A_{\mu\nu} + T^B_{\mu\nu} + T^\Phi_{\mu\nu} + T^\psi_{\mu\nu}) = 0, \tag{5}$$

$$\nabla^\mu F_{\mu\nu} = 0, \tag{6}$$

$$\nabla^\mu \big[(1 - \beta|\Phi|^2)^2 G_{\mu\nu} \big] - 2e^2 B_\nu \psi^2 = 0, \tag{7}$$

$$\Big[\nabla^2 - m^2 + \frac{1}{2}\beta(1 - \beta|\Phi|^2)G^2 \Big] \Phi = 0, \tag{8}$$

$$\big[\nabla^2 - (M^2 + e^2 B^2) \big] \psi = 0, \tag{9}$$

where

$$T^A_{\mu\nu} = \frac{1}{4}g_{\mu\nu}F^2 - F_{\mu\rho}F_\nu{}^\rho, \tag{10}$$

$$T^B_{\mu\nu} = (1 - \beta|\Phi|^2)^2 \Big(\frac{1}{4}g_{\mu\nu}G^2 - G_{\mu\rho}G_\nu{}^\rho \Big), \tag{11}$$

$$T^\Phi_{\mu\nu} = -2\nabla_{(\mu}\Phi\nabla_{\nu)}\Phi^* + g_{\mu\nu}(|\nabla\Phi|^2 + m^2|\Phi|^2), \tag{12}$$

$$T^\psi_{\mu\nu} = -2(\nabla_\mu\psi\nabla_\nu\psi + e^2 B_\mu B_\nu \psi^2)$$
$$+ g_{\mu\nu}\big[(\nabla_\mu\psi)^2 + (M^2 + e^2 B_\mu B^\mu)\psi^2 \big]. \tag{13}$$

The gravitational dual of the normal phase ($\psi = 0$) has been numerically constructed in Ref. [12], in which

the ansatz is

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{z^2}\left[-(1-z)p(z)U\mathrm{d}t^2+\frac{\mathrm{d}z^2}{(1-z)p(z)U}\right.\\
&\quad \left.+V_1\mathrm{d}x^2+V_2\mathrm{d}y^2\right],\\
A &= \mu(1-z)a\mathrm{d}t,\\
B &= \mu(1-z)b\mathrm{d}t,\\
\Phi &= \mathrm{e}^{\mathrm{i}kx}z^{3-\Delta}\phi,\\
\psi &= 0,
\end{aligned}
\tag{14}
$$

where $p(z)=1+z+z^2-\mu^2z^3/4$ and the scaling dimension of the scalar field $\Delta=3/2+(9/4+m^2)^{1/2}$. All the functions U,V_1,V_2,a,b and ϕ depend on the radial coordinate z only. Throughout this paper we set $m^2=-2$ so that $\Delta=2$. For a given coupling parameter β, each black hole solution is specified by four scaling-invariant parameters, i.e., the Hawking temperature $\hat{T}/\mu$ with $\hat{T}=(12-\mu^2)\mathrm{U}(1)/16\pi$, the lattice amplitude $\hat{\lambda}/\mu^{3-\Delta}$ with $\hat{\lambda}=\phi(0)$, the wave vector $\hat{k}/\mu$ and $b_0\equiv b(0)$, where without loss of generality we have set $a(0)=1$. For convenience, these quantities are abbreviated as T, λ and k in what follows.

The phase structure for the normal state has been investigated in detail in Ref. [12]. We briefly review its main features over the parameter space (λ,k,β,b_0) in the low temperature region, which is essential for us to explore the superconducting phase in the current work. In the ordinary Q-lattice background (λ,k) with $\beta=0$ and a single gauge field, a novel metal-insulator transition has been found by changing the parameters (λ,k) in Ref. [16]. Later, the specific phase diagram over the (λ,k) plane was given in Refs. [26] and [27]. Qualitatively, one finds that the region with large k and small λ falls into the metallic phase, while the region with small k and large λ falls into the insulating phase. This rule is also consistent with Mott's thought experiment, since small wavenumber k implies a larger lattice constant such that it becomes harder for electrons to hop to their neighbor sites, leading to an insulating phase. However, for the above insulating phases the hard gap is absent, which has been justified in the plot of optical conductivity [16, 26]. Remarkably, after introducing the interaction term with β in two-gauge formalism, we found in Ref. [12] that a hard gap can eventually emerge with the increase of the parameter β for an insulating phase. Next, we intend to investigate the key features of superconductivity over such Mott-like insulating phases with hard gaps.

In the holographic approach, the superconducting phase is achieved by finding non-trivial solutions of the charged scalar hair. Once its back reaction to the background is taken into account, we need to numerically solve the equations of motion (5)–(9). Without loss of generality, let us set $M^2=-2$. Then, beyond any details of equations, it is easy to see that the asymptotical

behavior of ψ at infinity is

$$
\psi=z\psi_1+z^2\psi_2.
\tag{15}
$$

Here, we treat ψ_1 as the source and ψ_2 as the expectation value of the scalar operator in dual quantum field theory. Since we expect the condensation will emerge without being sourced, we set $\psi_1=0$.

3   Condensation and phase structure

In this section we demonstrate the main numerical results. Superconductivity phase structures will be discussed first. After that, a connection between the critical temperature for condensation and the formation of a gap in the normal state will be shown. Numerical solutions to the background with scalar hair are obtained explicitly and it is verified that the phase transition is second order. Finally, the frequency behavior of optical conductivity in the superconducting phase is investigated in detail.

3.1   Phase structure

The charged scalar is expected to condensate when the Hawking temperature of the black brane drops to some critical temperature T_c, which reflects the instability of the AdS background with a violation of the usual BF bound. The condition for the occurrence of condensation depends on the charge of the scalar field as well as the background which is specified by the system parameters. In this section we will analyze the condition for condensation and obtain the critical temperature T_c for different system parameters.

The critical temperature for the formation of the superconducting phase can be estimated by finding static normalizable modes of the charged scalar field on a fixed background (14), which has been described in detail in Refs. [25, 26]. To this end, one may turn this problem into a positive self-adjoint eigenvalue problem for e^2 and so we rewrite the equation of motion for the charged scalar field in Eq. (9) as

$$
-(\nabla^2+2)\psi=-e^2B^2\psi,
\tag{16}
$$

which can be numerically solved once a specific metric of the background is given. Figure 1 illustrates the charge of the scalar field as the function of the critical temperature T_c for some representative values of the system parameters. First, in general, T_c is always increasing with the charge e. This tendency is the same as that in other holographic models as described in Refs. [25, 26]. It indicates that the increase of the charge makes the condensation easier. We shall fix $e=10$ throughout this paper without loss of generality. Second, we observe that T_c increases with β (b_0) when the other parameters are fixed (Fig. 1(a) and (b)) but T_c may be a non-monotonic function of λ or k (Fig. 1(c)), which implies there are

some interesting phenomena worth exploring. Thus we plot the critical temperature T_c as a function of lattice parameters (λ, k) in Fig. 2 and summarize our main observations as follows.

1) For large k, we find T_c rises and approximately saturates to a constant, which coincides with the phenomenon observed in the simplest superconductor model on the Q-lattice [26]. This is not surprising since in that region the hard gap is disappearing or the system simply enters a metallic phase. Also, when k is large the lattice effect is suppressed, and hence the system is similar to condensation on the AdS-RN background.

One remarkable phenomenon is observed in the small k region (Fig. 2(a) and (c)). That is, T_c rises as the wavenumber k decreases, with the other parameters fixed, which is contrary to the tendency observed in previous holographic superconductor model on the Q-lattice [26], in which the critical temperature always goes down with the decrease of wavenumber k until it approaches zero at tiny k, implying that no condensation could take place in such deep insulating phases [26]. This difference is significant because it indicates that in the current setup the superconductivity becomes *easier* as the system enters a deep insulating phase. This difference results from the involvement of the coupling term with non-zero β. In particular, the larger β is, the higher T_c is, as shown in Fig. 2(a). Physically, we know that this term is responsible for the formation of a hard gap when the system falls into an insulating phase. Therefore, we argue that the presence of a hard gap in the

insulating phase benefits the condensation of the scalar hair, reminiscent of the phenomenon observed in copper-oxide superconductors where the presence of a pseudogap causes the pre-pairing of electrons so as to make the superconductivity easier [28]. We have checked that this phenomenon is always observed in the small k region under the condition that the system falls into an insulating phase with non-zero β, independent of the values of the other parameters. We will further verify the connection between the formation of the hard gap and the superconductivity in the next subsection.

Interestingly, the critical temperature with vanishing k is much higher than that saturated constant for large k which would dual to a metallic phase. This phenomenon definitely deserves further investigation in future.

2) It is also interesting to plot T_c as a function of λ with other parameters fixed (Fig. 2(b) and (c)). For large k, T_c decreases monotonically with the increase of λ, as found in the simplest superconductor model on the Q-lattice [26]. For small k, however, T_c rises as λ increases! As we mentioned in the previous section, large λ always points to an insulating phase, but the key difference is that a hard gap presents in the small k region. Furthermore, for a fixed k with small value, we find the larger β makes the hard gap more evident, such that the corresponding T_c is higher.

Next we explicitly demonstrate that the formation of hard gap indeed leads to a higher critical temperature of superconductivity, by calculating the optical conductivity in the insulating phase.
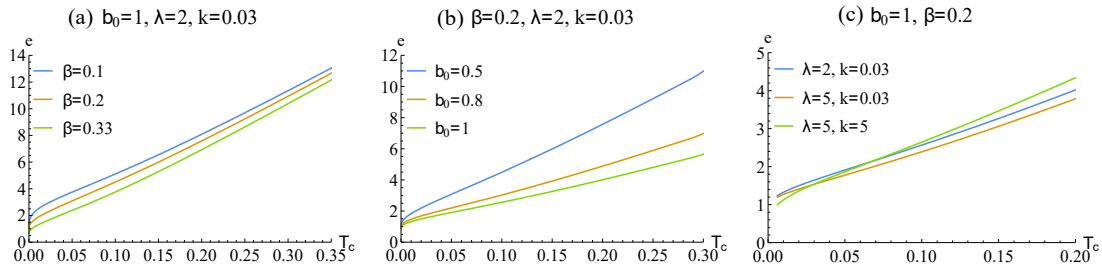

Fig. 1.   (color online) The charge of the scalar field as a function of the critical temperature T_c for some representative values of the system parameters.
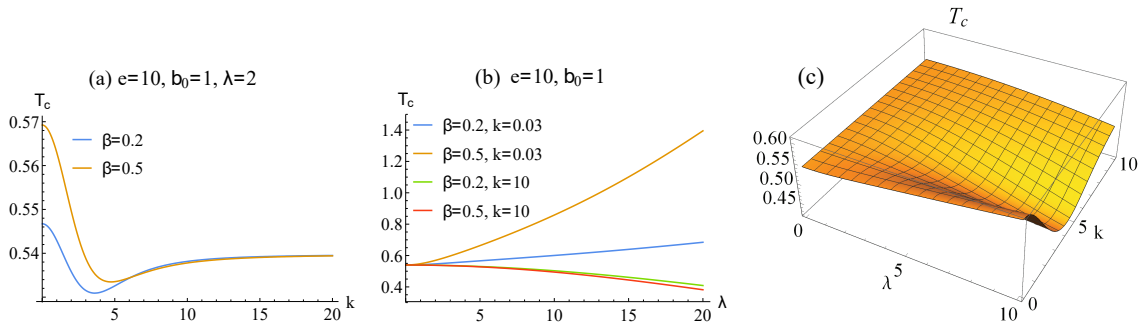

Fig. 2.   (color online) The critical temperature T_c as a function of system parameters: (a) the relation between T_c and k for selected parameters; (b) the relation between T_c and λ for selected parameters; and (c) a 3D plot of T_c as a function of λ and k for $b_0=1$ and $\beta=0.2$.

3.2 Critical temperature and gap in the normal state

In this subsection we will visualize our above statement, arguing that the presence of a hard gap in the insulating phase will make the condensation easier.

The connection between the formation of the gap and the formation of the condensation can be investigated by examining how critical temperature behaves when the gap in optical conductivity emerges with the variation of certain parameters. In what follows we fix $b_0 = 0.5$, *i.e.*, the chemical potential dual to the Maxwell field B, which means that we essentially work in the grand canonical ensemble. After that, we examine explicitly the relation between the gap and the condensation when changing system parameters β, λ, and k.

The optical conductivity of dual quantum field theory can be calculated by a linear perturbation theory in bulk geometry. To this end, we turn on the following self-consistent perturbations

$$\delta A_x = a_x(z) \mathrm{e}^{-\mathrm{i}\omega t}, \quad \delta B_x = b_x(z) \mathrm{e}^{-\mathrm{i}\omega t},$$
$$\delta g_{\mathrm{tx}} = h_{tx}(z) \mathrm{e}^{-\mathrm{i}\omega t}, \quad \delta \Phi = i \mathrm{e}^{\mathrm{i}kx} z^{3-\Delta} \varphi(z) \mathrm{e}^{-\mathrm{i}\omega t}. \quad (17)$$

Once the background solution is obtained, we can numerically solve the corresponding linearized perturbation equations with variables $(a_x(z), b_x(z), h_{tx}(z), \varphi(z))$ with the ingoing boundary conditions at the horizon, and read off the optical conductivity in response to the B field along the x-direction in terms of

$$\sigma(\omega) = \frac{\partial_z b_x(z)}{i\omega b_x(z)} \bigg|_{z=0}. \quad (18)$$

There are some key points to consider in the analysis:

1) We are only interested in the electrical response to the Maxwell field B, therefore on the boundary only $b_x(0)$ is turned on, with the other gauge field perturbation $a_x(0)$ set to 0.

2) To ensure that we are extracting the current-current correlator, the perturbations are required to satisfy an additional boundary condition $\varphi(0) - ik\lambda h_{tx}(0)/\omega = 0$, obtained from diffeomorphism and gauge transformation [16].

We demonstrate the numerical results of the phase diagram and the optical conductivity in Fig. 3. Figure 3(a) and (b) show that T_c increases with β, while the gap in optical conductivity becomes more evident as β increases. When varying parameter λ, similar phenomena are observed, as can be seen from Fig. 3(c) and (d). Finally, T_c increases with the decrease of k, and indeed the gap becomes more pronounced when k decreases, as shown in Fig. 3(e) and (f). To sum up, T_c increases when the gap becomes more evident. The gap in optical conductivity resembles the role of the pseudogap in the cuprates phase diagram.

We conclude that the presence of the hard gap in the insulating phase makes the condensation easier, indicating that at least in a holographic regime this kind of insulating phase with a hard gap is qualitatively different from all the insulating phases without a hard gap, as found in previous studies.

3.3 Condensation

So far, all the above discussions on the conditions of condensation are just based on the solutions to the eigenvalue problem of e^2 as described in Eq. (16), where each background is fixed without the condensation of scalar hair. Such an approximation is good enough for us to estimate the critical temperature T_c. Now to explicitly construct a background with scalar hair, we need to directly solve the coupled EOMs (5)-(9) with the ansatz (14), which can be done numerically with the standard spectral method. At the end of this section we demonstrate a result for the condensation of scalar hair $\sqrt{\langle O_2 \rangle}/T_c$ as a function of the temperature T/T_c in Fig.4.

In conventional BCS theory, the superconductivity phase transition is second order, which is described by $\sqrt{\langle O \rangle} \sim t^{1/4}$ where $t \equiv (1 - T/T_c)$. It is interesting to examine the order of the phase transitions demonstrated in our model. As a representative example, we show $\frac{\mathrm{dlog}(\sqrt{O_2}/\mu)}{\mathrm{dlog}(t)}$ vs $\log(t)$ as the inset in Fig. 4(a). It is readily seen that the superconductivity phase transition in our model is also second order, since $\frac{\mathrm{dlog}(\sqrt{O_2}/\mu)}{\mathrm{dlog}(t)} \sim 0.25$ uniformly.

3.4 Optical conductivity in the superconducting phase

In this subsection we investigate the frequency dependent behavior of the optical conductivity in the superconducting phase, with a focus on the influence of the hard gap in the insulating phase before the condensation takes place.

First, we illustrate the process of phase transition with the behavior of the optical conductivity. When the system transits from a normal phase to a superconducting phase, the DC conductivity changes from a finite value into a δ function. Thanks to the lattice structure which breaks the translational invariance in our formalism, this process now can be observed manifestly in optical conductivity. Nevertheless, a divergent DC conductivity cannot be captured by taking the limit $\omega \rightarrow 0$. Instead, the δ function can be reflected by $\mathrm{Im}\sigma(\omega) \sim \omega^{-1}$ in the low frequency region, in light of the Kramers-Kronig relations. Figure 5(a) demonstrates $\sigma(\omega)$ at four different temperatures across the phase transitions. The critical temperature $T_c \simeq 0.606$ at parameter $\beta = 4, k = 0.03, \lambda = 2, b_0 = 0.5, e = 10$. For insulating phases with $T = 0.632, 0.6062$, which are above T_c, it is found that in the low frequency region $\mathrm{dlogIm}\sigma(\omega)/\mathrm{dlog}(\omega) \sim 1$. This is expected since the
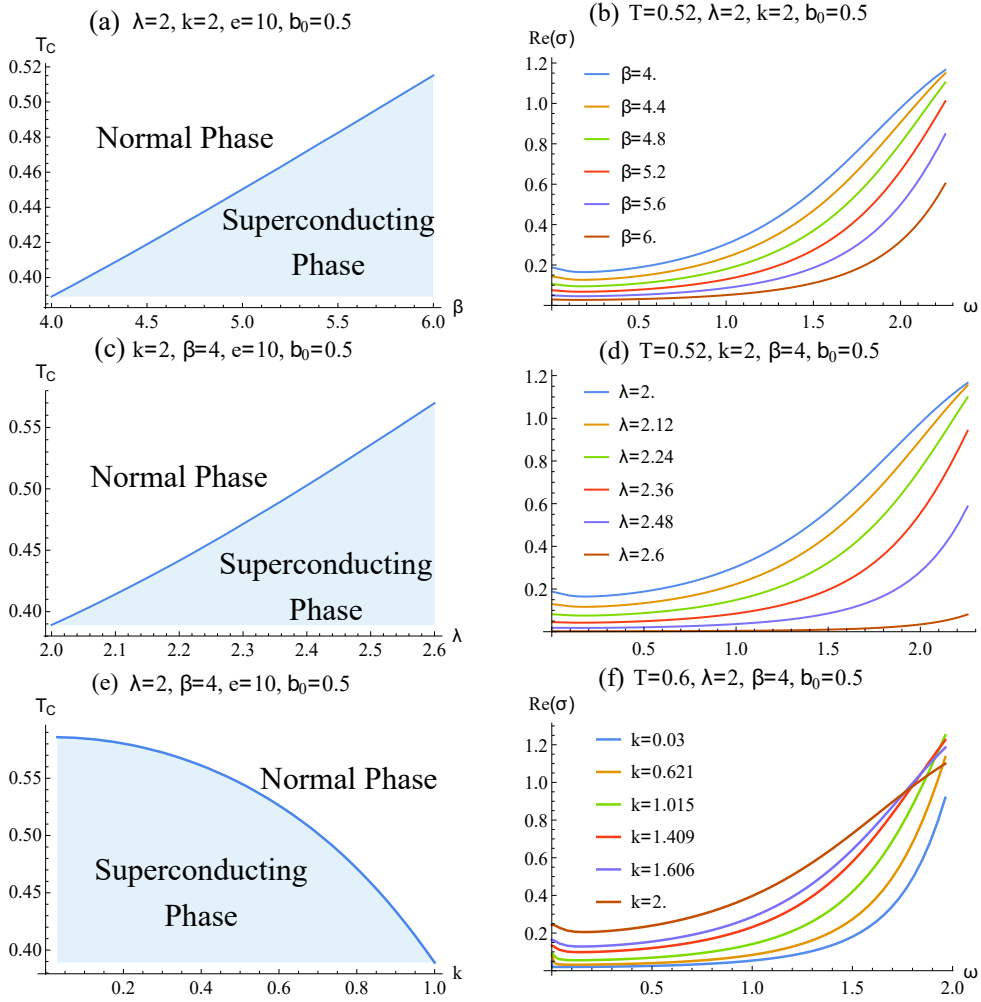
Fig. 3.   (color online) (a) The phase diagram for parameter β; (b) Re$\sigma(\omega)$ as a function of ω for several parameters of β; (c) the phase diagram for parameter λ; (d) Re$\sigma(\omega)$ as a function of ω for several parameters of λ; (e) the phase diagram for parameter k; (f) Re$\sigma(\omega)$ as a function of ω for several parameters of k.
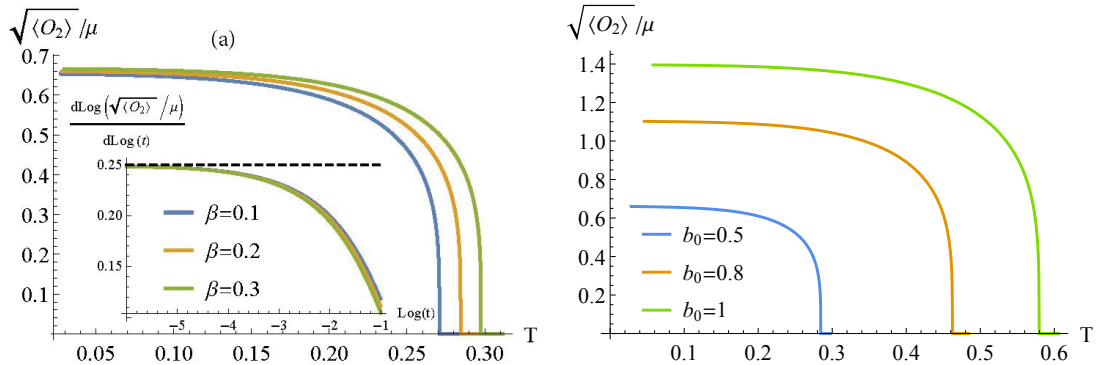

Fig. 4.   (color online) (a) The value of the condensation as a function of temperature with the given parameters $\lambda=2$, $k=0.03$, $b_0=0.5$, $e=10$ and several values of β. The inset is $\frac{\mathrm{d}\log(\sqrt{\langle O_2\rangle}/\mu)}{\mathrm{d}\log(t)}$ vs $\log(t)$ and the black dashed line is $\frac{\mathrm{d}\log(\sqrt{\langle O_2\rangle}/\mu)}{\mathrm{d}\log(t)}=0.25$. (b) The value of the condensation as a function of temperature with the given parameters $\lambda=2$, $k=0.03$, $\beta=0.2$, $e=10$ and several values of b_0.

imaginary part of Drude conductivity $\mathrm{Im}\sigma \sim \omega$[1]. For $T = 0.0612, 0.5985,$ which are below T_c, $\mathrm{dlogIm}\sigma(\omega)/\mathrm{dlog}(\omega) \sim -1$, i.e. $\mathrm{Im}\sigma(\omega) \sim \omega^{-1}$. Therefore, the different scaling behavior of $\mathrm{Im}\sigma$ clearly demonstrates the process of phase transition.

Second, we are interested in how the gap behaves with respect to the temperature below T_c. It is well-known in BCS theory, as well as in many holographic models, that the gap becomes more and more pronounced when the temperature drops. Figure 5(b) shows the real part of optical conductivity at several temperatures below T_c. It is easily seen that the gap becomes more and more evident when the temperature drops. This phenomenon is in agreement with both BCS theories and many other holographic superconductivity models.
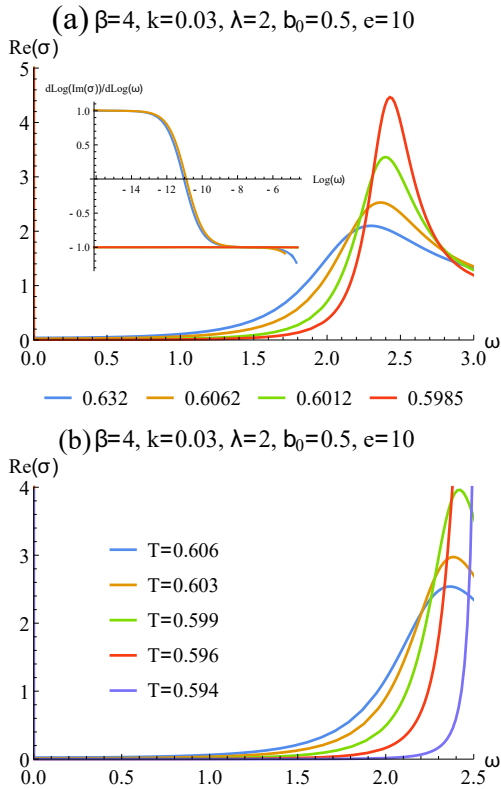

Fig. 5. (color online) (a) $\mathrm{Re}\sigma(\omega)$ at four different temperatures specified by the plot legend. The inset is $\mathrm{dlogIm}\sigma(\omega)/\mathrm{dlog}(\omega)$. (b) $\mathrm{Re}\sigma(\omega)$ at several temperatures below T_c.

4   Discussion

In this paper we have constructed a novel holographic superconductor based on a gravity theory with Q-lattices, in which an interacting term characterized by parameter β is involved such that a hard gap can

be observed in the insulating phase. In contrast to all the previous holographic models, in which the insulating phase suppresses the condensation of the scalar hair and the critical temperature becomes lower in comparison with that in the metallic phase, we have found that with the emergence of the hard gap, the insulating phase will benefit the condensation and the critical temperature becomes higher whenever the hard gap becomes more evident. The phase transition from an insulating phase to a superconducting phase has been explicitly justified by examining the behavior of the imaginary part of optical conductivity. All above phenomena imply that the hard gap in this model resembles the role of the pseudogap in cuprate phase diagrams. Therefore, this work may provide valuable insights into the mechanism of high T_c cuprates from the holographic side.

We believe that the fact of the presence of the hard gap in the insulating phase benefiting the superconductivity should be robust and general in the holographic approach, in the sense that it does not depend on the specific choice of parameter values in this model, nor on the specific formalism of the setup. Basically, we think the formation of a hard gap in the optical conductivity implies that more electrons are pre-paired in the dual field theory, such that the instability of the system can be induced at a higher temperature, leading to the condensation of the scalar hair.

The current setup in our model has one weak point that should be mentioned. As we have noticed and pointed out in Ref. [12], the existence of background solutions is strongly constrained by the values of the parameters. In particular, whenever an insulating phase with a hard gap is achieved, the bulk geometry dual to a superconducting phase can be obtained only in some region below the critical temperature. Numerical solutions do not exist when the temperature is decreased further. Although we could adjust the charge e as well as the other parameters to have solutions in an arbitrarily low temperature region, not all the parameters guarantee the existence of solutions in the zero temperature limit. Mathematically, it is not rare to encounter a gravitational system without domain wall solutions in certain ranges of parameters or boundary conditions [29]. From the dual theory side, when the hard gap is formed, the optical conductivity tends to vanish in the low frequency region. Further increasing β or decreasing the temperature would lead to negative conductivity, which is of course unphysical. In our model this unphysical phenomenon is not encountered due to the absence of solutions to the Einstein equations for these parameters. It is still desirable to obtain a system solvable for any given parameters and which exhibits all the interesting

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1) For both metallic and insulating phases, the coherent and incoherent behavior of the conductivity have been discussed in Ref. [12]. For an insulating phase the zero-frequency behavior of conductivity is not exactly Drude, but this linear relation with ω still holds.

behaviors revealed in our current work. We expect to improve this by adjusting the setup in the holographic models.

Along this direction, there is much worthy of further investigation. One avenue is to investigate the spectral function of a probe such as holographic fermions to see if the two fundamental features of Mott insulator could be observed, namely, the hard gap and the spectral weight transfer, and then observe its variations during the superconducting phase transition. Since the fermionic field only feels the bulk geometry as a probe, its spectral function would be very sensitive to the specific couplings of the fermion and background. We hope to explore this issue in the near future.

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