

Multiplicity fluctuation and correlation of mesons and baryons in ultra-relativistic heavy-ion collisions at the LHC^{*}

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Abstract: We study the multiplicity fluctuation and correlation of identified mesons and baryons formed at hadronization by the quark combination mechanism in the context of ultra-relativistic heavy-ion collisions. Based on the statistical method of free quark combination, we derive the two-hadron multiplicity correlations, including meson-meson and meson-baryon correlations, and take the effects of quark number fluctuation at hadronization into account by a Taylor expansion method. After including the decay contributions, we calculate the dynamical fluctuation observable ν_{dyn} for $K\pi$, $p\pi$ and Kp pairs and discuss what underlying physics can be obtained by comparing with data from Pb-Pb collisions at $\sqrt{s_{\text{NN}}}=2.76$ TeV and simulations from the HIJING and AMPT event generators.

Keywords: fluctuation and correlation, hadronization, quark combination, relativistic heavy ion collisions

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1 Introduction

At sufficiently high temperature and/or energy density, strongly interacting matter will undergo a phase transition from hadronic matter to a state in which quarks and gluons are not confined, the quark gluon plasma (QGP) [1]. Relativistic heavy-ion collisions serve as a laboratory to experimentally study the properties of QGP [2]. Dynamical fluctuations and correlations of (multi-)particle production carry important information on reaction dynamics, and are often used to study the properties of the phase transition between hadronic and partonic matter as well as the QCD critical point [3–20].

The multiplicity of the hadrons produced is one of the most basic quantities which reflects the reaction dynamics. The experimental data of event-averaged multiplicity of hadrons not only, by virtue of statistical models, give information on the volume and temperature of the system at chemical freeze-out [21–23], but also reveal the microscopic hadronization mechanism of the bulk quark system [24, 25]. Fluctuations of hadron multiplicities, and in particular, those of multiplicity ratios, carry sophisticated information about the dynamical properties of the hot quark matter, and (in particular) information about the confinement phase transition [13, 26, 27]. Many measurements of event-by-event particle ratio fluctuations have been carried out by the NA49 collabo-

ration in Pb-Pb collisions at the CERN Super Proton Synchrotron (SPS) [8–10], by the STAR collaboration in Au+Au collisions at the BNL Relativistic Heavy Ion Collider (RHIC) [11, 12], and by the ALICE collaboration in Pb-Pb collisions at the CERN Large Hadron Collider (LHC) [28].

We note that the available theoretical explanations of these data are usually based on thermal/statistical models or on direct simulations of popular event generators [14–19]. Explanations and predictions from different models of hadron production at different stages are needed, which will reveal the underlying physics of the experimental data from different viewpoints. In this paper, we study the multiplicity fluctuation and correlation of mesons and baryons created from hadronization using a quark (re-)combination mechanism. We focus on the effects of quark combination itself and those of quark number fluctuations at hadronization, and derive the multiplicity fluctuation and correlation of mesons and baryons in the quark combination mechanism. Applying our formulas, we calculate a dynamical fluctuation observable ν_{dyn} [26, 27] and discuss what underlying physics can be obtained by comparing with the data in Pb-Pb collisions at $\sqrt{s_{\text{NN}}}=2.76$ TeV and simulations from the HIJING and AMPT event generators [28]. Here, we only study the situation of zero baryon number density, in which the inputs or parameters are relatively few and

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easily fixed. Studies of Au+Au collisions at RHIC energies are left to forthcoming work.

The paper is organized as follows. Section 2 gives a detailed formulation for the inclusive multiplicity of identified hadrons and two-hadron multiplicity correlation in QCM. Section 3 shows how to include the effects of quark number fluctuation and correlation at hadronization. Section 4 takes the decay effects into account. Section 5 gives the numerical results of a dynamical quantity ν_{dyn} and discussions of different contributions, and a comparison with the experimental data and event generators. Section 6 summarises the work.

2 Hadronic multiplicity and multiplicity correlation in quark combination mechanism

In this section, we study the hadronic multiplicity and two-hadron multiplicity correlation in the quark combination mechanism (QCM). In QCM, a hadron is produced at hadronization by the combination of constituent quarks and antiquarks neighboring in phase space. These constituent quarks and antiquarks serve as the effective degrees of freedom for the strong-coupling quark-gluon system at hadronization. The mechanism has been applied with fair phenomenological success in explaining the data of hadronic transverse-momentum spectra, yields and longitudinal rapidity distributions [24, 25, 29–39]. In this paper, we do not intend to discuss the space-time details of the combination as done in Refs. [31, 32, 34, 38], but concentrate on the multiplicity properties of the identified hadrons based on a quark statistical approach with the effective constituent quark degrees of freedom.

2.1 Multiplicity of identified hadrons

For the hadronization of a quark system with given numbers of quarks and antiquarks denoted by N_f , with $f=u, d, s, \bar{u}, \bar{d}, \bar{s}$, we follow our previous work [40] and write the average multiplicities of identified hadrons directly produced by quark combination in the following form

$$\bar{N}_{B_i} = N_{B_i}^{(q)} \mathcal{P}_{q_1 q_2 q_3 \rightarrow B_i}, \quad (1)$$

$$\bar{N}_{M_i} = N_{M_i}^{(q)} \mathcal{P}_{q_1 \bar{q}_2 \rightarrow M_i}, \quad (2)$$

where $N_{B_i}^{(q)} = N_{\text{iter}} \prod_f \prod_{j=1}^{n_{f, B_i}} (N_f - j + 1)$ is the combination number of three quarks with specific flavors relating to B_i formation. n_{f, B_i} is the number of constituent quarks f contained in baryon B_i . N_{iter} is the iteration factor and is taken to be 1, 3, and 6 for the cases of three identical flavor, two different flavors and three different flavors contained in a baryon, respectively. Examples $N_p^{(q)} = 3N_u(N_u - 1)N_d$ and $N_{\Omega^-}^{(q)} = N_s(N_s - 1)(N_s - 2)$ show the evaluation of $N_{B_i}^{(q)}$. $\mathcal{P}_{q_1 q_2 q_3 \rightarrow B_i}$ denotes the

combination probability of $q_1 q_2 q_3 \rightarrow B_i$. The meson formula is similar. The combination number of specific quark-antiquark pairs for M_i formation is $N_{M_i}^{(q)} = \sum_k \omega_k \prod_f \prod_j^{n_{f, M_i, k}} (N_f - j + 1)$. Here, index f runs over all flavors of quarks and antiquarks. Index k takes into account the case of the flavor mixing for some mesons, e.g., π^0 is composed by $u\bar{u}$ and $d\bar{d}$ with equal weight $1/2$. k runs over all channels of flavor mixing and ω_k is the weight of the k -th channel. $n_{f, M_i, k}$ is the number of constituent (anti-)quarks f in the k -th channel and is taken to be 1 or 0. $\mathcal{P}_{q_1 \bar{q}_2 \rightarrow M_i}$ denotes the combination probability of $q_1 \bar{q}_2 \rightarrow M_i$.

The combination probabilities $\mathcal{P}_{q_1 q_2 q_3 \rightarrow B_i}$ and $\mathcal{P}_{q_1 \bar{q}_2 \rightarrow M_i}$ can be determined with a few parameters:

$$\mathcal{P}_{q_1 q_2 q_3 \rightarrow B_i} = C_{B_i} \frac{\bar{N}_B}{N_{qqq}}, \quad (3)$$

$$\mathcal{P}_{q_1 \bar{q}_2 \rightarrow M_i} = C_{M_i} \frac{\bar{N}_M}{N_{q\bar{q}}}. \quad (4)$$

Here, $\bar{N}_B = \sum_j \bar{N}_{B_j}$ is the average number of total baryons and $N_q = \sum_f N_f$ is total quark number. $N_{qqq} = N_q(N_q - 1)(N_q - 2)$ denotes the total possible number of three-quark combinations for baryon formation. Considering the flavor independence of the strong interaction, \bar{N}_B/N_{qqq} can be used to approximately denote the average probability of three quarks combining into a baryon. Factor C_{B_i} denotes the probability of forming B_i for a given $q_1 q_2 q_3$ combination which is already known to form a baryon. Equation (4) for mesons is similar. $\bar{N}_M = \sum_j \bar{N}_{M_j}$ is the average number of total mesons. $N_{\bar{q}}$ is the total number of antiquarks and $N_{q\bar{q}} = N_q N_{\bar{q}}$ is the total number of quark-antiquark pairs for meson formation. $\bar{N}_M/N_{q\bar{q}}$ is used to approximately denote the average probability of a quark and antiquark combining into a meson. C_{M_i} is the probability of forming M_i for a given flavor $q_1 \bar{q}_2$ combination which is already known to form a meson.

In this paper, we only consider the production of ground state $J^P = 0^-, 1^-$ mesons and $J^P = (1/2)^+, (3/2)^+$ baryons in the flavor SU(3) group. For mesons, we introduce a parameter $R_{V/P}$ to represent the ratio of the $J^P = 1^-$ vector mesons to the $J^P = 0^-$ pseudoscalar mesons of the same flavor composition, and we have

$$C_{M_i} = \begin{cases} \frac{1}{1+R_{V/P}} & \text{for } J^P = 0^- \text{ mesons} \\ \frac{R_{V/P}}{1+R_{V/P}} & \text{for } J^P = 1^- \text{ mesons.} \end{cases} \quad (5)$$

Similarly, we introduce a parameter $R_{O/D}$ to represent the ratio of $J^P = (1/2)^+$ octet to the $J^P = (3/2)^+$ decuplet baryons of the same flavor composition, and we

obtain

$$C_{B_i} = \begin{cases} \frac{R_{O/D}}{1+R_{O/D}} & \text{for } J^P = \left(\frac{1}{2}\right)^+ \text{ baryons} \\ \frac{1}{1+R_{O/D}} & \text{for } J^P = \left(\frac{3}{2}\right)^+ \text{ baryons,} \end{cases} \quad (6)$$

except that $C_\Lambda = C_{\Sigma^0} = R_{O/D}/(1+2R_{O/D})$, $C_{\Sigma^{*0}} = 1/(1+2R_{O/D})$, $C_{\Delta^{++}} = C_{\Delta^-} = C_{\Omega^-} = 1$. Parameters $R_{V/P}$ and $R_{O/D}$ are set to be 0.45 and 2.5, respectively. The number of constituent quarks is conserved at hadronization, which means

$$\overline{N}_M + 3\overline{N}_B = N_q, \quad (7)$$

$$\overline{N}_M + 3\overline{N}_{\bar{B}} = N_{\bar{q}}. \quad (8)$$

In Ref. [25], we obtained the empirical solution of \overline{N}_M , \overline{N}_B and $\overline{N}_{\bar{B}}$ for the hadronization of large quark system, which was tested against the RHIC data. As the net baryon number is negligible at LHC energies, the formula of baryon number is simply $\overline{N}_B \approx \overline{N}_{\bar{B}} \approx N_q/15$ and meson number is obtained by the above quark number conservation. In addition, the conservation of specific quark flavor is also satisfied,

$$\sum_{\alpha} n_{f,\alpha} \overline{N}_{\alpha} = N_f. \quad (9)$$

Index α denotes a hadron of kind α and $f = u, d, s, \bar{u}, \bar{d}, \bar{s}$ denotes the flavor of quarks and antiquarks.

2.2 Two-hadron multiplicity correlations

2.2.1 Two-baryon correlation

We start from the pair production of two baryons B_i and B_j

$$\overline{N}_{B_i B_j} = C_{B_i} C_{B_j} N_{B_i B_j}^{(q)} \frac{\overline{N}_B(N_B-1)}{N_{6q}}. \quad (10)$$

Here, $N_{B_i B_j}^{(q)}$ is the possible cluster number of six specific quarks relating to $B_i B_j$ joint formation, and is evaluated as $N_{B_i B_j}^{(q)} = N_{iter, B_i} N_{iter, B_j} \prod_f \prod_{k=1}^{n_{f, B_i} + n_{f, B_j}} (N_f - k + 1)$ where f runs over all quark flavors. $N_{6q} = \prod_{i=1}^6 (N_q - i + 1)$ is the total possible cluster number of six quarks. $\overline{N}_B(N_B-1)$ is the number of two-baryon pairs and $\overline{N}_B(N_B-1)/N_{6q}$ gives the average probability of six quarks combining into two baryons. We rewrite the term $C_{B_i} C_{B_j} N_{B_i B_j}^{(q)}/N_{6q} = P_{B_i} P_{B_j} (1 - A_{B_i B_j})$ where $P_{B_i} = \overline{N}_{B_i}/\overline{N}_B$ denotes the fraction of baryon i in total baryon production. $A_{B_i B_j}$ is a small quantity of the magnitude $\mathcal{O}(N_f^{-1})$. Using the relation $\overline{N}_B(N_B-1) = \overline{\sigma}_B^2 +$

$\overline{N}_B(\overline{N}_B-1)$, we have

$$\begin{aligned} \overline{C}_{B_i B_j} &= \overline{N}_{B_i} \overline{N}_{B_j} - \overline{N}_{B_i} \overline{N}_{B_j} \\ &= \overline{N}_{B_i B_j} + \delta_{i,j} \overline{N}_{B_i} - \overline{N}_{B_i} \overline{N}_{B_j} \\ &= P_{B_i} \overline{N}_B (\delta_{i,j} - P_{B_j}) \\ &\quad + P_{B_i} P_{B_j} [(1 - A_{B_i B_j}) \overline{\sigma}_B^2 - A_{B_i B_j} \overline{N}_B (\overline{N}_B - 1)]. \end{aligned} \quad (11)$$

The first term on the right-hand side is the result of a binomial distribution and is the leading term in the two-baryon multiplicity correlations. The second term on the right-hand side is quite small relative to the former. For more detailed discussions and numerical results on the above two-baryon correlation, we refer readers to Ref. [41]. The variance of total baryons $\overline{\sigma}_B^2$ is not determined analytically at present and we adopt a parameterization $\overline{\sigma}_B^2 \approx 0.36 \overline{N}_B$, according to the simulation of a quark combination model developed by the Shandong Group [24, 35].

For the baryon-antibaryon multiplicity correlation, we firstly write the pair production of baryon-antibaryon as

$$\overline{N}_{B_i \bar{B}_j} = C_{B_i} C_{\bar{B}_j} N_{B_i \bar{B}_j}^{(q)} \frac{\overline{N}_B \overline{N}_{\bar{B}}}{N_{3q3\bar{q}}}, \quad (12)$$

where $N_{B_i \bar{B}_j}^{(q)}$ is the possible number of clusters of three quarks and three antiquarks relating to $B_i \bar{B}_j$ joint formation, and is evaluated as $N_{B_i \bar{B}_j}^{(q)} = N_{iter, B_i} N_{iter, \bar{B}_j} \prod_f \prod_{k=1}^{n_{f, B_i} + n_{f, \bar{B}_j}} (N_f - k + 1)$, where index f runs over all flavors of quarks and antiquarks. $N_{3q3\bar{q}} = N_{qqq} N_{\bar{q}\bar{q}\bar{q}}$ and $\overline{N}_B \overline{N}_{\bar{B}} = \overline{\sigma}_B^2 + \overline{N}_B \overline{N}_{\bar{B}}$. Using the notation P_{B_i} and $P_{\bar{B}_j}$, we have

$$\overline{C}_{B_i \bar{B}_j} = \overline{N}_{B_i} \overline{N}_{\bar{B}_j} - \overline{N}_{B_i} \overline{N}_{\bar{B}_j} = P_{B_i} P_{\bar{B}_j} \overline{\sigma}_B^2. \quad (13)$$

2.2.2 Two-meson correlation

The pair production of two mesons M_i and M_j is written as

$$\overline{N}_{M_i M_j} = C_{M_i} C_{M_j} N_{M_i M_j}^{(q)} \frac{\overline{N}_M(N_M-1)}{N_{2q2\bar{q}}}. \quad (14)$$

Here $N_{M_i M_j}^{(q)}$ is the possible number of clusters of two specific quarks and two specific antiquarks relating to two-meson joint formation, and is evaluated as $\sum_{kl} \omega_k \omega_l \prod_f \prod_{m=1}^{n_{f, M_i, k} + n_{f, M_j, l}} (N_f - m + 1)$, where index f runs over all flavors of quarks and antiquarks. $N_{2q2\bar{q}} = N_q(N_q-1)N_{\bar{q}}(N_{\bar{q}}-1)$ is the total number of all $qq\bar{q}\bar{q}$ combinations. $\overline{N}_M(N_M-1)$ is the number of two-meson pairs and $\overline{N}_M(N_M-1)/N_{2q2\bar{q}}$ is the average probability of two quarks and two antiquarks combining into two mesons.

We rewrite the term $C_{M_i} C_{M_j} \frac{N_{M_i M_j}^{(q)}}{N_{2q2\bar{q}}} = P_{M_i} P_{M_j} (1 - A_{M_i M_j})$ where $P_{M_i} = \overline{N}_{M_i}/\overline{N}_M$ denotes the fraction of meson i in total meson production. $A_{M_i M_j}$ is a small quantity of the magnitude $\mathcal{O}(N_f^{-1})$. Using $\overline{N}_M(N_M-1) =$

$\bar{\sigma}_M^2 + \bar{N}_M(\bar{N}_M - 1)$, we have

$$\begin{aligned} \bar{C}_{M_i M_j} &= \overline{N_{M_i} N_{M_j}} - \bar{N}_{M_i} \bar{N}_{M_j} \\ &= P_{M_i} \bar{N}_M (\delta_{i,j} - P_{M_j}) \\ &\quad + P_{M_i} P_{M_j} [(1 - A_{M_i M_j}) \bar{\sigma}_M^2 - A_{M_i M_j} \bar{N}_M (\bar{N}_M - 1)]. \end{aligned} \quad (15)$$

This is quite similar to the two-baryon correlations in Eq. (11). The first term on the right-hand side is the form of the binomial distribution. The second term on the right-hand side is related to the fluctuation of global meson production and the effects of finite quark numbers by the $A_{M_i M_j}$ coefficient. We notice that the influence of the second term on two-meson correlation is obvious for some hadron species.

2.2.3 Baryon-meson multiplicity correlation

A similar procedure is applied to baryon-meson correlations. We firstly write the pair production of a baryon and a meson as

$$\bar{N}_{B_i M_j} = C_{B_i} C_{M_j} N_{B_i M_j}^{(q)} \frac{\overline{N_B N_M}}{N_{4q\bar{q}}}, \quad (16)$$

where $N_{B_i M_j}^{(q)}$ is the possible number of clusters of four specific quarks and an antiquark relating to $B_i M_j$ joint formation, and is evaluated as $N_{B_i M_j}^{(q)} = N_{iter, B_i} \sum_k \omega_k \prod_f \prod_{m=1}^{n_f, B_i + n_f, M_j, k} (N_f - m + 1)$. $N_{4q\bar{q}} = N_q(N_q - 1)(N_q - 2)(N_q - 3)N_{\bar{q}}$ is the total number of clusters of four quarks and an antiquark. We rewrite $\overline{N_B N_M} = -3\bar{\sigma}_B^2 + \bar{N}_B \bar{N}_M$ using the unitarity $\bar{N}_M + 3\bar{N}_B = N_q$ and rewrite the term $C_{B_i} C_{M_j} \frac{N_{B_i M_j}^{(q)}}{N_{4q\bar{q}}} = P_{B_i} P_{M_j} (1 - A_{B_i M_j})$, and we obtain

$$\begin{aligned} \bar{C}_{B_i M_j} &= \overline{N_{B_i} N_{M_j}} - \bar{N}_{B_i} \bar{N}_{M_j} \\ &= P_{B_i} P_{M_j} (1 - A_{B_i M_j}) (-3\bar{\sigma}_B^2 + \bar{N}_B \bar{N}_M) \\ &\quad - P_{B_i} P_{M_j} \bar{N}_B \bar{N}_M \\ &= -P_{B_i} P_{M_j} [3(1 - A_{B_i M_j}) \bar{\sigma}_B^2 + A_{B_i M_j} \bar{N}_B \bar{N}_M]. \end{aligned} \quad (17)$$

The antibaryon-meson correlation is obtained by taking the charge conjugation transformation of Eq. (17).

3 Effects of quark number fluctuation and correlation at hadronization

The quark system produced in heavy ion collisions always varies in size event-by-event, and we should take the effects of the fluctuation of quark numbers into account. Suppose the number of quarks and that of antiquarks follow a distribution $\mathcal{P}(\{N_f N_{\bar{f}}\}; \{\langle N_f \rangle, \langle N_{\bar{f}} \rangle\})$ around the event average $\langle N_f \rangle$ and $\langle N_{\bar{f}} \rangle$ with $f = u, d, s$. The event

average of a hadronic quantity A_h is then

$$\langle A_h \rangle = \sum_{\{N_f N_{\bar{f}}\}} \bar{A}_h \mathcal{P}(\{N_f N_{\bar{f}}\}; \{\langle N_f \rangle, \langle N_{\bar{f}} \rangle\}), \quad (18)$$

where \bar{A}_h is the result for given quark numbers and antiquark numbers. We expand \bar{A}_h as a Taylor series at the event average of quark numbers $\{\langle N_f \rangle, \langle N_{\bar{f}} \rangle\}$,

$$\begin{aligned} \bar{A}_h &= \bar{A}_h|_{\langle \cdot \rangle} + \sum_{f_1} \frac{\partial \bar{A}_h}{\partial N_{f_1}} \Big|_{\langle \cdot \rangle} \delta N_{f_1} \\ &\quad + \frac{1}{2} \sum_{f_1 f_2} \frac{\partial^2 \bar{A}_h}{\partial N_{f_1} \partial N_{f_2}} \Big|_{\langle \cdot \rangle} \delta N_{f_1} \delta N_{f_2} + \mathcal{O}(N_f^{-2}), \end{aligned} \quad (19)$$

where indexes f_1 and f_2 run over all flavors of quarks and antiquarks and $\delta N_{f_1} = N_{f_1} - \langle N_{f_1} \rangle$. The subscript $\langle \cdot \rangle$ denotes the evaluation at the event average. Substituting the above into Eq. (18), we get

$$\langle A_h \rangle = \bar{A}_h + \frac{1}{2} \sum_{f_1 f_2} \frac{\partial^2 \bar{A}_h}{\partial N_{f_1} \partial N_{f_2}} C_{f_1 f_2} + \mathcal{O}(\langle N_f \rangle^{-2}). \quad (20)$$

where $C_{f_1 f_2} = \langle \delta N_{f_1} \delta N_{f_2} \rangle$ is a two-body correlation function of quarks and antiquarks and we drop the subscript $\langle \cdot \rangle$ for short. Applying it to the multiplicity quantities N_α and $N_\alpha N_\beta$, we have, up to second order

$$\langle N_\alpha \rangle = \bar{N}_\alpha + \frac{1}{2} \sum_{f_1 f_2} (\partial_{12} \bar{N}_\alpha) C_{f_1 f_2}, \quad (21)$$

$$C_{\alpha\beta} = \bar{C}_{\alpha\beta} + \frac{1}{2} \sum_{f_1 f_2} [2\partial_1 \bar{N}_\alpha \partial_2 \bar{N}_\beta + \partial_{12} \bar{C}_{\alpha\beta}] C_{f_1 f_2}, \quad (22)$$

where we have used the abbreviation $\partial_1 \equiv \frac{\partial}{\partial N_{f_1}}$ and $\partial_{12} = \frac{\partial^2}{\partial N_{f_1} \partial N_{f_2}}$. Because multiplicity \bar{N}_α is an almost homogeneous function of quark numbers, and quark correlations $C_{f_1 f_2}$ are usually of the magnitude of $\langle N_{f_1} \rangle$ or $\langle N_{f_2} \rangle$, $(\partial_{12} \bar{N}_\alpha) C_{f_1 f_2}$ has the magnitude of \bar{N}_α / N_f , and therefore affects $\langle N_\alpha \rangle$ at $1/\langle N_f \rangle$ level. It is very small, as quark numbers are large. For two-hadron correlations, the effects of quark number fluctuation are of the magnitude of $\bar{N}_\alpha \bar{N}_\beta / N_q$ which is comparable with $\bar{C}_{\alpha\beta}$, and therefore will significantly influence two-hadron correlations.

As examples, in Fig. 1 we show the numerical results of variance and two-hadron correlation for a few hadrons. Quark numbers are taken as $\langle N_u \rangle = \langle N_d \rangle = \langle N_{\bar{u}} \rangle = \langle N_{\bar{d}} \rangle = 1137$ and $\langle N_s \rangle = \langle N_{\bar{s}} \rangle = 478$, which corresponds to the size of the quark system in the unit rapidity interval in 0-5% Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. Here, results directly by the combination, i.e., Eqs. (11), (15) and (17), are shown as solid circles. The open circles show the results of Eq. (22) after including only the effects of quark number fluctuations $C_{ff} = \langle N_f \rangle$, and the solid squares show the results of Eq. (22) after further including the flavor conservation of quarks and antiquarks

$C_{f\bar{f}} = \langle N_f \rangle$. We see that the effects of quark number fluctuation and correlation are non-trivial in general and vary with hadron species. Lattice QCD calculations [42–44] show that off-diagonal flavor susceptibilities of quark numbers in the vicinity of the (de-)confinement phase transition region are quite small compared with diagonal ones, $\chi_{us}/\chi_{ss} \approx -0.05$ and $\chi_{ud}/\chi_{uu} \approx -0.05$. Off-diagonal quark correlations $C_{f_1 f_2}$ and $C_{f_1 \bar{f}_2}$ are also usually small and therefore are not discussed here.

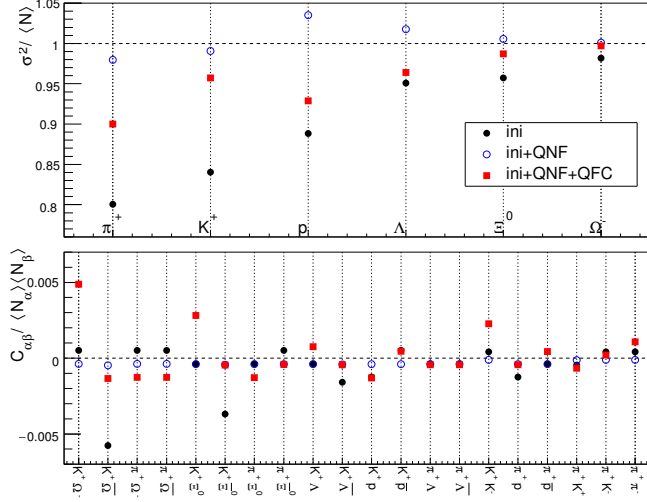


Fig. 1. (color online) Variance and two-hadron correlation for a few hadrons. Results by combination itself are shown as solid circles (ini); results after considering the quark number fluctuation (QNF) are shown as open circles (ini+QNF), and results after further considering the quark number flavor conservation (QFC) are shown as solid squares (ini+QNF+QFC).

4 Decay effects

The influence of resonance decay is usually important and complex. For a resonance α , the probability of the k -th decay channel is denoted as $D_{\alpha k}$ and is taken from the PDG [45]. The number of stable hadron a in this channel is denoted as $\omega_{\alpha k}^a$. The number of decay channels for α is denoted as d_α . The multiplicity of stable hadron a is

$$\langle N_a^{(f)} \rangle = \sum_{\alpha} \langle N_{\alpha}^{(i)} \rangle \left(\delta_{\alpha, a} + (1 - \delta_{\alpha, a}) \sum_{k=1}^{d_{\alpha}} D_{\alpha k} \omega_{\alpha k}^a \right), \quad (23)$$

where we use the superscript (i) to denote results for initially produced hadrons and (f) for final hadrons including decay contributions. For the two-hadron correlation,

$$C_{ab}^{(f)} = \sum_{\epsilon\sigma} C_{\epsilon\sigma}^{(i)} \left[\delta_{\epsilon, a} + (1 - \delta_{\epsilon, a}) \sum_{k=1}^{d_{\epsilon}} D_{\epsilon k} \omega_{\epsilon k}^a \right]$$

$$\begin{aligned} & \times \left[\delta_{\sigma, b} + (1 - \delta_{\sigma, b}) \sum_{k=1}^{d_{\sigma}} D_{\sigma k} \omega_{\sigma k}^b \right] \\ & + \sum_{\epsilon} \langle N_{\epsilon}^{(i)} \rangle (1 - \delta_{\epsilon, a}) (1 - \delta_{\epsilon, b}) \\ & \times \sum_{k, k'=1}^{d_{\epsilon}} D_{\epsilon k} (\delta_{k, k'} - D_{\epsilon k'}) \omega_{\epsilon k}^a \omega_{\epsilon k'}^b, \end{aligned} \quad (24)$$

which includes the superposition of two-body correlations of other hadrons which can decay into a and/or b . In particular, if a resonance ϵ can decay into both a and b , the $\langle N_{\epsilon}^{(i)} \rangle$ contribution term arises, the second term on the right-hand side, with a positive sign if a and b come from the same decay channel ($k=k'$) and a negative sign if a and b come from different decay channels ($k \neq k'$). Extensions of Eqs. (23) and (24) to cascade decays are straightforward but the formulas are too lengthy to be shown in this paper. We use the full decay formulas in practical calculations.

5 Dynamical fluctuation ν_{dyn} of $p\pi$, $K\pi$ and Kp pairs

In this section, we discuss an observable ν_{dyn} which is the combination of two-hadron correlations and is proposed as an effective probe of the dynamical fluctuations [26]. It takes the form $\langle \left(\frac{N_A}{\langle N_A \rangle} - \frac{N_B}{\langle N_B \rangle} \right)^2 \rangle$ and provides a measurement of the dynamical variance for the difference between the relative numbers of the two particle species A and B . Subtracting the base line of purely statistical fluctuations $\frac{1}{\langle N_A \rangle} + \frac{1}{\langle N_B \rangle}$, the generalized definition of ν_{dyn} is

$$\begin{aligned} \nu_{\text{dyn}, AB} &= \frac{\langle N_A(N_A - 1) \rangle}{\langle N_A \rangle^2} + \frac{\langle N_B(N_B - 1) \rangle}{\langle N_B \rangle^2} - 2 \frac{\langle N_A N_B \rangle}{\langle N_A \rangle \langle N_B \rangle} \\ &= \frac{\sigma_A^2 - \langle N_A \rangle}{\langle N_A \rangle^2} + \frac{\sigma_B^2 - \langle N_B \rangle}{\langle N_B \rangle^2} - 2 \frac{C_{AB}}{\langle N_A \rangle \langle N_B \rangle}. \end{aligned} \quad (25)$$

We see that ν_{dyn} will vanish for purely statistical fluctuations $\sigma^2 = \langle N \rangle$ without inter-particle correlation $C_{AB} = 0$. This observable has advantage of symmetry under the transposition of A and B and of independence of the detection efficiency.

We first calculate the $K\pi$ fluctuations $\nu_{\text{dyn}, K\pi}$, where K refers to $K^+ + K^-$ and π refers to $\pi^+ + \pi^-$. As discussed in Section 3, quark number fluctuation and correlation will obviously influence the variance and two-hadron correlations and therefore $\nu_{\text{dyn}, K\pi}$. In Fig. 2(a), we show $\nu_{\text{dyn}, K\pi}$ at different quark number fluctuation λ_1 and flavor conservation λ_2 . Decay contributions are also included. Here $\lambda_1 \equiv \sigma_f^2 / \langle N_f \rangle$ represents the variance of quark numbers with respect to the mean, and we take the same value for u , d , and s quarks. Pearson's correlation coefficient $\lambda_2 = C_{f\bar{f}} / \sigma_f \sigma_{\bar{f}}$ is used to describe the flavor conservation of quarks and antiquarks, and we also take the

same value for $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ pairs. The averaged quark numbers are taken as $\langle N_u \rangle = \langle N_d \rangle = \langle N_{\bar{u}} \rangle = \langle N_{\bar{d}} \rangle = 1819$ and $\langle N_s \rangle = \langle N_{\bar{s}} \rangle = 765$, which corresponds to the size of the quark system in the pseudo-rapidity interval $|\eta| \leq 0.8$ in 0-5% Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. We see that $\nu_{\text{dyn}, K\pi}$ increases with the increase of λ_1 and λ_2 . The increase of quark number fluctuation λ_1 will increase the magnitudes of the first and second terms of ν_{dyn} in Eq. (25) by the variance of hadronic multiplicity, see Fig. 1. The increase of λ_2 will mainly increase the hadronic pair correlation C_{K+K^-} and $C_{\pi^+\pi^-}$ and thus mainly increase the first and second terms of $\nu_{\text{dyn}, K\pi}$. For similar reasons, the results of $\nu_{\text{dyn}, Kp}$ and $\nu_{\text{dyn}, p\pi}$ also increase with λ_1 and λ_2 , where p refers to $p+\bar{p}$. In Fig. 2(b), we show the result of $\nu_{\text{dyn}, K\pi}$ by direct combination (ini), that by including quark number fluctuation and flavor conservation in case of $\lambda_1 = \lambda_2 = 1$, and that by further including decays, which illustrates these different contributions to the final $\nu_{\text{dyn}, K\pi}$.

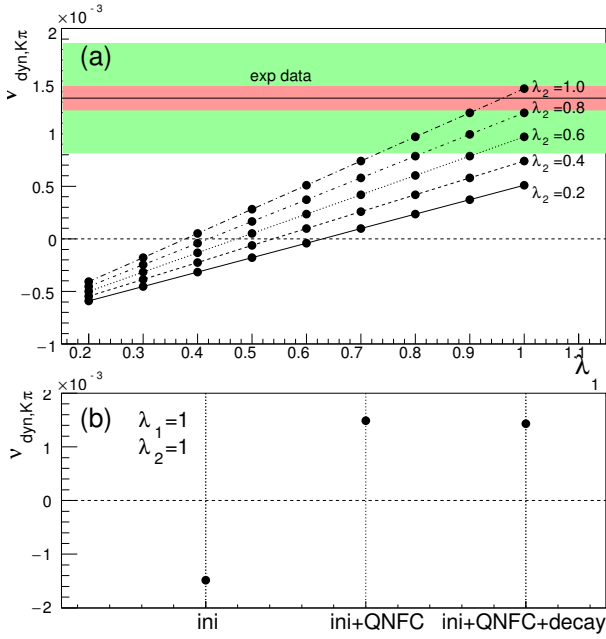


Fig. 2. (color online) (a) $\nu_{\text{dyn}, K\pi}$ at different values of quark number fluctuation λ_1 and flavor conservation λ_2 ; (b) the result of $\nu_{\text{dyn}, K\pi}$ obtained by considering combination itself, that obtained by including quark number fluctuation and flavor conservation (QNFC) at $\lambda_1 = \lambda_2 = 1$, and that obtained by further including the decays. The horizontal solid line with shadow bands in panel (a) is the value of $\nu_{\text{dyn}, K\pi}$ in central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [28].

The physical values of λ_1 and λ_2 in the context of central Pb-Pb collisions at the ALICE detector need to be discussed. On the one hand, because the pseudo-rapidity coverage $|\eta| \leq 0.8$ adopted by ALICE for ν_{dyn}

measurements is only a small fraction of the entire system ($y_{\text{beam}} > 8$) created in heavy-ion collisions at the LHC, $\lambda_1 \sim 1$ is reasonable in view of the Poisson distribution in grand-canonical ensembles. On the other hand, pseudo-rapidity coverage $|\eta| \leq 0.8$ is large for the conservation of quark flavor and we would expect $\lambda_2 \sim 1$, which is indicated from the observation that the radius of the measured charge balance is only about 0.45 [46] in central Pb-Pb collisions at the LHC. The experimental value of $\nu_{\text{dyn}, K\pi}$ [28] in 0-5% Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV is shown in Fig. 2(a), where the center value is shown as a horizontal solid line and statistical and systematic uncertainties are shown as shadow bands. We indeed see that the results are close to the data, as λ_1 and λ_2 take large values. However, we emphasize that such a comparison only serves as a reference, not as a decisive test of QCM, because the subsequent hadron re-scattering stage will also influence the dynamical fluctuation to a certain extent.

In Fig. 3, we show the multiplicity dependence of $\nu_{\text{dyn}, K\pi}$ as well as those of $\nu_{\text{dyn}, Kp}$ and $\nu_{\text{dyn}, p\pi}$, where p refers to $p+\bar{p}$. In QCM, hadronic variance σ_α^2 and covariance $C_{\alpha\beta}$ are all proportional to the hadronic multiplicity, which can be seen from Eqs. (11), (13), (15), and (17). Therefore, following the definition of Eq. (25), ν_{dyn} is inversely proportional to hadronic multiplicity and also $\langle dN_{\text{ch}}/d\eta \rangle$, and $\nu_{\text{dyn}} \times \langle dN_{\text{ch}}/d\eta \rangle$ for $K\pi$, Kp and $p\pi$ are almost unchanged if λ_1 and λ_2 keep constant. The solid lines in Fig. 3 show the results of QCM at fixed $\lambda_1 = 1$ and partial flavor conservation $\lambda_2 = 0.85$. The value of λ_2 is estimated by the measured charge balance function [46] via $\lambda_2 \approx \int_{-0.8}^{0.8} B(\delta\eta) d\delta\eta / \int_{-\infty}^{\infty} B(\delta\eta) d\delta\eta$. We notice that such inverse $\langle dN_{\text{ch}}/d\eta \rangle$ proportionality is one of the main properties of ν_{dyn} , and is also found in other models or event generators such as AMPT and HIJING, see the dashed lines and dotted lines in Fig. 3, respectively, which are taken from Ref. [28]. The experimental data of $\nu_{\text{dyn}, K\pi}$, $\nu_{\text{dyn}, Kp}$, and $\nu_{\text{dyn}, p\pi}$, which also exhibit such properties in general, within the statistical and systematic uncertainties, are shown in Fig. 3.

Comparison of the results of QCM, HIJING, AMPT and the experimental data brings us some useful understanding. Because the results of HIJING can be regarded as the superposition of independent pp collisions with string fragmentation, the comparison between the results of QCM and those of HIJING illustrates the possible effect for the change of hadronization mechanism related to QGP formation in relativistic heavy ion collisions. We see that in Fig. 3(a) the result of $\nu_{\text{dyn}, K\pi}$ (meson-meson pair) in QCM is slightly larger than that in HIJING, but in Fig. 3(b) and (c) those of $\nu_{\text{dyn}, pK}$ and $\nu_{\text{dyn}, p\pi}$ (baryon-meson pairs) in QCM are smaller than those in HIJING and are closer to the experimental data. The improved description for pK and $p\pi$ in QCM is mainly related to

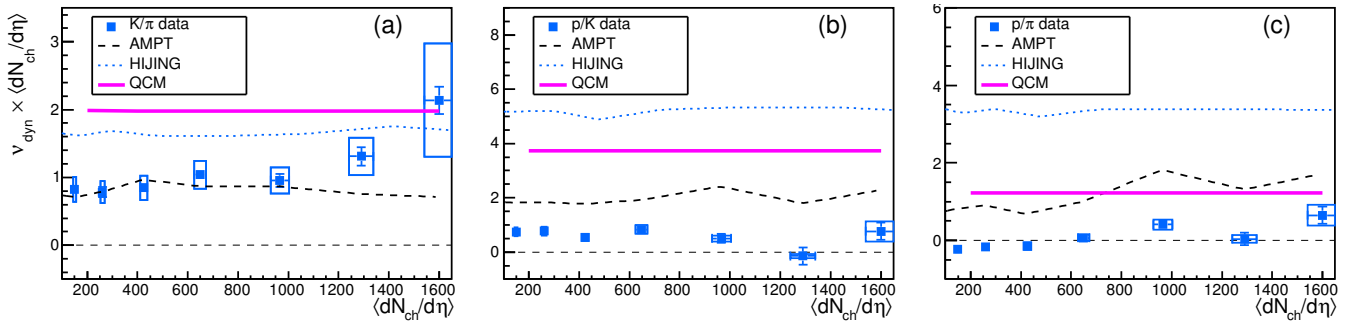


Fig. 3. (color online) Multiplicity dependence of dynamical fluctuations ν_{dyn} for $K\pi$, Kp and $p\pi$ pairs. The data in Pb-Pb collisions at $\sqrt{s_{\text{NN}}}=2.76$ TeV and results of AMPT and HIJING are taken from Ref. [28].

p production, which is because the baryon production in the low transverse momentum range in QCM is more phenomenologically successful than that in string fragmentation in the relativistic heavy-ion collisions where QGP is formed [38]. In addition, the comparison between results of the AMPT default version and those of HIJING indicates the effect of the hadronic re-scattering stage on ν_{dyn} . In contrast to HIJING, the AMPT default version further includes the evolution of the hadronic re-scattering stage. We see that the results of AMPT, the dashed lines in Fig. 3, are smaller than those of HIJING (dotted lines), and are closer to the data. This suggests that the hadronic re-scattering suppresses the ν_{dyn} of $K\pi$, pK and $p\pi$ to a large extent. Including this effect will also improve the QCM results to a certain extent in comparison with the experimental data, which deserves further study in future work.

6 Summary

In this paper, we have studied the second-order multiplicity fluctuation and correlation of identified mesons and baryons in the quark combination mechanism. We have built a preliminary framework in which the effects of different ingredients, such as quark combination itself, quark number fluctuation and correlation at hadroniza-

tion, can be separated and studied individually. Because the fluctuation and correlation of hadrons in the quark combination mechanism mainly depend on the constituent quark content of hadrons, there are lots of potentially interesting correlation properties among the results of different hadron species. These can be used to test the mechanism and, more importantly, obtain information about fluctuation and correlation for the quark system at hadronization, by virtue of the data of experimental observables. As an example, we have calculated a dynamical fluctuation observable ν_{dyn} for $K\pi$, $p\pi$ and Kp pairs in the context of Pb-Pb collisions at $\sqrt{s_{\text{NN}}}=2.76$ TeV, where we only consider the effects of quark number fluctuation and quark flavor conservation. In comparison with the experimental data and simulations of event generators, we find that the quark combination can reproduce the basic behavior of ν_{dyn} and hadronic re-scattering effects are not negligible. In forthcoming work, we will systematically consider other nontrivial effects besides hadronization, such as those of the hadronic re-scattering stage, finite acceptance, and nonzero baryon number density at lower energies. We will also carry out a systematic comparison with the new ν_{dyn} data of the LHC and also those of RHIC, and make final predictions for more hadron species such as $p\Lambda$, $K\Lambda$, $p\Xi$ and $\Lambda\Xi$.

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