Gottfried sum rule from maximum entropy method quark distributions with DGLAP evolution and with DGLAP evolution with GLR-MQ-ZRS corrections[*]

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Abstract: A new method to test the valence quark distribution of nucleons obtained from the maximum entropy method using the Gottfried sum rule by performing the DGLAP equations with GLR-MQ-ZRS corrections and the original leading-order/next-to-leading-order (LO/NLO) DGLAP equations is outlined. The test relies on knowledge of the unpolarized electron–proton structure function F_2^{ep} and the electron–neutron structure function F_2^{en} and the assumption that Bjorken scaling is satisfied. In this work, the original Gottfried summation value obtained by the integrals of the structure function at different Q^2 is in accordance with the theoretical value of $1/3$ under the premise of light-quark flavor symmetry of the nucleon sea, whether it results from dynamical evolution equations or from global quantum chromodynamics fits of PDFs. Finally, we present the summation value of the LO/NLO DGLAP global fits of PDFs under the premise of light-quark flavor asymmetry of the nucleon sea. According to analysis of the original Gottfried summation value with two evolution equations at different Q^2, we find that the valence quark distributions of nucleons obtained by using the maximum entropy method are effective and reliable.

Keywords: Gottfired sum rule, DGLAP-GLR-MQ-ZRS equations, DGLAP equations, deviation

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1   Introduction

Up to now, there exist numerous sum rules for unpolarized and polarized structure functions, some of which are rigorous results and others rely on more or less well justified assumptions [1]. The Adler sum rule [2] is exact and has no quantum chromodynamics (QCD) perturbative corrections, but its experimental verification is at a very low level of accuracy [3]. The constant 2 in the Adler sum rule is the result of local commutation relations of the time components of the hadronic weak current [4], which is based on the fundamental quark structure of the standard model. By contrast, the corresponding Gottfried sum rule [5] for charged lepton scattering was based merely on the valence quark picture and is modified both by perturbative and nonperturbative effects [6, 7]. The original Gottfried sum rule states that the integral over Bjorken variable x of a difference of electron–proton and electron–neutron structure functions is a constant $1/3$ under flavor symmetry in the

nucleon sea $(\bar{u}(x) = \bar{d}(x))$, which is independent of the negative four-momentum transfer squared Q^2. Some experimental results were achieved from electron and muon [8] scattering on isoscalar targets or on hydrogen target [9] deep inelastic scattering (DIS). For non-singlet Mellin moment neutrino and charged-lepton DIS, the $N = 1$ moments correspond to the Adler and Gottfried sum rules [5–7].

In this paper, we test the valence quark distributions of nucleons obtained from the maximum entropy method (MEM) by the original Gottfried sum rule using the DGLAP equation [10] with GLR-MQ-ZRS corrections (DGLAP-GLR-MQ-ZRS equations from the IMParton16 package) [11] at different Q^2 and compare these results with those obtained from the original DGLAP evolution equations and the latest global fits of parton distribution functions. The most important correction to the DGLAP evolution equations entails accounting for parton–parton recombination. In the IMParton16 package, we developed a dynamical parton model for the

origin of parton distributions and extended the initial evolution scale down to $Q^2 \sim 0.1$ GeV^2. For evolving the leading-order (LO) and next-to-leading-order (NLO) DGLAP equations, we use the modified Mellin transformation method by CANDIA [12] to calculate the original Gottfried summation value under the premise of light-quark flavor asymmetry of the nucleon sea ($\bar{u}(x) = \bar{d}(x)$). The starting scale for the LO and NLO evolution is $Q^2 = 1$ GeV^2. Finally, we give the summation value of the LO/NLO DGLAP latest global fits of parton distribution functions under the premise of light-quark flavor asymmetry of the nucleon sea ($\bar{u}(x) \neq \bar{d}(x)$). We find that the obtained summation values at different Q^2 are nearly consistent with experimental observations.

The organization of the paper is as follows. A nonperturbative initial input of valence quark distributions of nucleons obtained from the MEM is introduced in Section II. Section III discusses the Gottfried sum rule. Section IV presents comparisons of DGLAP-GLR-MQ-ZRS results with results from the LO/NLO DGLAP equations under the premise of light-quark flavor symmetry in the nucleon sea, as well as calculate the summation value of LO/NLO DGLAP latest global fits of parton distribution functions under the premise of light-quark flavor asymmetry in the nucleon sea. Finally, a summary is given in Section **5**.

2 Nonperturbative initial input from the quark–parton model

The quark model is a classification scheme for hadrons in terms of their valence quarks under the assumption that baryons are composed of three quarks and mesons of a quark and an antiquark. The solutions of the QCD evolution equations for parton distributions of the nucleon at high Q^2 depend on the initial parton distributions at low starting scale Q_0^2. According to the quark model, an ideal assumption is that the nucleon consists of only three valence quarks at extremely low Q_0^2. Hence, a nonperturbative initial input of the nucleon includes merely three valence quarks, which is the simplest input nucleon [13]. In the dynamical parton distribution function model, all sea quarks and gluons are QCD radioactively generated from valence quarks at high scale Q^2. The simple functional form to approximate the valence quark distribution is the time-honored canonical parametrization $f(x) = Ax^B(1-x)^C$ [14]. Therefore, the simplest parametrization of the naive nonperturbative input of the proton by using the MEM [15] is written as

$$
\begin{aligned}
u_v^p(x, Q_0^2) &= 7.191 x^{0.286}(1-x)^{1.359}, \\
d_v^p(x, Q_0^2) &= 13.068 x^{0.681}(1-x)^{3.026}.
\end{aligned}
\tag{1}
$$

In addition, the valence quark distributions of the free neutron obtained in previous work [16] is written as

$$
\begin{aligned}
u_v^n(x, Q_0^2) &= 16.579 x^{0.780}(1-x)^{3.267}, \\
d_v^n(x, Q_0^2) &= 8.678 x^{0.369}(1-x)^{1.511}.
\end{aligned}
\tag{2}
$$

By performing the DGLAP-GLR-MQ-ZRS evolution equations [11], one can determine the valence quark distributions of the nucleon at high Q^2 with the initial nonperturbative input obtained by using the MEM [15, 16]. We get the specific low starting scale $Q_0^2 = 0.0671$ GeV^2 for the naive nonperturbative input, by performing QCD evolution on the second moments of the valence quark distributions [17] and the measured moments of the valence quark distributions at a higher Q^2 [18]. The running coupling constant α_s for the leading order and the current quark masses are the parameters of perturbative QCD involved in the evolution equations [11, 15]. For evolving the LO and NLO DGLAP equations, we use the modified Mellin transformation method by CANDIA [12]. The starting scale for the LO and NLO evolution is $Q^2 = 1$ GeV^2.

3 Gottfried sum rule

In the proton, there are two up valence quarks (u_v) and one down valence quark (d_v). In fact, each quark distribution function $q_i(x)$ (i = u,d,s) always contains the sum of two parts, including the valence quark $q_i^{(v)}$ and the sea quark $q_i^{(s)}$ distribution function:

$$
q_i(x) = q_i^{(v)}(x) + q_i^{(s)}(x).
\tag{3}
$$

According to the definition of the distribution functions, the integrals of all distribution functions (quark and antiquark distribution functions $q_i(x)$ and $\bar{q}_i(x)$) within the proton should give the valence quark number. Therefore, the valence sum rules for the nonperturbative inputs are as follows:

$$
\begin{aligned}
\int_0^1 [u(x) - \bar{u}(x)] dx &= 2, \\
\int_0^1 [d(x) - \bar{d}(x)] dx &= 1. \\
\int_0^1 [s(x) - \bar{s}(x)] dx &= 0.
\end{aligned}
\tag{4}
$$

Through the transformation of Eq. (4), one can get

$$
\int_0^1 \left[\frac{2}{3}(u(x) - \bar{u}(x)) - \frac{1}{3}(d(x) - \bar{d}(x)) \right] dx = 1,
\tag{5}
$$

$$
\int_0^1 \left[\frac{2}{3}(d(x) - \bar{d}(x)) - \frac{1}{3}(u(x) - \bar{u}(x)) \right] dx = 0.
\tag{6}
$$

Equation (5) corresponds to the proton with a charge of 1. Equation (6) corresponds to the neutron with a charge of 0. The proton and the neutron are an isospin doublet, and up and down quarks are also isospin doublets, so the

distribution of the up quark in the neutron should be the same as that of the down quark in the proton.

According to the quark–parton model, the structure function of nucleon is written as

$$2xF_1(x) = F_2(x) = \sum_i e_i^2 x f_i(x), \qquad (7)$$

which is called the Callan–Gross expression [19]. In this equation, i is the flavor index, e_i is the electrical charge of the quark of flavor i (in units of the electron charge), and xf_i is the momentum fraction of the quark of flavor i. The structure functions of the proton and the neutron obtained from DIS of the charged lepton on protons and neutrons are, respectively,

$$\frac{1}{x}F_2^{ep}(x) = \frac{4}{9}[u(x)+\bar{u}(x)] + \frac{1}{9}[d(x)+\bar{d}(x)] + \frac{1}{9}[s(x)+\bar{s}(x)], \qquad (8)$$

$$\frac{1}{x}F_2^{en}(x) = \frac{4}{9}[d(x)+\bar{d}(x)] + \frac{1}{9}[u(x)+\bar{u}(x)] + \frac{1}{9}[s(x)+\bar{s}(x)]. \qquad (9)$$

For the proton, we can set

$$s_v(x) = \bar{s}_v(x) = \bar{u}_v(x) = \bar{d}_v(x) = 0,$$

$$u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{d}_s(x) = s_s(x) = \bar{s}_s(x) = \frac{1}{6}S(x),$$

$$S(x) = u_s(x) + d_s(x) + \bar{u}_s(x) + \bar{d}_s(x) + s_s(x) + \bar{s}_s(x). \qquad (10)$$

$S(x)$ is the sea quarks' sum of the proton, with the heavy quark's sea quark wave function neglected.

By bringing the constraints of Eqs. (3) and (10) into Eqs. (8) and (9), one can get

$$f(x) = \frac{1}{x}(F_2^{ep}(x) - F_2^{en}(x)) = \frac{1}{3}(u_v(x) - d_v(x)), \quad (11)$$

where $f(x)$ is a function of the Bjorken scaling variables x.

From Eq. (11), we see that the difference between the proton structure function F_2^{ep} and the neutron structure function F_2^{en} comes only from the contribution of the valence quarks, and the contribution of the sea quarks just cancels out. Therefore, the measurement of the proton and neutron structure functions will provide information about valence quarks. The integral of Eq. (11) with the constraints of Eq. (3) and Eq. (4) is as follows:

$$I = \int_0^1 \frac{dx}{x}(F_2^{ep}(x) - F_2^{en}(x)), \qquad (12)$$

where I is the integral summation value of Eq. (12). Theoretically, this integral value is a constant (1/3), which is called the original Gottfried sum rule [9] under flavor symmetry of the nucleon sea. In this paper, we use $I_i(Q^2)$ to represent the original Gottfried summation value from two evolution equations (the DGLAP-GLR-MQ-ZRS equations and the DGLAP equations) at different Q^2.

Gottfried studied high-energy electron–nucleon scattering, meson–nucleon reactions, and the spectroscopy of heavy-quark bound states. Then he proposed the Gottfried sum rule [5, 6, 9] for DIS to test the elementary quark model. The corresponding Gottfried sum rule for charged-lepton–nucleon DIS involved a form factor for the nucleon. Within the quark–parton model, the corresponding isospin sum rule in the case of charged-lepton–nucleon DIS is as follows:

$$I = \int_0^1 \frac{dx}{x}(F_2^{lp}(x) - F_2^{ln}(x))$$

$$= \int_0^1 dx \left[\frac{1}{3}(u_v(x) - d_v(x)) + \frac{2}{3}(\bar{u}(x) - \bar{d}(x)) \right]$$

$$= \frac{1}{3} - \frac{2}{3} \int_0^1 dx(\bar{d}(x) - \bar{u}(x)). \qquad (13)$$

If the nucleon sea were flavor symmetric, with $\bar{u}(x) = \bar{d}(x)$, one should have $I_G(Q^2) = 1/3$. If the nucleon sea were flavor asymmetric, namely, $\bar{u}(x) \neq \bar{d}(x)$, one should have $I_G(Q^2) \neq 1/3$. Moreover, this result is supported by the existing neutrino–nucleon DIS data [3] and the most detailed analysis of muon–nucleon DIS data of the NMC Collaboration [8]. It is worth noting that there are also some other works [20] on the light-quark flavor asymmetry deviation from the canonical value of 1/3 for the Gottfried sum rule.

4   Results and discussion

The DGLAP equations, which is based on the parton model and perturbative QCD theory, describe the evolution of quark and gluon densities with Q^2. The DGLAP-GLR-MQ-ZRS evolution equation is based on the DGLAP equation and mainly considers the parton recombination effect. Theoretical work on parton recombination was first proposed by Gribov, Levin, and Ryskin (GLR) [21], then Mueller and Qiu (MQ) put forward the recombination probabilities for gluons to go into gluons or into quarks in a low-density limit [22] and gave a detailed calculation. Finally, Zhu, Ruan, and Shen (ZRS) further presented a set of new and concrete evolution equations for parton recombination corrections [23].

It is worth noting that the number density of partons increases rapidly in the small x area. In a small x area, the number density of partons increases to a certain extent so that the quanta of partons overlap spatially. Therefore, parton–parton recombination becomes essential for small x area, which can effectively prevent the continuous increase of cross sections near their unitarity limit.

In fact, the GLR-MQ-ZRS corrections can be very effective in slowing down parton splitting at low scale $Q^2 < 1$ GeV². Up to now, ZRS have considered all the recombination functions for gluon–gluon, quark–gluon,

and quark–quark processes [23]. Because the gluon density is obviously greater than the quark density at small x, the gluon–gluon recombination effect is dominant in calculations [11]. Therefore, we use the simplified form of the DGLAP equations with GLR-MQ-ZRS corrections (DGLAP-GLR-MQ-ZRS equations) in the analysis [11].

To accurately test the validity of the DGLAP equations with GLR-MQ-ZRS corrections for the parton distribution function evolution at different Q^2, we perform the integral of Eq. (12), which is completely independent of Q^2 in theory. By applying the DGLAP-GLR-MQ-ZRS evolution equations, the quark distribution functions for the proton and the neutron (Eqs. (1) and (2)) are evolved to high Q^2, and the structure functions of the proton and the neutron, F_2^{p} and F_2^{n}, under different Q^2 are further calculated.
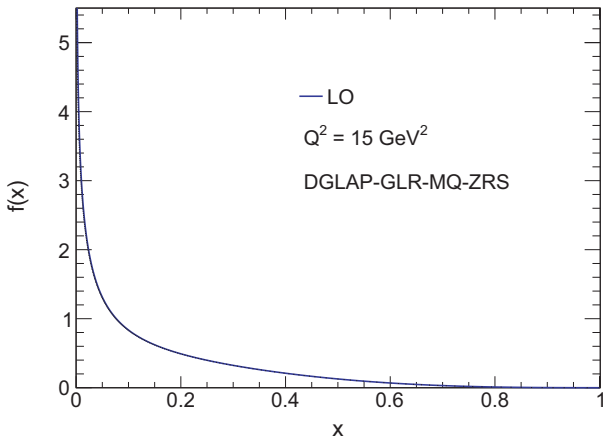

Fig. 1. (color online) $f(x)$ as a function of the Bjorken scaling variables x.

We take the DGLAP-GLR-MQ-ZRS equations as dynamical evolution equations to obtain the distribution of the right end of Eq. (11). The integral value of the area below the curve in Fig. 1 is 0.3333 at $Q^2 = 15$ GeV^2. It is obvious that the result from the DGLAP-GLR-MQ-ZRS equations is in good agreement with the theoretical value of 1/3 under light-quark flavor symmetry.

After that, we take the DGLAP equations as dynamical evolution equations to obtain the distribution of the right end of Eq. (11), the starting scale $Q_0^2 = 1$ GeV^2 for the LO and NLO evolution with naive nonperturbative input, which is from the modified Mellin transformation method by CANDIA [12]. By applying the DGLAP evolution equations, the quark distribution functions of the proton and the neutron from the MEM (Eqs. (1) and (2)) as initial input are evolved to high Q^2. Then one can get the original Gottfried summation value $I_{\mathrm{i}}(Q^2)$ of LO and NLO with light-quark flavor symmetry.

Figure 2 shows comparisons of the original Gottfried summation value $I_{\mathrm{i}}(Q^2)$ from the DGLAP-GLR-MQ-ZRS equations with results from DGLAP equations with

LO and NLO at different Q^2. The red solid line in Figure 2 represents the theoretical value of 1/3. Triangles, rhombuses, and stars represent the original Gottfried summation values at different Q^2 given by the DGLAP-GLR-MQ-ZRS evolution equations and the DGLAP evolution equations at LO and NLO, respectively.
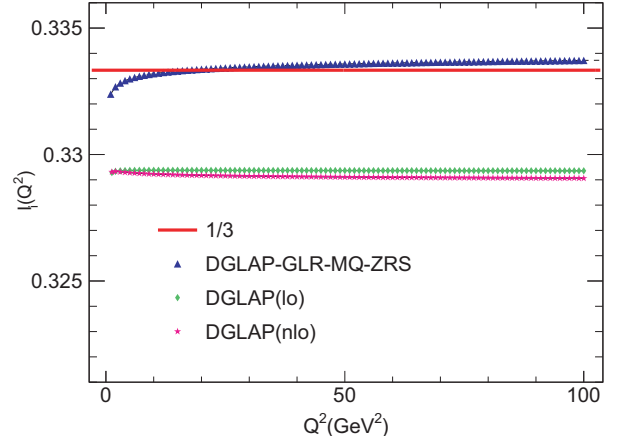

Fig. 2. (color online) Comparisons of original Gottfried summation value from DGLAP-GLR-MQ-ZRS equations (triangle) with results from DGLAP equations LO (rhombus) and NLO (star) at different Q^2 under the premise of light-quark flavour symmetry $\bar{u}(x) = \bar{d}(x)$.

It is apparent that the original Gottfried summation values of the DGLAP-GLR-MQ-ZRS equations have smaller deviations than the summation values of the LO and NLO DGLAP equations. However, a closer inspection reveals that the summation values from the DGLAP equations are not exactly equivalent to the theoretical value of 1/3 but are slightly smaller than 1/3. Moreover, the summation values from the NLO DGLAP equations are slightly smaller than the summation values of the LO DGLAP equations, which is from the α_{s}^2-level perturbative QCD correction. These corrections compared with the experimental analysis turn out to be small and cannot be responsible for the significant discrepancy between experimental results and the naive expectation of 1/3. It is noteworthy that the original quark–parton model expression for the original Gottfried sum rule is modified by perturbative QCD contributions when the nucleon sea was flavor symmetric in Ref. [7]. Furthermore, S.I.Alekhin et al. [27] take into account effects of possible nonperturbative in QCD expression for Gottfried sum rule and effects of nuclear corrections.

To more intuitively analyze the summation value under light-quark flavor symmetry and asymmetry of the nucleon sea, we compare the results from the dynamical evolution equations with the global fits from IMParton16 (Set B), MSTW (LO/NLO) [24], and CTEQ6l [25] and CT10 (NLO) [26], as shown in Fig. 3.

From Fig. 3, we can see that the results of the two evolution equations (DGLAP-GLR-MQ-ZRS/DGLAP equations) are basically consistent with the results with the global QCD fit MSTW08 (LO) (squares) and CTEQ6l (crosses) on the premise of flavor symmetry and are approximately equal to the theoretical value of 1/3. In the previous analysis, the summation value under flavor symmetry of the nucleon sea could not describe the deviation from the experimentally analyzed data. Consequently, we have to admit that this result from the quark-parton flavor-symmetric prediction is very naive.
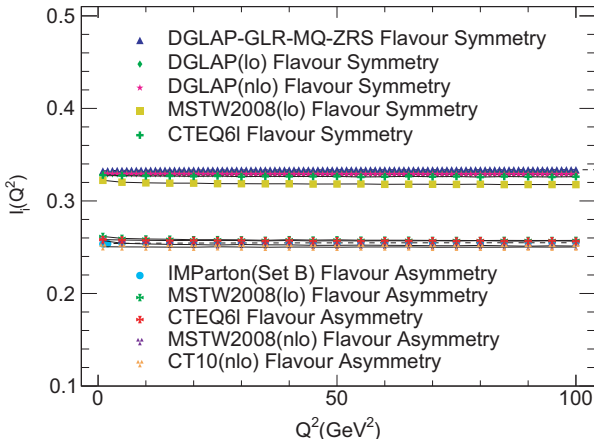

Fig. 3. (color online) Comparisons of summation value from two dynamics evolution equations (DGLAP-GLR-MQ-ZRS / DGLAP equations) with global QCD fit from IMParton16 (Set B), MSTW(LO / NLO) and CTEQ6l / CT10(NLO) at different Q^2 under the premise of light-quark flavour symmetry or asymmetry of the nucleon sea.

By analyzing the results of light-quark flavor symmetry, we find that it does not affect the necessity to introduce flavor asymmetry ($\bar{u}(x) \neq \bar{d}(x)$) for the description of the experimentally analyzed results for the Gottfried sum rule. Figure 3 shows the direct results from IMParton16 (Set B), MSTW (LO/NLO), and CTEQ61/CT10 (NLO) PDFs obtained by computing Eq. (12) with the use of Eq. (7). It clearly indicates violation of the theoretical value of 1/3 with light-quark asymmetry of the nucleon sea, which is in agreement with the experimental analysis. Moreover, one can find that the summation values from the NLO DGLAP global fits MSTW (NLO) and CT10 (NLO) PDFs by computing Eq. (12) with the use of Eq. (7) are slightly smaller than the LO DGLAP global fits MSTW (LO) and CTEQ6l PDFs, which are from the perturbative QCD correction.

5   Summary

In this work, the valence quark distribution function of the nucleon at low Q_0^2 obtained by using the MEM is used as nonperturbative initial input. The parton distributions of the nucleon are then evaluated dynamically at high Q^2 by using the DGLAP-GLR-MQ-ZRS equations and the LO and NLO DGLAP equations. Then, we get the unpolarized electromagnetic structure functions for the proton and the neutron, F_2^{p} and F_2^{n}. Through calculation of Eq. (12), one can further obtain the Gottfried summation value.

This is an interesting attempt to test the valence quark distribution function of the nucleon obtained by using the MEM via the Gottfried sum rule by performing the DGLAP-GLR-MQ-ZRS equations and the DGLAP equations. The original Gottfried summation value obtained by using Eq. (11) with different Q^2 is in accordance with the theoretical value of 1/3 under the light-quark flavor symmetry of the nucleon sea. It is apparent that the original Gottfried summation values of the DGLAP-GLR-MQ-ZRS equations have smaller deviations than the summation values of the LO/NLO DGLAP equations. Moreover, the summation value from the NLO DGLAP equations is slightly smaller than the summation value of the LO DGLAP equations, which is from the α_{s}^2-level perturbative QCD correction. The correction is small. It should be mentioned that the naive theoretical summation value equal to 1/3 is very preliminary compared with the existing experimental analysis results. Finally, we give the summation value from Fig. 3, which is not equal to 1/3 with NLO DGLAP evolution and the global fits from IMParton16 (Set B), MSTW (LO/NLO) [24], CTEQ6l [25] and CT10(NLO) [26] under light-quark flavor asymmetry. This result validates the necessity of introducing light-quark flavor asymmetry in the nucleon sea for the description of the experimental analysis results.

The Gottfried sum rule verifies the reliability of nonperturbative initial input of valence quark distributions from the starting low scale Q_0^2 by performing the DGLAP-GLR-MQ-ZRS equations. The DGLAP-GLR-MQ-ZRS equations based on the DGLAP equations with parton–parton recombination corrections is an important innovation, demonstrating that the nonlinear effects of parton–parton recombination are non-negligible at low Q^2. According to the results of the above analysis, the valence quark distribution functions of the nucleon obtained by using the MEM as initial input are valid and reliable.

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