Empirical formulas for proton decay half-lives: Role of nuclear deformation and *Q*-value

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Abstract: Two empirical formulas for the proton decay half-lives including nuclear deformation are proposed. The formula with parameter set I gives the logarithm of the proton decay half-lives as an explicit function of the orbital angular momentum with eight adjustable parameters, whereas that with set II represents the logarithm of the reduced half-lives as an implicit function of the angular momentum with seven adjustable parameters. Experimental data for the half-lives of 44 proton emitters in the ground state or isomeric state were used to obtain the parameters. The experimental and calculated Q values were used. Different sets of parameters were obtained for the ground state transition, isomeric state transition, and all transitions for both deformed and spherical nuclei. The best agreement with experimental data was observed for set I for deformed proton emitters with experimental Q values.

Keywords: proton emission, deformation, empirical formula

PACS: 23.60.+e **DOI:** 10.1088/1674-1137/42/10/104101

1 Introduction

The study of charged-particle radioactivity is one of the main fields in nuclear physics. Many theoretical and empirical models have been proposed to describe the charged-particle decay process [1–11]. The charged-particle radioactivity includes α decay, cluster decay, two-proton decay, and proton decay. After experimental confirmation of the proton radioactivity in 1970 by Jackson et al. [12], proton decay has been an interesting topic both experimentally and theoretically. Several experimental [12–20] and theoretical studies [21–43] have been performed to obtain the half-lives of spherical and deformed proton emitters for the ground and isomeric transitions. It is very important to have simple and accurate expressions for evaluation of the proton decay half-lives of both spherical and deformed proton emitters. Limited empirical formulas are available for this purpose [26, 27, 30, 31, 39, 44–46]. However, in Refs. [10, 47, 48], universal decay formulas based on the microscopic mechanism of charged particle emission were presented.

In Ref. [26], the centrifugal barrier, the structure of the decaying nucleus, and the corresponding preformation probability were taken into account. With this formula for deformed proton emitters, the experimental data lie approximately on two straight lines. The influence of the centrifugal barrier is completely included in the reduced half-life, which is defined as the half-life divided by an angular momentum function. In Ref. [27], an empirical model for calculation of the reduced half-life with one adjustable parameter was presented for systematic analysis of both the ground state and isomeric state proton transitions. By introducing the degree of nuclear deformation, $\delta=0.757\beta_2+0.273\beta_2^2$, the resulting data were grouped into two categories of nuclei, namely, largely prolate parent nuclei with degree of deformation $\delta\leqslant 0.1$ and those with other shapes. In Ref. [30], two formulas for spherical proton emission half-life calculation were presented. In these formulas, the angular momentum is included explicitly. Ref. [31] presents an analytical empirical formula for half-life calculation of spherical proton emitters with only one adjustable parameter, which is a complicated function of the Q value and angular momentum. In Ref. [45], two empirical formulas for calculation of the reduced half-life of spherical and deformed proton emitters were proposed, with three and four adjustable parameters, respectively. In the expression for deformed emitters, the quadrupole deformation parameter of the parent nuclei is included explicitly. In Ref. [39], an analytical expression with one adjustable parameter was proposed to obtain the half-life of deformed proton emission. The formula is a complicated function of the Q value and is explicitly dependent on the angular momentum, spectroscopic factor, and quadrupole deformation parameter of the parent nuclei. Recently, in Ref. [46], an

Received 18 April 2018, Published online 31 August 2018

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empirical formula was proposed for determining the half-life of spherical proton emitters with explicit dependence on the angular momentum and spectroscopic factor and four adjustable parameters.

Nuclear deformation actually plays an important role in calculation of the proton decay half-life. Moreover, the half-life is highly sensitive to the Q value and the orbital angular momentum. Ref. [30] found that when similar empirical formulas with different parameters are used, the half-life of the proton radioactivity is more sensitive to the Q value and angular momentum than that of α decay. Ref. [41] showed a notable dependence of the calculated half-life on the calculated and experimental Q values. For these reasons, we were motivated to propose two empirical formulas for half-life and reduced half-life calculation and investigate the role of the nuclear deformation and Q value on the ground state transition, isomeric state transition, and all transitions for both deformed and spherical nuclei.

This paper is organized as follows. In Sec. II, the theoretical formulas are introduced. The numerical results and discussions are given in Sec. III. Finally, the conclusions are presented.

2 Empirical formula

In order to show the dependence of different empirical formulas on the angular momentum, the available empirical formulas can be classified into two categories: half-life and reduced half-life.

2.1 Reduced half-life

As the first empirical formula, in Ref. [26], the following empirical relations were proposed for determining the reduced half-life of deformed proton emitters including the quadrupole deformation parameter of the parent nuclei.

$$\log_{10}(T^k_{\rm red}(s))=a_k(\chi-20)+b_k, \tag{1}$$

$$a_1=1.31,\quad b_1=-2.44\ Z<68$$

$$a_2=1.25,\quad b_2=-4.71\ Z>68$$

where $\chi=\sqrt{2}e^2(Z-1)\sqrt{\mu}Q^{-1/2}/\hbar$ is the Coulomb parameter, Z is the charge number of the parent nucleus, and μ is the reduced mass of the proton–daughter system.

In Ref. [27], the reduced half-life for deformed proton emitters was represented as

$$\log T_{\rm red}(s)=\alpha(\chi-\beta), \tag{2}$$

where $\chi=Z_{\rm d}\sqrt{\mu}Q^{-1/2}$ is the Coulomb parameter, $Z_{\rm d}$ is the charge number of the daughter nucleus, and μ is the reduced mass of the proton–daughter system. In this expression, the nuclear deformation of the parent nuclei is taken into account by introducing the deformation-dependent adjustable parameter g. Linear least-squares fitting procedures yielded $\alpha=0.327$, $\beta=7.27$, $g=0$, and $\sigma=0.30$ for the large prolate parent nuclei and $\alpha=0.377$, $\beta=20.0$, $g=0.12$, and $\sigma=0.29$ for the parent nuclei with other shapes.

In Ref. [45], the reduced half-life of deformed proton emitters was given as

$$\log(T_{\rm red}(s))=a\sqrt{\mu}ZQ^{-1/2}+b\sqrt{\mu}Z^{1/2}+c+d|\beta_2^3|, \tag{3}$$

where Z is the charge number of the daughter nuclei. For spherical emitters, one can fix $d=0$. By analyzing 39 experimental values consisting of the data for spherical and deformed emitters, the following parameter set was obtained: $a=0.407$, $b=-1.497$, $c=-17.660$, $d=27.269$, with a standard deviation $\sqrt{\bar{\sigma}^2}=0.384$.

2.2 Half-life

In Ref. [27], the proton decay half-life $[\tau=\log T_{\rm red}(s)]$ of deformed proton emitters was given as

$$\tau=\tau_0+\tau_1+\tau_2, \tag{4}$$

where τ_0,τ_1, and τ_2 are complicated functions of the Q value and angular momentum. These functions are responsible for the frequency of assaults on the barrier, the contribution from the overlapping barrier region, and that from the external separation barrier region, respectively. The g factor is the only adjustable parameter in this formula and appears in τ_1. Further, g is a function of the deformation parameter of the parent nuclei. This half-life for proton radioactivity including both the ground-state and isomeric proton transitions was obtained with a standard deviation $\sigma=0.34$.

The half-life formula for spherical proton emitters in Ref. [30] was given as

$$\log(T_{1/2}(s)) = (aZ+b)Q^{-1/2}+c +c_0\frac{l(l+1)}{\sqrt{(A-1)(Z-1)A^{-2/3}}}, \tag{5}$$

where Z and A are the charge and mass numbers of the parent nucleus, respectively. By performing a least-squares fit to the half-lives of the first 25 spherical proton emitters, the following parameters were obtained: $a=0.344$, $b=4.963$, $c=-31.125$, and $c_0=2.595$, with the average deviation $\bar{\sigma}=0.153$ between the experimental and fitted formulas. The set of parameters for the half-lives of deformed proton emitters was obtained by applying a least-squares fit to the experimental data of 11 nuclei with Z = 53–67 from Ref. [49]. These parameters are $a=0.364$, $b=4.647$, $c=-30.930$, and $c_0=2.624$. The average deviation is $\bar{\sigma}=0.323$.

The alternative formula for proton emission is written as

$$\log(T_{1/2}(s)) = a+bA^{1/6}Z^{1/2}+cZQ^{-1/2} +c_0\frac{l(l+1)}{\sqrt{(A-1)(Z-1)A^{-2/3}}}. \quad (6)$$

The following parameter sets were obtained: $a=-23.063, b=-0.422, c=0.417$, and $c_0=2.599$ for spherical proton emitters, with an average deviation $\bar{\sigma}=0.183$; and $a=-23.934, b=-0.394, c=0.438$, and $c_0=2.617$ for deformed proton emitters, with $\bar{\sigma}=0.316$. In Ref. [46], the following formula was obtained for spherical proton emitters:

$$\log(T_{1/2}(s)) = a+bA^{1/6}Z^{1/2}+cZQ^{-1/2} +dl(l+1)A^{-1/6}Z^{-1/2}-\log S_{\rm p}, \quad (7)$$

where Z, N, and A are the charge, neutron number, and mass number of the parent nucleus, respectively. $S_{\rm p}$ is the spectroscopic factor. The fitted coefficients are $a=-20.822$, $b=-0.532$, $c=0.415$, and $d=2.323$ with root-mean-square (rms) deviations of 0.139.

2.3 New formulas

To determine the proton decay half-life and reduced half-life for the ground state transition, isomeric state transition, and all transitions for both deformed and spherical nuclei, two formulas similar to the empirical formula of Ref. [46] are proposed as set I and set II, respectively. The spectroscopic factor does not change very much for spherical emitters but becomes complex, with a clear dependence upon the deformation parameter, for deformed emitters [22, 45]. Therefore, the effect of deformation of the proton emitter and the spectroscopic factor have been taken into account by introducing the quadrupole and hexadecapole deformation parameters (β_2, β_4) in these proposed formulas. These formulas are written as

$$\log(T_{1/2}^{(I)}(s)) = a+bA^{1/6}Z^{1/2}+cZQ^{-1/2} +dl(l+1)A^{-1/6}Z^{-1/2}+d_2|\beta_2|^{p_2}+d_4|\beta_4|^{p_4}, \quad (8)$$

and

$$\log(T_{\rm red}^{(II)}(s)) = a+bA^{1/6}Z^{1/2}+cZQ^{-1/2} +d_2|\beta_2|^{p_2}+d_4|\beta_4|^{p_4}, \quad (9)$$

where Z and A are the charge and mass number of the parent nucleus, respectively. The reduced half-life $T_{\rm red}$ is defined as [45]

$$T_{\rm red}=T_{1/2}{\rm e}^{-c_l\frac{\sqrt{1-x^2}}{x}}, \quad (10)$$

where

$$c_l=\frac{2\hbar l(l+1)}{\sqrt{2\mu Q R_{\rm C}^2}}, \quad x=\sqrt{R_{\rm t}/R_{\rm C}},$$

$$R_{\rm t}=1.225(1+A_{\rm d}^{1/3}), \text{ and } R_{\rm C}=\frac{e^2Z_{\rm d}}{Q}.$$

For more details about the theory, see Ref. [45].

Proton emission is energetically possible only when the Q value is positive and is given as

$$Q=\Delta M-(\Delta M_{\rm d}+M_{\rm p})+k(Z^{\epsilon}-Z_{\rm d}^{\epsilon}), \quad (11)$$

where $\Delta M, \Delta M_{\rm d}$, and $M_{\rm p}$ are the mass excesses of the parent nuclei, daughter nuclei, and emitted proton, respectively. The last term represents the screening effect of the atomic electrons. The Q values calculated using the mass excess values from the recent mass table of Audi et al. [50] were presented in Ref. [41]. One can use experimental data for the Q value from Ref. [19].

3 Results and discussion

The formulas introduced in the previous section are now applied to obtain the effects of deformation, the Q value, and the angular momentum on the proton decay half-life. The spectroscopic factor was not explicitly considered in the calculations, and its effect was taken into account implicitly in deformation-dependent terms.

In Figs. 1(a) and 1(b), the values of the deformation parameters β_2 and β_4 are shown for different proton emitters in the region $53\leqslant Z\leqslant 83$. These figures show that the proton emitters in this region are quite deformed.

In Table 1, the parameters $(a,b,c,d,d_2,p_2,d_4,p_4)$ for set I and parameters (a,b,c,d_2,p_2,d_4,p_4) for set II for 29 ground state transitions, 15 isomeric state transitions, and all transitions for 44 deformed proton emitters are presented. The upper and lower cases have been fitted with experimental [18, 19] and theoretical [41] Q values, respectively. The experimental half-life $\log T_{1/2}^{\rm exp}$ was taken from [18, 19]. The deformation parameters were taken from Ref. [51]. The rms error of the decimal logarithm of the proton decay half-life is evaluated as

$$\sqrt{\bar{\delta}^2}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}[\log T_{1/2}^{\rm exp.}(i)-\log T_{1/2}^{\rm cal.}(i)]^2}. \quad (12)$$

Table 2 lists the sets of parameters (a,b,c,d) for set I and (a,b,c) for set II obtained without considering the nuclear deformation parameters and by setting $d_2=d_4=0$ in Eqs. (8) and (9). The obtained data show that the nuclear deformation, angular momentum, and Q value significantly affect the rms value of the fitting process, and the lowest rms value is obtained for set I with the deformation parameters and $Q_{\rm exp}$.

In Table 3, the logarithm of the half-lives calculated with parameter sets I and II with experimental $Q_{\rm exp}$ values for the ground state and isomeric state transitions of deformed proton emitters are listed. The logarithms of the half-lives calculated with set I and set II ($\log T_{1/2}^{{\rm cal}(I)}$

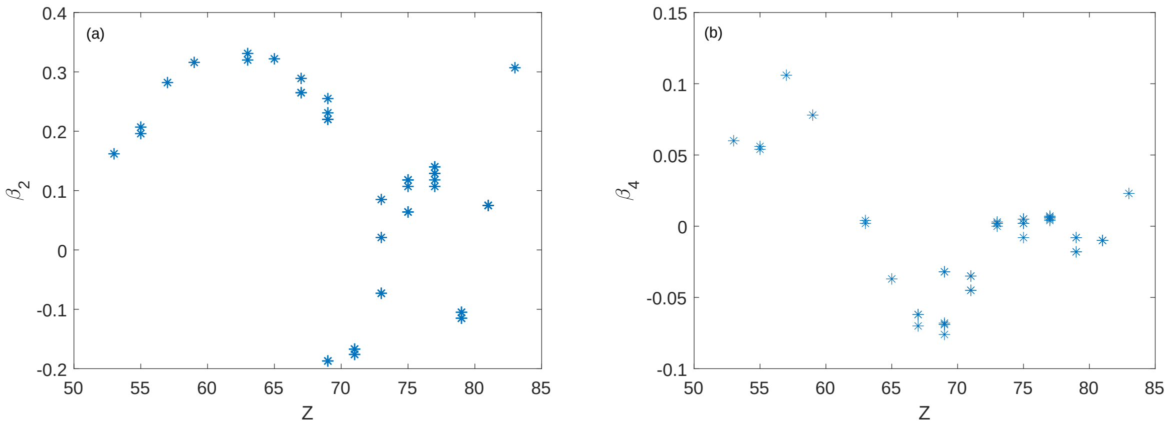

Fig. 1. (color online) (a) quadrupole, and (b) hexadecapole deformation parameters versus atomic number of proton emitters.

Table 1. Parameters of empirical formulas for ground state (29 Nuclei) transition, isomeric (15 Nuclei), and all transitions. Upper and lower sets have been obtained with $Q_{\rm exp}$ [19] and Q [41], respectively. The parameters (a,b,c,d,d_2,p_2,d_4,p_4) are listed for set I and (a,b,c,d_2,p_2,d_4,p_4) for set II.

transition	Set I	$\sqrt{\bar{\delta^2}}$	Set II	$\sqrt{\bar{\delta^2}}$
ground	(-20.154,-0.498,0.400,2.213,99.374,4.208,99.831,3.234)	0.320	(-21.415,-0.541,0.428,99.992,4.144,99.753,3.593)	0.361
isomeric	(-31.218,0.081,0.406,2.044,-2.467,0.666,99.995,1.809)	0.237	(-24.549,-0.194,0.367,8.097,1.877,-99.976,2.226)	0.295
all	(-21.171,-0.424,0.392,2.325,99.994,4.204,99.843,3.467)	0.364	(-22.009,-0.445,0.408,99.995,4.071,99.654,3.791)	0.393
ground	(-16.352,-0.452,0.333,1.332,99.068,4.256,99.509,3.951)	0.691	(-19.926,-0.594,0.416,99.995,4.049,18.656,4.724)	0.949
isomeric	(-25.798,-0.143,0.382,2.255,18.711,9.921,18.163,6.813)	0.663	(-11.825,-0.802,0.370,99.994,3.563,-99.997,1.411)	0.567
all	(-15.415,-0.512,0.338,1.890,99.937,4.358,-8.228,0.876)	0.776	(-16.793,-0.601,0.378,99.984,3.989,-17.125,1.098)	0.869

Table 2. Same as Table I, but without deformation. The parameters (a,b,c,d) are listed for set I and (a,b,c) for set II.

transition	Set I	$\sqrt{\bar{\delta^2}}$	Set II	$\sqrt{\bar{\delta^2}}$
ground	(-17.669,-0.591,0.393,2.136)	0.406	(-19.025,-0.651,0.427)	0.454
isomeric	(-27.410,-0.087,0.388,2.207)	0.261	(-25.580,-0.169,0.379)	0.353
all	(-18.948,-0.495,0.384,2.204)	0.435	(-19.723,-0.547,0.407)	0.485
ground	(-14.134,-0.536,0.327,1.269)	0.729	(-17.259,-0.713,0.414)	1.001
isomeric	(-25.635,-0.148,0.381,2.248)	0.666	(-24.255,-0.212,0.373)	0.693
all	(-16.989,-0.429,0.337,1.774)	0.828	(-18.363,-0.543,0.382)	0.952

Table 3. Comparison of the experimental [18, 19] and theoretical half-lives values for ground state and isomeric transitions. The Q-values are in MeV and half-lives are in seconds.

parent	(β_2,β_4)	l	$Q_{\rm exp}$	$Q_{\rm theo}$	$\log T_{1/2}^{\rm cal(I)}$	$\log T_{1/2}^{\rm cal(II)}$	$\log T_{1/2}^{\rm exp}$
^{109}I	(0.162,0.06)	2	0.827	0.831	-3.884	-3.999	-4.029
^{112}Cs	(0.196,0.054)	2	0.823	0.823	-3.096	-3.153	-3.301
^{113}Cs	(0.207,0.056)	2	0.976	0.985	-5.062	-5.260	-4.777
^{117}La	(0.282,0.106)	2	0.814	0.833	-1.862	-1.989	-1.623
^{121}Pr	(0.316,0.078)	2	0.900	0.904	-2.214	-2.285	-2.000
^{130}Eu	(0.331,0.004)	2	1.039	1.044	-2.649	-2.623	-3.046
^{131}Eu	(0.320,0.002)	2	0.959	0.964	-1.777	-1.687	-1.670
^{135}Tb	(0.322,-0.037)	3	1.200	1.194	-3.223	-3.232	-3.027
^{140}Ho	(0.289,-0.070)	3	1.106	1.091	-1.993	-2.001	-2.222
^{141}Ho	(0.265,-0.062)	3	1.190	1.195	-3.091	-3.162	-2.387
^{144}Tm	(0.255,-0.076)	5	1.725	1.711	-4.787	-4.855	-5.569
^{145}Tm	(0.231,-0.068)	5	1.753	1.755	-5.086	-5.162	-5.456
^{146}Tm	(0.220,-0.069)	5	1.210	0.916	-0.896	-0.603	-0.930
^{147}Tm	(-0.187,-0.032)	5	1.073	1.074	0.541	1.028	0.577
^{150}Lu	(-0.176,-0.045)	5	1.283	1.286	-1.276	-0.983	-1.194
^{151}Lu	(-0.167,-0.035)	5	1.253	1.256	-1.008	-0.674	-0.896
^{155}Ta	(0.021,0)	5	1.468	1.466	-2.573	-2.326	-2.538
^{156}Ta	(-0.073,0.002)	2	1.030	1.036	-0.597	-0.514	-0.609
^{157}Ta	(0.085,0.003)	0	0.947	0.957	-0.042	-0.016	-0.523
^{159}Re	(0.064,0.002)	5	1.816	1.607	-4.644	-4.606	-4.678
^{160}Re	(0.107,-0.008)	2	1.285	1.287	-3.084	-3.196	-3.060
^{161}Re	(0.118,0.005)	0	1.214	1.217	-2.986	-3.168	-3.357
^{164}Ir	(0.107,0.004)	5	1.844	1.577	-4.463	-4.431	-3.947
^{166}Ir	(0.129,0.006)	2	1.168	1.178	-1.249	-1.244	-0.818
^{167}Ir	(0.140,0.007)	0	1.096	1.087	-0.977	-1.041	-0.959
^{170}Au	(-0.105,-0.008)	2	1.488	1.489	-4.036	-4.240	-3.493
^{170}Au	(-0.115,-0.018)	0	1.464	1.451	-4.466	-4.793	-4.611
^{176}Tl	(0.075,-0.01)	0	1.282	1.271	-2.160	-2.332	-2.284
^{177}Tl	(0.075,-0.01)	0	1.180	1.168	-0.960	-1.053	-1.174
^{141m}Ho	(0.265,-0.062)	0	1.255	1.265	-5.785	-5.778	-5.180
^{146m}Tm	(0.220,-0.069)	5	1.140	1.056	-0.414	-0.187	-0.693
^{147m}Tm	(-0.187,-0.032)	2	1.133	1.135	-3.318	-3.351	-3.444
^{150m}Lu	(-0.176,-0.045)	2	1.306	1.307	-4.195	-4.516	-4.367
^{151m}Lu	(-0.167,-0.035)	2	1.332	1.387	-4.549	-4.731	-4.796
^{156m}Ta	(-0.073,0.002)	5	1.127	1.317	0.972	0.886	0.930
^{159m}Re	(0.064,0.002)	5	1.831	1.817	-4.431	-4.259	-4.695
^{161m}Re	(0.118,0.005)	5	1.338	1.387	-0.786	-0.625	-0.650
^{165m}Ir	(0.118,0.005)	5	1.733	1.737	-3.406	-3.162	-3.469
^{166m}Ir	(0.129,0.006)	5	1.340	1.348	-0.182	-0.138	-0.076
^{167m}Ir	(0.140,0.007)	5	1.261	1.264	0.617	0.648	0.875
^{170m}Au	(-0.105,-0.008)	5	1.77	1.753	-3.014	-3.004	-2.980
^{171m}Au	(-0.115,-0.018)	5	1.719	1.712	-2.641	-2.672	-2.654
^{177m}Tl	(0.075,-0.01)	5	1.984	1.955	-3.680	-3.922	-3.402
^{185m}Bi	(0.307,0.023)	0	1.624	1.614	-4.024	-4.027	-4.237

and $\log T_{1/2}^{\mathrm{cal}(II)}$, respectively) and the logarithms of the experimental half-lives $\log T_{1/2}^{\mathrm{exp}}$ show good agreement between the experimental values and the new formulas.

In order to compare the experimental data with the calculated values and analyze the dependence of the half-life on the nuclear deformation, Q value, and angular momentum, the following dimensionless parameters are defined:

$$r_1 = \log T_{1/2}^{(I)'} - \log T_{1/2}^{(\mathrm{sph})}, \tag{13}$$

$$r_2 = \log T_{1/2}^{(Q_{\mathrm{exp}})} - \log T_{1/2}^{(Q_{\mathrm{theo}})}, \tag{14}$$

$$r_3 = \log T_{1/2}^{(I)'} - \log T_{1/2}^{(II)'}, \tag{15}$$

$$r_4 = \log T_{1/2}^{(I)'} - \log T_{1/2}^{(\mathrm{exp})}, \tag{16}$$

where $T_{1/2}^{(I)'}$ and $T_{1/2}^{(\mathrm{sph})}$ are the half-lives calculated with the parameters in set I with Q_{exp} for all the transitions for deformed and spherical proton emitters, respectively. $T_{1/2}^{(Q_{\mathrm{exp}})}$ and $T_{1/2}^{(Q_{\mathrm{theo}})}$ are the half-lives calculated with the parameters in set I for all the transitions with Q_{exp} and Q_{theo}, respectively. $T_{1/2}^{(II)'}$ is the half-life calculated with the parameters in set II for all the transitions with Q_{exp}. $T_{1/2}^{(\mathrm{exp})}$ is the experimental half-life. Therefore, the parameter r_1 gives the deviation between the half-lives calculated with and without including the deformation parameters. The parameter r_2 gives the deviation between the half-lives of deformed proton emitters calculated with Q_{exp} and Q_{theo}. The parameter r_3 gives the deviation between the half-lives calculated using set I and set II. The parameter r_4 gives the deviation between the calculated and experimental half-lives.

Figures 2(a)–2(d) show the variation of the parameters r_1, r_2, r_3, and r_4 with the atomic number of the proton emitter. As can be seen, these parameters vary in ranges of $-0.5 \leqslant r_1 \leqslant 0.6$; $-0.5 \leqslant r_2 \leqslant 1$, except in four cases; $-0.4 \leqslant r_3 \leqslant 0.3$; and $-1 \leqslant r_4 \leqslant 1$. Fig. 2(a) shows that the deformation parameters play a significant role in the proton decay half-life formula. Fig. 2(b) shows a noticeable discrepancy between the half-lives calculated with Q_{exp} and Q_{theo}. Fig. 2(c) reveals small differences in the half-lives calculated with parameter sets I and II. Fig. 2(d) shows good agreement between the calculated half-lives and the experimental data.

The contribution of the deformation parameters is evaluated in Fig. 3. In Fig. 3(a), the half-lives calculated with the parameters in set I for all the transitions with

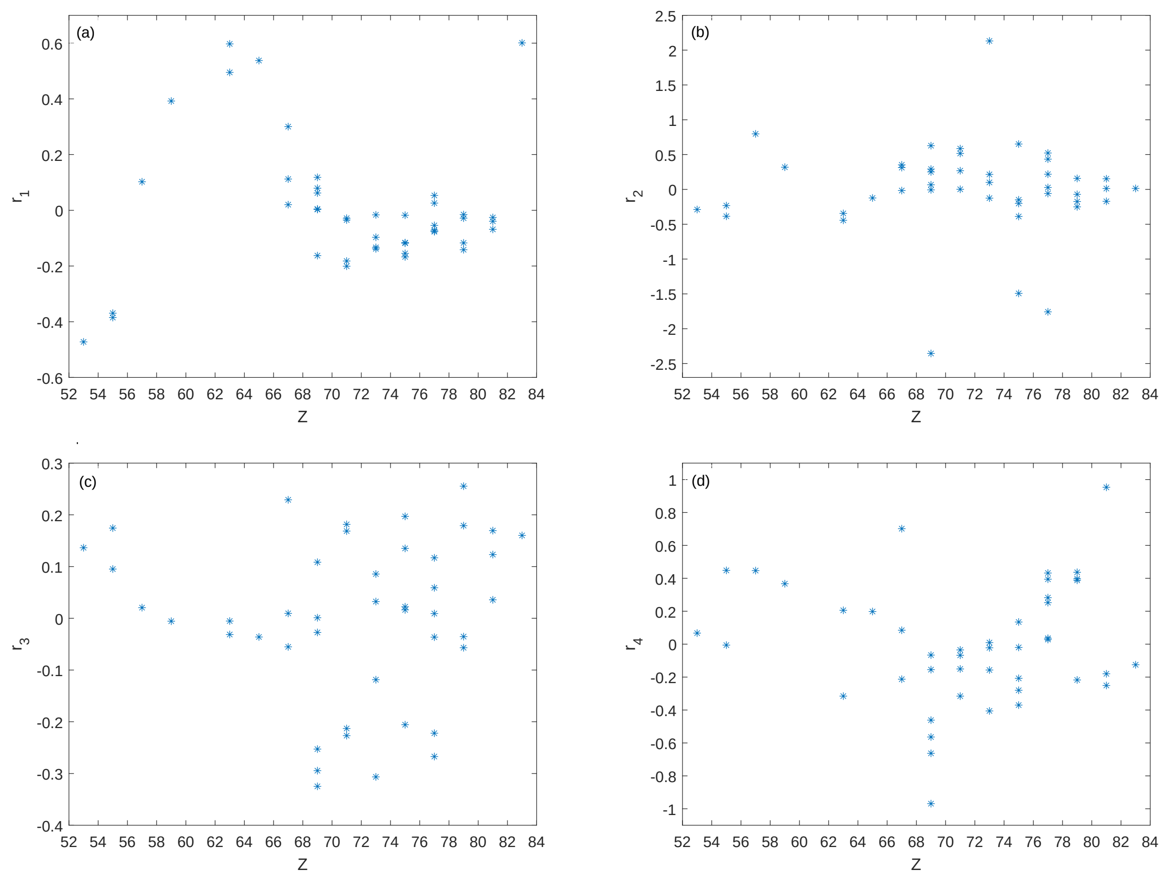

Fig. 2. (color online) r_1, r_2, r_3, and r_4 parameters versus atomic number of proton emitters.

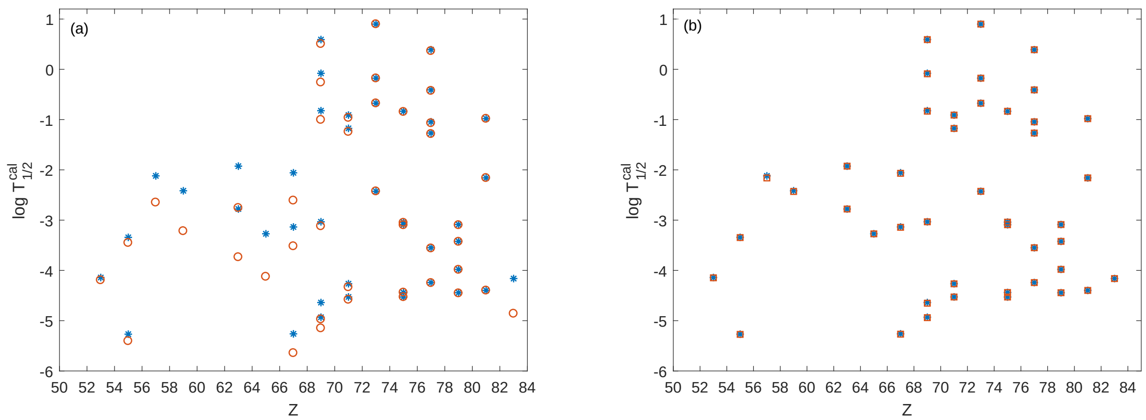

Fig. 3. (color online) Contribution of (a) quadrupole and hexadecapole deformation parameters and (b) hexadecapole deformation parameter in proton decay half-lives. The star, square, and circle symbols are calculated half-lives with parameters of set I with experimental Q-value for all proton emitters with full deformation parameters β_2 and β_4, with just β_2 by setting β_4=0, with setting β_2=β_4=0, respectively.

$Q_{\rm exp}$ and the complete set of deformation parameters β_2 and β_4 (stars) are compared with those calculated using the same formula but with $\beta_2=\beta_4=0$ (circles). This figure reveals that the nuclear deformation significantly affects the calculations. The maximum difference, approximately one unit in the logarithm of the half-life, is observed for the ^{117}La, ^{130,131}Eu, ^{135}Tb, and ^{185}Bi nuclei, which have large deformation parameters. In Fig. 3(b), the half-lives calculated with the complete set of deformation parameters (stars) are compared with those calculated using the same formula but with β_4=0 (squares). This figure shows that including the hexadecapole deformation in the calculations can affect the decay half-lives slightly. The maximum difference of approximately 0.04 is seen for ^{117}La nuclei. However, as can be seen in Fig. 1(b) and Table 3, the hexadecapole deformation parameter of most proton emitters is negligible or very small, except for ^{117}La.

Therefore, one can conclude that inclusion of the nuclear deformation parameters in the proton decay half-life formula, the use of $Q_{\rm exp}$, and the explicit or implicit dependence of the formula on the angular momentum have noticeable effects on the calculated half-lives.

It is worth noting that, by considering the nuclear deformation in Eq. (7),

$$\log(T(s)) = a+bA^{1/6}Z^{1/2}+cZQ^{-1/2} +dl(l+1)A^{-1/6}Z^{-1/2}-\log_{10}S_{\rm p}+d_2|\beta_2|^{p_2}, \tag{17}$$

and by using the data of Ref. [46] for the Q value and the spectroscopic factors of 26 proton emitters, the parameters of the proton decay half-life formula are obtained as $a=-27.477$, $b=-0.524$, $c=0.417$, $d=2.341$, $d_2=6.683$, $p_2=0.022$, with an rms error $\sqrt{\delta^2}=0.126$. The 13% reduction in the rms error reveals the role of nuclear deformation in the proton decay half-life formula.

4 Conclusion

By using calculated and experimental Q values, two empirical formulas with explicit (set I) and implicit (set II) dependence on the angular momentum were obtained for calculation of the half-lives of 44 deformed proton emitters in the ground state transition, isomeric transition, and all transitions. Twenty-four sets of parameters were obtained for both the deformed and spherical cases. Lower rms errors were obtained when the nuclear deformation, experimental Q value, and explicit function of the angular momentum were considered. The obtained results showed that the half-life of the proton radioactivity is highly sensitive to the Q value and angular momentum. Because of the strict and complicated dependence of set II on the Q value, the rms error decreased significantly when the experimental $Q_{\rm exp}$ value was considered. The calculated half-lives were in good agreement with experimental data.

We thank Dongdong Ni for valuable comments and discussions.

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