Comment on 'Note on $X(3872)$ production at hadron colliders and its molecular structure'*

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Abstract: We briefly comment on the paper by Albaladejo et al., Chinese Phys. C 41 121001, rejecting its conclusions.

The inequality (1) of the comment by Albaladejo et al. [1] was introduced in Ref. [2] in order to estimate the upper bound of the $X(3872)$ prompt production cross section at CDF without resorting to hadronic models for the calculation of the amplitude $\Psi(\boldsymbol{p}) = \langle X|D^0\bar{D}^{*0}(\boldsymbol{p})\rangle$. We are still convinced that this is the correct way to proceed and we do not see any progress made in Ref. [1].

To the effect of computing the cross section mentioned, we used the uncertainty principle to estimate the size of the allowed ball $\mathcal{R}$ in the space of relative $\boldsymbol{p}$ momenta in the center of mass of the generic $D^0\bar{D}^{*0}$ pair produced in a $pp(\bar{p})$ collision. The size of $\mathcal{R}$ must be compatible with the 'coalescence' of a meson pair into a loosely bound meson molecule. Hence the radius of the $\mathcal{R}$ ball is determined a priori and Eq. (3) in Ref. [2], once $\mathcal{R}$ is determined, simply reads

$$
\begin{aligned}
\sigma(p\bar{p}\rightarrow X) &\simeq \left|\int_{\mathcal{R}}\Psi(\boldsymbol{p})\langle D^0\bar{D}^{*0}(\boldsymbol{p})|\bar{p}p\rangle \mathrm{d}^3 p\right|^2 \\
&\lesssim \int_{\mathcal{R}}|\Psi(\boldsymbol{p})|^2\mathrm{d}^3 p \int_{\mathcal{R}}\left|\langle D^0\bar{D}^{*0}(\boldsymbol{p})|\bar{p}p\rangle\right|^2\mathrm{d}^3 p \\
&\lesssim \int_{\mathcal{R}}\left|\langle D^0\bar{D}^{*0}(\boldsymbol{p})|\bar{p}p\rangle\right|^2\mathrm{d}^3 p.
\end{aligned} \tag{1}
$$

Even if it were known how to compute the amplitude $\Psi(\boldsymbol{p})$ from first principles (which is not the case), the size of the $\mathcal{R}$ ball in momentum space should be understood on the basis of physical arguments as the ones reported below, suggesting its radius to be $\bar{p}\sim 20$ MeV. These arguments involve square moduli of the amplitudes only.

We first briefly analyze the case of the deuteron, where more experimental information is available. The attractive Yukawa potential can be parameterized as

$$
V = -g\frac{\mathrm{e}^{-r/r_0}}{r}, \tag{2}
$$

where $r_0\sim 1/m_\pi = 1.4$ fm and

$$
g = \frac{f_{\pi N}^2}{4\pi}, \tag{3}
$$

with $f_{\pi N} \approx 2.1$, as can be computed by solving the Schrödinger problem to get a binding energy of 2.2 MeV[1]. The quantum version of the virial theorem is

$$
2\overline{T} = \left(\Psi, \sum_{i=1}^{3}r_i\frac{\partial V}{\partial r_i}\Psi\right) = -\overline{V} + \frac{g}{r_0}\overline{\mathrm{e}^{-r/r_0}}, \tag{4}
$$

therefore the mean $\overline{E} = \overline{T} + \overline{V}$ is

$$
\overline{E} = -\frac{\bar{p}^2}{2\mu} + \frac{g}{r_0}\overline{\mathrm{e}^{-r/r_0}}, \tag{5}
$$

where μ is the reduced mass of the bound state.

1) The following results are also found using a square well potential and are well documented in the literature.

Both the radius, $\overline{r} = 2.1$ fm [3], and the binding energy, $\overline{E} \simeq -2.2$ MeV, are known for the deuteron. Equation (5) then gives $\bar{p}_D \approx 105$ MeV. It is this that determines the radius of the ball $\mathcal{R}$, significantly smaller than the value given in Ref. [1], $\bar{p} \gtrsim 300$ MeV.

The $X(3872)$ is a more extreme case. The binding energy is found below $|\overline{E}| \lesssim 0.1$ MeV and, as commented in a large number of papers (see reviews in Ref. [4] and references therein), for such a small binding energy the expected size of the state is $\bar{r} \gtrsim 1/\sqrt{2\mu|\overline{E}|} \sim 10$ fm. This allows us to neglect the second term in Eq. (5), finding $\bar{p} \approx 20$ MeV, in agreement with the radius of the region $\mathcal{R}$ used in Ref. [2]. We have checked that the value of g cannot be large enough to spoil the approximation used. This can be done either extracting g from the $D^* \rightarrow D\pi$ decay rate [5] or by solving the bound state problem.

In contrast, Albaladejo et al. [1], to estimate the radius $\bar{p}$ of $\mathcal{R}$ corresponding to the production of $X(3872)$, assume that $\langle D^0 \bar{D}^{*0}(\boldsymbol{p})|\bar{p}p\rangle$ in Eq. (1) is almost independent of $\boldsymbol{p}$ and study the quantity

$$I(\bar{p}) = \int_{\mathcal{R}} \Psi(\boldsymbol{p}) \mathrm{d}^3 p, \qquad (6)$$

seeking the $\bar{p}$ value such that $I(\bar{p})$ becomes constant from there on. As an example, the normalizable wave function for shallow bound states used by Artoisenet and Braaten [6] to describe the $X(3872)$ molecule decreases like p^{-2} so that, for large $\bar{p}$

$$I(\bar{p}) \sim \bar{p}, \qquad (7)$$

which indeed does not indicate any region at all. This makes the approach in Ref. [1] totally useless. To circumvent this obvious problem, a cutoff Λ is introduced to manipulate the wave-function $\Psi(\boldsymbol{p}) \rightarrow \Psi_\Lambda(\boldsymbol{p})$ in Eq. (6).

This cannot be the way to determine the size of $\mathcal{R}$ which, in our view, must be obtained with arguments involving the binding energy and the interaction coupling constant, independently of any educated guesses on the explicit form of $\Psi(\boldsymbol{p})$. One should also notice that the ad hoc treatment of the cutoff introduces a change of sign in $\Psi(\boldsymbol{p})$ depending on Λ (see Fig. 1 of Albaladejo et al. [1]) — as if the amplitude for projecting the state $|D\bar{D}^*\rangle$ onto the observed $|X\rangle$ could go through some spurious zeroes. Also, the S-wave wave function of the deuteron in Fig. 2 of Ref. [1] displays zeroes for some particular choices of the cutoff and the model, in contradiction with the fact that the ground state wave function should not present any node.

Disregarding for a moment all these adverse considerations on Ref. [1], the bare minimum one can conclude from it is that deuterons should be produced equally or more abundantly than molecular $X(3872)$ resonances. For a definitive test one should, then, compare the production of these two particles at high transverse momenta, say, $p_\perp > 15$ GeV, where the $X(3872)$ is copiously observed at CMS.

Extrapolation of the fits shown in Ref. [8] suggests an extremely low deuteron production cross section at high transverse momenta, in agreement with our estimate of the size $\bar{p}$ of the deuteron as a pn molecule and in contrast with the large observed cross-section for $X(3872)$ production. There are currently no available data for deuteron production at such high transverse momenta; however, measurements might be possible at ALICE and LHCb Run II.

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