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Asymptotic-de sitter inflation in the light of the planck data*

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Abstract: Planck 2015 data, emphasize that the background geometry during inflation is not pure de Sitter, but from the slow variation of Hubble parameter during the inflationary era, it can be quasi-de Sitter. This motivates us to consider an Asymptotic-de Sitter mode function for reconstructing of initial mode and primordial power spectrum of curvature perturbation. Using Markov Chain Monte Carlo (MCMC) method together with applying recent observational constraints from the Cosmic Microwave Background (CMB) data for the parameterized asymptotic initial mode in term of c, show some deviation from Bunch-Davies mode ($c=1$). Based on Planck 2015 data release the amplitude of scalar perturbations in 68% confidence level is $10^9 A_s = 2.94^{+0.42}_{-0.42}$ and deviation from Bunch-Davies mode is ~ 0.05, i.e. $c \sim 1.05$. In this parametrization, the CMB power spectrum of our model shows more red-tilt in comparison with ΛCDM model. Furthermore, we found upper limits for tensor-to-scaler ratio with different pivot scales.

Keywords: inflation, cosmic microwave background, initial state

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1 Introduction

Observation of the Cosmic Microwave Background Radiation (CMBR) give us a good opportunity to test the new theoretical models of early universe. The inflationary scenario is a robust theoretical framework which prepares a physically causal mechanism for large-scale structure formation [1]. During the period of inflation, very small causally connected regions are expanded exponentially due to the inflationary expansion of the universe [2–5]. In this way, inflation magnifies amplitude of all tiny quantum fluctuations and therefore, the frequencies of fluctuations are shifted to the red part of the spectrum.

One of the main predictions of this theory is primordial curvature perturbation which leads to an imprint on the observed CMBR. Indeed, the pattern of the CMB angular power spectrum is dependent on the particular inflationary model. Therefore, by considering a primordial power spectrum, one can calculates numerically the angular power spectrum of the CMB, and thus a parameterized primordial curvature spectrum can be compared with CMB data [6]. For example, the power-law parametrization of power spectrum with a running index is the simplest parameterized model which is mostly consistent with the CMB data. Other parameterizations, motivated by the theoretical models or observed data such as broken power spectrum [7, 8]and cut-off at large scales [7, 9, 10] are suggested.

On the other hand, the initial state of primordial fluctuations can affect their statistical properties such as power spectrum. Motivated by this fact together with observational data released by WMAP and Planck, some studies reveal that the initial state effects may be from pre-inflationary evolution [11–13]; or from a non-singular bounce as studied in [14, 15]; or from trans-Planckian physics [16, 17], or from the string theory effects [18, 19] and so on.

In general, it is assumed that these tiny quantum fluctuations begin in a minimum energy state which is called Bunch-Davies (BD)vacuum state [20]. Since, we have not enough information about the pre inflationary era, there is no exact reason for the selection of initial BD vacuum [21–24]. In addition to these studies, the CMB anisotropy measured by the Planck, offers an approximately de Sitter geometry for unknown physics era [25]. So, it is claimed that one can assumes, the initial state of quantum fluctuations deviates from BD state.

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Now, the key question is, how much is the deviation from the pure de Sitter geometry? The Answer of this important question, stimulates us to probe the shape of new initial mode function, named Asymptotic-de Sitter mode. First time, this mode is proposed based on considering an expansion form of Hankel function up to second order with respect to $1/k\tau$ [26].

Afterward, the authors in [27, 28], parameterized this mode in terms of Hankel function index i.e. ν, and discussed on asymptotic method for selection of the general form of mode functions in the inflationary background. Another possibility (i.e. excited state) considered in [29, 30], showed that this non-trivial initial mode of field equation which is introduced for nearly de Sitter background, leads to the higher order corrections in primordial scalar power spectrum or appears the cut-off scale in it [31].

It is well known, considering a model by strong deviation from de Sitter background, leads to the lack of cosmological inflation idea. So, the new mode is parameterized in a way that it approaches asymptotically to the standard de Sitter mode in the first approximation. It is clear that a specific initial time τ_0, at the beginning of inflation, the choice of initial mode is completely arbitrary. But, the quasi-de Sitter form of background of the inflationary expansion, confirmed by recent observation, motivates us to apply this mode. Actually, our mode is defined for early time limit and it can use as a correction to the BD mode. In addition, it was proved that this mode do not generate any back-reaction effect in this limit [28].

In this work, in order to constrain our model parameters and reconstruct the power spectrum of curvature perturbations and Asymptotic-de Sitter mode, we perform MCMC analysis using the recent Planck 2015 temperature data. The results show that, our CMB spectra has more red-tilt compared with ΛCDM model, which can be due to the increase in fluctuations correlation, which is predicted by our model [32]. Also, unlike the large tensor-to-scalar ratio value ($r\sim0.2$) released by BICEP2, we have found that r value for our tensor mode spectra, i.e. $r_{0.05}<0.059$ and $r_{0.002}<0.006$, is confirmed by Planck 2015 results.

Our work is organized as follows: In Sec. 2 the cosmological perturbation theory and general Asymptotic-de Sitter mode is briefly reviewed. In Sec. 3 we derive the parameterized power spectrum of curvature perturbation. Afterward, we compare model with Planck TT data and ΛCDM model. In Sec. 4 the results of the CMB analysis and upper bound to tensor-to-scalar ratio are given. Finally, we conclude in Sec. 5.

2 Cosmological perturbations with asymptotic-de sitter mode function

2.1 Quantum fluctuations in inflationary background

From the perspective of effective field theory, unknown physics effects as trans-Planckian effects, can appear in the background geometry as well as the initial state of quantum fluctuations of scalar field and its evolution. So, we can make departure from homogeneous universe and consider the perturbed metric with two more degrees of freedom, $\Phi(\tau,\mathbf{x})$ and $\Psi(\tau,\mathbf{x})$. The line element of perturbed metric based on these functions can be written as

$$\begin{aligned}\mathrm{d}s^2&=-(1+2\Phi)\mathrm{d}t^2+a^2(t)(1-2\Psi)\mathrm{d}x^2\\&=a^2(\tau)(-(1+2\Phi)\mathrm{d}\tau^2+(1-2\Psi)\mathrm{d}x^2),\end{aligned}\tag{1}$$

where Φ and Ψ coincide with gauge-invariant potentials in the conformal-Newtonian gauge. On the other hand the inflaton field fluctuates about its homogeneous background part during inflation as

$$\phi(\tau,x)=\phi_0+\delta\phi(\tau,x),\tag{2}$$

where ϕ_0 is homogeneous part of the field. Because of Einstein field equation, the fluctuations of metric and perturbed part of inflaton field are related to each other by using Mukhanov variables v and z in gauge invariant formalism in the following form

$$v=a\delta\phi+z\Phi,\tag{3}$$

where $z=a\phi_0/H$. It is considered, the dynamics of quantum fluctuations are governed by following action [2]

$$S=\frac{1}{2}\int\mathrm{d}^3x\mathrm{d}\tau\left((v')^2-(\nabla v)^2+\frac{z''}{z}v^2\right).\tag{4}$$

So the equation of motion in Fourier space for primordial scalar perturbations in gauge invariant formalism is

$$v''_k+\left(k^2-\frac{z''}{z}\right)v_k=0,\tag{5}$$

where prime denotes derivative with respect to conformal time τ and $v_k(\tau)$ is the Fourier mode of quantum field

$$\hat{v}(\tau,x)=\int\frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\left(\hat{a}_k v_k(\tau)\mathrm{e}^{\mathrm{i}k.x}+\hat{a}^\dagger_k v^*_k(\tau)\mathrm{e}^{-\mathrm{i}k.x}\right),\tag{6}$$

where $\hat{a}_k$ ($\hat{a}^\dagger_k$) is annihilation (creation) operator.

2.2 Power spectrum of curvature perturbation with BD mode

Considering pure de Sitter geometry during inflation for space-time expansion leads to the following equation for time evolution of mode function

$$v''_k+\left(k^2-\frac{2}{\tau^2}\right)v_k=0.\tag{7}$$

In order to coincide solutions with the BD mode, the initial conditions i.e. $\tau\to-\infty, v_k(\tau)\to\frac{e^{ik\tau}}{\sqrt{k}}$ and normalization condition $W[u_k^*,u_k']=-i$ are imposed to them. Then the solution of (7), named BD mode, takes the following form

$$v_k^{\mathrm{BD}}=\frac{1}{\sqrt{2k}}\left(1-\frac{i}{k\tau}\right)\mathrm{e}^{-\mathrm{i}k\tau}. \tag{8}$$

The power spectrum of curvature perturbation in gauge invariant formalism on super-horizon scales is

$$P_{\mathcal{R}}(k)=\frac{k^3}{2\pi^2}\left|\frac{v_k}{z}\right|^2. \tag{9}$$

So, for BD mode

$$P_{\mathcal{R}}(k)=\left(\frac{H}{2\pi}\right)^2\left(\frac{H}{\dot{\phi}_0}\right)^2. \tag{10}$$

2.3 Asymptotic-de Sitter modes for scalar perturbation

One of the main aspects of the initial condition for inflation is the choice of the initial vacuum state at the beginning of the inflationary phase. Indeed, the vacuum state can be fixed by choosing proper initial mode function. In the limit of quasi-de Sitter inflation the quantity $\frac{z''}{z}$ is not equal to $\frac{2}{\tau^2}$, and one can rewrite the mode equation (5) as

$$v_k''+\left(k^2-\frac{\nu^2-1/4}{\tau^2}\right)v_k=0. \tag{11}$$

So the general solution for $v_k(\tau)$ is

$$v_k=\frac{\sqrt{\pi\tau}}{2}\left(A_kH_\nu^{(1)}(|k\tau|)+B_kH_\nu^{(2)}(|k\tau|)\right). \tag{12}$$

For $\nu=\frac{3}{2}$ we have pure de Sitter mode up to first order of $1/|k\tau|$, but for $\nu\approx\frac{3}{2}$ and the early time limit of the universe $|k\tau|\gg1$, we can use the expansion form of Hankel function up to the higher orders of $1/|k\tau|$ [26, 28, 29]. So, we generate a whole family of modifications of the usual de Sitter mode function in terms of the parameter c and conformal time τ as [28]

$$v_k^{\mathrm{gen}}=\frac{A_k\mathrm{e}^{-\mathrm{i}k\tau}}{\sqrt{2k}}\left(1-\frac{ic}{k\tau}-\frac{d}{k^2\tau^2}+\dots\right)+\frac{B_k\mathrm{e}^{\mathrm{i}k\tau}}{\sqrt{2k}}\left(1+\frac{ic}{k\tau}-\frac{d}{k^2\tau^2}+\dots\right), \tag{13}$$

where by considering initial condition and Wronskian of solutions we obtain the positive frequency mode function which is called Asymptotic-de Sitter Mode [28] as

$$v_k^{\mathrm{Asymp}}=\frac{\mathrm{e}^{-\mathrm{i}k\tau}}{\sqrt{2k}}\left(1-\frac{ic}{k\tau}-\frac{d}{k^2\tau^2}\right). \tag{14}$$

Substitution of this solution in (11) concludes that

$$c=\frac{4\nu^2-1}{8},\quad d=\frac{c(c-1)}{2}. \tag{15}$$

As we can see, the general mode (14) is a function of both parameters τ and c, so the choice of initial vacuum not only depend on conformal time τ, but also depend on the value of c (type of the geometry [28]).

3 Reconstruction of asymptotic-de Sitter mode

3.1 Parametrization of primordial curvature perturbation

In order to reconstruct the primordial power spectrum of curvature perturbation, at first we should calculate $P_{\mathcal{R}}$ by substituting new mode function (14) into relation (9). As a result we can find primordial power spectrum of scalar perturbation as

$$P_{\mathcal{R}}(k)=\frac{H^2}{8\pi^2\epsilon M_{\mathrm{Pl}}^2}\left[c+\frac{d^2}{(k\tau_*)^2}\right], \tag{16}$$

by consideration of $\Lambda=-kH\tau_*$ with condition $H\ll\Lambda<M_{\mathrm{Pl}}$ that τ_* is an initial fixed time that first used for calculation of power spectrum [34], we can rewrite

$$P_{\mathcal{R}}(k)=\frac{H^2}{8\pi^2\epsilon M_{\mathrm{Pl}}^2}\left[c+d^2\left(\frac{H}{\Lambda}\right)^2\right].$$

On the other hand, in the quasi- de Sitter inflation the Hubble parameter is scale dependent as

$$\frac{H}{H_*}\sim\left(\frac{k}{k_*}\right)^{-\epsilon}, \tag{17}$$

where H_* is the Hubble parameter correspond to the particular scale k_*, when the perturbation with this scale exits the horizon [33]. In practice, we choose k_* as a pivot scale which is the largest observable scales exited the horizon at first. Substituting (17) into (16) yields

$$P_{\mathcal{R}}(k)=\frac{H^2}{8\pi^2\epsilon M_{\mathrm{Pl}}^2}\left[c+\left(\frac{d^2H_*^2}{\Lambda^2}\right)\left(\frac{k}{k_*}\right)^{-2\epsilon}\right]. \tag{18}$$

In next step, we can rewrite the formula (18) in form of a parameterized power spectrum

$$P_{\mathcal{R}}(k)=\frac{A_S}{[c+\Gamma_*d^2]}\left[c+\Gamma_*d^2\left(\frac{k}{k_*}\right)^{-\xi}\right], \tag{19}$$

where $A_S=\frac{H^4}{4\pi^2\dot{\phi}_0^2}$ is the scale invariant power spectrum and $\Gamma_*=\left(\frac{H_*}{\Lambda}\right)^2$ and $\xi=2\epsilon$. Within this parametrization, we employ four free parameters i.e. A_S,c,Γ_*,ξ which the parameter c has a main role among them. Indeed, the main goal of this study is to explore a tiny deviation from pure de Sitter geometry (i.e. $c=1$) in the inflationary background, which as expected is fond by estimation of the parameter c.

3.2 CMB power spectrum of scalar modes

The CMB power spectrum are given by [6]

$$C_\ell = (4\pi)^2 \int \mathrm{d}k k^2 T_\ell(k) T'_\ell(k) P_{\mathcal{R}}(k). \tag{20}$$

To calculate power spectrum for our model we need to solve Einstein-Boltzmann set of equations. For this purpose we have modified the publicly available Einstein-Boltzmann code CAMB [35] to compute power spectrum of anisotropies with modified primordial power spectrum which is parameterized according to (19). In Fig. 1 we see temperature anisotropy power spectrum of scalar modes $\mathcal{D}_\ell = \ell(\ell+1)C_\ell/2\pi$ in comparison with Planck data and ΛCDM model. Obviously model shows consistency with observation but has greater ISW power spectrum than ΛCDM.

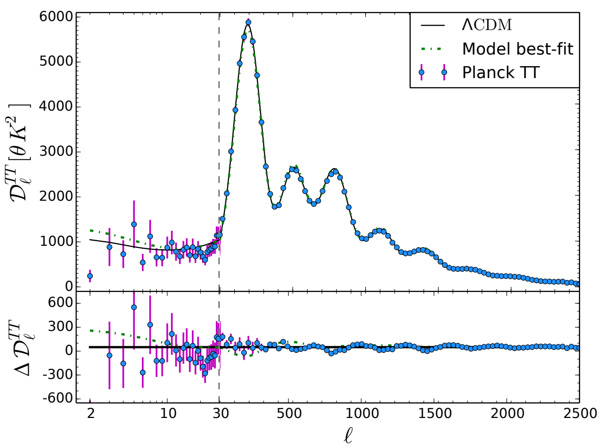

Fig. 1. (color online) Upper panel: Power spectrum of temperature anisotropy C_ℓ for model in comparison with ΛCDM model and Planck TT data. Our model shows more red-tilt than ΛCDM for small multiples ℓ. Lower panel: Residue of model with respect to ΛCDM and observational data.

3.3 CMB power spectrum of tensor mode perturbations

Unlike the E-mode component of the CMB temperature anisotropy, the B-mode polarization anisotropy is sourced by tensor perturbation. As discussed in [2] , the tensor mode function of perturbations are also given by Hankel function. So, a similar analysis to scalar mode can be repeated to achieve tensor perturbations spectra by employing Asymptotic-de Sitter mode function. In the usual single-field inflation, the tensor mode spectrum is in this form

$$P_h = \frac{2}{M_{\rm Pl}} P_\phi, \tag{21}$$

where $P_\phi = \frac{k^3}{2\pi^2}|v_k|^2$ is the scalar mode power spectrum. Substituting (14) into definition of P_ϕ, yields

$$P_h = \frac{H^2}{2\pi^2 M_{\rm Pl}^2}\left[c+\Gamma_* d^2\left(\frac{k}{k_*}\right)^{-\xi}\right]. \tag{22}$$

By parametrizing this result same as the scalar mode (i.e. (19)) we have

$$P_h = \frac{A_T}{[c+\Gamma_* d^2]}\left[c+\Gamma_* d^2\left(\frac{k}{k_*}\right)^{-\xi}\right]. \tag{23}$$

In Fig. 2 we have shown tensor power spectrum for different values of tensor-to-scalar ratio r.

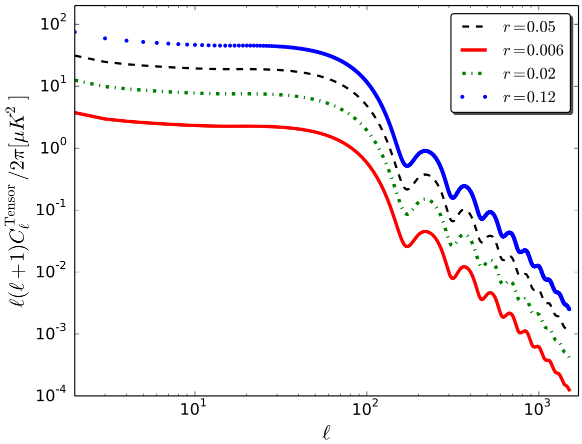

Fig. 2. (color online) The tensor power spectrum of model with different values of tensor-to-scalar ratio.

4 Estimation of cosmological parameters

We explore the parameter space by Monte Carlo code for Cosmological Parameter extraction CosmoMC [36] connected to the modified version of Einstein-Boltzmann code CAMB.

4.1 Observational constraints from Planck data

As it mentioned in previous sections we modified CAMB code to calculate power spectrum. To estimate parameters we joined CAMB program with Monte Carlo code CosmoMC.

The Markov Chain Monte Carlo (MCMC) method is an iterative algorithm that sample parameter space from a prior distribution on given parameters, by constructing a Markov chain, that samples the desired posterior distribution.

CosmoMC is a Fortran MCMC code for exploring cosmological parameter space. The code does compute accurate theoretical matter and CMB power spectra with the aid of CAMB code. It then produces a set of chain files that includes the chain values of the cosmological parameters requested, and it comes together with a Python tool, GetDist, that analyzes the chain files [37].

We assume spatially flat geometry for background and set pivot scale to $k_* = 0.05$ Mpc^{-1} in our analysis. To determine best-fit values and confidence ranges we used CMB temperature fluctuations angular power spectra from Planck 2015.

Our parameter space contains 8 parameters

$$\Theta_k: \quad \Omega_b h^2, \Omega_c h^2, \theta, A_S, \Gamma_* \xi, c, \tau_{\rm opt}$$

The best-fit values of cosmological parameters, and their 1σ marginalized limits from Planck are reported in Table 2. It is clear that, (Table 2) early-time observation (CMB) gives parameters with more accuracy.

Table 1. Prior on parameter space, used in the posterior analysis in this paper.

parameter	prior	shape of PDF
$\Omega_b h^2$	[0.005−0.100]	Top-Hat
$\Omega_c h^2$	[0.001−0.990]	Top-Hat
c	[0.85−1.15]	Top-Hat
θ	[0.5−10]	Top-Hat
Γ_0	[0.92−1.1]	Top-Hat
ξ	[0.01−0.1]	Top-Hat
$\ln(10^{10} A_s)$	[2.000−4.000]	Top-Hat
τ_{opt}	[0.01−0.8]	Top-Hat

Table 2. Bayesian 68% confidence limits for model based on Planck TT with k_*=0.05 Mpc^{-1}.

parameter	Planck TT
$\Omega_b h^2$	$0.02159^{+0.00018}_{-0.00028}$
$\Omega_c h^2$	$0.11182^{+0.00095}_{-0.0024}$
Ω_m	$0.270^{+0.015}_{-0.017}$
$100\theta_{MC}$	$1.04129^{+0.00088}_{-0.00083}$
$10^9 A_S$	$2.94^{+0.42}_{-0.42}$
c	$1.0535^{+0.0089}_{+0.0010}$
$10^4\Gamma_*$	$1.01109^{-0.00068}_{-0.013}$
ξ	$0.02952^{+0.00057}_{-0.0016}$
τ_{opt}	$0.207^{+0.035}_{-0.022}$
r	<0.0588

Table 3. Bayesian 68% confidence limits for model based on Planck TT with k_*=0.002 Mpc^{-1}.

parameter	Planck TT
$\Omega_b h^2$	$0.02157^{+0.00018}_{-0.00026}$
$\Omega_c h^2$	$0.11211^{+0.00062}_{-0.0027}$
Ω_m	$0.278^{+0.045}_{-0.024}$
$100\theta_{MC}$	$1.04092^{+0.00089}_{-0.00085}$
$10^9 A_S$	$2.94^{+0.45}_{-0.55}$
c	$1.0608^{+0.0014}_{-0.0019}$
$10^4\Gamma_*$	$1.00972^{+0.00078}_{-0.00027}$
ξ	$0.0324^{+0.017}_{-0.0041}$
τ_{opt}	$0.207^{+0.032}_{-0.024}$
r	<0.00625

For $k_* = 0.05$ Mpc^{-1} we have Table 2 and For $k_* = 0.002$ Mpc^{-1} we have Table 3.

4.2 Upper bound on r

Now, we are going to imply the Planck constraint on tensor mode perturbations. As a definition, the standard primordial tensor-to-scalar ratio at the pivot scale is

$$r = \frac{P_h(k_*)}{P_{\mathcal{R}}(k_*)}, \tag{24}$$

where $P_h(k_*) = A_T = \frac{2H^2}{\pi^2 M_{\rm Pl}^2}$ and $P_{\mathcal{R}}(k_*) = A_S$ [5]. The constraints on tensor-to-scaler ratio based on Planck full mission data by assuming two cases, with $k_* = 0.05$ Mpc^{-1} is:

$$r_{0.05} < 0.059, \tag{25}$$

and with k_*=0.002 Mpc^{-1} is:

$$r_{0.002} < 0.006. \tag{26}$$

Planck team in 2015 data release reported an upper limit for this ratio $r_{0.002}<0.11$ [25] which our results is consistent with their result.

5 Summary and conclusions

In this paper we examined the cosmological inflationary model with a asymptotic-de Sitter geometry. Indeed, we reconstructed the shape of primordial curvature spectra and Asymptotic-de Sitter mode function corresponding to quantum fluctuations. To check consistency of model with cosmological observations a maximum likelihood analysis was performed by using publicly available MCMC code CosmoMC. Primordial power spectrum of model is parameterized with c, A_s, Γ_*, ξ and from Planck 2015 data the best-fit of parameters are: $\Omega_b h^2 = 0.02159^{+0.00018}_{-0.00028}$, $\Omega_c h^2 = 0.11182^{+0.00095}_{-0.0024}$, $10^9 A_s = 2.94^{+0.42}_{-0.42}$, $c = 1.0535^{+0.0089}_{+0.0010}$, $10^4\Gamma_* = 1.01109^{-0.00068}_{-0.013}$ and ξ=$0.02952^{+0.00057}_{-0.0016}$. Also upper bound on tensor to scalar ratio is $r<0.0588$.

The results indicated that the initial mode, depended to parameter c, has a slight deviation i.e. $\sim 0.05-0.06$, from Bunch-Davies mode. Conclusively, the new CMB anisotropy spectrum shows more red-tilt in comparison with scale invariant CMB spectrum. This discrepancy between new spectrum and ΛCDM in the low ℓ limit ($\ell<10$) is sensible because, amplitude of primordial fluctuations predicted by our model is higher than the other one. Moreover, by applying Planck constraint on the free parameters of our model, we achieve to the new limit for tensor to scalar ratio during inflation when the scales of perturbations is equal to pivot scale.

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