

ρ meson impact parameter distributions^{*}

Bao-Dong Sun(孙保东)^{1,2;1)} Yu-Bing Dong(董宇兵)^{1,2,3}

¹Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

²School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China

³Theoretical Physics Center for Science Facilities (TPCSF), CAS, Beijing 100049, China

Abstract: In this paper, the ρ meson impact parameter dependent parton distributions and the impact parameter dependent form factors are introduced and discussed. By employing a Gaussian form wave packet, we calculate the impact parameter distributions of the ρ meson based on a light-cone constituent quark model.

Keywords: ρ meson, impact parameter distribution, light-cone approach

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1 Introduction

It is common believed that the usual parton distributions (PDFs) can only give the longitudinal information of a hadron target in the deep inelastic scattering (DIS) processes, while the generalized parton distributions (GPDs) have the promising ability to shade light on the transverse information, which gives rise to the idea of “quark/gluon imaging” of hadrons [1]. Moreover, the impact parameter distributions (IPDs), obtained by the Fourier transform of GPDs with respect to the transverse momentum transfer, may show some information about the transverse impact space position of partons [2]. This impact parameter representation is useful in processes such as high-energy scattering and hard processes [3]. It is also argued that, in position space, IPDs play a similar role to the charge distributions, and are, thus, very promising for understanding the hadron internal structures.

As we know, $G_C(Q^2)$ is the form factor of the conserved local current, and is thus independent of the renormalization scale μ . It can be obtained through the sum rules from GPDs, which by definition are probed in hard processes [3]. In the case of Fourier transforms of GPDs, Burkardt pointed out that, when $\xi = 0$, the Fourier transforms of GPDs have the interpretation of a density of partons with longitudinal momentum fraction x , localized at \mathbf{b}_\perp relative to the transverse center in the impact parameter space, which is allowed by the Heisenberg uncertainty principle [4, 5]. Due to the significance

of the form factors in the impact parameter space, many theoretical works have been devoted to study the IPDs of pions, kaons and nucleons [5–15].

It should be mentioned that our recent work [16] gave a discussion of the ρ meson unpolarized GPDs in momentum space with a Light-Cone Constituent Quark Model (LCCQM). The form factors and some other low-energy observables of the ρ meson were calculated and our numerical results agreed with the previous publications and some experimental data [17]. In the literature, the constituent quark model is also used to describe the form factors of pions, nucleons, deuterons, etc. [18–20]. Moreover, the contributions from the valence and non-valence regimes to the form factors and generalized parton distributions were discussed and analyzed in detail. In addition, the reduced matrix elements, which are the moments of the DIS structure functions, were also estimated and the obtained values were compatible with the available lattice calculation at the same scale ratio [21]. In general, our numerical results for the unpolarized GPDs [16] were reasonable and satisfying. Therefore, in this work, we extend the phenomenological model to study the IPDs of the ρ meson and to calculate the impact parameter dependent PDFs of $q(x, \mathbf{b}_\perp)$ and $q(\mathbf{b}_\perp)$ and the form factors of $q^{C,M,Q}(x, \mathbf{b}_\perp)$ and $q^{C,M,Q}(\mathbf{b}_\perp)$.

The paper is organized as follows. In Section 2, the framework of the impact parameter dependent PDFs is presented. In Section 3, we discuss the wave packets and the cutoff for the numerical calculation. The definitions of the impact parameter dependent FFs are given in Sec-

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1) E-mail: sunbd@ihep.ac.cn



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tion 4. Our numerical results for the PDFs and FFs in the impact parameter space are shown in Section 5, and Section 6 gives a short summary and conclusion.

2 Impact parameter dependent PDFs

When considering the nucleon GPDs without helicity

$$\begin{aligned}
 q_N(x, \mathbf{b}_\perp) &= |\mathcal{N}|^2 \int \frac{d^2 \mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2 \mathbf{p}'_\perp}{(2\pi)^2} \times \langle p^+, \mathbf{p}'_\perp, \lambda | \left[\int \frac{dz^-}{4\pi} \bar{q}(-\frac{z^-}{2}, \mathbf{b}_\perp) \gamma^+ q(\frac{z^-}{2}, \mathbf{b}_\perp) e^{-ixp^+ z^-} \right] | p^+, \mathbf{p}_\perp, \lambda \rangle \\
 &= |\mathcal{N}|^2 \int \frac{d^2 \mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2 \mathbf{p}'_\perp}{(2\pi)^2} H_q(x, \xi=0, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp \cdot (\mathbf{p}_\perp - \mathbf{p}'_\perp)} \\
 &= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, 0, -\Delta_\perp^2) e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} \\
 &= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H_q(x, 0, -\Delta_\perp^2) \\
 &= q_N(x, b),
 \end{aligned} \tag{1}$$

where the normalization factor \mathcal{N} satisfies $|\mathcal{N}|^2 \int \frac{d^2 \mathbf{p}_\perp}{(2\pi)^2} = 1$, and $\Delta_\perp = |\Delta_\perp| = \sqrt{\Delta_x^2 + \Delta_y^2}$ and $b = |\mathbf{b}_\perp| = \sqrt{b_x^2 + b_y^2}$. Cylindrical symmetry is applied in the last but one step and J_0 is the Bessel function of the first kind $J_\nu(z)$ with $\nu = 0$. The parton distribution depends on transverse impact space position \mathbf{b}_\perp only through its norm b being the consequence of the longitudinal polarization. In the third step the integral turns to the total and transverse momentum transfer, i.e., $d^2 \mathbf{p}_\perp d^2 \mathbf{p}'_\perp = d^2 \Delta_\perp d^2 \mathbf{P}_\perp$, with $\Delta_\perp = \mathbf{p}'_\perp - \mathbf{p}_\perp$ and $\mathbf{P}_\perp = (\mathbf{p}'_\perp + \mathbf{p}_\perp)/2$, and using the fact that GPD H is independent of total transverse momentum \mathbf{P}_\perp . Ignoring the helicity flip, the spin projection λ can be dropped. In the forward limit, namely $\xi = 0$, we have $t = (p' - p)^2 = -\Delta_\perp^2$.

Note that Hoodbhoy [23] has already pointed out the DIS structure function F_1 , F_2 , g_1 , and g_2 of spin-1 targets can be precisely measured in the same way as that of spin-1/2 targets. Analogous to the fact that the structure function F_1 connects to GPD H_q for spin-1/2 targets, we simply assume F_1 connects to the GPD H_1^q for spin-1 targets as well. As shown by Eqs. (37~39) in Ref. [16], the isospin combination implies that

$$\int_{-1}^1 dx H_i^u(x, \xi, t) = \int_{-1}^1 dx H_i^{I=1}(x, \xi, t). \tag{2}$$

Hereafter we omit the label of quark flavor u and isospin $I=1$ for simplicity. Due to the similar roles of H_q and H_1 , we introduce the impact parameter dependent PDF for spin-1 targets (for the u quark),

$$q(x, b) = \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) H_1(x, 0, -\Delta_\perp^2), \tag{3}$$

One can further define the total parton distribution in

the impact parameter space as

$$q(b) = \int_0^1 dx q(x, b). \tag{4}$$

Notice that $\int d^2 \mathbf{b}_\perp q(x, b) = H_1(x, 0, 0)$, which is equal to the usual PDF $q(x)$ in the forward limit $t = \Delta^2 \rightarrow 0$. Therefore, $q(x, b)$, the Fourier transform of the GPD $H_1(x, \xi=0, -\Delta_\perp^2)$ w.r.t. $-\Delta_\perp^2$, can be identified, in analogy to the nucleon case, with the probability of finding a quark with longitudinal momentum fraction x and transverse impact space position \mathbf{b}_\perp in the ρ meson.

It should be emphasized that in Ref. [2], the nucleon impact parameter dependent PDF q_N was proved to satisfy the positive constraints for the so-called ‘‘good’’ quark field. In our model calculation, the phenomenological vertexes (see Eq. (24) in Ref. [16]) involve the loop momentum (k), and the form of the vertexes is fixed according to the constraints from isospin symmetry. Our sophisticated model cannot simply reproduce the procedure of Ref. [2] to fold the correlation function into a norm of a quantity (see Eq. (23) of Ref. [2]). Therefore, the positive constraint for $q(x, b)$ with a realistic model calculation needs to be proven further.

3 Wave packets

The Fourier transform of a plane wave is not well defined, thus, one can start with the wave packets instead of the plane wave. In the non-relativistic limit, the Fourier transform of the charge form factor $G_C(Q^2)$ can be interpreted as the charge distribution in the transverse direction. In other words, as long as the wave packets peak sharply at some point in position space, by taking the non-relativistic limit, the Fourier transform of the

charge distribution equals the form factor. By the way, a Gaussian weighting factor was also adopted in a recent lattice QCD calculation [24], in order to suppress the unphysical oscillatory behaviour. The oscillation is due to the finite lattice size and nucleon momentum. The result in the small Bjorken $x (< 0.3)$ region is changed by weighting. In Ref. [25], the Gaussian ansatz is also applied to shape the hadron when calculating generalized distribution amplitudes of the pion pair production process.

Moreover, as pointed out by Burkardt [2, 4], the interpretation of the Fourier transform of the form factor as the charge distribution may receive relativistic corrections in the rest frame. However, such a problem may disappear in either Breit frame or infinite momentum frame (IMF). In the relativistic case, the transform receives relativistic corrections when the wave packet is localized with a size smaller than the Compton wavelength of the system. In the IMF, the relativistic correction can be managed to be very small, and therefore, the wave packet does not change the interpretation, as long as the wave packets are set slowly varying w.r.t. Δ_{\perp} . To be specific, the width of the wave packets must be much larger than a typical QCD scale Λ_{QCD} (~ 0.23 GeV). For a Gaussian form wave packet, one gets $\sigma \ll 1/\Lambda_{\text{QCD}} \sim 3/M$, with M being the ρ meson mass.

On the other hand, as Diehl [5] has discussed, a real hadron is an extended object and is smeared out by an amount σ . From the experimental viewpoint, there is a largest measured value $|t|_{\text{max}}$ and thus there is the accuracy of the measurement $\sigma \sim (|t|_{\text{max}})^{-1/2}$. According to the observations and to the limit of the effect from unmeasured values of t , a Gaussian form wave packet can also be reasonably introduced. Thus we have

$$\begin{aligned} & \int \frac{d^2 \mathbf{p}_{\perp} d p^+}{(2\pi)^2 p^+} p^+ \delta(p^+ - p_0^+) G\left(\mathbf{p}_{\perp}, \frac{1}{\sigma^2}\right) |p, \lambda\rangle \\ & \sim \int \frac{d^2 \mathbf{p}_{\perp}}{(2\pi)^2} \exp\left(-\frac{\mathbf{p}_{\perp}^2 \sigma^2}{2}\right) |p^+, \mathbf{p}_{\perp}, \lambda\rangle, \end{aligned} \quad (5)$$

where $G(\mathbf{p}_{\perp}, 1/\sigma^2) = \exp(-\mathbf{p}_{\perp}^2 \sigma^2/2)$ and the mixed state is modified to be

$$\begin{aligned} |p^+, \mathbf{b}_{\perp}, \lambda\rangle_{\sigma} &= \mathcal{N}_{\sigma} \int \frac{d^2 \mathbf{p}_{\perp}}{(2\pi)^2} e^{-i\mathbf{b}_{\perp} \cdot \mathbf{p}} G\left(\mathbf{p}_{\perp}, \frac{1}{\sigma^2}\right) |p^+, \mathbf{p}_{\perp}, \lambda\rangle \\ &\stackrel{\sigma \rightarrow 0}{=} |p^+, \mathbf{b}_{\perp}, \lambda\rangle, \end{aligned} \quad (6)$$

where the normalization factor \mathcal{N}_{σ} satisfies $|\mathcal{N}_{\sigma}|^2 \int \frac{d^2 \mathbf{p}_{\perp}}{(2\pi)^2} = 1$ and $\lim_{\sigma \rightarrow 0} \mathcal{N}_{\sigma} = \mathcal{N}$. Note that our normalization of states is different from that in Ref. [5]. This action will add two Gaussian functions in the expression, $G(\mathbf{p}_{\perp}, \frac{1}{\sigma^2})$ and $G(\mathbf{p}'_{\perp}, \frac{1}{\sigma^2})$, into the definition of $q(x, b)$ (see Eq. (2)). We can still change variables to remove the dependence of \mathbf{P}_{\perp} , which leaves only one $G(\Delta_{\perp}, \frac{1}{\sigma^2})$. Consequently, the definition of the impact parameter dependent PDF

is modified to be

$$\begin{aligned} q_{\sigma}(x, b) &= \int_0^{\infty} \frac{\Delta_{\perp} d\Delta_{\perp}}{2\pi} J_0(b\Delta_{\perp}) G\left(\Delta_{\perp}, \frac{2}{\sigma^2}\right) H_1(x, 0, -\Delta_{\perp}^2) \\ &= \int_0^{\infty} \frac{\Delta_{\perp} d\Delta_{\perp}}{2\pi} J_0(b\Delta_{\perp}) e^{-\Delta_{\perp}^2 \sigma^2/4} H_1(x, 0, -\Delta_{\perp}^2), \end{aligned} \quad (7)$$

and

$$q_{\sigma}(b) = \int_0^1 dx q_{\sigma}(x, b). \quad (8)$$

Reference [5] also argued that in order to give a well-defined (positive, or without sign flip) longitudinal momentum p^3 , $|\mathbf{p}_{\perp}| \ll p^+$ is required. However, as one can see in Eq. (5), \mathbf{p}_{\perp} and p^+ are separated in the wave packet and thus this requirement actually does not affect the result of the integrals. This can also be seen from the property of GPDs. In the forward limit, $H(x, 0, -\Delta_{\perp}^2)$ is not affected by this requirement either. Moreover, Ref. [26] emphasized that since the longitudinal momentum is p^+ in the front form, one needs not to go to infinite momentum along the moving direction, and not to impose the constraint on the p^3 component either.

According to the above discussions, the relation $\sigma \sim (|t|_{\text{max}})^{-1/2}$ inspires us to introduce a cutoff (Δ_0) of the momentum transfer in the integral as well

$$q(x, b, \Delta_0) = \int_0^{\Delta_0} \frac{\Delta_{\perp} d\Delta_{\perp}}{2\pi} J_0(b\Delta_{\perp}) H_1(x, 0, -\Delta_{\perp}^2), \quad (9)$$

and

$$q(b, \Delta_0) = \int_0^1 dx q(x, b, \Delta_0). \quad (10)$$

This assumption is supported by a comparison between the results of the integrals with a wave packet, $q_{\sigma}(b)$ (width $\sigma \sim 1/\Delta_0$) and the one with a cutoff $q(b, \Delta_0)$. This will be shown in Section 5.

4 Impact parameter dependent FFs

We emphasize that the unpolarized impact parameter dependent PDFs are proposed to describe the transverse distribution of unpolarized partons in an unpolarized target. As shown in previous sections, the IPDs can be obtained through Fourier transform of the unpolarized GPD H_1 . We notice that the conventional charge, magnetic dipole and quadrupole FFs are the integrals of the linear combination of H_i . This motivates us to explore the possibility of obtain the IPDs with respect to the three FFs. The sum rules relating to the GPDs and the FFs G_i are [27]

$$\begin{aligned} \int_{-1}^1 dx H_i(x, \xi, t) &= G_i(t) \quad (i=1, 2, 3), \\ \int_{-1}^1 dx H_i(x, \xi, t) &= 0 \quad (i=4, 5), \end{aligned} \quad (11)$$

where G_i^q are the FFs in the decomposition of the local current. The FFs $G_{C,M,Q}$ can be expressed in terms of $G_{1,2,3}$ as [28]

$$\begin{aligned} G_C(t) &= G_1(t) + \frac{2}{3}\eta G_Q(t), \\ G_M(t) &= G_2(t), \\ G_Q(t) &= G_1(t) - G_2(t) + (1+\eta)G_3(t), \end{aligned} \quad (12)$$

where $\eta = -t/4M^2$. Together with Eq. (11), one can obtain $G_{C,M,Q}$ directly from GPDs $H_{1,2,3}$. This allows us to bypass the well-known ambiguity of the angular condition [29]. With the above two equations, one can get the relations

$$\begin{aligned} G_C(t) &= \int_{-1}^1 dx \left[H_1(x, \xi, t) + \frac{2}{3}\eta \left[H_1(x, \xi, t) \right. \right. \\ &\quad \left. \left. - H_2(x, \xi, t) + (1+\eta)H_3(x, \xi, t) \right] \right], \\ G_M(t) &= \int_{-1}^1 dx H_2(x, \xi, t), \\ G_Q(t) &= \int_{-1}^1 dx \left[H_1(x, \xi, t) - H_2(x, \xi, t) + (1+\eta)H_3(x, \xi, t) \right]. \end{aligned} \quad (13)$$

Notice that by taking $\xi = 0$ and $\eta = -t/4M^2 = \Delta_\perp^2/4M^2$, one can get quantities similar to the integrands in Eq. (1). We have the impact parameter dependent FFs

$$\begin{aligned} q_\sigma^C(x, b) &= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) e^{-\Delta_\perp^2 \sigma^2/4} \\ &\quad \times \left[H_1(x, 0, -\Delta_\perp^2) + \frac{2}{3} \frac{\Delta_\perp^2}{4M^2} [H_1(x, 0, -\Delta_\perp^2) \right. \\ &\quad \left. - H_2(x, 0, -\Delta_\perp^2) + \left(1 + \frac{\Delta_\perp^2}{4M^2}\right) H_3(x, 0, -\Delta_\perp^2)] \right], \end{aligned} \quad (14)$$

$$\begin{aligned} q_\sigma^M(x, b) &= \frac{1}{G_M(0)} \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) \\ &\quad \times e^{-\Delta_\perp^2 \sigma^2/4} H_2(x, 0, -\Delta_\perp^2), \end{aligned} \quad (15)$$

$$\begin{aligned} q_\sigma^Q(x, b) &= \frac{1}{G_Q(0)} \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) e^{-\Delta_\perp^2 \sigma^2/4} \\ &\quad \times [H_1(x, 0, -\Delta_\perp^2) - H_2(x, 0, -\Delta_\perp^2) \\ &\quad + (1 + \frac{\Delta_\perp^2}{4M^2}) H_3(x, 0, -\Delta_\perp^2)], \end{aligned} \quad (16)$$

and

$$q_\sigma^{C,M,Q}(b) = \int_0^1 dx q_\sigma^{C,M,Q}(x, b). \quad (17)$$

Comparing the impact parameter dependent FFs, Eq. (14), with the impact parameter dependent PDFs,

Eq. (7), we introduce the ‘‘difference’’ quantities

$$\begin{aligned} q_\sigma^{QC}(x, b) &= \int_0^\infty \frac{\Delta_\perp d\Delta_\perp}{2\pi} J_0(b\Delta_\perp) e^{-\Delta_\perp^2 \sigma^2/4} \\ &\quad \times \left(\frac{2}{3} \frac{\Delta_\perp^2}{4M^2} \right) \left[H_1(x, 0, -\Delta_\perp^2) - H_2(x, 0, -\Delta_\perp^2) \right. \\ &\quad \left. + \left(1 + \frac{\Delta_\perp^2}{4M^2}\right) H_3(x, 0, -\Delta_\perp^2) \right], \end{aligned} \quad (18)$$

$$q_\sigma^{QC}(b) = \int_0^1 dx q_\sigma^{QC}(x, b), \quad (19)$$

which receive the contribution from the quadrupole moment. The ‘‘difference’’ quantities satisfy

$$\begin{aligned} q_\sigma^{QC}(x, b) &= q_\sigma^C(x, b) - q_\sigma(x, b), \\ q_\sigma^{QC}(b) &= q_\sigma^C(b) - q_\sigma(b). \end{aligned} \quad (20)$$

It is clear that the impact parameter dependent PDFs relate to the impact parameter dependent FFs and

$$\int_0^1 dx \int_{-\infty}^\infty d^2\mathbf{b} q_\sigma^{C,M,Q}(x, b) = 1. \quad (21)$$

Thus, it is possible to interpret q_σ^C , q_σ^M and q_σ^Q as the percentage of the contributions to the charge (normalized to 1), magnetic dipole μ_ρ and quadrupole moment Q_ρ respectively, from the parton with the longitudinal momentum fraction x and transverse impact space position \mathbf{b}_\perp .

5 Results

In our previous work [16] with a light-cone constituent quark model, we took the two model parameters of the constituent mass $m = 0.403$ GeV and regulator mass $m_R = 1.61$ GeV, and we calculated the GPDs of the ρ meson. In our LCCQM, we introduced an effective Lagrangian for the ρ - $q\bar{q}$ interaction with a phenomenological vertex Γ^u and a Bethe-Salpeter amplitude. By integrating the minus component of the quark momentum k^- analytically and rest of the components numerically, we obtained the GPDs and FFs of the ρ meson.

In this work, we simply extend the calculation to the impact parameter dependent PDFs $q(b)$ and impact parameter dependent FFs $q_\sigma^{C,M,Q}(b)$. Figure 1 gives the $q(b)$ with a wave packet, $q_\sigma(b)$, and with a cutoff on the momentum transfer, $q(b, \Delta_0)$, respectively. The comparison shows that the cutoff (Δ_0) has a similar effect as the wave packet with width $\sigma \sim 1/\Delta_0$. Of course, we expect that the prediction of the constituent quark model is reasonable only in the region of $|t|^{1/2} \lesssim 2$ GeV and when the momentum transfer becomes larger the constituent quark model fails. The width of the wave packet is also constrained by the uncertainty principle: to have a valid probability interpretation of the initial and final states, the position dispersion ($\sim \sigma$) cannot be smaller than the

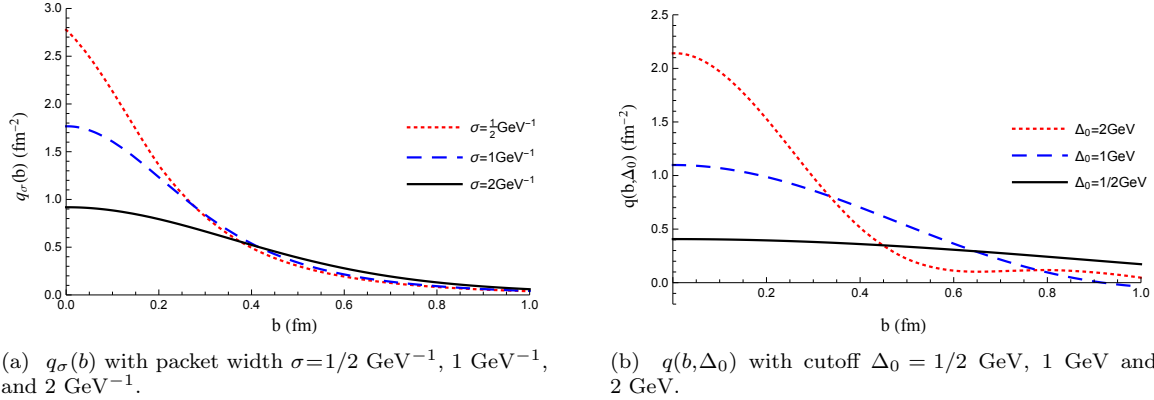


Fig. 1. (color online) The impact parameter dependent PDF $q(b)$ with (a) a wave packet and (b) a cutoff on the momentum transfer.

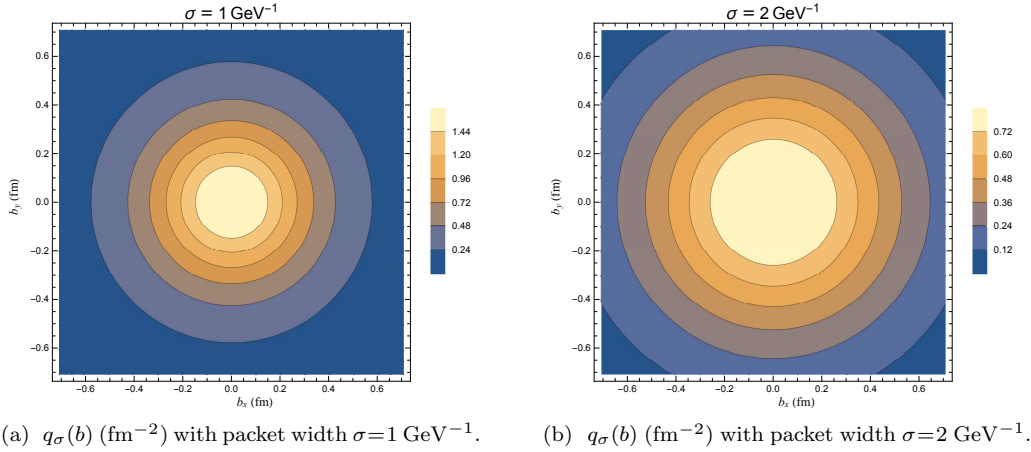


Fig. 2. (color online) Contour plots of the impact parameter dependent PDF $q(b)$ with a wave packet.

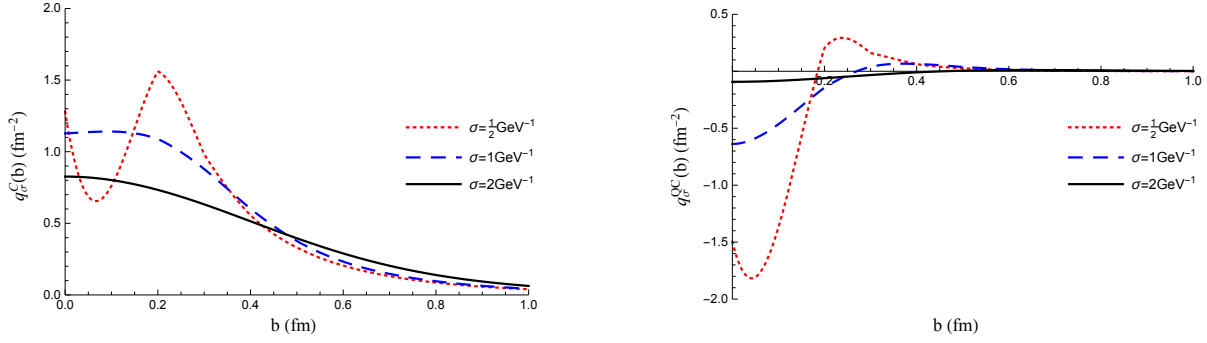
Compton wavelength. In the later content, our numerical results in Fig. 3(a) agree with this point of view.

Figure 2 gives the contour plots of the impact parameter dependent PDF $q_\sigma(b)$ with $\sigma = 1 \text{ GeV}^{-1}$ and 2 GeV^{-1} . Since we choose the polarization in the z direction, the parton distribution is invariant under rotation around the z direction. We see that as σ becomes smaller, the wave functions of the initial and final states get closer to a plane wave, and the parton distribution also becomes more transversely localized in the position space, as shown in Fig. 1 and Fig. 2.

Figures 3 and 4 give the impact parameter dependent FFs $q_\sigma^{C,M,Q}(b)$ and $q_\sigma^{QC}(b)$ with $\sigma = 1/2 \text{ GeV}^{-1}$, 1 GeV^{-1} , 2 GeV^{-1} respectively. Figure 4 shows that, as the wave packet becomes more sharply localized (σ decreases), the contributions are concentrated more in the small b_\perp region for both the magnetic dipole μ_ρ and quadrupole moment Q_ρ . For the impact parameter charge density, Fig. 3(a), the distributions with σ less than about 1 GeV^{-1} become obscure due to the oscillation. As we argued before, the ρ meson is an extended object and its Compton wavelength

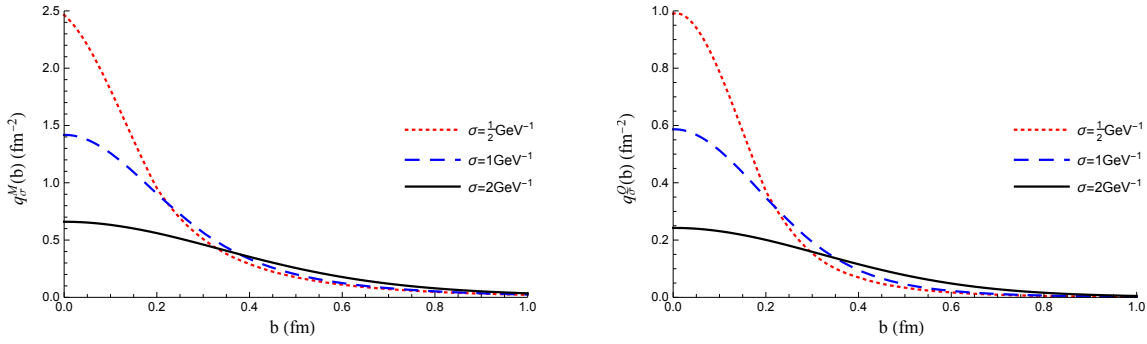
is $1/m_\rho = 1.3 \text{ GeV}^{-1}$. The position dispersion $\langle \Delta x \rangle = \sigma$ in the case of the Gaussian wave packet. The uncertainty principle ($\langle \Delta x \rangle \langle \Delta p \rangle \geq 1/2$ in natural units) gives the constraint that, to maintain the probability interpretation of the states, the position dispersion $\langle \Delta x \rangle$ should not be smaller than the Compton wavelength. Otherwise, localizing a wave packet to less than its Compton wavelength in size will in general induce various relativistic corrections [4]. With the help of Figs. 1 and 3(b), and Eq. (20), the oscillation in $q_\sigma^C(b)$ can be explained as the behaviour of $q_\sigma^{QC}(b)$ which is related to the quadrupole moment. From the experimental aspect, since the ρ meson quadrupole moment is small, this phenomenon is hard to determine.

Figures 5 and 6 show the numerical result of $q_\sigma(x, b)$ and $q_\sigma^{C,M,Q,QC}(x, b)$ with $\sigma = 1 \text{ GeV}^{-1}$ and $x = 1/10, 3/10$ and $1/2$ respectively. When $x \leq 1/10$, $q_\sigma^C(x, b)$ has negative values as $b < 0.4 \text{ fm}$ (see Fig. 5(b)), due to the oscillation of $q_\sigma^{QC}(x, b)$ (see Fig. 5(c)). In the small x region (like $x < 1/10$ in our case), it is believed that the contribution of the gluon GPDs becomes more important, which is beyond the scope of the present model.



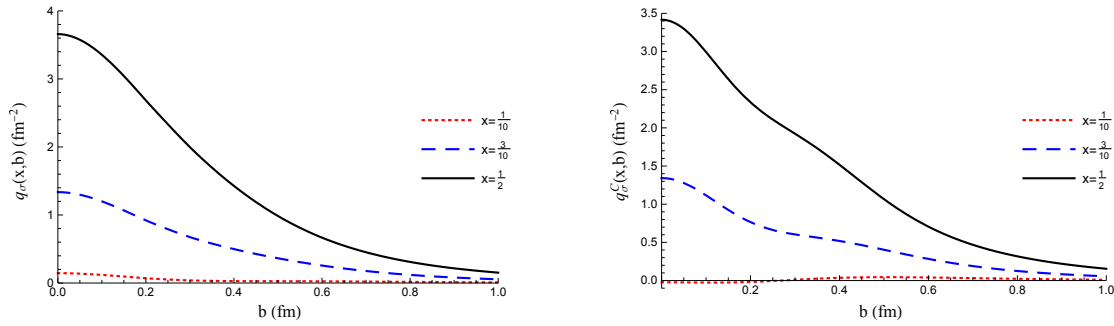
(a) $q_\sigma^C(b)$ with packet width $\sigma=1/2 \text{ GeV}^{-1}$, 1 GeV^{-1} , and 2 GeV^{-1} . (b) $q_\sigma^{QC}(b)$ with packet width $\sigma=1/2 \text{ GeV}^{-1}$, 1 GeV^{-1} , and 2 GeV^{-1} .

Fig. 3. (color online) The impact parameter dependent FFs $q_\sigma^C(b)$ and $q_\sigma^{QC}(b)$ with $\sigma=1/2 \text{ GeV}^{-1}$, 1 GeV^{-1} and 2 GeV^{-1} .

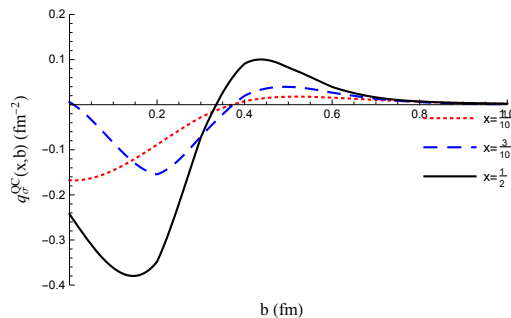


(a) $q_\sigma^M(b)$ with packet width $\sigma=1/2 \text{ GeV}^{-1}$, 1 GeV^{-1} , and 2 GeV^{-1} . (b) $q_\sigma^Q(b)$ with packet width $\sigma=1/2 \text{ GeV}^{-1}$, 1 GeV^{-1} , and 2 GeV^{-1} .

Fig. 4. (color online) The impact parameter dependent FFs $q_\sigma^{M,Q}(b)$ with $\sigma=1/2 \text{ GeV}^{-1}$, 1 GeV^{-1} , and 2 GeV^{-1} .



$q_\sigma(x,b)$ with $\sigma=1 \text{ GeV}^{-1}$ and $x=1/10, 3/10, \text{ and } 1/2$. $q_\sigma^C(x,b)$ with $\sigma=1 \text{ GeV}^{-1}$ and $x=1/10, 3/10 \text{ and } 1/2$.



$q_\sigma^{QC}(x,b)$ with $\sigma=1 \text{ GeV}^{-1}$ and $x=1/10, 3/10 \text{ and } 1/2$.

Fig. 5. (color online) The impact parameter dependent PDFs $q_\sigma(x,b)$ and FFs $q_\sigma^{C,QC}(x,b)$ with $\sigma=1 \text{ GeV}^{-1}$ and $x=1/10, 3/10 \text{ and } 1/2$.

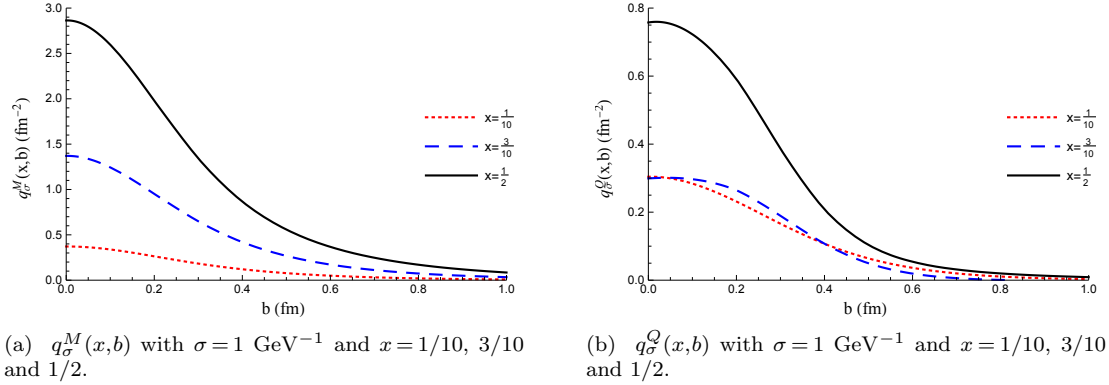


Fig. 6. (color online) The impact parameter dependent FFs $q_{\sigma}^{M,Q,Q^C}(x,b)$ with $\sigma = 1 \text{ GeV}^{-1}$ and $x = 1/10, 3/10$ and $1/2$.

The symmetry around $x \sim 1/2$ of the parton distributions, implied by the isospin symmetry, is not satisfied well due to this reason. In addition, we found, from Fig. 6, that the distributions approximately remain the same in $q_{\sigma}^Q(x,b)$ when $1/10 \leq x \leq 3/10$.

6 Summary and conclusions

In this work, analogous to the definition of the pion and nucleon impact parameter dependent PDFs, we introduce the ρ meson impact parameter dependent PDFs ($q(x,b)$ and $q(b)$) and impact parameter dependent FFs ($q^{C,M,Q}(x,b)$ and $q^{C,M,Q}(b)$). By employing the LC-CQM, as we have done previously, we carried out the numerical calculation of those quantities for the first time. We believe that $q^{C,M,Q}(x,b)$ may be interpreted as the percentages of the contributions to the charge (normalized to 1), magnetic dipole μ_{ρ} , and quadrupole moment Q_{ρ} , respectively, from a parton with a longitudinal

momentum fraction x and a transverse impact space position \mathbf{b}_{\perp} . Considering the facts that the ρ meson is an extended object and there exists a largest measured value of momentum transfer in realistic measurements, a Gaussian form wave packet is employed in our numerical calculation. Our numerical results show that the wave packet approach plays a similar effect to the cutoff in the integral, which is due to the validity of the constituent quark model. Our numerical results for impact parameter charge distributions also show that the width of the Gaussian wave packet should be larger than the Compton wavelength. We expect that this approach is needed in a phenomenological model calculation in order to remove the possible negative values of the impact parameter charge distributions $q_{\sigma}^C(x,b)$, which cannot be understood by the density interpretation.

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