A model to explain the angular distribution of J/ψ and $\psi(2S)$ decay into $\Lambda\overline{\Lambda}$ and $\Sigma^0\overline{\Sigma^0}$

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Abstract: BESIII data show a particular angular distribution for the decay of J/ψ and $\psi(2S)$ mesons into $\Lambda\overline{\Lambda}$ and $\Sigma^0\overline{\Sigma}^0$ hyperons: the angular distribution of the decay $\psi(2S) \to \Sigma^0\overline{\Sigma}^0$ exhibits an opposite trend with respect to the other three channels: $J/\psi \to \Lambda\overline{\Lambda}$, $J/\psi \to \Sigma^0\overline{\Sigma}^0$ and $\psi(2S) \to \Lambda\overline{\Lambda}$. We define a model to explain the origin of this phenomenon.

1   Introduction

Since their discovery, charmonia, i.e., $c\bar{c}$ mesons, have become unique tools for extending our knowledge of strong interaction dynamics at low and medium energies. In the case of lightest charmonia, their decay mechanisms can only be studied by means of effective models, since, due to their low-energy regime, these processes are beyond the perturbative description of quantum chromodynamics.

We study the decays of J/ψ and $\psi(2S)$ mesons into baryon-antibaryon pairs $B\overline{B} = \Lambda\overline{\Lambda}$, $\Sigma^0\overline{\Sigma}^0$. The differential cross section of the process $e^+e^- \to \psi \to B\overline{B}$ has the well known parabolic expression in $\cos\theta$ [1]

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta} \propto 1 + \alpha_B \cos^2\theta,$$

where α_B is the so-called polarization parameter and θ is the baryon scattering angle, i.e., the angle between the outgoing baryon and the beam direction in the e^+e^- center-of-mass frame. As pointed out in Ref. [2], only the decay $J/\psi \to \Sigma^0\overline{\Sigma}^0$ has a negative polarization parameter α_B.

Figures 1 and 2 show the BESIII data [3] for the angular distribution of the four decays: $J/\psi \to \Lambda\overline{\Lambda}$, $J/\psi \to \Sigma^0\overline{\Sigma}^0$, and $\psi(2S) \to \Lambda\overline{\Lambda}$, $\psi(2S) \to \Sigma^0\overline{\Sigma}^0$.

2   Amplitudes and branching ratios

The Feynman amplitude for the decay $\psi \to B\overline{B}$ can be written in terms of the strong magnetic and Dirac form factors as

$$\mathcal{M}_{\psi \to B\overline{B}} = -i e^\mu_\psi \overline{u}(p_1)\Gamma_\mu v(p_2)$$

where the matrix Γ_μ is defined in Eq. (A2), ϵ^μ_ψ is the polarization vector of the ψ meson, and the four-momenta follow the labelling of Eq. (A1). The branching ratio (BR) is given by the standard form for the two-body decay

$$\mathcal{B}_{\psi \to B\overline{B}} = \frac{1}{\Gamma_\psi} \frac{1}{8\pi} \overline{\left|\mathcal{M}_{\psi \to B\overline{B}}\right|^2} \frac{|\vec{p}_1|}{M_\psi^2},$$

where Γ_ψ is the total width of the ψ meson. Using the mean value of the modulus squared of the amplitude,

Fig. 1.   (color online) Angular distribution of the baryon for the J/ψ decays into $\Lambda\bar{\Lambda}$ (upper panel) and $\Sigma^0\bar{\Sigma}^0$ (lower panel).

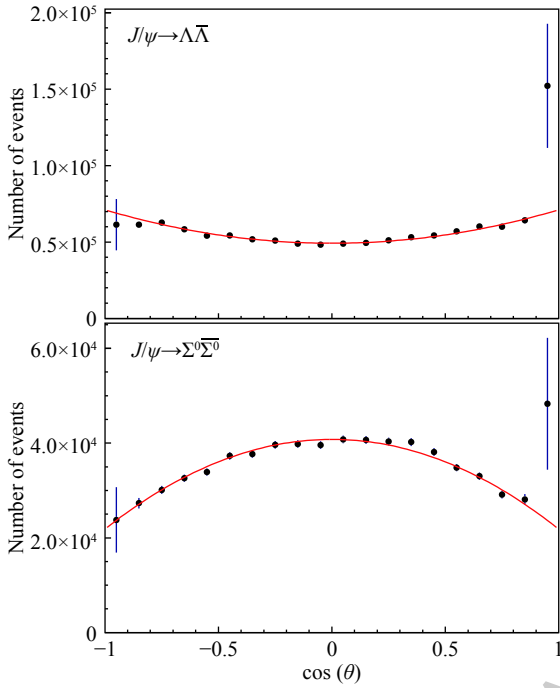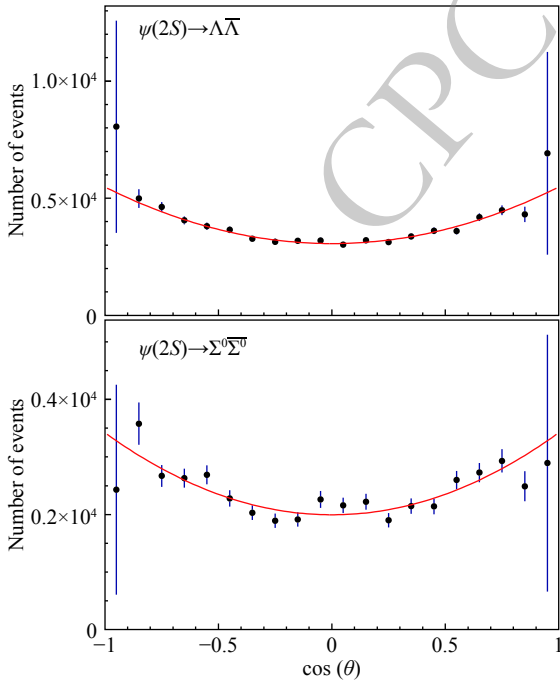

Fig. 2.   (color online) Angular distribution of the baryon for the $\psi(2S)$ decays into $\Lambda\bar{\Lambda}$ (upper panel) and $\Sigma^0\bar{\Sigma}^0$ (lower panel).

written in terms of the Sachs couplings,

$$\overline{|\mathcal{M}_{\psi\to B\bar{B}}|^2} = \frac{4}{3}M_\psi^2\left(|g_M^B|^2 + \frac{2M_B^2}{M_\psi^2}|g_E^B|^2\right).$$

we obtain the BR as

$$\mathcal{B}_{\psi\to B\bar{B}} = \frac{M_\psi\beta}{12\pi\Gamma_\psi}\left(|g_M^B|^2 + \frac{2M_B^2}{M_\psi^2}|g_E^B|^2\right). \tag{1}$$

Since it does not depend on α_B, it cannot be used to determine the polarization parameter.

The above expression for BR can be written as the sum of the moduli squared of two amplitudes

$$\mathcal{B}_{\psi\to B\bar{B}} = |A_M^B|^2 + |A_E^B|^2, \tag{2}$$

where, comparing with Eq. (1),

$$A_M^B = \sqrt{\frac{M_\psi\beta}{12\pi\Gamma_\psi}}g_M^B, \quad A_E^B = \sqrt{\frac{M_\psi\beta}{6\pi\Gamma_\psi}}\frac{M_B}{M_\psi}g_E^B.$$

It follows that the polarization parameter of Eq. (A3) can also be written as

$$\alpha_B = \frac{1 - 2|A_E^B|^2/|A_M^B|^2}{1 + 2|A_E^B|^2/|A_M^B|^2}.$$

3 Effective model

The $SU(3)$ baryon octet states can be described in matrix notation as follows [4]

$$O_B = \begin{pmatrix} \Lambda/\sqrt{6} + \Sigma^0/\sqrt{2} & \Sigma^+ & p \\ \Sigma^- & \Lambda/\sqrt{6} - \Sigma^0/\sqrt{2} & n \\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{pmatrix},$$

$$O_{\bar{B}} = \begin{pmatrix} \bar{\Lambda}/\sqrt{6} + \bar{\Sigma}^0/\sqrt{2} & \bar{\Sigma}^+ & \bar{\Xi}^+ \\ \bar{\Sigma}^- & \bar{\Lambda}/\sqrt{6} - \bar{\Sigma}^0/\sqrt{2} & \bar{\Xi}^0 \\ \bar{p} & \bar{n} & -2\bar{\Lambda}/\sqrt{6} \end{pmatrix},$$

where the first matrix is for baryons and the second for antibaryons. We can consider J/ψ and $\psi(2S)$ mesons as $SU(3)$ singlets. In view of the $SU(3)$ symmetry, the zero level Lagrangian density should have the $SU(3)$ invariant form $\mathcal{L}^0 \propto \text{Tr}(B\bar{B})$. Moreover, we consider two sources of $SU(3)$ symmetry breaking: the quark mass and the EM interaction. The first can be parametrized by introducing the spurion matrix [5]

$$S_m = \frac{g_m}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

where g_m is the effective coupling constant. This matrix describes the mass breaking effect due to the mass difference between s and u and d quarks, where the $SU(2)$ isospin symmetry is assumed, so that $m_u = m_d$. This $SU(3)$ breaking is proportional to the 8th Gell-Mann matrix λ_8. The EM breaking effect is related to the fact that the photon coupling to quarks, described by the four-current

$$\frac{1}{2}\bar{q}\gamma^\mu\left(\lambda_3 + \lambda_8/\sqrt{3}\right)q,$$

is proportional to the electric charge. This effect can be parametrized using the following spurion matrix

$$S_e = \frac{g_e}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

where g_e is the effective EM coupling constant.

The most general $SU(3)$ invariant effective Lagrangian density is given by [5]

$$\mathcal{L} = g\mathrm{Tr}(OO_{\bar{B}}) + d\mathrm{Tr}(\{O, O_{\bar{B}}\}S_e) + f\mathrm{Tr}([OO_{\bar{B}}]S_e)$$
$$+ d'\mathrm{Tr}(\{O, O_{\bar{B}}\}S_m) + f'\mathrm{Tr}([O, O_{\bar{B}}]S_m),$$

where g, d, f, d' and f' are coupling constants. We can extract the Lagrangians describing the J/ψ and $\psi(2S)$ decays into $\Lambda\overline{\Lambda}$ and $\Sigma^0\overline{\Sigma}^0$

$$\mathcal{L}_{\Sigma^0\bar{\Sigma}^0} = (G_0 + G_1)\Sigma^0\bar{\Sigma}^0, \mathcal{L}_{\Lambda\bar{\Lambda}} = (G_0 - G_1)\Lambda\bar{\Lambda}, \quad (3)$$

where G_0 and G_1 are combinations of coupling constants, i.e.,

$$G_0 = g, \quad G_1 = \frac{d}{3}(2g_m + g_e).$$

Using the structure of Eq. (2), the BRs can be expressed in terms of the electric and magnetic amplitudes as

$$\mathcal{B}_{\psi \to \Sigma^0\bar{\Sigma}^0} = |A_E^\Sigma|^2 + |A_M^\Sigma|^2,$$
$$\mathcal{B}_{\psi \to \Lambda\bar{\Lambda}} = |A_E^\Lambda|^2 + |A_M^\Lambda|^2.$$

Moreover, as obtained in Eq. (3), the amplitudes can be further decomposed as combinations of leading, E_0 and M_0, and sub-leading terms, E_1 and M_1, with opposite relative signs, i.e.,

$$\mathcal{B}_{\psi \to \Sigma^0\bar{\Sigma}^0} = |E_0 + E_1|^2 + |M_0 + M_1|^2$$
$$= |E_0|^2 + |E_1|^2 + 2|E_0||E_1|\cos(\rho_E)$$
$$+ |M_0|^2 + |M_1|^2 + 2|M_0||M_1|\cos(\rho_M),$$
$$\mathcal{B}_{\psi \to \Lambda\bar{\Lambda}} = |E_0 - E_1|^2 + |M_0 - M_1|^2$$
$$= |E_0|^2 + |E_1|^2 - 2|E_0||E_1|\cos(\rho_E)$$
$$+ |M_0|^2 + |M_1|^2 - 2|M_0||M_1|\cos(\rho_M),$$

where ρ_E and ρ_M are the phases of the ratios E_0/E_1 and M_0/M_1.

4   Results

In this work, we have used the data from precise measurements [3, 6] of the branching ratios and polarization parameters, reported in Table 1, based on the events collected with the BESIII detector at the BEPCII collider. These data are in agreement with the results of other experiments [7-11]. Since for each charmonium state we have six free parameters (four moduli and two relative phases) and only four constrains (two BRs and two polarization parameters), we fix the relative phases ρ_E and ρ_M. The values $\rho_E = 0$ and $\rho_M = \pi$ appear as phenomenologic-

ally favored by the data. Indeed, (largely) different phases would give negative, and hence unphysical, values for the moduli $|E_0|$, $|E_1|$, $|M_0|$ and $|M_1|$. Moreover, as shown in Fig. 5, where the four moduli for J/ψ and $\psi(2S)$ are represented as functions of the phases with $\rho_E \in [-\pi/2, \pi/2]$ and $\rho_M \in [\pi/2, 3\pi/2]$, the obtained results are quite stable, and the central values $\rho_E = 0$, $\rho_M = \pi$ maximise the hierarchy between the moduli of leading, E_0 and M_0, and sub-leading amplitudes, E_1 and M_1. These values for $|E_0|$, $|E_1|$, $|M_0|$ and $|M_1|$ are reported in Table 2 and shown in Fig. 3. The corresponding values of $|g_E|$, $|g_M|$ are reported in Table 3 and shown in Fig. 4. The large sub-leading J/ψ amplitudes $|E_1|$, $|M_1|$ (see Table 2 and Fig. 3) are responsible for the inversion of the $|g_E^B|$, $|g_M^B|$ hierarchy (see Fig. 4 and Table 3).
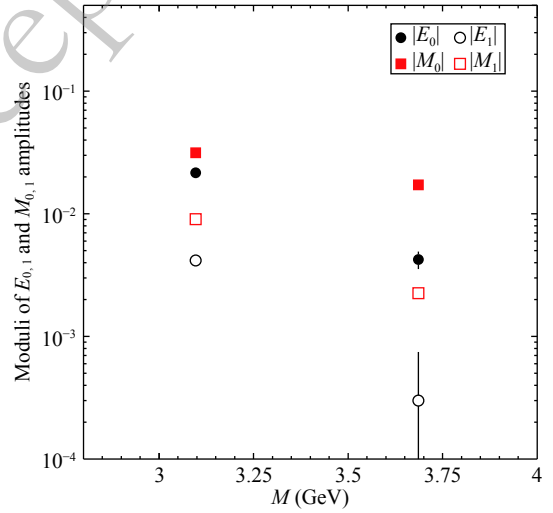

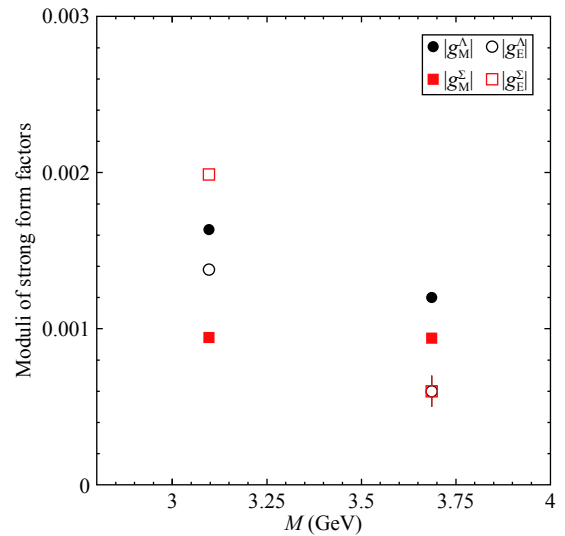Fig. 3.   (color online) Moduli of the parameters from Table 2 as function of the charmonium state mass M.

Fig. 4.   (color online) Moduli of the parameters from Table 3 as function of the charmonium state mass M.
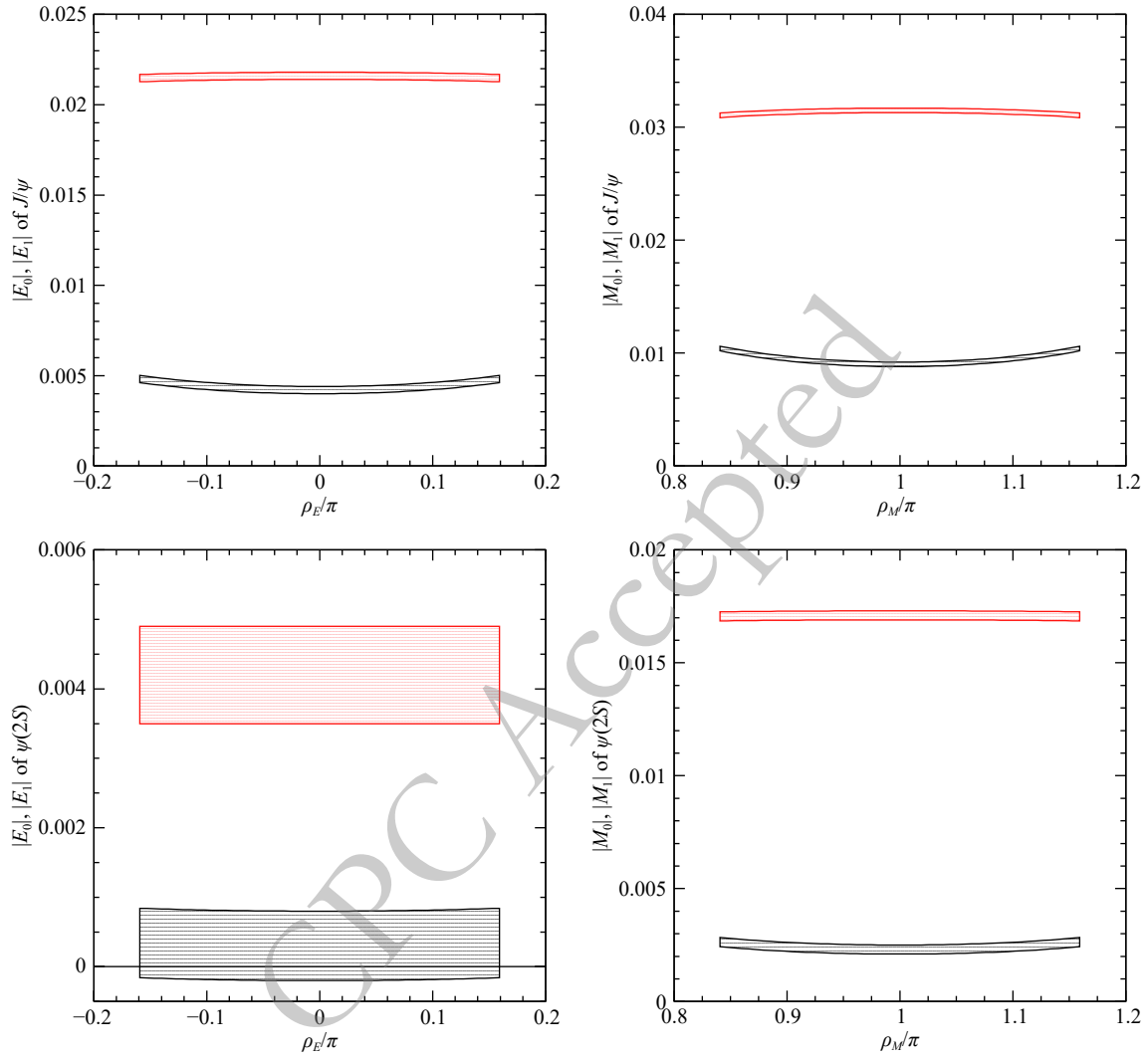
Fig. 5.   (color online) Red and black bands represent moduli of leading and sub-leading amplitudes, respectively. The vertical width indicates the error. Top left: moduli of amplitudes E_0 and E_1 for J/ψ. Top right: moduli of amplitudes M_0 and M_1 for J/ψ. Bottom left: moduli of amplitudes E_0 and E_1 for $\psi(2S)$. Bottom right: moduli of amplitudes M_0 and M_1 for $\psi(2S)$.

Table 1.   Branching ratios and polarization parameters from Ref. [3]. The value of α_B for the decay $J/\psi \to \Lambda\overline{\Lambda}$ is from Ref. [6].

Decay	BR	Pol. par. α_B
$J/\psi \to \Sigma^0\overline{\Sigma}^0$	$(11.64 \pm 0.04) \times 10^{-4}$	-0.449 ± 0.020
$J/\psi \to \Lambda\overline{\Lambda}$	$(19.43 \pm 0.03) \times 10^{-4}$	0.461 ± 0.009
$\psi(2S) \to \Sigma^0\overline{\Sigma}^0$	$(2.44 \pm 0.03) \times 10^{-4}$	0.71 ± 0.11
$\psi(2S) \to \Lambda\overline{\Lambda}$	$(3.97 \pm 0.03) \times 10^{-4}$	0.824 ± 0.074

Table 2.   Moduli of the leading and sub-leading amplitudes.

Ampl.	J/ψ	$\psi(2S)$
$|E_0|$	$(2.16 \pm 0.02) \times 10^{-2}$	$(0.42 \pm 0.07) \times 10^{-2}$
$|E_1|$	$(0.42 \pm 0.02) \times 10^{-2}$	$(0.03 \pm 0.05) \times 10^{-2}$
$|M_0|$	$(3.15 \pm 0.02) \times 10^{-2}$	$(1.72 \pm 0.02) \times 10^{-2}$
$|M_1|$	$(0.90 \pm 0.02) \times 10^{-2}$	$(0.23 \pm 0.02) \times 10^{-2}$

Table 3.   Moduli of the strong Sachs form factors

FFs	J/ψ	$\psi(2S)$
$|g_E^\Sigma|$	$(1.99 \pm 0.04) \times 10^{-3}$	$(0.6 \pm 0.1) \times 10^{-3}$
$|g_M^\Sigma|$	$(0.94 \pm 0.02) \times 10^{-3}$	$(0.94 \pm 0.02) \times 10^{-3}$
$|g_E^\Lambda|$	$(1.37 \pm 0.04) \times 10^{-3}$	$(0.6 \pm 0.1) \times 10^{-3}$
$|g_M^\Lambda|$	$(1.64 \pm 0.03) \times 10^{-3}$	$(1.20 \pm 0.02) \times 10^{-3}$

5   Conclusions

The Λ and Σ^0 angular distributions can be explained using an effective model with the $SU(3)$-driven Lagrangian

$$\mathcal{L}_{\Sigma^0\overline{\Sigma}^0 + \Lambda\overline{\Lambda}} = (G_0 + G_1)\Sigma^0\overline{\Sigma}^0 + (G_0 - G_1)\Lambda\overline{\Lambda}.$$

The interplay between the leading G_0 and sub-lead-

ing G_1 contributions to the decay amplitude determines the sign and value of the polarization parameter α_B.

In particular, the different behavior of the $J/\psi \to \Sigma^0\overline{\Sigma}^0$ angular distribution is due to the large values of the subleading amplitudes $|E_1|$ and $|M_1|$. This implies that the $SU(3)$ mass breaking and EM effects, which are responsible for these amplitudes, play a different role in the dynamics of the J/ψ and $\psi(2S)$ decays.

Appendix A: Production cross section

We consider the decay of a charmonium state, a $c\bar{c}$ vector meson ψ, produced via e^+e^- annihilation, into a baryon-antibaryon $B\overline{B}$ pair, i.e., the process

$$e^-(k_1) + e^+(k_2) \to \psi(q) \to B(p_1) + \overline{B}(p_2), \qquad (A1)$$

where the 4-momenta are given in parentheses. The Feynman diagram is shown in Fig. A1 and the corresponding amplitude is

$$\mathcal{M}_{e^+e^-\to\psi\to B\overline{B}} = -ie^2 J_B^\mu \, D_\psi\left(q^2\right) \bar{v}(k_2)\gamma_\mu u(k_1),$$

where $J_B^\mu = \bar{u}(p_1)\Gamma^\mu v(p_2)$ is the baryonic four-current, $D_\psi\left(q^2\right)$ is the ψ propagator, which includes the γ-ψ electromagnetic (EM) coupling, and $\bar{v}(k_2)\gamma_\mu u(k_1)$ is the leptonic four-current. The four-momenta follow the labelling of Eq. (A1). The Γ^μ matrix can be written as [15]
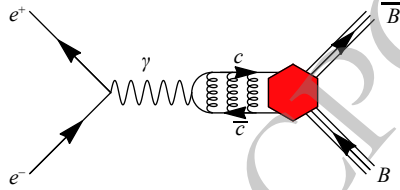

Fig. A1.   (color online) Feynman diagram of the process $e^+e^- \to \psi \to B\bar{B}$, the red hexagon represents the $\psi B\bar{B}$ coupling.

$$\Gamma^\mu = \gamma^\mu f_1^B + \frac{i\sigma^{\mu\nu}q_\nu}{2M_B} f_2^B, \qquad (A2)$$

where M_B is the baryon mass and f_1^B and f_2^B are constant form factors that we call "strong" Dirac and Pauli couplings; they weigh the vector and tensor parts of the $\psi B\overline{B}$ vertex[1]. We introduce the strong electric and magnetic Sachs couplings [16]

$$g_E^B = f_1^B + \frac{M_\psi^2}{4M_B^2} f_2^B, \quad g_M^B = f_1^B + f_2^B,$$

that have the structure of the EM Sachs form factors [17]. M_ψ is the mass of the charmonium state. The four quantities f_1^B, f_2^B, g_E^B and g_M^B are in general complex numbers. The differential cross section of the process $e^+e^- \to \psi \to B\overline{B}$ in the e^+e^- center-of-mass frame, in terms of the two Sachs couplings, reads

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta}{2M_\psi^2}\left(|g_M^B|^2 + \frac{4M_B^2}{M_\psi^2}|g_E^B|^2\right)\left(1 + \alpha_B\cos^2\theta\right),$$

where

It is interesting to note that the angular distributions of $\Sigma^0(1385)$ and $\Sigma^\pm(1385)$, measured by BESIII [12, 13], show the same Σ^0 behavior.

The process $e^+e^- \to J/\psi \to \Sigma^+\overline{\Sigma}^-$ is currently under investigation [14]. The behavior of its angular distribution could add important information to the knowledge of the J/ψ decay mechanism.

$$\beta = \sqrt{1 - \frac{4M_B^2}{M_\psi^2}}$$

is the velocity of the out-going baryon at the ψ mass, θ is the scattering angle, and the polarization parameter α_B is given by

$$\alpha_B = \frac{M_\psi^2\,|g_M^B|^2 - 4M_B^2|g_E^B|^2}{M_\psi^2\,|g_M^B|^2 + 4M_B^2\,|g_E^B|^2}, \qquad (A3)$$

$\alpha_B \in [-1,1]$ and depends only on the modulus of the ratio g_E^B/g_M^B, e.g., Fig. A2 shows the behavior of α_Λ in the case of $\psi = J/\psi$ as function of $|g_E^\Lambda|/|g_M^\Lambda|$. The strong Sachs and Dirac and Pauli couplings are related through α_B. Let us consider three special cases. With maximum positive polarization, $\alpha_B = 1$, the strong electric Sachs coupling vanishes, i.e.,
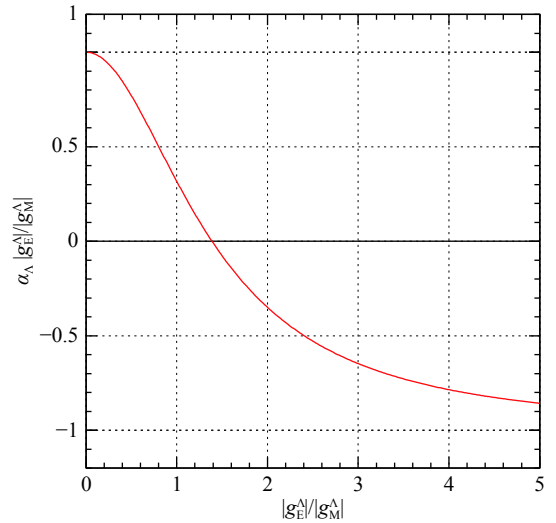

Fig. A2.   (color online) Polarization parameter α_Λ for $\psi = J/\psi$ as function of the ratio $|g_E^\Lambda|/|g_M^\Lambda|$. The masses are from Ref. [18].

$$\alpha_B = 1 \;\to\; g_E^B = 0, \;\; f_1^B = -\frac{M_\psi^2}{4M_B^2} f_2^B,$$

$$g_M^B = f_1^B\left(1 - \frac{4M_B^2}{M_\psi^2}\right) = f_2^B\left(1 - \frac{M_\psi^2}{4M_B^2}\right),$$

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1) When the non-constant matrix Γ^μ is introduced to describe the EM coupling $\gamma B\overline{B}$, the tensor term contains also the anomalous magnetic moment, that, in this case where Γ^μ parametrises the strong vertex $\psi B\overline{B}$, has been embodied in the strong Pauli coupling.

the relative phase between f_1^B and f_2^B is $i\pi$, and the ratio of the moduli is $M_\psi^2/(4M_B^2)$.

With maximum negative polarization we have

$$\alpha_B = -1 \quad \rightarrow \quad g_M^B = 0, \ f_1^B = -f_2^B,$$

$$g_E^B = f_1^B\left(1 - \frac{M_\psi^2}{4M_B^2}\right) = f_2^B\left(\frac{M_\psi^2}{4M_B^2} - 1\right),$$

so that in this case the strong magnetic Sachs coupling vanishes, the relative phase between f_1^B and f_2^B is $-i\pi$ and the ratio of the moduli is one.

Finally, in the case with no polarization, $\alpha_B = 0$, we obtain the modulus of the ratio between the Sachs couplings

$$\alpha_B = 0 \quad \rightarrow \quad \frac{|g_E^B|}{|g_M^B|} = \frac{M_\psi}{2M_B}.$$

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