Forward-backward asymmetries in $\Lambda_b \to \Lambda l^+ l^-$ in the Bethe-Salpeter equation approach*

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Abstract: Using the Bethe-Salpeter equation (BSE), we investigate the forward-backward asymmetries ($A_{\rm FB}$) in $\Lambda_b \to \Lambda l^+ l^- (l = e, \mu, \tau)$ in the quark-diquark model. This approach provides precise form factors that are different from those of quantum chromodynamics (QCD) sum rules. We calculate the rare decay form factors for $\Lambda_b \to \Lambda l^+ l^-$b and investigate the (integrated) forward-backward asymmetries in these decay channels. We observe the integrated $A^l_{\rm FB}$, $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda e^+ e^-) \simeq -0.1371$, $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) \simeq -0.1376$, and $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda \tau^+ \tau^-) \simeq -0.1053$; the hadron side asymmetries $\bar{A}^h_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) \simeq -0.2315$; the lepton-hadron side asymmetries $\bar{A}^{lh}_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) \simeq 0.0827$; and the longitudinal polarization fractions $\bar{F}_L(\Lambda_b \to \Lambda \mu^+ \mu^-) \simeq 0.5681$.

Keywords: rare decay, Bethe-Salpeter equation, heavy flavor physics

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I. INTRODUCTION

The decays of hadrons involving the flavor changing neutral current (FCNC) transition such as $\Lambda_b \to \Lambda l^+ l^-$ can provide essential information about the inner structure of hadrons, reveal the nature of the electroweak interaction, and provide model-independent information about physical quantities such as Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The rare decay $\Lambda_b \to \Lambda \mu^+ \mu^-$ was first observed by the CDF collaboration in 2011 [1]. Some experimental progress on $\Lambda_b \to \Lambda l^+ l^-$ was also achieved [2–5], and the radiative decay $\Lambda_b \to \Lambda \gamma$ was observed in 2019 [3] by the LHCb collaboration. The LHCb collaboration determined the forward-backward asymmetries ($A^l_{\rm FB}$) of the decay $\Lambda_b \to \Lambda \mu^+ \mu^-$ to be $A^l_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) = -0.05 \pm 0.09$ (stat) ± 0.03 (syst), $A^h_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) = -0.29 \pm 0.09$ (stat) ± 0.03 (syst), and $F_{\rm L}(\Lambda_b \to \Lambda \mu^+ \mu^-) = 0.61^{+0.11}_{-0.14} \pm 0.03$ (syst) at the low dimuon invariant mass squared range $15 < q^2 < 20$ GeV^2 in 2015 [4]. However, these numbers were updated in 2018 to $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) = -0.39 \pm 0.04$ (stat) ± 0.01 (syst), $A^h_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) = -0.3 \pm 0.05$ (stat) ± 0.02 (syst), and $\bar{A}^{lh}_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) = 0.25 \pm 0.04$ (stat) ± 0.01 (syst) in the same invariant mass squared region [5]. Note that $A^l_{\rm FB}$ is significantly lager than the previous one. In this study, we investigate the $A_{\rm FB}$ of $\Lambda_b \to \Lambda l^+ l^-$ in the Bethe-Salpeter equation (BSE) approach. Theoretically, only a few studies have been conducted on $A_{\rm FB}(\Lambda_b \to \Lambda l^+ l^-)$ [6–17]. References [6] ([7]) provided the integrated forward-backward asymmetries $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) = -0.13$ (-0.12) and $\bar{A}_{\rm FB}(\Lambda_b \to \Lambda \tau^+ \tau^-) = -0.04$ (-0.03), whereas the results of Ref. [8] were $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda e^+ e^-) = 1.2 \times 10^{-8}$, $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda \mu^+ \mu^-) = 8 \times 10^{-4}$, and $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda \tau^+ \tau^-) = 9.6 \times 10^{-4}$. Ref. [10] analyzed the differential $\bar{A}_{\rm FB}(\Lambda_b \to \Lambda l^+ l^-)$ in the heavy quark limit. Using the nonrelativistic quark model, Ref. [11] investigated the lepton-side forward-backward asymmetries $\bar{A}^l_{\rm FB}(\Lambda_b \to \Lambda l^+ l^-)$. In the quark-diquark model, Ref. [12] investigated the lepton-side forward-backward asymmetries $A_{\rm FB}$, the hadron-side forward-backward asymmetries $A^h_{\rm FB}$, and the hadron-lepton forward-backward asymmetries $A^{hl}_{\rm FB}$. In an approach of the light-cone sum rules, Refs. [13, 14] investigated the rare decays of $\Lambda_b \to \Lambda \gamma$ and $\Lambda_b \to \Lambda l^+ l^-$. Ref. [15] investigated the phenomenological potential of the rare decay $\Lambda_b \to \Lambda l^+ l^-$ with a subsequent, self-analyzing $\Lambda_b \to N\pi$ transition. With the form factors (FFs) ex-

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tracted from a constituent quark model, Ref. [16] investigated the rare weak dileptonic decays of the Λ_b baryon. Ref. [17] studied $\mathcal{B}_1 \to \mathcal{B}_2 l^+ l^-$ ($\mathcal{B}_{1,2}$ are spin 1/2 baryons) with the $SU(3)$ flavor symmetry. The FFs of $\Lambda_b \to \Lambda$ differ in different models. Generally, the number of independent FFs of $\Lambda_b \to \Lambda$ can be reduced to 2 when working in the heavy quark limit [18],

$$\langle \Lambda(p)|\bar{s}\Gamma b|\Lambda_b(v)\rangle = \bar{u}_\Lambda (F_1(q^2) + F_2(q^2)\not{v})\Gamma u_{\Lambda_b}(v), \quad (1)$$

where $\Gamma = \gamma_\mu$, $\gamma_\mu\gamma_5$, $q^\nu\sigma_{\nu\mu}$, and $q^\nu\sigma_{\nu\mu}\gamma_5$, q^2 is the square of the transformed momentum. The FF ratio $R(q^2) = F_2(q^2)/F_1(q^2)$ was considered a constant in many studies assuming the same shape for F_1 and F_2, and it was derived from quantum chromodynamics (QCD) sum rules in the framework of the heavy quark effective theory [6]. For example, in Refs. [6, 7] the q^2 dependence of FF F_i $(i=1,2)$ were given as follows:

$$F_i(q^2) = \frac{F_i(0)}{1 - aq^2 + bq^4}, \quad (2)$$

where a and b are constants. Using experimental data for the semileptonic decay $\Lambda_c \to \Lambda e^+ \nu_e$ ($m_\Lambda^2 \le q^2 \le m_{\Lambda_c}^2$), the CLEO collaboration provided the ratio $R = -0.35 \pm 0.04$ (stat) ± 0.04 (syst) [19]. In Ref. [20], the authors investigated $\Lambda_b \to \Lambda\gamma$ obtaining $R = -0.25 \pm 0.14 \pm 0.08$. In Refs. [6, 7, 21], the authors investigated the baryonic decay $\Lambda_b \to \Lambda l^+ l^-$ and obtained $R = -0.25$. In Ref. [22], the relation $F_2(q^2)/F_1(q^2) \approx F_2(0)/F_1(0)$ was given. However, according to the pQCD scaling law [23–25], the FFs should not have the same shape. Using Stech's approach, Ref. [26] obtained the FF ratio $R(q^2) \propto -1/q^2$. From the data in Ref. [27], we can estimate the value of R and observe that it changes from -0.83 to -0.32, which is not a constant. In our previous studies [28, 29], we observed that the ratio R is not a constant in the Λ_b rare decay in a large momentum region in which we did not consider the long distance contributions because they have a small effect on the FFs of this decay [30, 31]. In these studies, Λ_b (Λ) was considered a bound state of two particles: a quark and a scalar diquark. This model has been used to study many heavy baryons [32]. Using the kernel of the BSE, including scalar confinement and one-gluon-exchange terms and the covariant instantaneous approximation, we obtained the Bethe-Salpeter (BS) wave functions of Λ_b and Λ [28, 29]. In this study, we recalculate the FFs of $\Lambda_b \to \Lambda$ in this model.

The remainder of this paper is organized as follows. In Sec. II, we derive the general FFs and $A_{\rm FB}$ for $\Lambda_b \to \Lambda l^+ l^-$ in the BS equation approach. In Sec. III, the numerical results for $A_{\rm FB}$ and $\bar{A}_{\rm FB}$ of $\Lambda_b \to \Lambda l^+ l^-$ are provided. Finally, the summary and discussion are presented in Sec. V.

II. THEORETICAL FORMALISM

A. BSE for $\Lambda_b(\Lambda)$

As shown in Fig. 1, following our previous research, the BS amplitude of $\Lambda_b(\Lambda)$ in momentum space satisfies the integral equation [28, 29, 33–39]

$$\chi_P(p) = S_{\rm F}(\lambda_1 P + p)\int \frac{{\rm d}^4 q}{(2\pi)^4} K(P,p,q)\chi_P(q) S_D(\lambda_2 P - p), \quad (3)$$

where $K(P,p,q)$ is the kernel, which is defined as the sum of the two particles irreducible diagrams, $S_{\rm F}$ and S_D are the propagators of the quark and scalar diquark, respectively. $\lambda_{1(2)} = m_{q(D)}/(m_q + m_D)$, where $m_{q(D)}$ is the mass of the quark (diquark), and P is the momentum of the baryon.

We assume the kernel has the following form:

$$-{\rm i}K(P,p,q) = I \otimes I V_1(p,q) + \gamma_\mu \otimes (p_2 + q_2)^\mu V_2(p,q), \quad (4)$$

where V_1 results from the scalar confinement, and V_2 is from the one-gluon-exchange diagram. According to the potential model, V_1 and V_2 have the following forms in the covariant instantaneous approximation ($p_l = q_l$) [28, 29, 37–39]:

$$\tilde{V}_1(p_t - q_t) = \frac{8\pi\kappa}{[(p_t-q_t)^2 + \mu^2]^2} - (2\pi)^2\delta^3(p_t - q_t) \times \int \frac{{\rm d}^3 k}{(2\pi)^3}\frac{8\pi\kappa}{(k^2+\mu^2)^2}, \quad (5)$$

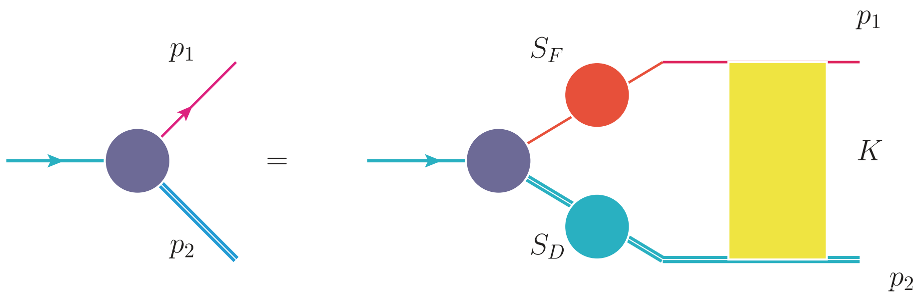

Fig. 1. (color online) BS equation for $\Lambda_b(\Lambda)$ in momentum space (K is the interaction kernel)

$$\tilde{V}_2(p_t - q_t) = -\frac{16\pi}{3}\frac{\alpha_{\text{seff}}^2 Q_0^2}{[(p_t-q_t)^2+\mu^2][(p_t-q_t)^2+Q_0^2]}, \tag{6}$$

where μ is a small parameter; to avoid the divergence in numerical calculation, this parameter is considered to be sufficiently small such that the results are not sensitive to it. The parameters κ and α_{seff} are related to scalar confinement and the one-gluon-exchange diagram, respectively. q_t is the transverse projection of the relative momentum along the momentum P, which is defined as $p_l = \lambda_1 P - v\cdot p$, $p_t^\mu = p^\mu - (v\cdot p)p^\mu$ $(v^\mu = P^\mu/M)$, $q_t^\mu = q^\mu - (v\cdot q)v^\mu$, and $q_l = \lambda_2 P - v\cdot q$. The second term of $\tilde{V}_1$ is introduced to avoid infrared divergence at the point $p_t = q_t$, and μ is a small parameter to avoid the divergence in numerical calculations. Analyzing the electromagnetic FFs of the proton, $Q_0^2 = 3.2$ GeV^2 was observed to provide consistent results with the experimental data [40].

The propagators of the quark and diquark can be expressed as follows:

$$S_F(p_1) = \mathrm{i}\,\not{v}\left[\frac{\Lambda_q^+}{M-p_l-\omega_q+\mathrm{i}\epsilon}+\frac{\Lambda_q^-}{M-p_l+\omega_q-\mathrm{i}\epsilon}\right], \tag{7}$$

$$S_D(p_2) = \frac{\mathrm{i}}{2\omega_D}\left[\frac{1}{p_l-\omega_D+\mathrm{i}\epsilon}-\frac{1}{p_l+\omega_D-\mathrm{i}\epsilon}\right], \tag{8}$$

where $\omega_q = \sqrt{m^2-p_t^2}$ and $\omega_D = \sqrt{m_D^2-p_t^2}$, M is the mass of the baryon, and $\Lambda^\pm$ are the projection operators, which are defined as

$$2\omega_q\Lambda_q^\pm = \omega_q \pm \not{v}(\not{p}_t+m), \tag{9}$$

and satisfy the following relations:

$$\Lambda_q^\pm\Lambda_q^\pm = \Lambda_q^\pm,\ \Lambda_q^\pm\Lambda_q^\mp = 0. \tag{10}$$

Generally, we require two scalar functions to describe the BS wave function of $\Lambda_b(\Lambda)$ [33–35],

$$\chi_P(p) = (f_1(p_t^2)+\not{p}_t f_2(p_t^2))u(P), \tag{11}$$

where $f_i, (i=1,2)$ are the Lorentz-scalar functions of p_t^2, and $u(P)$ is the spinor of a baryon.

Defining $\tilde{f}_{1(2)} = \int\frac{\mathrm{d}p_l}{2\pi}f_{1(2)}$, and using the covariant instantaneous approximation, the scalar BS wave functions satisfy the following coupled integral equations:

$$\tilde{f}_1(p_t) = \int\frac{\mathrm{d}^3q_t}{(2\pi)^3}M_{11}(p_t,q_t)\tilde{f}_1(q_t)+M_{12}(p_t,q_t)\tilde{f}_2(q_t), \tag{12}$$

$$\tilde{f}_2(p_t) = \int\frac{\mathrm{d}^3q_t}{(2\pi)^3}M_{21}(p_t,q_t)\tilde{f}_1(q_t)+M_{22}(p_t,q_t)\tilde{f}_2(q_t), \tag{13}$$

where

$$\begin{aligned} M_{11}(p_t,q_t) =& \frac{(\omega_q+m)(\tilde{V}_1+2\omega_D\tilde{V}_2)-p_t\cdot(p_t+q_t)\tilde{V}_2}{4\omega_D\omega_q(-M+\omega_D+\omega_q)} \\ &-\frac{(\omega_q-m)(\tilde{V}_1-2\omega_D\tilde{V}_2)+p_t\cdot(p_t+q_t)\tilde{V}_2}{4\omega_D\omega_c(M+\omega_D+\omega_q)}, \end{aligned} \tag{14}$$

$$\begin{aligned} M_{12}(p_t,q_t) =& \frac{-(\omega_q+m)(q_t+p_t)\cdot q_t\tilde{V}_2+p_t\cdot q_t(\tilde{V}_1-2\omega_D\tilde{V}_2)}{4\omega_D\omega_c(-M+\omega_D+\omega_c)} \\ &-\frac{(m-\omega_q)(q_t+p_t)\cdot q_t\tilde{V}_2-p_t\cdot q_t(\tilde{V}_1+2\omega_D\tilde{V}_2)}{4\omega_D\omega_q(M+\omega_D+\omega_q)}, \end{aligned} \tag{15}$$

$$\begin{aligned} M_{21}(p_t,q_t) =& \frac{(\tilde{V}_1+2\omega_D\tilde{V}_2)-(-\omega_q+m)\left(1+\dfrac{q_t\cdot p_t}{p_t^2}\right)\tilde{V}_2}{4\omega_D\omega_q(-M+\omega_D+\omega_q)} \\ &-\frac{-(\tilde{V}_1-2\omega_D\tilde{V}_2)+(\omega_q+m)\left(1+\dfrac{q_t\cdot p_t}{p_t^2}\right)\tilde{V}_2}{4\omega_D\omega_q(M+\omega_D+\omega_q)}, \end{aligned} \tag{16}$$

$$\begin{aligned} &M_{22}(p_t,q_t) \\ &=\frac{(m-\omega_q)(\tilde{V}_1+2\omega_D\tilde{V}_2))p_t\cdot q_t-p_t^2(q_t^2+p_t\cdot q_t)\tilde{V}_2}{4p_t^2\omega_D\omega_q(-M+\omega_D+\omega_q)} \\ &\quad-\frac{(m+\omega_q)(-\tilde{V}_1-2\omega_D\tilde{V}_2))p_t\cdot q_t+p_t^2(q_t^2+p_t\cdot q_t)\tilde{V}_2}{4p_t^2\omega_D\omega_q(M+\omega_D+\omega_q)}. \end{aligned} \tag{17}$$

When the mass of the b quark approaches infinity [32], the propagator of the b quark satisfies the relation $\not{v}S_{\mathrm{F}}(p_1) = S_{\mathrm{F}}(p_1)$ and can be reduced to

$$S_{\mathrm{F}}(p_1) = \mathrm{i}\frac{1+\not{v}}{2(E_0+m_D-p_l+\mathrm{i}\epsilon)}, \tag{18}$$

where $E_0 = M-m-m_D$ is the binding energy. Thus, the BS wave function of Λ_b has the form $\chi_P(v) = \phi(p)u_{\Lambda_b}(v,s)$, where $\phi(p)$ is the scalar BS wave function [32], and the BS equation for Λ_b can be replaced by

$$\begin{aligned}\phi(p) =& -\frac{\mathrm{i}}{(E_0+m_D-p_l+\mathrm{i}\epsilon)(p_l^2-\omega_D^2)} \\ &\times\int\frac{\mathrm{d}^4q}{(2\pi)^4}(\tilde{V}_1+2p_l\tilde{V}_2)\phi(q).\end{aligned} \tag{19}$$

Generally, we can take E_0 to be about -0.14 GeV and κ to be about 0.05 GeV^3 [28, 29].

B. Asymmetries of $\Lambda_b \to \Lambda l^+ l^-$ decays

In the Standard Model, the $\Lambda_b \to \Lambda l^+ l^-$ $(l = e,\mu,\tau)$ transitions are described by $b \to s l^+ l^-$ at the quark level. The Hamiltonian for the decay of $b \to s l^+ l^-$ is given by

$$\mathcal{H}(b \to s l^+ l^-) = \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[C_9^{\rm eff} \bar{s}\gamma_\mu (1-\gamma_5) b \bar{l}\gamma^\mu l - \mathrm{i} C_7^{\rm eff} \bar{s} \frac{2 m_b \sigma_{\mu\nu} q^\nu}{q^2} (1+\gamma_5) b \bar{l}\gamma^\mu l + C_{10} \bar{s}\gamma_\mu (1-\gamma_5) b \bar{l}\gamma^\mu \gamma_5 l \Big], \tag{20}$$

where G_F is the Fermi coupling constant, α is the fine structure constant at the Z mass scale, V_{ts} and V_{tb} are the CKM matrix elements, q is the total momentum of the lepton pair, and C_i $(i = 7, 9, 10)$ are the Wilson coefficients. $C_7^{\rm eff} = -0.313$, $C_9^{\rm eff} = 4.334$, $C_{10} = -4.669$ [41–43]. The relevant matrix elements can be parameterized in terms of the FFs as follows:

$$\begin{aligned}
\langle \Lambda(P')|\bar{s}\gamma_\mu b|\Lambda_b(P)\rangle &= \bar{u}_\Lambda(P')(g_1\gamma^\mu + \mathrm{i} g_2 \sigma^{\mu\nu} q_\nu + g_3 q_\mu) u_{\Lambda_b}(P),\\
\langle \Lambda(P')|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b(P)\rangle &= \bar{u}_\Lambda(P')(t_1\gamma^\mu + \mathrm{i} t_2 \sigma^{\mu\nu} q_\nu + t_3 q^\mu)\gamma_5 u_{\Lambda_b}(P),\\
\langle \Lambda(P')|\bar{s} i\sigma^{\mu\nu} q^\nu b|\Lambda_b(P)\rangle &= \bar{u}_\Lambda(P')(s_1\gamma^\mu + \mathrm{i} s_2 \sigma^{\mu\nu} q_\nu + s_3 q^\mu) u_{\Lambda_b}(P),\\
\langle \Lambda(P')|\bar{s} i\sigma^{\mu\nu}\gamma_5 q^\nu b|\Lambda_b(P)\rangle &= \bar{u}_\Lambda(P')(d_1\gamma^\mu + \mathrm{i} d_2 \sigma^{\mu\nu} q_\nu + d_3 q^\mu)\gamma_5 u_{\Lambda_b}(P),
\end{aligned} \tag{21}$$

where $P(P')$ is the momentum of the $\Lambda_b(\Lambda)$, $q^2 = (P-P')^2$ is the transformed momentum squared, and g_i, t_i, s_i, andd_i $(i = 1,2,$ and $3)$ are the transition FFs, which are Lorentz scalar functions of q^2. The Λ_b and Λ states can be normalized as follows:

$$\langle \Lambda(P')|\Lambda(P)\rangle = 2E_\Lambda (2\pi)^3 \delta^3(P-P'), \tag{22}$$

$$\langle \Lambda_b(v',P')|\Lambda_b(v,P)\rangle = 2v_0 (2\pi)^3 \delta^3(P-P'). \tag{23}$$

Comparing Eq. (1) with Eq. (21), we obtain the following relations:

$$\begin{aligned}
g_1 &= t_1 = s_2 = d_2 = \left(F_1 + \sqrt{r} F_2\right),\\
g_2 &= t_2 = g_3 = t_3 = \frac{1}{m_{\Lambda_b}} F_2,\\
s_3 &= F_2(\sqrt{r}-1), \quad d_3 = F_2(\sqrt{r}+1),\\
s_1 &= d_1 = F_2 m_{\Lambda_b}(1 + r - 2\sqrt{r}\omega),
\end{aligned} \tag{24}$$

where $r = m_\Lambda^2/m_{\Lambda_b}^2$ and $\omega = (M_{\Lambda_b}^2 + M_\Lambda^2 - q^2)/(2M_{\Lambda_b} M_\Lambda) = v\cdot P'/m_\Lambda$. The transition matrix for $\Lambda_b \to \Lambda$ can be expressed in terms of the BS wave functions of Λ_b and Λ:

$$\langle \Lambda(P')|\bar{d}\Gamma b|\Lambda_b(P)\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \bar{\chi}_{P'}(v')\Gamma \chi_P(p) S_D^{-1}(p_2). \tag{25}$$

When $\omega \neq 1$, we can obtain the following expression by substituting Eqs. (11) and (19) into Eq. (25):

$$F_1 = k_1 - \omega k_2, \tag{26}$$

$$F_2 = k_2, \tag{27}$$

where

$$k_1(\omega) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} f_1(p')\phi(p) S_D^{-1}(p_2), \tag{28}$$

$$k_2(\omega) = \frac{1}{1-\omega^2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} f_2(p') p_t' \cdot v \phi(p) S_D^{-1}. \tag{29}$$

The decay amplitude of $\Lambda_b \to \Lambda l^+ l^-$ can be rewritten as follows:

$$\begin{aligned}
\mathcal{M}(\Lambda_b \to \Lambda l^+ l^-) = \frac{G_F \lambda_t}{2\sqrt{2}\pi} [&\bar{l}\gamma_\mu l \{\bar{u}_\Lambda [\gamma_\mu (A_1 + B_1 + (A_1 - B_1)\gamma_5)\\
&+ \mathrm{i}\sigma^{\mu\nu} p_\nu (A_2 + B_2 + (A_2 - B_2)\gamma_5)] u_{\Lambda_b}\}\\
&+ \bar{l}\gamma_\mu \gamma_5 l \{\bar{u}_\Lambda [\gamma^\mu (D_1 + E_1 + (D_1 - E_1)\gamma_5)\\
&+ \mathrm{i}\sigma^{\mu\nu} p_\nu (D_2 + E_2 + (D_2 - E_2)\gamma_5)\\
&+ p^\mu (D_3 + E_3 + (D_3 - E_3)\gamma_5)] u_{\Lambda_b}\}],
\end{aligned} \tag{30}$$

where A_i, B_i,D_j, andE_j $(i = 1,2$ and $j = 1,2,3)$ are defined as follows:

$$\begin{aligned}
A_i &= \frac{1}{2}\left\{ C_9^{\rm eff}(g_i - t_i) - \frac{2C_7^{\rm eff} m_b}{q^2}(d_i + s_i) \right\},\\
B_i &= \frac{1}{2}\left\{ C_9^{\rm eff}(g_i + t_i) - \frac{2C_7^{\rm eff} m_b}{q^2}(d_i - s_i) \right\},\\
D_j &= \frac{1}{2} C_{10}(g_j - t_j), \ E_j = \frac{1}{2} C_{10}(g_j + t_j).
\end{aligned} \tag{31}$$

In the physical region $(\omega = (m_{\Lambda_b}^2 + m_\Lambda^2 - q^2)/(2m_{\Lambda_b} m_\Lambda))$, the decay rate of $\Lambda_b \to \Lambda l^+ l^-$ is obtained as follows:

$$\frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda l^+ l^-)}{\mathrm{d}\omega \mathrm{d}\cos\theta} = \frac{G_F^2 \alpha^2}{2^{14}\pi^5 m_{\Lambda_b}} |V_{tb} V_{ts}^*|^2 v_l \sqrt{\lambda(1,r,s)} \mathcal{M}(\omega,\theta), \tag{32}$$

where $s = 1 + r - 2\sqrt{r}\omega$, $\lambda(1,r,s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $v_l = \sqrt{1 - \frac{4m_l^2}{s m_{\Lambda_b}^2}}$, and the decay amplitude is expressed as follows [44]:

$$\mathcal{M}(\omega,\theta) = \mathcal{M}_0(\omega) + \mathcal{M}_1(\omega)\cos\theta + \mathcal{M}_2(\omega)\cos^2\theta, \tag{33}$$

where θ is the polar angle, as shown in Fig. 2.

$$\begin{aligned}
\mathcal{M}_0(\omega) =& 32 m_l^2 m_{\Lambda_b}^4 s(1+r-s)(|D_3|^2+|E_3|^2) + 64 m_l^2 m_{\Lambda_b}^3 (1-r-s)\mathrm{Re}(D_1^* E_3 + D_3 E_1^*) \\
&+ 64 m_{\Lambda_b}^2 \sqrt{r}(6m_l^2 - M_{\Lambda_b}^2 s)\mathrm{Re}(D_1^* E_1) + 64 m_l^2 m_{\Lambda_b}^3 \sqrt{r}(2m_{\Lambda_b} s\mathrm{Re}(D_3^* E_3) + (1-r+s)\mathrm{Re}(D_1^* D_3 + E_1^* E_3)) \\
&+ 32 m_{\Lambda_b}^2 (2m_l^2 + m_{\Lambda_b}^2 s)\Big\{(1-r+s) m_{\Lambda_b} \sqrt{r}\mathrm{Re}(A_1^* A_2 + B_1^* B_2) \\
&- m_{\Lambda_b}(1-r-s)\mathrm{Re}(A_1^* B_2 + A_2^* B_1) - 2\sqrt{r}(\mathrm{Re}(A_1^* B_1) + m_{\Lambda_b}^2 s\mathrm{Re}(A_2^* B_2))\Big\} \\
&+ 8 m_{\Lambda_b}^2 \Big[4m_l^2(1-r-s) + m_{\Lambda_b}^2((1+r)^2 - s^2)\Big](|A_1|^2 + |B_1|^2) \\
&+ 8 m_{\Lambda_b}^4 \Big\{4m_l^2[\lambda + (1+r-s)s] + m_{\Lambda_b}^2 s[(1-r)^2 - s^2]\Big\}(|A_2|^2 + |B_2|^2) \\
&- 8 m_{\Lambda_b}^2 \Big\{4m_l^2(1+r-s) - m_{\Lambda_b}^2[(1-r)^2 - s^2]\Big\}(|D_1|^2 + |E_1|^2) \\
&+ 8 m_{\Lambda_b}^5 s v^2 \Big\{-8 m_{\Lambda_b} s\sqrt{r}\mathrm{Re}(D_2^* E_2) + 4(1-r+s)\sqrt{r}\mathrm{Re}(D_1^* D_2 + E_1^* E_2) \\
&- 4(1-r-s)\mathrm{Re}(D_1^* E_2 + D_2^* E_1) + m_{\Lambda_b}[(1-r)^2 - s^2](|D_2|^2 + |E_2|^2)\Big\},
\end{aligned} \tag{34}$$

$$\begin{aligned}
\mathcal{M}_1(\omega) =& -16 m_{\Lambda_b}^4 s v_l \sqrt{\lambda}\Big\{2\mathrm{Re}(A_1^* D_1) - 2\mathrm{Re}(B_1^* E_1) + 2m_{\Lambda_b}\mathrm{Re}(B_1^* D_2 - B_2^* D_1 + A_2^* E_1 - A_1^* E_2)\Big\} \\
&+ 32 m_{\Lambda_b}^5 s v_l \sqrt{\lambda}\Big\{m_{\Lambda_b}(1-r)\mathrm{Re}(A_2^* D_2 - B_2^* E_2) + \sqrt{r}\mathrm{Re}(A_2^* D_1 + A_1^* D_2 - B_2^* E_1 - B_1^* E_2)\Big\},
\end{aligned} \tag{35}$$

$$\mathcal{M}_2(\omega) = 8 m_{\Lambda_b}^6 s v_l^2 \lambda(|A_2|^2 + |B_2|^2 + |E_2|^2 + |D_2|^2) - 8 m_{\Lambda_b}^4 v_l^2 \lambda(|A_1|^2 + |B_1|^2 + |E_1|^2 + |D_1|^2). \tag{36}$$

The lepton-side forward-backward asymmetry, A_{FB}, is defined as

$$A_{FB} = \frac{\int_0^1 \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2 \mathrm{d}z}\mathrm{d}z - \int_{-1}^0 \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2 \mathrm{d}z}\mathrm{d}z}{\int_{-1}^1 \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2 \mathrm{d}z}\mathrm{d}z}, \tag{37}$$

where $z = \cos\theta$. The "naively integrated" observables are obtained using [17]

$$\langle X \rangle = \frac{1}{q_{\max}^2 - q_{\min}^2} \int_{q_{\min}^2}^{q_{\max}^2} X(q^2)\mathrm{d}q^2. \tag{38}$$

We define the integrated A_{FB} as

$$\bar{A}_{FB} = \int_{\hat{q}_{\min}}^{\hat{q}_{\max}} \mathrm{d}\hat{q}^2 A_{FB}(\hat{q}^2). \tag{39}$$

where $\hat{q}^2 = q^2/M_{\Lambda_b}^2$. With the aid of the helicity amplitudes of $\Lambda_b \to \Lambda l^+ l^-$, we can also calculate the hadron forward-backward asymmetry, the lepton-hadron side asymmetry, and the fraction of longitudinally polarized dileptons.

The hadron forward-backward asymmetry has the form

$$A_{FB}^h(q^2) = \frac{\alpha_\Lambda}{2} \frac{\frac{v_l^2}{2}(\mathcal{H}_P^{11} + \mathcal{H}_P^{22} + \mathcal{H}_{L_P}^{11} + \mathcal{H}_{L_P}^{22}) + \frac{3m_l^2}{q^2}(\mathcal{H}_P^{11} + \mathcal{H}_{L_P}^{11} + \mathcal{H}_{S_P}^{22})}{\mathcal{H}_{tot}}. \tag{40}$$

The lepton-hadron side asymmetry has the form

$$A_{FB}^{lh}(q^2) = -\frac{3}{4}\frac{\alpha_\Lambda}{2}\frac{v_l \mathcal{H}_U^{12}}{\mathcal{H}_{tot}}. \tag{41}$$

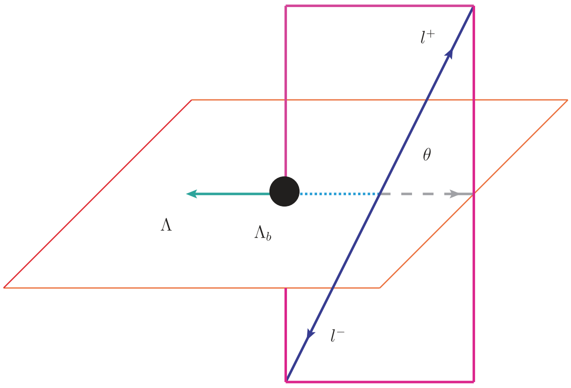

Fig. 2. (color online) Definition of the angle θ in the decay $\Lambda_b \to \Lambda l^- l^+$.

The fraction of the longitudinally polarized dileptons is expressed by

$$F_L(q^2) = \frac{\frac{v_l^2}{2}(\mathcal{H}_L^{11} + \mathcal{H}_L^{22}) + \frac{m_l^2}{q^2}(\mathcal{H}_U^{11} + \mathcal{H}_L^{11} + \mathcal{H}_S^{22})}{\mathcal{H}_{\rm tot}}. \tag{42}$$

In Eqs. (40 –42), $\mathcal{H}_X^{mm'}(X = U, L, S, P, L_P, S_P, m = 1,2)$ represent different helicity amplitudes, and $\mathcal{H}_{\rm tot}$ is the total helicity amplitude, $\alpha_\Lambda = 0.642 \pm 0.013$. The explicit expression for $\mathcal{H}_X^{mm'}$ is provided in Ref. [12].

III. NUMERICAL ANALYSIS AND DISCUSSION

In this section, we perform a detailed numerical analysis of $A_{\rm FB}(\Lambda_b \to \Lambda l^+ l^-)$. In this study, we take the masses of baryons as $m_{\Lambda_b} = 5.62$ GeV and $m_\Lambda = 1.116$ GeV [45], and the masses of quarks as $m_b = 5.02$ GeV and $m_s = 0.516$ GeV [34, 35, 39]. The variable ω changes from 1 to 2.617, 2.614, 1.617 for e, μ, τ, respectively.

Solving Eqs. (12) and (19) for Λ and Λ_b, we can obtain the numerical solutions of their BS wave functions. In Table 1, we provide the values of $\alpha_{\rm seff}$ for different values of κ for Λ and Λ_b with $E_0 = -0.14$ GeV.

From Table 1, we observe that the value of $\alpha_{\rm seff}$ is weakly dependent on the value of κ. In Fig. 3, we plot the FFs and FF ratio $R(\omega)$. From this figure, we observe that $R(\omega)$ varies from -0.75 to -0.25 in our model. In Ref. [27], $R(\omega)$ varied from -0.42 to -0.83 in the same ω region, which is in agreement with our result and the estimated value from Refs. [28, 29] mentioned in the Introduction. In the range of $2.43 \le \omega \le 2.52$ (corresponding to $M_\Lambda^2 \le q^2 \le M_{\Lambda_c}^2$), $R(\omega)$ is about -0.25. In the same ω region, assuming the FFs have the same dependence on q^2, the CLEO collaboration measured $R = -0.35 \pm 0.04 \pm 0.04$ in the limit $m_c \to +\infty$. These results are in good agreement with our research in the same ω region.

In Table 2, we provide $\bar{A}_{\rm BF}^l$, $\bar{A}_{\rm FB}^{lh}$, $\bar{A}_{\rm FB}^h$, and $\bar{F}_L$ for $\Lambda_b \to \Lambda\mu^+\mu^-$ and compare our results with those of other studies. We can observe that these asymmetries differ significantly in different models. Considering these differences, $\bar{A}_{\rm FB}^l$ changes between -0.30 and 0, $\bar{A}_{\rm FB}^{lh}$ is about 0.1, $\bar{A}_{\rm FB}^h$ is about -0.25, and $\bar{F}_L$ changes from 0.3 to 0.6. Without including the long distance contribution, Ref. [6] provided the integrated forward-backward asymmetry $\bar{A}_{\rm BF}^l(\Lambda_b \to \Lambda\mu^+\mu^-) = -0.1338$. The result of Ref. [7] was $\bar{A}_{\rm BF}^l(\Lambda_b \to \Lambda\mu^+\mu^-) = -0.13(-0.12)$ in the QCD sum rule

Table 1. Values of $\alpha_{\rm seff}$ for Λ and Λ_b for different κ values.

κ/GeV^3	Λ	Λ_b
0.045	0.559	0.775
0.047	0.555	0.777
0.049	0.551	0.778
0.051	0.547	0.780
0.053	0.544	0.782
0.055	0.540	0.784

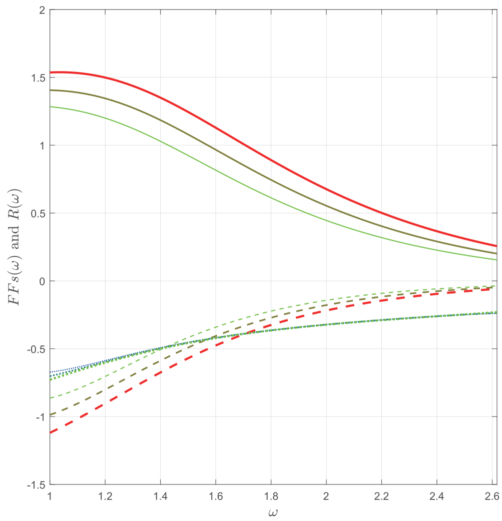

Fig. 3. (color online) Values of F_1 (solid line), F_2 (dash line) and $R(\omega)$ (dot line) as a function of ω (the lines become thicker with the increase in κ).

Table 2. Longitudinal polarization fractions and forward-backward asymmetries for $\Lambda_b \to \Lambda\mu^+\mu^-$.

	$\bar{A}_{\rm FB}^l$	$\bar{A}_{\rm FB}^{lh}$	$\bar{A}_{\rm FB}^h$	$\bar{F}_L$
[6, 7]	-0.13	–	–	0.5830
[8]	8.0×10^{-4}	–	–	–
[12]	-0.286	0.101	-0.288	0.525
[13]	$-0.0122^{+0.0142}_{-0.0073}$	–	–	–
[15]	-0.29 ± 0.05	$0.13^{+0.22}_{-0.03}$	-0.26 ± 0.03	0.4 ± 0.1
[17]	$-0.04^{+0.00}_{-0.01}$	–	–	$0.34^{+0.03}_{-0.02}$
our work	-0.1376 ± 0.0001	0.0576	-0.1613 ± 0.0001	0.3957 ± 0.0002

approach (pole model). Using the covariant constituent quark model with (without) the long distance contribution, Ref. [8] obtained the result $\bar{A}^l_{\rm BF}(\Lambda_b \to \Lambda\mu^+\mu^-) = 1.7\times10^{-4}(8\times10^{-4})$.

For $q^2 \in [15,20]$ GeV^2, the LHCb collaboration provided $A^l_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+) = -0.05\pm0.09$ in 2015, which was updated to $A^l_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+) = -0.39\pm0.04$ three years later [4, 5]. In our study, in the same region, the value of $A^l_{\rm BF}(\Lambda_b \to \Lambda\mu^-\mu^+)$ changes from -0.44 to -0.35, which is in good agreement with the most recent experimental data of the LHCb collaboration. With the latest high-precision lattice QCD calculations in the same region, Ref. [46] obtained the values $A^l_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+) = -0.344$ in the large ς_u and small ς_d regions (ς_u, ς_d are model parameters [47]) and $A^l_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+) = -0.24$ in the large ς_d and small ς_u regions. In Fig. 4, we plot the q^2-dependence of $A^l_{\rm FB}(\Lambda_b \to \Lambda e^-e^+)$, $A^l_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+)$, and $A^l_{\rm FB}(\Lambda_b \to \Lambda\tau^-\tau^+)$. From Fig. 4, we can observe that $A^l_{\rm FB}(\Lambda_b \to \Lambda\mu^+\mu^-)$ is in good agreement with the lattice QCD calculation in the entire q^2 region [48]. The results of other references results are also shown in Table 3. In Fig. 5, we plot the q^2-dependence of $A^h_{\rm FB}(\Lambda_b \to \Lambda e^-e^+)$, $A^h_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+)$, and $A^h_{\rm FB}(\Lambda_b \to \Lambda\tau^-\tau^+)$, respectively. For $q^2 \in [15,20]$ GeV^2, the LHCb collaboration obtained the value for $\Lambda_b \to \Lambda\mu^-\mu^+$ as -0.29 ± 0.07, which is in good agreement our result $-0.2304 \sim -0.0685$. The results of other references results are also shown in Table 3. In Fig. 6, we plot the q^2-dependence of $A^{lh}_{\rm FB}(\Lambda_b \to \Lambda e^-e^+)$, $A^{lh}_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+)$, and $A^{lh}_{\rm FB}(\Lambda_b \to \Lambda\tau^-\tau^+)$, respectively. Ref. [12] obtained the value $A^{lh}_{\rm FB}(\Lambda_b \to \Lambda\mu^-\mu^+) = 0.145$, which is agreement with our results $0.1257 \sim 0.1555$ in the region $q^2 \in [15,20]$ GeV^2. In Fig. 7, we plot the q^2-dependence of $F_L(\Lambda_b \to \Lambda e^-e^+)$, $F_L(\Lambda_b \to \Lambda\mu^-\mu^+)$, and $F_L(\Lambda_b \to \Lambda\tau^-\tau^+)$, respectively. In the region $q^2 \in [15,20]$ GeV^2, the LHCb collaboration obtained the value $F_L(\Lambda_b \to \Lambda\mu^-\mu^+) = 0.61^{+0.11}_{-0.14}$, which is close to our result of$0.3398 \sim 0.4530$. The results of other references results are also shown in Table 3. From these figures, we observe that all these asymmetries are not very sensitive to the parameters κ and E_0 in our model.

Ref. [17] obtained the naively integrated values $\langle A^l_{\rm FB}\rangle = -0.19^{+0.00}_{-0.01}$ and $\langle F_L\rangle = 0.6\pm0.02$ for $\Lambda_b \to \Lambda\mu^+\mu^-$, whereas in our paper, these values are -0.1976 and 0.5681, respectively. Our results are very close to those of Ref. [17]. In our paper, we obtain $\bar{A}^l_{\rm FB} = -0.0708\pm 0.0001(-0.0590\pm0.0001)$ and $\bar{A}^h_{\rm FB} = -0.1604\pm0.0001$ (-0.1541 ± 0.0002) for $\Lambda_b \to \Lambda e^+e^-(\Lambda_b \to \Lambda\tau^+\tau^-)$. The values given in Ref. [8] are $\bar{A}^l_{\rm FB} = 1.2\times10^{-8}(9.6\times10^{-4})$ and $\bar{A}^h_{\rm FB} = -0.321(-0.259)$, and Refs. [13] and [7] provide $\bar{A}^l_{\rm FB} = -0.0067$ and $\bar{A}^l_{\rm FB} = -0.04$ for $\Lambda_b \to \Lambda\tau^+\tau^-$. Comparing the values in these theoretical approaches, we observe that the asymmetries may vary widely among the

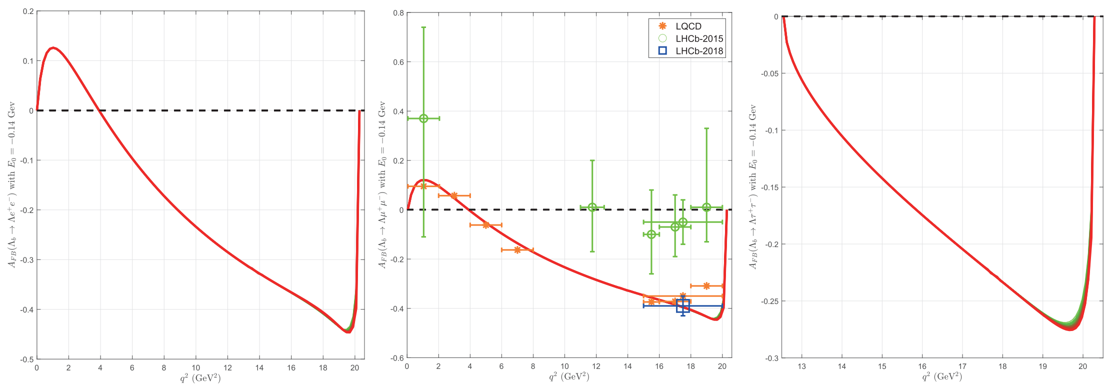

Fig. 4. (color online) Values of $A_{\rm FB}(\Lambda_b \to \Lambda l^+l^-)$ as a function of q^2 for different values of κ as shown in Table 1.

Table 3. Longitudinal polarization fractions and forward-backward asymmetries for $\Lambda_b \to \Lambda\mu^+\mu^-$ in $q^2 \in [15,20]$ GeV^2.

–	$A^l_{\rm FB[15,20]}$	$A^{lh}_{\rm FB[15,20]}$	$A^h_{\rm FB[15,20]}$	$F_{L[15,20]}$
LHCb [4, 5]	-0.39 ± 0.04	–	-0.29 ± 0.07	$0.61^{+0.11}_{-0.14}$
[6, 7]	$-0.40\sim-0.25$	–	–	$0.37\sim0.62$
[8]	$-0.24\sim-0.13$	–	>-0.308	–
[12]	-0.40	0.145	-0.29	0.38
[13]	$-0.075\sim-0.017$	–	–	–
[17]	$-0.34^{+0.01}_{-0.02}$	–	–	$0.4^{+0.01}_{-0.02}$
[48]	$-0.350(13)$	–	-0.2710 ± 0.0092	0.409 ± 0.013
our work	$-0.44\sim-0.35$	$0.1257\sim0.1555$	$-0.2304\sim-0.0685$	$0.3398\sim0.4530$

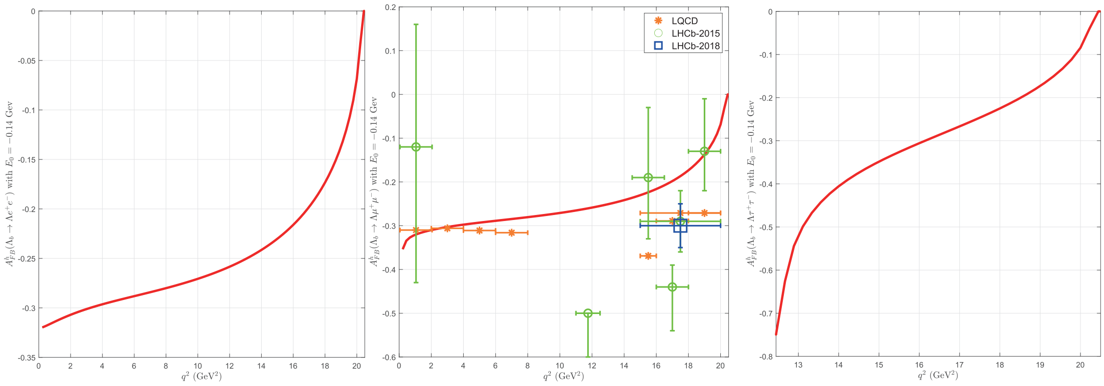

Fig. 5. (color online) Values of $A^h_{\rm FB}(\Lambda_b\to\Lambda l^+l^-)$ as a function of q^2 for different values of κ as shown in Table 1.

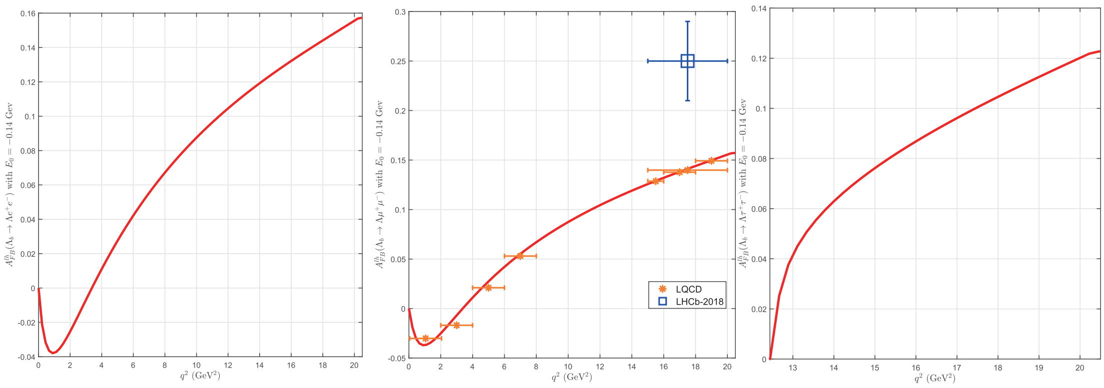

Fig. 6. (color online) Values of $A^h_{\rm FB}(\Lambda_b\to\Lambda l^+l^-)$ as a function of q^2 for different values of κ as shown in Table 1.

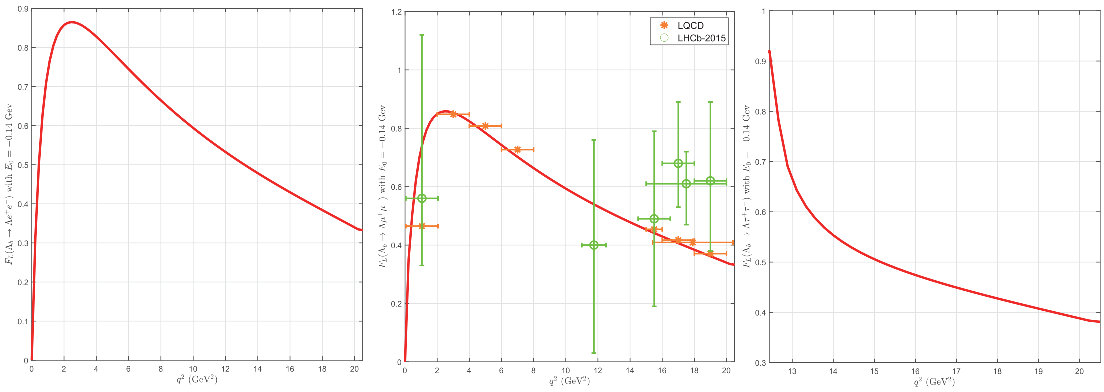

Fig. 7. (color online) Values of $F_L(\Lambda_b\to\Lambda l^+l^-)$ as a function of q^2 for different values of κ as shown in Table 1.

theoretical models because the FFs in these models are different.

IV. SUMMARY AND CONCLUSIONS

In this study, we use the BSE to study the forward-backward asymmetries in the rare decays $\Lambda_b\to\Lambda l^+l^-$ in a covariant quark-diquark model. In this picture, $\Lambda_b(\Lambda)$ is considered a bound state of a $b(s)$-quark and a scalar diquark.

We establish the BSE for the quark and scalar diquark system and then derive the FFs of $\Lambda_b\to\Lambda$. We solve the BS equation of this system and then provide the values of the FFs and R. We observe that the ratio R is not a constant, which is in agreement with Ref. [26] and the pQCD scaling law [23–25]. Using these FFs, we calculate the forward-backward asymmetries $A^l_{\rm FB}$, $A^{lh}_{\rm FB}$, and$A^h_{\rm FB}$ and longitudinal polarization fractions F_L and the integrated forward-backward asymmetries $\bar{A}^l_{\rm FB}$, $\bar{A}^{lh}_{\rm FB}$, and$\bar{A}^h_{\rm FB}$ as well as$\bar{F}_L$ for $\Lambda_b\to\Lambda l^+l^-(l=e,\,\mu,\,\tau)$. Comparing with other theoretical studies, we observe that the FFs are different; thus, these asymmetries are different. The long distance contributions are not included in this paper. They will be considered in our future research to compare the experimental data more exactly.

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