


Decay properties of $D_{s0}^*(2317)^+$ as a conventional $c\bar{s}$ meson*

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Abstract: Taking $D_{s0}^*(2317)^+$ as a conventional $c\bar{s}$ meson, we calculate its dominant strong and electromagnetic decays in the framework of the Bethe-Salpeter method. Our results are $\Gamma(D_{s0}^{*+} \rightarrow D_s^+\pi^0) = 7.83_{-1.55}^{+1.97}$ keV and $\Gamma(D_{s0}^{*+} \rightarrow D_s^{*+}\gamma) = 2.55_{-0.45}^{+0.37}$ keV. The contributions of the different partial waves from the initial and final state wave functions to the decay width are also calculated, and we find that the relativistic corrections in both decay processes are very large.

Keywords: $D_s(2317)$, relativistic, Bethe-Salpeter equation

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I. INTRODUCTION

The particle $D_{s0}^*(2317)^+$ was discovered in the invariant mass distribution of $D_s^+\pi^0$ by the BABAR collaboration in 2003 [1]. According to an analysis of experimental data, it is proposed that its isospin and spin parity quantum number are $I(J^P) = 0(0^+)$, its mass is $M = 2317.8 \pm 0.5$ MeV, and its full width is $\Gamma < 3.8$ MeV [2]. Since its discovery, its small width and low mass [3] have aroused great interest among experimentalists [4–6] and high-energy physics theorists [7–22]. Experts have not fully understood the internal structure of $D_{s0}^*(2317)$; it has been interpreted as a traditional $c\bar{s}$ state [7–12], while others think it might be an exotic meson state, such as a D-K bound state [17–19, 23], $c\bar{s}q\bar{q}$ tetraquark state [13–15], or $D_s\pi$ quasibound state [16], or a mixture of a $c\bar{s}$ meson and a $c\bar{s}q\bar{q}$ tetraquark [21, 22], $c\bar{s}$ meson combined with quark-antiquark and meson-meson interpolators [24], $c\bar{s}$ meson with the effect of the nearby DK threshold taken into account by employing the corresponding four-quark operators [25], $c\bar{s}$ meson using interpolating fields of different structures [26], etc. A more detailed description of exotic hadrons can be found in Ref. [27].

In order to determine its internal structure, further study of its production and decay is needed, especially its

decay, which is important for revealing its internal composition, because different internal structures will lead to different decay behaviors and branching ratios. If it is a traditional meson, the $D_{s0}^*(2317)$ mass is lower than the threshold of $D_s + \eta$ or $D_{u,d} + K$; thus, these possible Okubo-Zweig-Iizuka (OZI)-allowed decay channels are kinematically forbidden. Cho and Wise proposed the $\eta - \pi^0$ mixing mechanism and the origin of isospin violating effects from the mass splitting of u and d quarks [28]. In this case, $D_{s0}^*(2317)^+$ is converted first into $D_s^+\eta$ and then into $D_s^+\pi^0$ by the mixing, *i.e.*, $D_{s0}^*(2317)^+ \rightarrow D_s^+\eta \rightarrow D_s^+\pi^0$. Therefore, the strong decay channel $D_{s0}^*(2317)^+ \rightarrow D_s^+\pi^0$ is the dominant decay channel of this particle. This conclusion is supported by experimental data [2]. $D_{s0}^*(2317)$ has another permissible decay channel, *i.e.*, the electromagnetic (EM) decay channel $D_{s0}^*(2317) \rightarrow D_s^*\gamma$. Normally, EM decay is far narrower than strong decay, but because it is not affected by isospin violation, the EM decay width may be less different from the strong decay width.

There have been discussions of its strong and EM decays using different models [3, 7–10, 13, 29]. Among them, Godfrey found a large branching ratio to $D_s^*\gamma$ for $D_{s0}^*(2317)$; Bardeen *et al.* [8] studied $D_s + \pi$, $D_s + 2\pi$, and EM transitions from the full chiral theory; Colangelo and

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Fazio used the method based on heavy quark symmetries and the vector meson dominance ansatz to study the strong and EM decays; Zhu *et al.* used the $3P_0$ model [12] and the light-cone QCD sum rule (LCQSR) [30] to study the strong decays of $D_{s0}^*(2317)$; Liu *et al.* studied the strong pionic and radiative decays in the Constituent Quark Meson model [31]; Wang calculated the corresponding strong coupling constant in the framework of the LCQSR [32] and the radiative decay with the assumption of vector meson dominance; Guo *et al.* studied the strong decay by constructing an effective chiral Lagrangian under the assumption of a hadronic molecule [33]; and Liu *et al.* [34] studied the strong decay under the assumption of a DK molecule in lattice QCD.

In these studies, there is a lack of the relativistic method. Therefore, in this paper, we assume $D_{s0}^*(2317)$ as the $c\bar{s}$ traditional meson and study its strong and EM decays under the Bethe-Salpeter (BS) framework. The BS equation is a formally exact equation to describe the relativistic bound state [35, 36]. Using the BS method, we have studied the strong decays [37, 38] and electromagnetic decays [39, 40] of some particles, and the results are in good agreement with the experimental data. In this paper, we will show the advantages of the relativistic BS method, which can help reveal why the decay width of $D_{s0}^*(2317)$ is so small, especially the radiative electromagnetic decay width.

The remainder of this paper is organized as follows. We show the hadronic matrix elements and the formula for the two-body strong decay width of $D_{s0}^*(2317) \rightarrow D_s\pi^0$ through $\eta-\pi^0$ mixing in Section II. EM decay is presented in Section III. We show and compare our results with experimental data and other theoretical approaches in literature and summarize our results in Section IV.

II. TWO-BODY STRONG DECAY OF $D_{s0}^*(2317)$

In this section, we give the formula for calculating the hadronic transition matrix element and the strong decay, as well as the relativistic wave functions used.

A. Hadronic transition matrix element

When both quarks in the initial state appear in the final state meson, the strong decay of $D_{s0}^*(2317)^+$ can be expressed by a Feynman diagram, as shown in Fig. 1. The transition matrix element of the two-body strong decay $D_{s0}^*(2317)^+ \rightarrow D_s^+\eta$ can be expressed as (we compute the $D_{s0}^*(2317)^+ \rightarrow D_s^+\eta$ process first, and then we import $\eta-\pi^0$ mixing for $D_s^+\eta \rightarrow D_s^+\pi^0$) [41]

$$\begin{aligned} & \langle D_s^+(P_f)\eta(P')|D_{s0}^{*+}(P) \rangle \\ &= \int d^4x e^{iP'x} (M_\eta^2 - P'^2) \langle D_s^+(P_f)|\Phi_\eta(x)|D_{s0}^{*+}(P) \rangle, \end{aligned} \quad (1)$$

where, P , P_f , and P' represent the momenta of $D_{s0}^*(2317)^+$, D_s^+ , and η , respectively. $\Phi_\eta(x)$ represents the field of η . The light meson field is expressed as the derivative of the axial-vector flow divided by the decay constant f_P of the light pseudoscalar meson according to the PCAC relation:

$$\Phi_\eta(x) = \frac{1}{M_\eta^2 f_P} \partial^\mu (\bar{s}\gamma_\mu\gamma_5 s). \quad (2)$$

Figure 1 can be approximately converted into Fig. 2 by using Eq. (1).

By combining Eqs. (1) and (2), we obtain:

$$\begin{aligned} & \langle D_s^+(P_f)\eta(P')|D_{s0}^{*+}(P) \rangle \\ &= \frac{M_\eta^2 - P'^2}{M_\eta^2 f_P} \int d^4x e^{iP'x} \langle D_s^+(P_f)|\partial^\mu (\bar{s}\gamma_\mu\gamma_5 s)|D_{s0}^{*+}(P) \rangle. \end{aligned} \quad (3)$$

By using partial integration and applying the low-energy theorem, we can obtain the form of the transition matrix element in the momentum space. It can be written as

$$\begin{aligned} & \langle D_s^+(P_f)\eta(P')|D_{s0}^{*+}(P) \rangle \\ &= \frac{M_\eta^2 - P'^2}{M_\eta^2 f_P} \int d^4x e^{iP'x} \langle D_s^+(P_f)|\partial^\mu (\bar{s}\gamma_\mu\gamma_5 s)|D_{s0}^{*+}(P) \rangle \\ &= \frac{-iP'^\mu (M_\eta^2 - P'^2)}{M_\eta^2 f_P} \int d^4x e^{iP'x} \langle D_s^+(P_f)|\bar{s}\gamma_\mu\gamma_5 s|D_{s0}^{*+}(P) \rangle \\ &\approx (2\pi)^4 \delta^4(P - P_f - P') \frac{-iP'^\mu}{f_P} \langle D_s^+(P_f)|\bar{s}\gamma_\mu\gamma_5 s|D_{s0}^{*+}(P) \rangle. \end{aligned} \quad (4)$$

Then, the transition amplitude for the process $D_{s0}^*(2317)^+ \rightarrow D_s^+ + \eta$ is

$$T = \frac{-iP'^\mu}{f_P} \langle D_s^+(P_f)|\bar{s}\gamma_\mu\gamma_5 s|D_{s0}^{*+}(P) \rangle. \quad (5)$$

In the Mandelstam form [42], the transition amp-

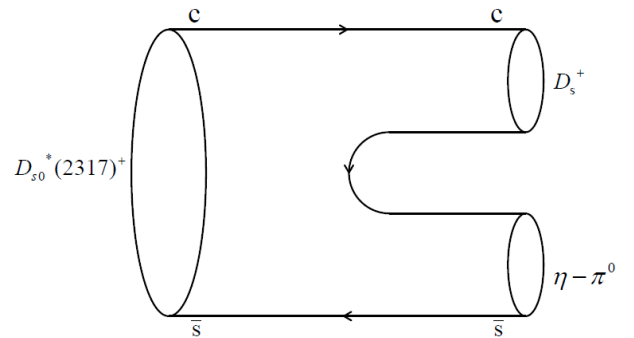


Fig. 1. Original strong decay diagram for $D_{s0}^*(2317)^+ \rightarrow D_s^+\pi^0$.

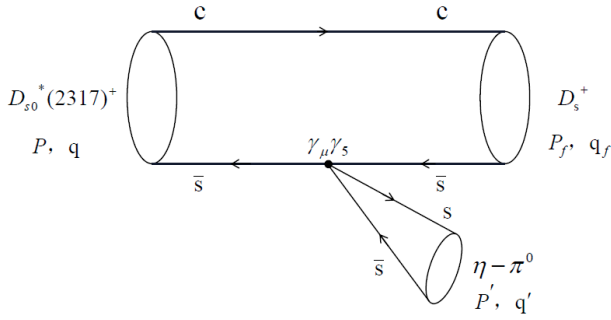


Fig. 2. Strong decay diagram for $D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0$ after approximation.

litude can be written as an overlapping integral of the initial and final meson wave functions [43]:

$$\begin{aligned} \mathcal{M} &= \langle D_s^+(P_f) | \bar{s} \gamma_\mu \gamma_5 s | D_{s0}^{*+}(P) \rangle \\ &= \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 q_f}{(2\pi)^4} \text{Tr} [\bar{\chi}_{P_f}(q_f) S_1^{-1} \chi_P(q) \gamma_\mu \gamma_5] \\ &\quad \times (2\pi)^4 \delta^4(p_1 - p_{1f}) \\ &\approx \int \frac{d^3 q_\perp}{2\pi^3} \text{Tr} \left[\bar{\varphi}_{P_f}^{++}(q_\perp - \alpha_1 P_{f\perp}) \frac{\not{P}}{M} \varphi_P^{++}(q_\perp) \gamma_\mu \gamma_5 \right], \quad (6) \end{aligned}$$

where $\chi_r(q)$ and $\chi_{P_f}(q_f)$ represent the relativistic BS wave functions of $D_{s0}^*(2317)^+$ and D_s^+ , respectively. q and q_f represent the internal relative momenta of the initial and final mesons, respectively. $p_1, p_2, p_{1f},$ and p_{2f} represent the momenta of the quarks and anti-quarks of the initial and final mesons, respectively. S_1 represents the propagator of the quarks. M represents the mass of $D_{s0}^*(2317)^+$, and $\alpha_1 = \frac{m_c}{m_c + m_s}$ with the quark mass m_c and the anti-quark mass m_s ; $\varphi_{P_f}^{++}$ and φ_P^{++} represent the positive-energy wave functions of D_s^+ and $D_{s0}^*(2317)^+$, respectively. $P_{f\perp}$ is defined as $P_{f\perp}^\mu = P_f^\mu - \left(\frac{P \cdot P_f}{M^2} \right) P^\mu$ (similar definition to q_\perp), and $\bar{\varphi}_{P_f}^{++}$ is defined as $\gamma_0 (\varphi_{P_f}^{++})^\dagger \gamma_0$.

B. Relativistic wave functions

In our model, we use the modified BS method based on the constituent quark model to give the wave functions for J^P or J^{PC} states. Here, we will directly give the corresponding positive energy wave functions of initial and final states.

1. 0^+ state D_{s0}^{*+}

The relativistic positive energy Salpeter wave function for the $J^P = 0^+$ state $D_{s0}^*(2317)^+$ (3P_0 state), can be written as

$$\varphi_{0^+}^{++}(P, q_\perp) = A_1 \not{q}_\perp + A_2 \frac{\not{P} \not{q}_\perp}{M} + A_3 + A_4 \frac{\not{P}}{M}, \quad (7)$$

where the A_i ($i = 1, 2, 3, 4$) are related to the original radial wave functions f_1 and f_2 of the 0^+ wave function [44], quark mass m_i , quark energy ω_i ($i = c, s$), and meson mass M :

$$A_1 = \frac{1}{2} \left(f_1 + f_2 \frac{m_c + m_s}{\omega_c + \omega_s} \right), \quad A_2 = \frac{\omega_c + \omega_s}{m_c + m_s} A_1,$$

$$A_3 = q_\perp^2 \frac{\omega_c + \omega_s}{m_c \omega_s + m_s \omega_c} A_1, \quad A_4 = \frac{m_s \omega_c - m_c \omega_s}{m_c + m_s} A_1.$$

We should note that the wave function of a meson is not a pure wave; it actually includes different partial waves. For example, in the positive energy wave function, the A_1 and A_2 terms are P waves, which provide the main non-relativistic contribution, while the A_3 and A_4 terms are S waves, which give the main relativistic corrections [45].

2. 0^- state D_s^+

The relativistic positive energy Salpeter wave function for D_s^+ (1S_0 state) with $J^P = 0^-$ can be written as

$$\varphi_{0^-}^{++}(P, q_\perp) = \left[B_1 \frac{\not{P}}{M} + B_2 + B_3 \not{q}_\perp + B_4 \frac{\not{q}_\perp \not{P}}{M} \right] \gamma_5, \quad (8)$$

where the parameters B_i are functions of the radial wave functions g_1 and g_2 for the 0^- state [44]:

$$B_1 = \frac{M}{2} \left(g_1 + g_2 \frac{m_c + m_s}{\omega_c + \omega_s} \right), \quad B_2 = \frac{\omega_c + \omega_s}{m_c + m_s} B_1,$$

$$B_3 = -\frac{m_c - m_s}{m_c \omega_s + m_s \omega_c} B_1, \quad B_4 = \frac{\omega_c + \omega_s}{m_c \omega_s + m_s \omega_c} B_1.$$

We point out that the B_1 and B_2 terms are S waves, which are the main partial waves of the meson, and the B_3 and B_4 terms are P waves, which provide the main relativistic corrections [45].

C. Decay width of strong decay

The two-body decay width formula can be expressed (import $\eta - \pi^0$ mixing) as

$$\Gamma = \frac{1}{8\pi} \frac{|\vec{P}_f|}{M^2} \frac{1}{2J+1} \sum_\lambda \left| \frac{T t_{\pi\eta}}{m_\pi^2 - m_\eta^2} \right|^2, \quad (9)$$

where $|\vec{P}_f| = \sqrt{[M^2 - (M_f - m_\pi)^2][M^2 - (M_f + m_\pi)^2]} / (2M)$, which is the three-momenta of the final meson D_s^+ . $M, M_f,$ and m_π represent the masses of the initial state and

two final states, respectively. J represents the spin quantum number of the initial meson, which is 0 in this case. $t_{\pi\eta}$ is the mixing matrix entry of $\eta - \pi^0$, and $t_{\pi\eta} = \langle \pi^0 | \mathcal{H} | \eta \rangle = -0.003 \text{ GeV}^2$ [46] is chosen.

III. EM DECAY OF $D_{s0}^*(2317)^+$

Similarly, we give the Feynman diagram of the EM decay of $D_{s0}^*(2317)^+$, corresponding to the two subplots of Fig. 3:

A. EM transition matrix element

The transition matrix element of the EM decay $D_{s0}^*(2317)^+ \rightarrow D_s^* \gamma$ can be written as

$$\langle D_s^*(P_f, \epsilon) \gamma(k, \epsilon_0) | D_{s0}^{*+} \rangle = (2\pi)^4 \delta^4(P - P_f - k) \epsilon_{0\mu} \mathcal{M}^\mu, \quad (10)$$

where ϵ_0 and ϵ represent the polarization vectors of the photon and final meson, respectively. P , P_f , and k represent the momenta of the initial meson, final meson, and photon, respectively.

M represents the invariant amplitude, which is expressed as

$$\begin{aligned} \mathcal{M}^\mu = & \int \frac{d^4 q}{(2\pi)^4} \frac{d^4 q_f}{(2\pi)^4} \text{Tr} \left[\bar{\chi}_{P_f}(q_f) Q_1 \gamma^\mu \chi_P(q) (2\pi)^4 \right. \\ & \times \delta^4(p_2 - p_2') S_2^{-1}(p_2) \\ & \left. + \bar{\chi}_{P_f}(q_f) (2\pi)^4 \delta^4(p_1 - p_1') S_1^{-1}(p_1) \chi_P(q) Q_2 \gamma^\mu \right]. \end{aligned} \quad (11)$$

Here, Q_1 and Q_2 represent the electric charges (in unit of e) of quarks and anti-quarks, respectively. We use the instantaneous approximation to obtain the following form of the amplitude [43]:

$$\begin{aligned} \mathcal{M}^\mu = & \int \frac{d^3 q}{(2\pi)^3} \text{Tr} \left[Q_1 \bar{\varphi}_{P_f}^{++}(q_\perp + \alpha_2 P_{f\perp}) \gamma^\mu \varphi_P^{++}(q_\perp) \frac{\not{P}}{M} \right. \\ & \left. + Q_2 \bar{\varphi}_{P_f}^{++}(q_\perp - \alpha_1 P_{f\perp}) \frac{\not{P}}{M} \varphi_P^{++} \gamma^\mu \right], \end{aligned} \quad (12)$$

where M represents the mass of $D_{s0}^*(2317)^+$, and $q_{f\perp} = q_\perp + \alpha_2 P_{f\perp}$ and $q_{f\perp} = q_\perp - \alpha_1 P_{f\perp}$ correspond to two different processes. $\alpha_1 = \frac{m_c}{m_c + m_s}$ and $\alpha_2 = \frac{m_s}{m_c + m_s}$ are substituted into the above formula with the quark mass m_c and the anti-quark mass $m_{\bar{s}}$, respectively.

B. Relativistic wave function for 1^- state D_s^*

The relativistic positive energy Salpeter wave function for D_s^* (3S_1 state) can be written as

$$\begin{aligned} \varphi_{1^-}^{++}(P, q_\perp) = & C_1 \not{\epsilon} + C_2 \not{\not{P}} + C_3 (q_\perp \not{\epsilon} - q_\perp \cdot \epsilon) \\ & + C_4 (\not{P} \not{q}_\perp - \not{P} q_\perp \cdot \epsilon) \\ & + q_\perp \cdot \epsilon (C_5 + C_6 \not{P} + C_7 \not{q}_\perp + C_8 \not{q}_\perp \not{P}), \end{aligned} \quad (13)$$

where we define the parameters C_i using the radial wave functions f_i ($i = 3, 4, 5, 6$) for a 1^- state [44]:

$$\begin{aligned} C_1 = & \frac{M}{2} \left(f_5 - f_6 \frac{\omega_c + \omega_s}{m_c + m_s} \right), \\ C_2 = & -\frac{1}{2} \frac{m_c + m_s}{\omega_c + \omega_s} \left(f_5 - f_6 \frac{\omega_c + \omega_s}{m_c + m_s} \right), \\ C_3 = & \frac{M}{2} \frac{\omega_s - \omega_c}{m_c \omega_s + m_s \omega_c} \left(f_5 - f_6 \frac{\omega_c + \omega_s}{m_c + m_s} \right), \\ C_4 = & \frac{1}{2} \frac{\omega_c + \omega_s}{\omega_c \omega_s + m_c m_s - q_\perp^2} \left(f_5 - f_6 \frac{\omega_c + \omega_s}{m_c + m_s} \right), \\ C_5 = & \frac{1}{2M} \frac{m_c + m_s}{\omega_c \omega_s + m_c m_s + q_\perp^2} \left[M^2 \left(f_5 - f_6 \frac{m_c + m_s}{\omega_c + \omega_s} \right) \right. \\ & \left. + q_\perp^2 \left(f_3 + f_4 \frac{m_c + m_s}{\omega_c + \omega_s} \right) \right], \\ C_6 = & \frac{1}{2M^2} \frac{\omega_c + \omega_s}{\omega_c \omega_s + m_c m_s + q_\perp^2} \left[M^2 \left(f_5 - f_6 \frac{m_c + m_s}{\omega_c + \omega_s} \right) \right. \\ & \left. + q_\perp^2 \left(f_3 + f_4 \frac{m_c + m_s}{\omega_c + \omega_s} \right) \right], \\ C_7 = & \frac{1}{2M} \left(f_3 + f_4 \frac{m_c + m_s}{\omega_c + \omega_s} \right) - \frac{f_6 M}{m_c \omega_s + m_s \omega_c}, \\ C_8 = & \frac{1}{2M^2} \frac{\omega_c + \omega_s}{m_c + m_s} \left(f_3 + f_4 \frac{m_c + m_s}{\omega_c + \omega_s} \right) \\ & - f_5 \frac{\omega_c + \omega_s}{(m_c + m_s)(\omega_c \omega_s + m_c m_s - q_\perp^2)}. \end{aligned}$$

The positive energy wave function $\varphi_{1^-}^{++}(P, q_\perp)$ for D_s^* includes three partial waves: the C_1 and C_2 terms are S waves; the C_3 , C_4 , C_5 , and C_6 terms are P waves; and the C_7 and C_8 terms are D waves [45].

IV. RESULTS AND DISCUSSION

In this paper, the following masses of constituent quarks are adopted: $m_c = 1.62 \text{ GeV}$ and $m_s = 0.50 \text{ GeV}$. We take the meson mass to have the following values: $M_{D_{s0}^+} = 2.317 \text{ GeV}$, $M_{D_s^*} = 1.968 \text{ GeV}$, $M_{D_s} = 2.112 \text{ GeV}$, $M_\eta = 0.548 \text{ GeV}$, and $M_{\pi^0} = 0.135 \text{ GeV}$ [2]. The decay constant is $f_\pi = 0.130 \text{ GeV}$.

A. Numerical results for decay widths

1. Strong decay

Our relativistic prediction result for the strong decay of $D_{s0}^*(2317)^+$ is

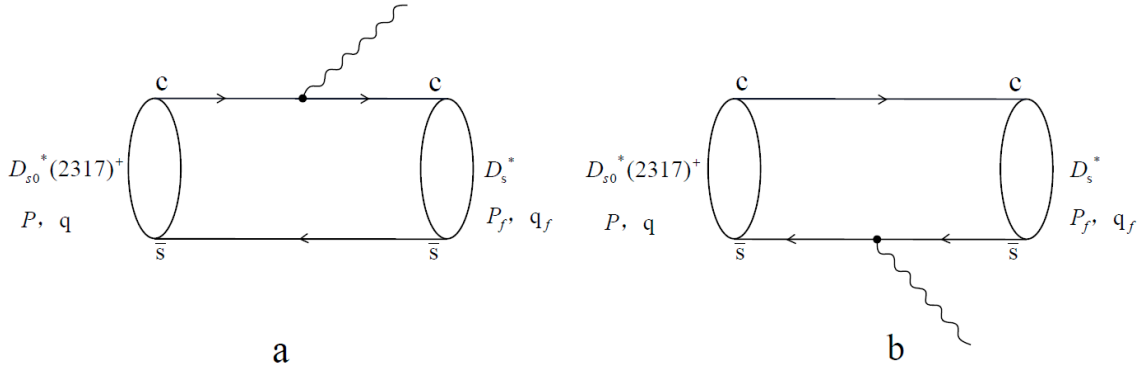


Fig. 3. Feynman diagram responsible for the EM decay $D_{s0}^*(2317)^+ \rightarrow D_s^{*+}\gamma$.

$$\Gamma(D_{s0}^{*+} \rightarrow D_s^{*+}\pi^0) = 7.83_{-1.55}^{+1.97} \text{ keV}, \quad (14)$$

where the theoretical uncertainties are also shown. These uncertainties are calculated by varying all the input parameters simultaneously within $\pm 5\%$ of the central value, and the largest variation is taken.

For comparison, we list our prediction and those of other groups in Table 1. As shown, our prediction $\Gamma(D_s^{*+}\pi^0) = 7.83_{-1.55}^{+1.97} \text{ keV}$ is consistent with the theoretical results in Refs. [8, 9, 31, 47]. Among them, Ref. [9] is based on the heavy quark symmetries and vector meson dominance ansatz; Ref. [31] is based on the framework of the Constituent-Quark-Meson (CQM) model; and Ref. [8] is based on full chiral theory. We also refer to Ref. [47], in which the QCD sum rules were used to analyze the strong decay process under the hypothesis of the four-quark state. In Table 1, the last two predictions of Refs. [33, 34], which assume that $D_{s0}^*(2317)^+$ is a DK hadronic molecule, differ significantly from our prediction and others, indicating that the strong decay may be crucial to test the nature of $D_{s0}^*(2317)^+$.

2. EM decay

Our relativistic prediction of the EM decay of $D_{s0}^*(2317)^+$ is

$$\Gamma(D_{s0}^{*+} \rightarrow D_s^{*+}\gamma) = 2.55_{-0.45}^{+0.37} \text{ keV}. \quad (15)$$

Our result and those from other theoretical groups are presented in Table 2 for comparison. Our result is slightly larger than those from Refs. [7, 9, 31] (within a reasonable margin of error); consistent with the prediction in Ref. [32], which was calculated in the framework of the light-cone QCD sum rules; and slightly smaller than the prediction in Ref. [48], which assumes that $D_{s0}^*(2317)^+$ is a DK molecule.

B. Contributions of different partial waves

As mentioned previously, the complete relativistic wave function for a J^P state is not a pure wave. It includes different partial waves [45]: both the non-relativistic main part and the relativistic correction terms are included. We calculate the contributions (to the decay width) of different partial waves in the initial and final states. The results for strong decay are presented in Table 3, and those for EM decay are presented in Table 4. In the tables, "complete" means that the complete wave function is used, "S wave" means that only the S partial wave makes a contribution and other partial waves are ignored.

1. Strong decay

From Table 3, we can see that the main contribution of the 0^+ state D_{s0}^{*+} comes from its P wave, which is the non-relativistic part, and the relativistic correction (S wave) makes a small contribution. Meanwhile, for the 0^- state D_s^+ , the main contribution comes from its S partial wave, which is the non-relativistic part, and the re-

Table 1. Strong decay widths (in units of keV) of $D_{s0}^*(2317)^+$.

Decay channel	Our work	[9]	[31]	[7]	[8]	[47]	[33]	[34]
$\Gamma(D_{s0}^{*+} \rightarrow D_s^{*+}\pi^0)$	$7.83_{-1.55}^{+1.97}$	6	3.68–8.71	~10	7.74	6 ± 2	180 ± 110	133 ± 22

Table 2. EM decay widths (in unit of keV) of $D_{s0}^*(2317)^+$.

Decay channel	Our work	[9]	[31]	[7]	[32]	[48]
$\Gamma(D_{s0}^{*+} \rightarrow D_s^{*+}\gamma)$	$2.55_{-0.45}^{+0.37}$	1	~1.1	1.9	1.3–9.9	3.7 ± 0.3

Table 3. Strong decay widths (keV) of different partial waves for $D_{s0}^{*+} \rightarrow D_s^+ \pi^0 (0^+ \rightarrow 0^-)$.

0^+	0^-		
	<i>complete</i>	<i>S wave</i> (B_1, B_2)	<i>P wave</i> (B_3, B_4)
<i>complete</i>	$7.83^{+1.97}_{-1.55}$	$16.9^{+2.52}_{-2.11}$	$1.74^{+0.08}_{-0.09}$
<i>P wave</i> (A_1, A_2)	$11.3^{+2.43}_{-1.95}$	$21.8^{+2.94}_{-2.50}$	$1.70^{+0.08}_{-0.10}$
<i>S wave</i> (A_3, A_4)	0.32 ± 0.01	0.30 ± 0.01	$2.27^{+0.04}_{-0.03} \times 10^{-4}$

Table 4. EM decay widths (keV) of different partial waves for $D_{s0}^{*+} \rightarrow D_s^* \gamma (0^+ \rightarrow 1^-)$.

0^+	1^-			
	<i>complete</i>	<i>S wave</i> (C_1, C_2)	<i>P wave</i> (C_3, C_4, C_5, C_6)	<i>D wave</i> (C_7, C_8)
<i>complete</i>	$2.55^{+0.37}_{-0.45}$	0.16 ± 0.05	$1.39^{+0.13}_{-0.15}$	$1.99^{+0.25}_{-0.26} \times 10^{-4}$
<i>P wave</i> (A_1, A_2)	$4.26^{+0.52}_{-0.61}$	$0.83^{+0.15}_{-0.17}$	$1.35^{+0.10}_{-1.13}$	$7.2^{+2.46}_{-2.29} \times 10^{-5}$
<i>S wave</i> (A_3, A_4)	0.21 ± 0.01	0.26 ± 0.02	$3.49^{+4.84}_{-2.95} \times 10^{-4}$	$5.11^{+1.66}_{-0.36} \times 10^{-4}$

lativistic correction part (*P* partial wave) makes a very small contribution.

In the strong decay, If we only keep the non-relativistic wave functions and ignore the other partial waves for both the initial and final states, we obtain the non-relativistic result

$$\Gamma_0(D_{s0}^{*+} \rightarrow D_s^+ \pi^0) = 21.8^{+2.94}_{-2.50} \text{ keV}, \quad (16)$$

which is far larger than the relativistic one of $\Gamma = 7.83^{+1.97}_{-1.55}$ keV, indicating that the relativistic correction in this transition is very large.

2. EM decay

Table 4 presents the contributions of different partial waves in the EM decay $D_{s0}^{*+} \rightarrow D_s^* \gamma$. The main contribution of D_{s0}^{*+} comes from its *P wave*, which is its dominant partial wave. The final state D_s^* is a 1^- vector. It is usually denoted as a 3S_1 state in a non-relativistic method, which means that it is an *S wave* dominated state. We have shown that in addition to the dominant *S wave*, its wave function also includes *P* and *D* partial waves, and they provide the relativistic corrections. From Table 4, we can see that the contribution of its dominant *S* partial wave is suppressed, while the *P wave* gives the main contribution, indicating that the relativistic correction is dominant in this EM decay. We can see this if we take the non-relativistic limit without relativistic correction. In Ta-

ble 4, if only the *P wave* in the initial state and the *S wave* in the final state make contributions, the non-relativistic result is obtained:

$$\Gamma_0(D_{s0}^{*+} \rightarrow D_s^* \gamma) = 0.83^{+0.15}_{-0.17} \text{ keV}, \quad (17)$$

which is far smaller than the relativistic one of $\Gamma = 2.55^{+0.37}_{-0.45}$ keV.

C. SUMMARY

In this paper, taking the particle $D_{s0}^*(2317)^+$ as the conventional $c\bar{s}$ meson, we calculate its main decay processes in the framework of the Bethe-Salpeter method, *i.e.*, the strong decay and electromagnetic decay. Our results are $\Gamma(D_{s0}^{*+} \rightarrow D_s^+ \pi^0) = 7.83^{+1.97}_{-1.55}$ keV and $\Gamma(D_{s0}^{*+} \rightarrow D_s^* \gamma) = 2.55^{+0.37}_{-0.45}$ keV. Because of the low mass, $D_{s0}^*(2317)^+$ has no direct OZI-allowed strong decay channel. Its strong decay channel $D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0$ violates the isospin conservation, and the $\eta - \pi^0$ mixing is needed. Owing to this mixing effect, the strong decay width is relatively small.

In addition, we calculated the contributions of different partial waves. For the decay of $D_{s0}^{*+} \rightarrow D_s^+ \pi^0$, the main contribution comes from the dominant partial waves of initial and final states, while for the $D_{s0}^{*+} \rightarrow D_s^* \gamma$ decay, the main contribution comes from the dominant partial wave in the initial state and the small *P* partial wave in the final state. In both cases, the relativistic corrections are very large.

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