Higgs-like (pseudo)scalars in AdS$_4$, marginal and irrelevant deformations in CFT$_3$, and instantons on S^3

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Abstract: Employing a 4-form ansatz of 11-dimensional supergravity over a non-dynamical $AdS_4\times S^7/Z_k$ background and setting the internal space as an S^1 Hopf fibration on CP^3, we obtain a consistent truncation. The (pseudo)scalars, in the resulting scalar equations in Euclidean AdS$_4$ space, may be considered to arise from (anti)M-branes wrapping around the internal directions in the (Wick-rotated) skew-whiffed M2-brane background (as the resulting theory is for anti-M2-branes), thus realizing the modes after swapping the three fundamental representations $\mathbf{8}_s$, $\mathbf{8}_c$, and $\mathbf{8}_v$ of $SO(8)$. Taking the backreaction on the external and internal spaces, we obtain the massless and massive modes, corresponding to exactly marginal and marginally irrelevant deformations on the boundary CFT$_3$, respectively. Subsequently, we obtain a closed solution for the bulk equation and compute its correction with respect to the background action. Next, considering the Higgs-like (breathing) mode $m^2=18$, having all supersymmetries as well as parity and scale-invariance broken, solving the associated bulk equation with mathematical methods, specifically the Adomian decomposition method, and analyzing the behavior near the boundary of the solutions, we realize the boundary duals in the $SU(4)\times U(1)$-singlet sectors of the ABJM model. Then, introducing the new dual deformation $\Delta_+=3,6$ operators made of bi-fundamental scalars, fermions, and $U(1)$ gauge fields, we obtain the $SO(4)$-invariant solutions as small instantons on a three-sphere with the radius at infinity, which correspond to collapsing bulk bubbles leading to big-crunch singularities.

Keywords: AdS$_4$/CFT$_3$ correspondence, scalar equations in AdS$_4$, instantons, marginal and irrelevant operators, (anti)M-branes

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I. INTRODUCTION

Euclidean solutions with finite actions, known as instantons, as non-perturbative phenomena are crucial in various branches of physics from quantum corrections to classical system behavior to early universe cosmology.[1)] In a series of recent studies - refer to [1−5] - we have presented numerous such solutions in the context of AdS$_4$/CFT$_3$ correspondence, the best model of which being the ABJM [6].

Specifically, the ABJM action describes the world-volume of N intersecting M2-branes on an Z_k orbifold of C^4 (four complex coordinates), with the orbifold acting as $X^A\to \mathrm{e}^{\frac{2\pi \mathrm{i}}{k}}X^A$, where $A=1,2,3,4$. In the 't Hooft large N limit with fixed $\lambda=N/k$, the 11-dimensional (11D) supergravity (SUGRA) over $AdS_4\times S^7/Z_k$ is valid when $N\gg k^5$. Moreover, based on the orbifold, the subgroup $SU(4)\times U(1)\equiv H$ of the original $SO(8)\equiv G$ remains. The 3D boundary theory is a special case of the $U(N)_k\times U(N)_{-k}$ Chern-Simon (CS) gauge theory with $\mathcal{N}=6$ supersymmetry (SUSY) and matter fields (scalars Y^A and fermions ψ_A) in bi-fundamental representations (reps) ($\mathbf{4}_1$ and $\bar{\mathbf{4}}_{-1}$) of H.

Here, by maintaining the ABJM background geometry unchanged and considering a 4-form ansatz of the 11D SUGRA composed of the ABJM internal $CP^3\ltimes S^1/Z_k$ space elements and scalars in the external 4D Euclidean anti-de Sitter ($EAdS_4$) space, associated with probe (anti)M-branes wrapped around mixed directions in the (M2-branes) anti-M2-branes background resulting in the anti-M2-branes theory, we will have a *consistent truncation* such that only H-singlet fields remain in the trun-

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1) Because our universe may now be or in the past have been in a quasi-stable vacuum state, tunneling from that state to a stable one is interesting. In fact, a false vacuum corresponding to the local minimum of the potential of a scalar field is unstable and decays through tunnelling meditated by instantons or bounces.

cated theory and all dependencies on the internal 7D space are omitted in resulting equations.[1)]

However, instantons as topological objects should not backreact on the background geometry. To this end, we solve the truncated equations in $EAdS_4$ along with the equations resulting from zeroing the energy-momentum (EM) tensors of the Einstein's equations that result in equations for massless and massive bulk (pseudo)scalars, which in turn correspond to exact and irrelevant marginal deformation of the dual boundary theory. In addition, we consider a Higgs-like mode ($m^2 = 18$), also known as *breathing mode*; refer to [8, 10−12]. To obtain the solutions for the bulk scalar equations, we employ the usual mathematical methods, especially the Adomian Decomposition Method (ADM) [13], for solving the Nonlinear Partial Differential Equations (NPDEs).

Next, after analyzing the bulk solutions near the boundary and dual symmetries, we propose the corresponding dual operators to deform the boundary action and obtain the solutions. In fact, because the bulk setups and solutions break all SUSYs, $\mathcal{N} = 8 \to 0$, as well as parity- and scale-invariance, to realize the boundary duals, we swap the three fundamental reps $\mathbf{8}_s$, $\mathbf{8}_c$, and $\mathbf{8}_v$ of $SO(8)$. Accordingly, we could realize the H-singlet scalars and pseudoscalars in the mass spectrum of the 11D SUGRA on the background geometry after the branching $G \to H$, corresponding to the H-singlet boundary operators. Meanwhile, because the scale symmetry is violated owing to the mass term in the equations and their extreme nonlinearity, the solutions should preserve the $SO(4)$ symmetry of the original isometry $SO(4,1)$ of $EAdS_4$. Keeping a singlet sector of the boundary ABJM action,[2)] with only one scalar, one fermion, and $U(1)\times U(1)$ part of the gauge group, we obtain such an $SO(4)$-invariant solutions with finite actions, which are always small instantons triggering instabilities on a three-sphere with radius r at infinity (S^3_∞), and describe big crunch singularities in the bulk.

The remainder of this paper is organized as follows. In Sec. II, we present the 11D SUGRA background, including the 4-form ansatz, and equations for (pseudo) scalars in $EAdS_4$. Accordingly, we emphasize that, in the anti-M2-branes background, a (pseudo)scalar becomes Higgs-like, spontaneously breaking the symmetry and making the main equation homogeneous. In subsection II.A, we consider the backreaction; that is, after computing the EM tensors of the Einstein's equations in Appendix A, setting them to zero, and solving the resulting scalar equations with the main one in the bulk, we obtain the solvable PDEs for the massless and massive (pseudo)scalars from taking the backreaction on the external and internal spaces, respectively. Next, in subsection II.B, we present an exact solution for the equations in the previous subsection and compute its corrections with respect to the background action. In Sec. III, we employ the known differential equation solution methods to solve the main Higgs-like NPDE and obtain solutions near the boundary. In particular, in subsection III.A, we use the ADM (details of which are presented in Appendix B) to obtain the appropriate solutions for near the boundary analyses of the Higgs-like mode $m^2 = 18$, up to the third order of the perturbative series expansion. In Sec. IV, we first discuss the dual symmetries due to the bulk setups, equations, and solutions. Next, we briefly present the spectrum of 11D SUGRA over $AdS_4 \times S^7/Z_k$ and verify whether we can find the desired H-singlet scalars and pseudoscalars among various generations after swapping the fundamental reps of $SO(8)$ for gravitino and branching of $G \to H$. Then, we present the basic elements of AdS_4/CFT_3 correspondence for (pseudo)scalars needed for our boundary analyses. In Sec. V, we look for dual solutions in the ABJM-like 3D field theories. Accordingly, in subsections V.A and V.B, we consider marginal and irrelevant deformations with the new H-singlet $\Delta_+ = 3, 6$ operators, corresponding to the massless (when taking the backreaction) and massive bulk states, respectively, and find $SO(4)$-invariant solutions with finite actions as instantons. In addition, based on the boundary solutions, we confirm the state-operator correspondence, match the bulk-boundary parameters, and determine an unknown scalar function in a bulk solution based on the correspondence. Meanwhile, note that with a marginal triple-trace deformation of a dimension-one operator composed of bi-fundamental scalars, we could build the tri-critical $O(N)$ model and obtain the Fubini-like instantons. Furthermore, in subsection V.B, we confirm the Bose-Fermi (BF) duality between a deformation with the latter $\Delta_+ = 3$ operator (in fact the massless Regular Boson (RB) model) and that with a $\Delta_+ = 6$ operator composed of bi-fundamental fermions (in fact the massless Critical Fermion (CF) model) at least at the level of solutions and correspondence. Eventually, in Sec. VI, we present a summary along with comments on the solutions, physical interpretations, connections with other studies, and related issues.

II. FROM 11D SUPERGRAVITY TO 4D GRAVITY EQUATIONS

We start with the 4-form ansatz [3)]

1) It is discussed in [7] that a consistent truncation includes the singlet fields under the internal isometry group, by setting to zero non-singlets ones; see also [8]. Particularly, a consistent truncation of M-theory over $AdS_4 \times S^7$ to the 4D $\mathcal{N} = 8$ $SO(8)$ gauged supergravity is presented in [9] including a special case, where just the graviton and a scalar potential is retained, as is the case here; For a newer look, see [10].

2) As discussed in [4], the boundary solutions might also realize in singlet sectors of 3D $U(N)$ and $O(N)$ CS matter theories.

3) see also [1].

$$G_4 = f_1 G_4^{(0)} + R^4 \mathrm{d} f_2 \wedge J \wedge e_7 + R^4 f_3 J^2 \tag{1}$$

for 11D SUGRA over $AdS_4 \times S^7/Z_k$ when the internal space is considered as a $U(1)$ bundle on CP^3, where $G_4^{(0)} = \mathrm{d}\mathcal{A}_3^{(0)} = N\mathcal{E}_4$ holds for the ABJM [6] background with $N = (3/8)R^3$ units of flux quanta on the internal space, $R = 2R_{\rm AdS}$ is the AdS curvature radius, $\mathcal{E}_4$ is the unit-volume form on AdS_4, J is the Kähler form on CP^3, e_7 is the seventh vielbein[1)] (of the internal space), and f_i's with $i = 1,2,3$ are scalar functions in bulk coordinates.

Taking the ansatz (1) from the Bianchi identity and Euclidean 11D equation

$$\mathrm{d}G_4 = 0, \qquad \mathrm{d} *_{11} G_4 - \frac{\mathrm{i}}{2} G_4 \wedge G_4 = 0, \tag{2}$$

we get

$$f_3 = f_2 \pm \frac{C_2}{R}, \qquad \bar{f}_1 = \mathrm{i}\frac{3}{16} R^5 f_3^2 \pm \mathrm{i}\frac{3}{8} C_3 R^3, \tag{3}$$

where C_i's are real constants and $N f_1 \equiv \bar{f}_1$. Note that the plus and minus signs on the last term of the RHS equation indicate considering the Wick-rotated (WR) and skew-whiffed (SW) backgrounds, respectively; the ABJM background is realized with $C_3 = 1$. In addition, from Eq. (2), using the relationship presented in (3), we get

$$\Box_4 f_3 - \frac{1}{R_{\rm AdS}^2}(1 \pm 3C_3) f_3 - 6 f_3^3 = 0, \tag{4}$$

[2)]where $*_4 \mathrm{d}(*_4 \mathrm{d} f_3) = \Box_4$ is the $EAdS_4$ Laplacian. Accordingly, we use the following conventions

$$*_4\mathbf{1} = \frac{R^4}{16}\mathcal{E}_4, \quad *_7\mathbf{1} = \frac{R^7}{3!}J^3 \wedge e_7 = R^7 \mathcal{E}_7, \quad *_7(J \wedge e_7) = \frac{R}{2}J^2. \tag{5}$$

Next, from (3) and (4), we write [3)]

$$\Box_4 f_2 - m^2 f_2 \mp \delta f_2^2 - \lambda f_2^3 \mp F = 0, \tag{6}$$

[4)] where

$$m^2 = \frac{4}{R^2}\left(1 \pm 3C_3 + \frac{9}{2}C_2^2\right), \quad \delta = \frac{18}{R}C_2,$$
$$\lambda = 6, \quad F = \frac{4}{R^3}\left(C_2 \pm 3C_2 C_3 + \frac{3}{2}C_2^3\right). \tag{7}$$

To make Eq. (6) homogeneous (that is $F = 0$), for cases other than $C_2 = 0$, we have to set

$$\delta^2 = 27 m^2 \quad \texttt{OR} \quad C_2^2 = \frac{\mp 6 C_3 - 2}{3} \tag{8}$$

Hence,

$$m^2 R_{\rm AdS}^2 = -2(1 \pm 3C_3) \equiv -2\bar{m}^2 R_{\rm AdS}^2, \tag{9}$$

where $\bar{m}^2$ is indeed the squared mass of f_3 in (4). To have physically permissible (non-imaginary) masses in this case, we have only to consider the SW version with $C_3 \geq 1/3$. As a result, the SW ABJM background is realized with $C_3 = 1$; then, $m^2 R_{\rm AdS}^2 = +4$ ($C_2 = 2/\sqrt{3}$).

In addition, as noticed in [5], $\pm(C_2/2) = \pm\sqrt{-\bar{m}^2/\lambda}$ are in fact homogenous vacua; thus, the (pseudo)scalar is Higgs-like, and the LHS relation in (3) imposes spontaneous symmetry breaking, where f acts as fluctuation around the homogeneous vacua.

A. Taking backreaction and resulting equations

To take backreaction, we should first compute the EM tensors of the corresponding Einstein's equations, the details of which are presented in Appendix A. In fact, because we are looking for instantons that, as topological objects, should not backreact on the background geometry, we solve the main bulk equations using the equations presented in Appendix A, resulting from setting the EM tensors to zero.

Accordingly, we observe that Eq. (A7) is solved with (6) by taking

$$\Box_4 f_2 = 0, \tag{10}$$

which means taking the backreaction of the external AdS_4 space on the background geometry yields the *massless* $m^2 R_{\rm AdS}^2 = 0$ bulk (pseudo)scalar replying to the boundary *exactly marginal* operators.[5)]

In the same manner, noting that the Eq. (A9) is the same as the main one (6), from solving the Eqs. (A8) as

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1) It is noted that the vielbein is along the fiber direction when we view S^7/Z_k as S^1 fibration over CP^3.

2) Note that the so-called ϕ^4 coupling constant here is $\lambda_4 = 3$ (given that $\lambda = 2\lambda_4$); see for instance [14] also for discussions on the zeroing of the scalar third-order self-interaction.

3) See [4, 5, 15] for similar equations.

4) It should be noted that the plus sign in front of the terms containing C_2 shows that the true vacuum is placed on the right-hand side (RHS) of the false vacuum in the corresponding double-well potential, and vice versa for the minus sign.

5) See [3, 4, 16, 17] for discussions on such a correspondence.

well as (A7) and (A8) with (6), that is taking the backreaction of the internal (indeed CP^3) and entire 11D space, we get

$$\Box_4 f_2 - \frac{2}{R^2} f_2 = \pm \frac{2C_2}{R^3}, \tag{11}$$

$$\Box_4 f_2 - \frac{8}{9R^2} f_2 = \pm \frac{8C_2}{9R^3}, \tag{12}$$

respectively, with $m^2 R^2_{\rm AdS} = 1/2, 2/9$ corresponding to the *marginally irrelevant* $\Delta_\pm = 3/2 \pm \sqrt{11}/2, 3/2 \pm \sqrt{(89/9)}/2$ boundary operators, of which we encountered the former recently in [4].

B. A solution for the case with backreaction

One may solve Eqs. (11) and (12) using the usual mathematical methods, such as separation in variable. However, a well-known closed solution for the equations- leaving out the inhomogeneous terms that do not contribute to the dynamics [1)]- reads [18,19]

$$f_0(u,\vec{u}) = \bar{C}_{\Delta_+}\left(\frac{u}{u^2 + (\vec{u}-\vec{u}_0)^2}\right)^{\Delta_+},$$
$$\bar{C}_{\Delta_+} = \frac{\Gamma(\Delta_+)}{\pi^{3/2}\Gamma(\nu)}. \tag{13}$$

where Δ_+ (Δ_-) is the larger (smaller) root of $m^2 = \Delta(\Delta-3)$ in AdS_4, with $\Delta_\pm = 3/2 \pm \nu$, $\sqrt{9+4m^2} = 2\nu$, and we use the $EAdS_4$ metric

$$ds^2_{\rm EAdS_4} = \frac{R^2}{4u^2}\left(du^2 + dx^2 + dy^2 + dz^2\right), \tag{14}$$

noting $\vec{u} = (x,y,z)$ in upper-half Poincar é coordinates; therefore,

$$\Box_4 f = \frac{4u^2}{R^2}\left(\partial_i\partial_i + \partial_u\partial_u - \frac{2}{u}\partial_u\right) f. \tag{15}$$

Further, because the bulk solutions including the backreaction correspond to variants of marginal operators, for simplicity, we consider the instanton solution for (10) and compute its correction with respect to the background action. To this end, as the background geometry does not change, we use the right parts of the bosonic action of 11D SUGRA in Euclidean space as

$$S^E_{11} = -\frac{1}{4\kappa^2_{11}}\int\left(G_4 \wedge *_{11} G_4 - \frac{\rm i}{3}\mathcal{A}_3 \wedge G_4 \wedge G_4\right), \tag{16}$$

where $\kappa^2_{11} = 9\pi\mathcal{G}_{11} = \frac{1}{4\pi}(2\pi l_p)^9$, and κ_{11}, $\mathcal{G}_{11}$ and l_p are the 11D gravitational constant, Newton's constant, and Plank length, respectively.

Next, from the ansatz expressed in (1), using (5), we write its 11D dual 7-form as

$$G_7 = \frac{8}{3}R^3\bar{f}_1 J^3 \wedge e_7 - \frac{R^5}{2} *_4 df_2 \wedge J^2 + \frac{R^7}{8} f_3 \mathcal{E}_4 \wedge J \wedge e_7; \tag{17}$$

and

$$G_4 = d\mathcal{A}_3, \quad \mathcal{A}_3 = \tilde{\mathcal{A}}^{(0)}_3 + R^4 (f_3 J \wedge e_7),$$
$$\tilde{G}^{(0)}_4 = d\tilde{\mathcal{A}}^{(0)}_3 = \bar{f}_1 \mathcal{E}_4. \tag{18}$$

By placing the latter relations in (16), using (3) (noting that, for $C_2 = 0$, $f_2 = f_3$), we get

$$\tilde{S}^E_{11} = -\frac{R^9}{16\kappa^2_{11}}\int\Big[-\frac{3}{2}C_3^2\mathcal{E}_4 + df_3 \wedge *_4 df_3 + \frac{R^2}{2}f_3^2\mathcal{E}_4$$
$$+\frac{3}{8}R^4 f_3^4 \mathcal{E}_4 + {\rm i}\frac{4}{3R}d\left(f_3^2\mathcal{A}^{(0)}_3\right)\Big] \wedge J^3 \wedge e_7, \tag{19}$$

where the first term on the RHS is the contribution of the ABJM background realized with $C_3 = 1$, as one may see from the second term in the RHS relation expressed in (3); the last (surface) term, as a total derivative, does not contribute to the equations, and hence, we discard it.

Then, to compute the action expressed in (19) based on the solution presented in (13) with $\Delta_+ = 3$, we use

$$\mathcal{E}_4 = -\frac{du}{u^4} \wedge dx \wedge dy \wedge dz,$$
$$\texttt{vol}_7 = \frac{R^7}{3!}\int J^3 \wedge e_7 = \frac{\pi^4 R^7}{3k},$$
$$\kappa^2_{11} = \frac{16}{3}\left(\frac{\pi^{10}R^9}{3k^3}\right)^{1/2}, \tag{20}$$

and the 3D spherical coordinates, setting $|\vec{u}-\vec{u}_0| = r$. As a result, the finite contribution of the action, after integrating on the external space coordinates, in the unit 7D internal volume reads

$$\tilde{S}^{\rm corr.}_{11} = -\hat{c}\sqrt{\frac{k^3}{R}}\frac{1}{\epsilon^6}\left(1 + \check{c}\frac{R^2}{\epsilon^6}\right), \tag{21}$$

where $\hat{c} \simeq 0.000016$ and $\check{c} \simeq 0.0033$; because of singularities, we have included $\epsilon > 0$ as a cutoff parameter to evade the infinity of integrals with respect to (wrt) u; see [20]. Meanwhile, we note that, for finite k and R, the contribution expressed in (21) is small.

1) Note also that to adjust (11) and (12) with the main Eqs. (6) with (7), we have to set $C_2 = 0$.

III. SOLUTIONS FOR THE HIGGS-LIKE SCALAR EQUATION

The Higgs-like (pseudo)scalar equation of (6), with (8) and (15), reads [1)]

$$\left[\partial_i\partial_i+\partial_u\partial_u-\frac{2}{u}\partial_u-\frac{m^2}{u^2}\right]f(u,\vec{u})+\frac{1}{u^2}\left[3\sqrt{3}m\,f(u,\vec{u})^2-6\,f(u,\vec{u})^3\right]=0. \quad (22)$$

For its linear part, using the spherical coordinates with $r=|\vec{u}|$, discarding the angular parts, and separating variables, i.e., $f_0(u,r)=f(r)g(u)$, we can write

$$\left[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-k^2\right]f(r)=0,$$
$$\left[\frac{d^2}{du^2}-\frac{2}{u}\frac{d}{du}-\frac{m^2}{u^2}+k^2\right]g(u)=0; \quad (23)$$

with combinations of Hyperbolic and Bessel (or with $k=i\kappa$, Trigonometric and Modified Bessel) functions as solutions for $f(r)$ and $g(u)$, respectively.[2)]

Then, one may use the leading order (LO) solutions to obtain the higher-order solutions of the full NPDE. The resulting solutions always reproduce the right behavior of (pseudo)scalars near the boundary as

$$f(u\to 0,r)\approx\alpha(r)u^{\Delta_-}+\beta(r)u^{\Delta_+}. \quad (25)$$

In contrast, one may employ an ansatz as follows:

$$f(u,r)=F(\xi),\quad \xi=u^{1/2}\,f(r), \quad (26)$$

which turns (22) into the following NODE

$$\left[\frac{d^2}{d\xi^2}-\frac{5}{\xi}\frac{d}{d\xi}-\frac{4m^2}{\xi^2}\right]F(\xi)-\frac{4}{\xi^2}\mathcal{F}(F(\xi))=0, \quad (27)$$

where we define

$$\mathcal{F}(F(\xi))\equiv -3\sqrt{3}m\,F(\xi)^2+6\,F(\xi)^3. \quad (28)$$

As a result, the appropriate part of a perturbative solution for (27), up to the first or next-to-leading order (NLO), reads [3)]

$$f^{(1)}(u,r)=\sum_{l=-}^{+}C_l\left(u\,f(r)^2\right)^{\Delta_l}, \quad (30)$$

where C_l's are real constants.

Similarly, with $\xi=r/u$ (the so-called self-similar reduction method via the scale-invariance of variables; see, for instance, [22]), Eq. (22) turns into

$$\left[\left(\xi^2+1\right)\frac{d^2}{d\xi^2}+\frac{(2+4\xi^2)}{\xi}\frac{d}{d\xi}-m^2\right]F(\xi)-\mathcal{F}(F(\xi))=0. \quad (31)$$

A solution for the linear part of the latter equation is in terms of Legendre functions, and from this, one may build perturbative series solutions for higher-orders; for such a solution, see [4]. Alternatively, we can use

$$F(\xi)=e^{\int G(\xi)d\xi},\quad \frac{1}{F(\xi)}\frac{dF(\xi)}{d\xi}=G(\xi), \quad (32)$$

which turns Eq. (31) into the following first-order *Riccati equation*:

$$\left(\xi^2+1\right)\left[\frac{dG(\xi)}{d\xi}+G(\xi)^2\right]+\frac{1}{\xi}(2+4\xi^2)G(\xi)-m^2=0. \quad (33)$$

For massive modes, a common series solution for the latter equation, keeping the normalizable term appropriate for the corresponding boundary analyses of AdS_4/CFT_3, reads

$$f_0(u,r)=\tilde{C}_{\Delta_+}\left(\frac{u}{r}\right)^{\Delta_+}, \quad (34)$$

from which one may build higher-order solutions. For example, for the mode $m^2=18$ that we consider, a series expansion around $u=0$, up to NLO, reads

$$f^{(1)}(u,r)=\left[\check{C}_{\Delta_-}\ln\left(\frac{r}{u}\right)\right]\left(\frac{u}{r}\right)^{\Delta_-=-3}+\hat{C}_{\Delta_+}\left(\frac{u}{r}\right)^{\Delta_+=6} \quad (35)$$

with real constants $\check{C}_{\Delta_-}$ and $\hat{C}_{\Delta_+}$ – when doing boundary analyses, we return to this solution as well.

1) From now on, we use $f_2\equiv f$ and the plus sign for the f^2 term in the equations.

2) An interesting solution for the r part is $f(r)\sim e^{-r}/r$, which might be considered as constrained instantons; see for instance [21] and also [4], where we discussed a similar solution in a 3D boson model. Another interesting solution is

$$f(r)=\tilde{C}_1 r^{\ell}+\frac{\tilde{C}_2}{r^{\ell+1}}, \quad (24)$$

where $\tilde{C}_1$ and $\tilde{C}_2$ are constants and $l(l+1)=k^2$.

3) It is recalled that we generally use

$$f^{(n)}=\sum_{n=0}^{n}f_n. \quad (29)$$

A. Solutions of the equation for $m^2 = 18$ with ADM

Here, we employ the ADM formulation, as shown in Appendix 8, to build series solutions appropriate for near the boundary analyses of the specific Higgs-like mode $m^2 = 18$. This mode could be realized with $C_3 = 17/3$ in the WR version of (4) (equally for (6) in addition to $C_2 = 0$) and with $C_3 = 10/3$ and $C_2 = \sqrt{6}$ in the SW version of (6) for $F = 0$. As a result, the series solutions of these equations about $u = 0$, with the initial or near the boundary data from (B1) with $\Delta_+ = 6$ and the Adomian polynomials (B5) with $\delta = 0$ and $\delta = 9\sqrt{6}$, respectively, up to NNNLO, wrt (29), read

$$f^{(3)}(u,r) = -5 f(r)\Big[1 + 20\ln(u) + 400\ln(u)^2 - 8000\ln(u)^3\Big]u^6 + O(u^8), \tag{36}$$

$$f^{(3)}(u,r) = -20 f(r) u^6 + \frac{17965}{43904}\left(\frac{\mathrm{d}^2 f(r)}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}f(r)}{\mathrm{d}r}\right)u^8 + O(u^{10}). \tag{37}$$

Meanwhile, from near the boundary behavior of the closed solution of (13),

$$f_0(u \to 0, r) \approx \bar{C}_{\Delta_+}\left(\frac{u}{r^2}\right)^{\Delta_+}, \tag{38}$$

we consider $f(r) = \bar{C}_6/r^{12}$ to rewrite the series solutions clearly.

Moreover, we can use (B8) with (B9) and near the boundary behavior of the closed solution of (B7), yielding

$$g_0(u \to 0, r) = \frac{2}{\sqrt{3}}\frac{b_0}{(a_0^2 - b_0^2 + r^2)}\left[1 - \frac{2a_0}{(a_0^2 - b_0^2 + r^2)}u\right], \tag{39}$$

as the initial data, which might also be obtained from the LHS relation in (B1), in the ADM, to obtain approximate solutions. As a result, we arrive at a series solution about $u = 0$, up to the first iteration of ADM or NLO of the expansion, as

$$f^{(1)}(u,r) = \sum_{\Delta_+=1}^{6}\frac{\mathcal{H}_{\Delta_+}(r, a_0, b_0, m)\,u^{\Delta_+}}{(a_0^2 - b_0^2 + r^2)^{\Delta_+}}, \tag{40}$$

where $\mathcal{H}_{\Delta_+}(r, a_0, b_0, m)$ is a polynomial of its arguments; in particular, for the term corresponding to the bulk mode $m^2 = 18$, it becomes

$$\mathcal{H}_6 = -\frac{64}{\sqrt{3}}a_0^3 b_0^3. \tag{41}$$

IV. DUAL SYMMETRIES, MASS SPECTRUM AND CORRESPONDENCE

First, we remind that the truncation here is consistent, considering that our ansatz (1) is H-singlet, given that e_7, J and the (pseudo)scalars in resulting equations respect the same symmetry. Second, the setups here are as if we add ℓ probe (anti)M-branes to the (WR)SW M2-branes background and so, the resultant theory is for anti-M2-branes with the quiver gauge group of $SU(N+\ell)_k \times SU(N)_{-k}$. Indeed the (anti)M-branes wrap around mixed internal and external directions, breaking all SUSY's and parity.[1)] and that to realize the latter we focus on $U(1) \subset U(\ell)$ part of the gauge group (in the large k limit) and keep G as a spectator- a so-called novel Higgs mechanism; see for instance [25].[2)] Third, the bulk settings break the inversion (and so, the special conformal transformation K_μ) symmetry and scale-invariance (denoted by the dilation operator D) because of the mass and non-linear terms in the bulk action and translational-invariance (denoted by the translation operator P_μ) due to the non-constant solutions.[3)] As a result, the conformal symmetry $SO(4,1)$ (as the isometry of $EAdS_4$) breaks into $SO(4)$, which in turn includes six generators consisting of three Lorentz transformations (denoted by the operator $L_{\mu\nu}$) and $R_\mu \approx \left(K_\mu + a^2 P_\mu\right)$[4)] corresponding to rotations on S^3, where a is the scale parameter. The four generators of the broken symmetries (translations and scale transformations)- and therefore the four free parameters a (or b_0) and $\vec{u}_0$- move $SO(4)$- symmetric ($SO(3,1)$ in Lorentzian signature) bubble around the 4D bulk.

In contrast, the mass spectrum of 11D SUGRA over $AdS_4 \times S^7/Z_k$[5)] includes three generations of scalars $(0_1^+, 0_2^+, 0_3^+)$ and two generations of pseudoscalars $(0_1^-, 0_2^-)$.

1) According to [23], when all components of the 11D 4-form are in the internal space, such a thing happens as well; and according to [24], with such G_4, the resulting solutions are unstable.

2) The same result could be inferred from the idea of the fractional (anti)M2-branes as probe (anti)M5-branes wrapped around S^3/Z_k (S^1 fibration over CP^1); see [26].

3) Meantime, although because of the breaking of scale-invariance, the boundary operators obtain anomalous dimensions (due to corrections to the bulk tree-level diagrams and presence of interactions), here we consider their bare dimensions in quenched approximation.

4) It is noticeable that with the bulk solution (B7), $S_\mu \approx \left(K_\mu - b_0^2 P_\mu\right)$ is used instead.

5) For original works on the spectrum, see for example [27−31] among many others and [32] as a comprehensive review until then with references therein and also [33, 34] as well as [35−37] for newer looks.

In fact, the massless multiplet ($n=0$) includes a graviton ($\mathbf{1}$), a gravitino ($\mathbf{8}_s$), 28 spin-1 fields ($\mathbf{28}$), 56 spin-$\frac{1}{2}$ fields ($\mathbf{56}_s$), 35 scalars ($\mathbf{35}_v$) of 0_1^+ arising from the external components ($\mathcal{A}_{\mu\nu\rho}$), and 35 pseudoscalars ($\mathbf{35}_c$) of 0_1^- arising from the internal components ($\mathcal{A}_{mnp}$), without any H-singlet under the branching $G\to H$ for scalars ($\mathbf{35}_v\to\bar{\mathbf{10}}_{-2}\oplus\mathbf{10}_2\oplus\mathbf{15}_0$) and pseudoscalars ($\mathbf{35}_c\to\mathbf{10}_{-2}\oplus\bar{\mathbf{10}}_2\oplus\mathbf{15}_0$).[1] In massive or higher KK multiplets ($n>0$), the massless ($m^2=0$) pseudoscalar and scalar set in $\mathbf{840}'_s$ of 0_1^- with $n=2$ and $\mathbf{1386}_v$ of 0_1^+ with $n=4$ of G, again without any H-singlet under the branching. In the same manner, the massive ($m^2=18$) pseudoscalar sets in $\mathbf{840}'_c$ of 0_2^- with $n=4$ and $\mathbf{75075}_{vc}$ of 0_1^- with $n=6$ of G, while as scalar, it sets in $\mathbf{30940}_v$ of 0_1^+ with $n=10$ and $\mathbf{23400}_v$ of 0_3^+ with $n=6$, as well as $\mathbf{1}$ of 0_2^+ with $n=2$ of G, again without any H-singlet under the branching, except for the last one $\mathbf{1}(0,0,0,0)\to\mathbf{1}_0[0,0,0]$.

However, because of the triality of G[2], one can exchange its three inequivalent reps $\mathbf{8}_v$, $\mathbf{8}_s$, and $\mathbf{8}_c$. In fact, to find the desired singlet modes and realize SUSY breaking in the boundary theory, we swap the three reps[3]. Therefore, swapping $\mathbf{8}_s\leftrightarrow\mathbf{8}_c$ and keeping $\mathbf{8}_v$ constant, which means exchanging spinors(supercharges) with fermions and keeping scalars unchanged, the massless and massive pseudoscalar reps change accordingly without any H-singlet under the branching of the resulting reps, while the scalar reps do not change. In the same manner, after swapping $\mathbf{8}_s\leftrightarrow\mathbf{8}_v$ and keeping $\mathbf{8}_c$ constant, which means exchanging spinors with scalars and keeping fermions unchanged, the resulting reps of both modes as pseudoscalar do not include any H-singlet under the branching. However, we have $\mathbf{1386}_s$ and $\mathbf{30940}_s$, $\mathbf{23400}_s$ from the massless and massive scalar modes, respectively, while rep $\mathbf{1}$ of 0_2^+ with $n=2$ of G remains the same as before, with the latter swapping. For the latter reps, the branching $G\to H$ reads

$$\mathbf{30940}_s\to\mathbf{1}_0\oplus\mathbf{20}'_0\oplus\mathbf{105}_0\oplus\mathbf{336}_0\oplus\mathbf{825}'_0\oplus\mathbf{1716}'_0\oplus\mathbf{3185}_0\oplus\ldots,$$
$$\mathbf{23400}_s\to\mathbf{1}_0\oplus\mathbf{15}_0\oplus3(\mathbf{20}'_0)\oplus\mathbf{84}_0\oplus3(\mathbf{105}_0)\oplus2(\mathbf{175}_0)\oplus\mathbf{336}_0\oplus\mathbf{729}_0\oplus2(\mathbf{735}_0)\oplus\mathbf{3640}_0\oplus\ldots,\tag{42}$$

where we have only written $U(1)$-neutral reps. The corresponding reps for $\mathbf{1386}_s$ remain the same as the first four terms of the reps above for $\mathbf{30940}_s$ under the branching. As a result, we see that, after exchanging $\mathbf{s}\leftrightarrow\mathbf{v}$, the desired H-singlet rep ($\mathbf{1}_0$) occurs for both massless and massive (pseudo)scalars we consider here.

In contrast, a bulk (pseudo)scalar with near the boundary behaviour of (25) could be quantized with either the Neumann or alternate ($\delta\beta=0$) boundary condition for the masses in the range of $-9/4\le m^2\le-5/4$ or the Dirichlet or standard ($\delta\alpha=0$) boundary condition that can in turn be applied to any mass (see for instance [40, 41]), while the regularity (that Δ_+ is real) and stability require that the mass is above the Breitenlohner–Freedman (BF) bound $m^2\ge m_{\rm BF}^2=-9/4$ [42, 43]. As a result, for the massless and massive modes, only mode β is normalizable; α and β have holographic expositions as source and vacuum expectation value of the one-point function of the operator Δ_+, and vice versa for the operator Δ_-. Then, we write the Euclidean AdS/CFT dictionary as

$$\langle O_{\Delta_+}\rangle_\alpha=-\frac{\delta W[\alpha]}{\delta\alpha}=\beta,\quad \langle O_{\Delta_-}\rangle_\beta=-\frac{\delta\tilde{W}[\beta]}{\delta\beta}=\alpha,$$
$$\tilde{W}[\beta]=-W[\alpha]-\int \mathrm{d}^3\vec{u}\;\alpha(\vec{u})\beta(\vec{u}),\tag{43}$$

where $W[\alpha]$ ($\tilde{W}[\beta]$) is the generating functional of the connected correlator of the operator O_{Δ_+} (O_{Δ_-}) on the usual (dual) boundary CFT$_3$ with Δ_+ (Δ_-) quantization.

V. DUAL SOLUTIONS IN BOUNDARY 3D FIELD THEORIES

The bulk setups with the symmetries discussed in the previous section, including parity breaking, are dual to the boundary CS $O(N)$ or $U(N)$ interacting vector models.[4] However, we usually consider elements of ABJM's model with, depending on the case, only one scalar (say $Y=\varphi=h(r)\mathbf{I}_N$, with $h(r)$ as the scalar profile) or fermion (say ψ) [5] resulting in zero scalar and fermion potentials, and the following deformation:

$$\mathcal{L}^{(p)}=\mathcal{L}_{CS}^{+}-\mathtt{tr}\left(\mathrm{i}\bar{\psi}\gamma^kD_k\psi\right)-\mathtt{tr}\left(D_kY^\dagger D^kY\right)-\mathcal{W}_{\Delta}^{(p)},\tag{44}$$

where the CS Lagrangian reads

$$\mathcal{L}_{CS}^{+}=\frac{\mathrm{i}k}{4\pi}\,\varepsilon^{ijk}\,\mathtt{tr}\left(A_i^+\partial_jA_k^++\frac{2i}{3}A_i^+A_j^+A_k^+\right),\tag{45}$$

which is attributed to the remaining $U(1)$ part of the ori-

1) It is noticeable that after the Hopf reduction $S^7/Z_k\to CP^3\ltimes S^1/Z_k$, just the neutral states under $U(1)$ remain in the spectrum [38], and the states with odd n on S^7 are excluded.

2) Look at [39] for related studies with the triality.

3) It is reminded that $\mathbf{8}_s\to\mathbf{1}_{-2}\oplus\mathbf{1}_2\oplus\mathbf{6}_0$, $\mathbf{8}_c\to\mathbf{4}_{-1}\oplus\bar{\mathbf{4}}_1$, $\mathbf{8}_v\to\bar{\mathbf{4}}_{-1}\oplus\mathbf{4}_1$ under the branching $G\to H$.

4) It is noticeable that according to [44], nonlinear Higher-Spin gauge theories violating parity in AdS_4 correspond to nonlinear interacting 3D boundary CFTs.

5) The singlet (pseudo)scalar or fermion we consider could be taken from decomposing the eight (pseudo)scalars or fermions as $X^I\to(\Phi^n,\Phi,\bar{\Phi})$, with Φ representing either ψ or Y, $I,J...=(1,...6,7,8)=(n,7,8)$ and $\Phi=\Phi^7+i\Phi^8$, $\Phi^\dagger=\bar{\Phi}$, transforming in the rep $(\mathbf{6}_0,\mathbf{1}_2,\mathbf{1}_{-2})$ under $SO(8)\to SU(4)_R\times U(1)_b$.

ginal quiver gauge group discussed in Section 4[1]. $D_k\Phi = \partial_k\Phi + \mathrm{i}A_k\Phi - \mathrm{i}\Phi\hat{A}_k$, $F_{ij} = \partial_i A_j - \partial_j A_i + \mathrm{i}\left[A_i, A_j\right]$, and $\mathcal{W}_\Delta^{(p)}$, whose integral is W, as depicted in (43), which stands for (with p marking) deformations we make with various H-singlet operators.

A. Marginal deformations and solutions for the massless state

For the bulk solutions in subsections II.A and II.B, arising from taking the backreaction, which correspond to (exactly and irrelevant [2]) marginal operators, except for the $\Delta_+ = 3$ operators of $O_3^{(a)} = \mathtt{tr}(\varphi\bar{\varphi})^3$, $O_3^{(b)} = \mathtt{tr}(\varphi\bar{\varphi})\,\mathtt{tr}(\psi\bar{\psi})$, $O_3^{(c)} = \mathtt{tr}(A\wedge F)$, and $O_3^{(d)} = \mathtt{tr}(\varphi\bar{\varphi})\varepsilon^{ij}F_{ij}^+$ already considered in [2−5, 16, 17, 45], we include two new ones:

$$O_3^{(e)} = \mathtt{tr}(\varphi\bar{\varphi})^2\varepsilon^{kij}\varepsilon_{ij}A_k^+, \quad O_3^{(f)} = \mathtt{tr}(\psi\bar{\psi})\varepsilon^{kij}\varepsilon_{ij}A_k^+. \tag{46}$$

Next, we consider the following deformation:

$$\mathcal{W}_3^{(abf)} = \lambda_6 O_3^{(a)} + \hat{\lambda}_6 O_3^{(b)} + \check{\lambda}_6 O_3^{(f)}, \tag{47}$$

where the λ's are the coupling constants. We set $\alpha = 1$ for now. Then, if we take both CS terms $\mathcal{L}_{CS} + \hat{\mathcal{L}}_{CS}$ instead of $\mathcal{L}_{CS}^+$ in (44), after some mathematical manipulations on the resultant scalar $\bar{\varphi} = \varphi^\dagger$, fermion $\bar{\psi}$, and gauge A_k^+ field equations, we get

$$\partial_k\partial^k\varphi - 3\lambda_6\varphi^5 = 0, \tag{48}$$

$$\mathrm{i}\bar{\psi}\gamma^k\partial_k\psi + 2\bar{\psi}\gamma^k\psi A_k^+ + \frac{\mathrm{i}k}{4\pi}\varepsilon^{ijk}F_{ij}^+A_k^+ = 0, \tag{49}$$

where $Y = Y^\dagger$ and $A_i^- = 0$ are also set. Subsequently, a closed solution for (48) reads

$$h = \left(\frac{1}{g_6}\right)^{1/4}\left(\frac{a}{a^2 + (\vec{u} - \vec{u}_0)^2}\right)^{1/2}, \tag{50}$$

where $g_6 \equiv -\lambda_6$. Accordingly, by employing the ansatz

$$A_k^+ = \varepsilon_{kij}\varepsilon^{ij}A^+(r), \tag{51}$$

where $A^+(r)$ is a scalar function on the boundary, a solution for (49) reads

$$A^+ = \frac{3}{4}\left(\frac{a}{a^2 + (\vec{u} - \vec{u}_0)^2}\right), \tag{52}$$

$$\psi = \tilde{a}\left(\frac{a + \mathrm{i}(\vec{u} - \vec{u}_0).\vec{\gamma}}{\left[a^2 + (\vec{u} - \vec{u}_0)^2\right]^{\varsigma = 3/2}}\right)\chi, \tag{53}$$

where $\vec{\gamma} = (\sigma_2, \sigma_1, \sigma_3)$ are the Euclidean gamma matrices, and χ with $\chi^\dagger\chi = 1$ is a constant dimensionless spinor[3]. Finally, from computing the corresponding boundary action

$$S_{(3)}^{\text{modi.}} = -\int\big((\partial_i\varphi)^2 - 2\,\mathtt{tr}(\bar{\psi}\gamma^3\psi)A_3^+ + \lambda_6\varphi^6 + \hat{\lambda}_6\varphi^2\,\mathtt{tr}(\psi\bar{\psi}) + 12\check{\lambda}_6 A^+\,\mathtt{tr}(\psi\bar{\psi})\big) \tag{54}$$

based on the solutions (50), (52), and (53) and setting the couplings equal to 1 and $\tilde{a} = a = a^\dagger$ for simplicity, we get

$$S_{(3)}^{\text{modi.}} = -36\int_0^\infty \frac{\pi a^3 r^2}{(a^2 + r^2)^3}\,\mathrm{d}r = -\frac{9}{4}\pi^2, \tag{55}$$

[4] which is finite, indicating an instanton with size $a \geq 0$ at the origin ($\vec{u}_0 = 0$) of a three-sphere with radius r at infinity (S_∞^3).

Consequently, as a basic test of the correspondence, wrt (43), we have

$$\langle O_3^{(a,b,f)}\rangle_\alpha = a_1\left(\frac{a}{a^2 + (\vec{u} - \vec{u}_0)^2}\right)^3, \tag{56}$$

with $a_1, a_2, \ldots$ being the boundary constants, as compatible with near the boundary behavior of (34) with $\Delta_+ = 3$ wrt (25), in the limit of $a \to 0, r \to \infty$. Meanwhile, compared with the bulk closed solution of (13), this boundary solution may be considered as an instanton sitting at the conformal point of $u = a$.[5]

Note also that, with $\hat{\lambda}_6 = \check{\lambda}_6 = 0$ in (47) and including

1) We may also take the $U(1)\times U(1)$ part of the gauge group, with the gauge fields $A_i^\pm \equiv (A_i \pm \hat{A}_i)$, noting that the fundamental fields of ABJM are neutral wrt A_i^+ (diagonal $U(1)$) and A_i^- acts as baryonic symmetry; and since our (pseudo)scalars are neutral, we set $A_i^- = 0$. As a result, we will also examine the sum of $\mathcal{L}_{CS}$ (for A_i) and $\hat{\mathcal{L}}_{CS}$ (for $\hat{A}_i$) instead of $\mathcal{L}_{CS}^+$ in the boundary analyzes.

2) See footnote 4) on this page.

3) See [46] for a similar ansatz.

4) Note that we could take $r = |\vec{u} - \vec{u}_0|$ (or $|x - x_0|$) with $\vec{u}_0$ (or x_0) as an arbitrary origin.

5) In fact, we already discussed in [3, 4] the potential $-\lambda_6(\varphi^2)^3$ of the tri-critical $O(N)$ or $U(N)$ model that is unbounded from below and so, there are instabilities near the potential extrema and tunneling mediated by Fubini-like instantons of the size a and locations $\vec{u}_0$. Meantime, for any positive value of λ_6, the corresponding operator is not exactly marginal but it becomes quantum irrelevant; see [47−50]. On the other hand, we saw in subsection 2.1 when taking the backreaction of the whole 11D space, the resultant bulk solutions corresponded to marginally irrelevant deformations. In other words, by quantum corrections and the breaking of conformal invariance, an exactly marginal configuration may change to a marginally irrelevant one.

a mass-deformation term ($m_b^2\,\mathrm{tr}(\varphi\bar{\varphi})$),[1)] we have, in general, the RB model (look at also at [4]), whose $\bar{\varphi}$ equation reads

$$\left(\partial_i\partial^i - m_b^2\right)h + 3\,g_6\,h^5 = 0. \tag{57}$$

Solutions for its free massive equation are available in terms of (modified) Bessel functions; an explicit expression is stated as

$$h_c(r) \cong \frac{a_2}{\sqrt{m_b}}\,\frac{\mathrm{e}^{-m_b r}}{r}, \tag{58}$$

which satisfies the condition $h_c(r\to\infty)\to 0$, resulting in a finite action. Solutions for the interaction equation could be obtained in the context of constrained instantons; see [21, 54–56]. In fact, considering (58) as the initial data, one may employ perturbative methods and get solutions with a simple structure, such as $h\sim 1/r$. Thus, we have the single-operator correspondence $\langle O_3^{(a)}\rangle_\alpha \sim 1/r^6$ with the typical near the boundary solution of (34) for $\Delta_+ = 3$.

In particular, if we use only the operator $O_3^{(e)}$ to deform the action of (44), discarding its fermion kinetic term, the equations for $\bar{\varphi}$ and A_k^+ read

$$\partial_i\partial^i\varphi - 2\varphi\,\mathrm{tr}(\varphi\bar{\varphi})\,\varepsilon^{kij}\varepsilon_{ij}A_k^+ = 0, \tag{59}$$

$$\frac{\mathrm{i}k}{4\pi}\varepsilon^{kij}F_{ij}^+ - \mathrm{tr}(\varphi\bar{\varphi})\,\varepsilon^{kij}\varepsilon_{ij} + \mathrm{i}\left[\varphi\left(\partial^k\bar{\varphi}\right) - \left(\partial^k\varphi\right)\bar{\varphi}\right] = 0, \tag{60}$$

respectively. Next, with $\varphi=\bar{\varphi}$, from the last two equations, we can write

$$\partial_i\partial^i h(r) = 0 \Rightarrow h(r) = a_3 + \frac{a_4}{r}, \tag{61}$$

while for the gauge field, we may use the ansatz (51) with $A^+(r)\sim 1/r^2$. Thus, with $a_3=0$, the basic correspondence $\langle O_3^{(e)}\rangle_\alpha \sim 1/r^6$ is realized, with near the boundary solution of (34) with $\Delta_+=3$ wrt (25). In contrast, if $\varphi\neq\bar{\varphi}$, which is allowed due to being in Euclidean space, and explicitly with

$$\varphi = h(r)\,I_N, \quad \varphi^\dagger = a_5\,I_N, \tag{62}$$

from Eqs. (59) and (60), we get

$$\partial_k\left(\varepsilon^{kij}F_{ij}^+\right) = 0, \quad A_k^+ = \mathrm{i}\,\partial_k \ln h. \tag{63}$$

The latter solution is reminiscent of the duality ($A_k = \eta_{kj}\partial^j h/h$) between the instanton solution of the pure $SU(2)$ from Yang-Mills theory [57],

$$A_k \approx \eta_{kj}\frac{(x-x_0)^j}{a^2+(x-x_0)^2} \Rightarrow F_{ij}\approx \eta_{ij}\left(\frac{a}{a^2+(\vec{u}-\vec{u}_0)^2}\right)^2, \tag{64}$$

with η_{ij} as 't Hooft symbols [58], and the $SO(4)$-invariant solution of the so-called φ^4 model, expressed as [2)]

$$\nabla^2 h + \lambda_4 h^3 = 0 \Rightarrow h = \sqrt{\frac{8}{\lambda_4}}\left(\frac{a}{a^2+(\vec{u}-\vec{u}_0)^2}\right). \tag{65}$$

As a result, with the latter solutions, we have the same correspondence as (56) for $O_3^{(e)}$.

B. Irrelevant deformations and solutions for the massive state

For the Higgs-like (pseudo)scalar $m^2=18$, except for the $\Delta_+=6$ operators introduced in [2], including $O_6^{(b)} = (O_3^{(b)})^2$ (a double-trace deformation), we consider a few new ones. The operator $O_6^{(a)} = \mathrm{tr}(\psi\bar{\psi})^3$, first considered in [4], is interesting in that it can also be taken in the CF model. In fact, if we consider the deformation

$$\mathcal{W}_6^{(a)} = m_f\,O_2^{(a)} + \tilde{g}_6\,\alpha\,O_6^{(a)}, \tag{66}$$

where $O_2^{(a)} = \mathrm{tr}(\psi\bar{\psi})$, by excluding the scalar kinetic term in (44), the $\bar{\psi}$ equation reads

$$\mathrm{i}\gamma^k\partial_k\psi + m_f\,\psi + 3\,\tilde{g}_6\,\alpha\,\psi\,\mathrm{tr}(\psi\bar{\psi})^2 = 0. \tag{67}$$

The solution of (53) with $\tilde{a} = a/(\tilde{g}_6)^{1/4}$ is also valid for the latter equation provided that

$$\alpha = \mathrm{tr}(\psi\bar{\psi})^{-3/2} \tag{68}$$

for the massless case ($m_f=0$) and $m_f \to \tilde{\alpha}(\vec{u}) = \mathrm{tr}(\psi\bar{\psi})^{1/2}$ for the massive case. Thus, the deformation might in fact be considered as a triple-trace one. As a result,

$$\langle O_6^{(a)}\rangle_\alpha = \left(\frac{\tilde{a}}{a^2+(\vec{u}-\vec{u}_0)^2}\right)^6, \tag{69}$$

1) For studies on massive deformations of ABJM model, see [51–53].

2) See for instance [59–65].

which, wrt (25), corresponds to (40) for $\Delta_+=6$, considering (41), $a_0^2-b_0^2=a^2$, and $-a_0 b_0 \sim \tilde{a}^2$. Moreover, we can obtain an explicit profile for $f(r)$ in (30) from this solution. In fact, according to the above discussions, with $\Delta_\pm=6,-3$ and the correspondence rules of (43), we get

$$f(r)=\left[\frac{\tilde{a}}{a^2+r^2}\right]^{1/2}, \tag{70}$$

where $C_-=\sqrt{C_+}$, with $C_+=1$ for simplicity.

It is also interesting to check the BF duality (or 3D Bosonization)- see for instance [66, 67]- from our setups attributed to RB and CF models [1)] at the level of the solutions. Indeed, under the BF duality, the coupling of $\tilde{g}_6\,\mathrm{tr}(\psi\bar{\psi})^3\sim\tilde{g}_6\sigma_f^3$, where σ_f is the so-called Hubbard-Stratonovich field, is mapped into the coupling of $g_6\,\mathrm{tr}(\varphi\bar{\varphi})^3$ ($\mathcal{W}_3^{(a)}$ of (47)); see [68, 69].[2)] In this regard, from the solutions of the boson model (50) and fermion model (53), we have

$$\mathrm{tr}(\psi\bar{\psi})=\left(\frac{1}{\tilde{g}_6}\right)\left(\frac{a}{a^2+(\vec{u}-\vec{u}_0)^2}\right)^2=\mathrm{tr}(\varphi\bar{\varphi})^2, \tag{71}$$

where $\tilde{g}_6\leftrightarrow g_6$, thus realizing the BF duality with $\psi\leftrightarrow\varphi^2$ or $\psi\leftrightarrow\varphi$ when including α in the fermion model.

In this way, we now examine two new operators

$$O_6^{(c)}=\mathrm{tr}(\psi\bar{\psi})^2\varepsilon^{ij}F_{ij}^+,\quad O_6^{(d)}=\mathrm{tr}(\psi\bar{\psi})\,\mathrm{tr}(F_{ij}^+F^{+ij}); \tag{72}$$

with the associated deformations

$$\mathcal{W}_6^{(q)}=\alpha O_6^{(q)}, \tag{73}$$

where $q=c,d,...,h$ from now on. Next, discarding the scalar kinetic term of (44), with $\mathcal{W}_6^{(c)}$, the fermion $\bar{\psi}$ and gauge A_i^+ equations read

$$\mathrm{i}\gamma^k\partial_k\psi+2\alpha\psi\,\mathrm{tr}(\psi\bar{\psi})\varepsilon^{ij}F_{ij}^+=0, \tag{74}$$

$$\frac{\mathrm{i}k}{4\pi}\varepsilon^{kij}F_{ij}^++2\bar{\psi}\gamma^k\psi=0, \tag{75}$$

respectively, reminding that the second term on the LHS of (75) exists when we include both CS terms (for A_i and $\hat{A}_i$) in (44) and that $F_{ij}^-=0, A_i^-=0$ is set. With only the CS term of (45), the ansatz

$$A_\mu^+=\omega_{\mu\nu}x^\nu A(r),\quad \omega_{\mu\nu}=\begin{cases}1 & :\ \nu>\mu,\\ 0 & :\ \nu=\mu,\ \ \mu,\nu\neq i,j,\end{cases} \tag{76}$$

for the $U(1)$ gauge field, with μ,ν for the boundary indices as well and $A(r)$ as another boundary scalar function, we obtained the following desired solution (see also [5]):

$$A(r)=\frac{a_6+4a_7 r}{4r^4}\Rightarrow\varepsilon^{ij}F_{ij}^+\equiv F^+=\frac{a_6}{r^4}. \tag{77}$$

In this case, a solution for ψ is taken from (53) with $a=0$ and $\tilde{a}=\frac{\mathrm{i}}{2}\sqrt[3]{\frac{4}{5}}$. As a result,

$$\langle O_6^{(c)}\rangle_\alpha=\frac{a_6\tilde{a}^4}{r^{12}}\cong f(r), \tag{78}$$

with $f(r)$ in (37). Accordingly, one can also adjust $\bar{C}_6=a_6\tilde{a}^4$ of (38) wrt (25).

However, combining (74) and (75), we get

$$\gamma^k\partial_k\psi+\frac{16\pi}{k}\alpha\,\mathrm{tr}(\psi\bar{\psi})^2\gamma^3\psi=0, \tag{79}$$

where taking the third component of the gamma matrices is for compatibility with the solution we take for ψ, which in turn reads from (53) with $\tilde{a}=a\sqrt[1/2]{\frac{-3\mathrm{i}k}{16\pi}}$. Thus, from (74), we have

$$F_{ij}^+=a_8\varepsilon_{ij}\left(\frac{a}{a^2+(\vec{u}-\vec{u}_0)^2}\right)^2, \tag{80}$$

where $a_8=(3\pi\mathrm{i}/k)^{1/2}$, reminding that $F^+(r\to\infty)\to 0$.[3)] As a result,

$$\langle O_6^{(c)}\rangle_\alpha=\frac{3}{2}\frac{a^2\tilde{a}^2}{\left[a^2+(\vec{u}-\vec{u}_0)^2\right]^6}, \tag{81}$$

which, for $\Delta_+=6$, can be made to correspond to (40) with (41) and to (13) with an instanton at the conformal point of $u=a$.

Moreover, to confirm the instanton nature of the Euclidean solutions, we compute the value of the corresponding action as follows:

$$S_{(6c)}=\int\mathcal{W}_6^{(c)}\,\mathrm{d}^3\vec{u}\quad\Rightarrow S_{(6c)}^{\mathrm{modi.}}=\frac{3\pi^2}{8}\frac{a}{\tilde{a}}, \tag{82}$$

1) See [4], where we have used these models in more detail.

2) Indeed, a double-trace deformation of the latter takes the RB model to the CF model; see the deformation (66).

3) It is noticable that the A_i^+ equation of (75) and the solution of (53) result in zero magnetic charge or flux, $\Phi=\oint_{S^3_\infty}F^+=0$; see also [16].

where we have used the result of the integral presented in (55) and the same interpretation.

Similarly, for the deformation $\mathcal{W}_6^{(d)}$ of (73), discarding the scalar kinetic term of (44) and taking both CS terms, the fermion $\bar{\psi}$ and gauge A_i^+ equations read

$$\mathrm{i}\gamma^k\partial_k\psi+\alpha\psi\,\mathrm{tr}(F^+_{ij}F^{+ij})=0, \tag{83}$$

$$\frac{\mathrm{i}k}{4\pi}\varepsilon^{kij}F^+_{ij}+2\bar{\psi}\gamma^k\psi+4\alpha\,\mathrm{tr}(\psi\bar{\psi})\partial_jF^{+jk}=0. \tag{84}$$

Then, using (68), solutions for the fermion and gauge fields are read from (53) with $\tilde{a}=a\sqrt[1/2]{\frac{-9\mathrm{i}k}{8\pi}}$ and from (80) with $a_8=1$. However, if we set $\alpha=1$ in the equations, a solution for ψ is read from (53) with $\varsigma=0$ instead of $3/2$ along with the gauge solution (80) with $a_8=1/a$ to obtain $\tilde{a}$ the same as before. As a result, we have

$$\langle O_6^{(d)}\rangle_\alpha=\frac{a^2\tilde{a}^2}{\left[a^2+(\vec{u}-\vec{u}_0)^2\right]^3}, \tag{85}$$

which corresponds to the bulk near the boundary solution of (35) with $\check{C}_{-3}=0$ and $\hat{C}_6=a^2\tilde{a}^2$, in the limit of $a\to0, r\to\infty$(see footnote 5 on page 6).

Another operator we consider is

$$O_6^{(e)}=\mathrm{tr}(\varphi\bar{\varphi})^4\varepsilon^{ij}F^+_{ij}; \tag{86}$$

we deform the action of (44), discarding its fermion term, with (73) having $q=e$. As a result, the scalar $\bar{\varphi}$ equation reads

$$\partial_i\partial^i\varphi-4\alpha\varphi\,\mathrm{tr}(\varphi\bar{\varphi})^3\varepsilon^{ij}F^+_{ij}=0, \tag{87}$$

where the gauge A_i^+ equation is the same as (60) apart from omitting the middle term on the LHS. Next, with $\varphi=\bar{\varphi}$, we can obtain for the gauge part a similar solution to (80) with $a_8=1$. Then, taking $\alpha\sim\mathrm{tr}(\varphi\bar{\varphi})^{-3}$ and $F^+\sim h^4$, we obtain a similar solution to (50) for h with $g_6=1$; thus, the same correspondence as (69) with $a=\tilde{a}$ for $O_6^{(e)}$ is confirmed.

However, when $\varphi\neq\bar{\varphi}$ we take (62). Subsequently, from (87) and (60) without the middle term on the LHS of the latter, we can write

$$\partial_k\left(\varepsilon^{kij}F^+_{ij}\right)-\frac{16\pi}{k}\alpha\,\mathrm{tr}(\varphi\bar{\varphi})^4\varepsilon^{ij}F^+_{ij}=0. \tag{88}$$

Then, using the ansatz (76), $F^+=-2\left(6A(r)+2r\acute{A}(r)\right)$, with $a_5=\sqrt[1/2]{\frac{-3\sqrt{3}k}{16\pi}}$ and $\alpha=1$, we get

$$\frac{\mathrm{d}^2A(r)}{\mathrm{d}r^2}+\left(\tilde{h}(r)+\frac{4}{r}\right)\frac{\mathrm{i}A(r)}{\mathrm{i}r}+\frac{3}{r}\tilde{h}(r)A(r)=0, \tag{89}$$

for which we can write the following solution:

$$h^4(r)\equiv\tilde{h}(r)=\frac{n}{r}\Rightarrow A(r)=\frac{a_7}{r^3}+\frac{a_9}{r^n}\Rightarrow F^+ =\frac{4a_9}{r^n}(n-3), \tag{90}$$

with n being a real number. As a result, we have

$$\langle O_6^{(e)}\rangle_\alpha=\frac{4na_5^4a_9}{r^{n+1}}(n-3), \tag{91}$$

which, with $n=5$ wrt (25), corresponds to the normalizable part of the bulk solution (35), after adjusting the constants of both sides. In the same manner, with $n=11$, the expression can be made to correspond to $f(r)$ in (37).

Among other similar operators, if we use any of the following three operators

$$O_6^{(f)}=\mathrm{tr}(\varphi\bar{\varphi})\mathrm{tr}(\psi\bar{\psi})^2\varepsilon^{ijk}\varepsilon_{ij}A^+_k,$$
$$O_6^{(g)}=\mathrm{tr}(\varphi\bar{\varphi})^2\mathrm{tr}(\psi\bar{\psi})\varepsilon^{ij}F^+_{ij},$$
$$O_6^{(h)}=\mathrm{tr}(\varphi\bar{\varphi})\mathrm{tr}(\psi\bar{\psi})\varepsilon^{ijk}F^+_{ij}A^+_k, \tag{92}$$

to deform (44) wrt (73), the fermion solution may correspond to (53) with $a=0$, scalar solution may correspond to (61), gauge solution may correspond to $F^+(r)$ in (77), and $A^+(r)\sim1/r^2$ (for $O_6^{(f)}$) according to the ansatz of (51). Consequently, as a primary test of the correspondence, $\langle O_6^{(f,g,h)}\rangle_\alpha\sim1/r^{12}$ matches with the bulk solution (37) and (38) for $\Delta_+=6$ as before.

VI. SUMMARY AND COMMENTS

In this study, we started from 11D SUGRA with a fixed background geometry of $AdS_4\times S^7/Z_k$ and dynamical 4-form ansatz and obtained a consistent truncation such that the resulting scalar equations in the external $EAdS_4$ space do not include any dependence on the internal space ingredients, and the associated (pseudo)scalars are H-singlets. In addition, as the solutions are poised to probe (anti)M-branes wrapped around the three internal directions $CP^1\ltimes S^1/Z_k$ in the (WR)SW background, they break all SUSYs and parity, and the resultant theory holds for anti-M2-branes. The scale-invariance was also broken due to the mass terms and nonlinearities of the equations. Taking the backreaction, we obtained the massless ($m^2=0$) and massive ($m^2=1/9, 2/9$) modes corresponding to the exactly marginal and marginally irrelevant operators on the 3D boundary, respectively. Then, we demonstrated a closed solution for the resultant equation and computed its correction to the bulk background action. Moreover, for the NPDE equation of the Higgs-like ($m^2=18$) mode, arising from spontaneous symmetry breaking, we employed the ADM method and arrived at interesting series solutions appropriate for near the boundary analyses.

In order to realize supersymmetry and parity breaking, as well as the H-singlet bulk (pseudo)scalars, we swapped the three fundamental reps of $SO(8)$ and observed that, under the branching of $G\rightarrow H$, such (pseudo)scalars were realized. Because of the bulk symmetries, the boundary duals could come off in the singlet sectors of ABJM-like models, from which we built some new marginal and irrelevant operators composed of a scalar, fermion, and $U(1)$ gauge field. Having said that, we observed that solutions with finite actions and $SO(4)$ symmetry on a three-sphere at infinity could be obtained. After that, we confirmed the state-operator correspondence, adjusted the bulk and boundary parameters, and specified the unknown functions in the bulk from the boundary solutions. In addition, we confirmed the existence of a BF duality ($\varphi\leftrightarrow\psi$) between RB and CF models in terms of the solutions and correspondence.

In order to further confirm the results and reconcile with previous studies conducted by others and their applications, a few more points are worth mentioning. First, we remind that the instantons considered here are mainly attributed to the unbounded boundary potential from below and have also dual interpretations in the form of Coleman-de Luccia (CdL) bounces [70] mediating the false-vacuum decay and formation of true-vacuum bubble within it. According to [71], such AdS_4 bubbles collapse and eventually end in a big crunch singularity.[1)] Second, our bulk solutions (not) considering the backreaction correspond to (irrelevant) marginally irrelevant deformations (see also the footnote 4 on page 8) and are consistent with the result reported in [75] that states field theories on dS_3 with $SO(3,1)$-invariant solutions and irrelevant deformations are dual to vacuum decays and cosmic singularities in AdS_4. Third, we notice the probe (anti)M2-branes wrapped around S^3/Z_k that result in domain-walls interpolating among different vacua [76]. According to [77], a domain-wall at $u=0$ separates two degenerate AdS_4 vacua. Indeed, with conformal-invariance breaking, we deal with the problem on constant-u patches, where the boundary is dS_3 in the Lorentzian signature. Fourth, such a truncation is interesting in some cosmological (inflationary and bouncing) models[2)]. Specifically, our almost degenerate double-well scalar potential from (6) accepts *bounce* solutions; thus, it is possible to address the problem from that point of view and provide interesting analyses.

APPENDIX A: EM TENSORS AND RESULTING EQUATIONS

We use the Einstein's equations

$$\mathcal{R}_{MN}-\frac{1}{2}g_{MN}\mathcal{R}=8\pi\mathcal{G}_{11}T^{G_4}_{MN}, \tag{A1}$$

where

$$T^{G_4}_{MN}=\frac{1}{4!}\left[4G_{MPQR}G_N^{PQR}-\frac{1}{2}g_{MN}G_{PQRS}G^{PQRS}\right], \tag{A2}$$

and the capital $M,N,\ldots$, small $m,n,\ldots$, and Greek $\mu,\nu,\ldots$ indices are for the entire 11D, 6D internal CP^3, and 4D external AdS_4 spaces, respectively.

Next, using the conventions and performing computations similar to those expressed in Appendix B of [3], we get

$$G_{PQRS}G^{PQRS}=96\left[\frac{8}{3R^8}f_1^2+\frac{R^2}{32}(\partial_\mu f_2)(\partial^\mu f_2)+\frac{1}{8}f_3^2\right], \tag{A3}$$

$$G_{\mu PQR}G_\nu^{PQR}=\frac{64}{R^8}f_1^2g_{\mu\nu}+\frac{3R^2}{4}(\partial_\mu f_2)(\partial_\nu f_2), \tag{A4}$$

$$G_{mPQR}G_n^{PQR}=\left[2f_3^2+\frac{R^2}{4}(\partial_\mu f_2)(\partial^\mu f_2)\right]g_{mn}, \tag{A5}$$

$$G_{7PQR}G_7^{PQR}=\frac{3R^2}{4}(\partial_\mu f_2)(\partial^\mu f_2)\,g_{77}, \tag{A6}$$

with a 4! factor for all terms.

Then, by plugging (A3) with (A4), (A5), and (A6) back into (A2); using (3) with the conventions of (7); taking the traces; and using the Euler-Lagrange equation, we finally get

$$\Box_4 f_2+4m^2f_2\pm4\delta f_2^2+4\lambda f_2^3\pm4F=0, \tag{A7}$$

1) We remind that CFTs with unbounded potentials from below have observables that evolve to infinity in finite time, and their bulk duals are gravities coupled to (pseudo)scalars with potentials coming from consistent truncations of supergravity, as it stands here; see for instance [72−74].

2) See, for instance, [78] with references therein.

$$\Box_4 f_2 + \left(3m^2 - \frac{8}{R^2}\right) f_2 \pm 3\delta f_2^2 + 3\lambda f_2^3 \pm 3F = \pm\frac{8C_2}{R^3}, \tag{A8}$$

$$\Box_4 f_2 - m^2 f_2 \mp \delta f_2^2 - \lambda f_2^3 \mp F = 0, \tag{A9}$$

for the external AdS_4 space, internal CP^3, and eleventh S^1/Z_k components, respectively, noting that (A9) is the same as the main expression in Eq. (6).

APPENDIX B: BASICS OF ADM FOR SOLVING THE SCALAR EQUATION

The Adomian decomposition method or the inverse operator method [13] is a mathematical method specially used to solve NPDEs; see [79]. Because we are looking for solutions near the boundary ($u=0$), we use

$$f_0(0,r) = f(0,r) - u f_u(0,r), \quad f(0,r) = f(r)u^{\Delta_+} \tag{B1}$$

as the initial data for the second order NPDE (22), corresponding to the Dirichlet boundary condition. Accordingly, from the main equation, we can write [1)]

$$\Box_4 f_0 - m^2 f_0 = 0, \tag{B2}$$

with the closed solution of (13), and

$$\Box_4 f_{n+1} - m^2 f_{n+1} = \sum_{n=0}^{\infty} A_n, \tag{B3}$$

where the nonlinear terms are written as the sum of Adomian polynomials A_n's,

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\mathcal{F}\left(\sum_{n=0}^{n}\lambda^n f_n\right)\right]_{\lambda=0}, \quad n = 0,1,2,\ldots, \tag{B4}$$

with $\mathcal{F}(f)$ for the nonlinear terms of (28). As a result,

$$A_0 = 6f_0^3 - \delta f_0^2, \quad A_1 = 18 f_0^2 f_1 - 2\delta f_0^2 f_1,$$
$$A_2 = 18 f_0^2 f_2 + 18 f_0 f_1^2 - 2\delta f_0 f_2 - \delta f_1^2, \ldots, \tag{B5}$$

where $\delta = 3\sqrt{3}m$. Accordingly, a series solution up to the nth order of the iteration processes may be written according to (29).

In contrast, with $f = (u/R_{\rm AdS})g$ and $R_{\rm AdS} = 1$ from (22), we can write

$$(\partial_i\partial_i + \partial_u\partial_u) g_0 - 6g_0^3 = 0, \tag{B6}$$

with the exact solution of

$$g_0(u,\vec{u}) = \frac{2}{\sqrt{3}}\left(\frac{b_0}{-b_0^2 + (u+a_0)^2 + (\vec{u}-\vec{u}_0)^2}\right), \tag{B7}$$

which is indeed for the so-called conformally coupled (pseudo)scalar $m^2 = -2$ in the SW version of (4) with $C_3 = 1$- with a_0, b_0 being physically meaningful constants and $|\vec{u}-\vec{u}_0| \equiv r$ when using the spherical coordinates- and

$$(\partial_i\partial_i + \partial_u\partial_u)\, g_{n+1} - 6g_{n+1}^3 = \sum_{n=0}^{\infty} A_n, \tag{B8}$$

with the Adomian polynomials

$$A_0 = \frac{(2+m^2)}{u^2} g_0 - \frac{3\sqrt{3}m}{u} g_0^2,$$
$$A_1 = \frac{(2+m^2)}{u^2} g_1 - \frac{6\sqrt{3}m}{u} g_0 g_1, \ldots, \tag{B9}$$

to obtain series solutions near the boundary $u = 0$.

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