Light quarkonium and charmonium mass shifts in an unquenched quark model*

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Abstract: The unquenched quark model for the light quarkonium and charmonium states is explored in this study. The quark-pair creation operator in the ${}^3P_0$ model, which combines the two-quark and four-quark components, is modified by considering the effects of the created quark pair's energy. Furthermore, the separation between the created quark pair and valence quark pair is modified. All the wave functions, including those for the mesons and the relative motion between two mesons, are obtained by solving the corresponding Schrödinger equation using the Gaussian expansion method. The aim of this study is to find a new set of parameters that can accurately describe the mass spectrum of low-lying light quarkonium and charmonium states. Moreover, certain exotic states, such as $X(3872)$, can be described well in the unquenched quark model.

Keywords: the unquenched quark model, Gaussian expansion method, light quarkonium and charmonium states

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I. INTRODUCTION

In a nonrelativistic valence quark model, a baryon comprises three quarks, and a meson comprises quark-antiquark. The model has successfully described the properties of low-lying hadrons and hadron-hadron interactions. For example it is successfully applied to heavy quarkonia, such as bottomonium and charmonium [1−18], and also, to a certain extent, light mesons [19−21]. With the progresses of experiments, increasingly more new exotic hadrons have been reported by experimental collaborations since 2003. These exotic states cannot be effectively described by the valence quark model, which poses a significant challenge for the quark model.

For example, the measured mass of the second P-wave charmonium state $X(3872)$ [22] is 100 MeV lower than the predicted mass by the quark model for $\chi_{c_1}(2P)$. Furthermore, its decay width is $\leq$ 1 MeV. Additionally, similar problems are observed for the charmed meson states $D^*_{s_0}(2317)$ [23] and $D_{s_1}(2460)$ [24]. These puzzling issues have led theorists to refer to them as "exotic states." Various explanations, such as multi-quark states, hybrid states, and gluonic excitations, have been proposed.

To describe these exotic hadrons in the quark model, the model should be extended. By considering that the quark number is not a conserved quantity and quark pairs $q\bar{q}$ can be excited in a vacuum, a new quark model, termed as the unquenched quark model (UQM), has been developed. The wave functions of meson and baryon in UQM can be expressed as follows:

$$|\text{Meson}>= |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}g\rangle + ... \quad (1)$$

$$|\text{Baryon}>= |qqq\rangle + |qqqq\bar{q}\rangle + |qqqg\rangle + ... \quad (2)$$

The first term represents the wave function in the nonrelativistic valence quark model. The second and third terms consider the quark pairs and the gluon excitation in the vacuum. As a preliminary phase of the development of the UQM, only the first two terms—the valence term and the valence with quark-antiquark excitation —are considered in the model.

To date, there have been many theoretical studies ex-

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ploring the effects of quark-antiquark pair excitation on the properties of hadrons. For example, in a recent article published in "Nature" [25], scientists presented the asymmetry in the momentum distribution of antimatter quarks, indicating evidence of matter-antimatter asymmetry within the proton. Regarding the Roper resonance $N(1440)$, the latest results suggest that it is a radially excited state of the proton core surrounded by a 20% meson cloud [26]. Furthermore, by considering meson-baryon coupling effects, Kenta Miyahara *et al.* proposed that $\Lambda(1405)$ is a mixture of three-quark and five-quark states wherein $\bar{K}N$ is the dominant component [27].

For heavy-light systems, Beveren *et al.* considered the DK coupling channel effects in the $c\bar{s}$ system and performed calculations on the mass of $D^*_{s0}(2317)$, which provided a good explanation for the experimental data [28]. In a previous study by Albaladejo *et al.* [29], the influence of $D^{(*)}K$ meson-meson coupling channels on P-wave $c\bar{s}$ states was considered to examine the internal structure of $D^*_{s0}(2317)$ and $D_{s_1}(2460)$. They suggested that these particles are predominantly composed of a four-quark structure mixed with a quark-antiquark component.

For the heavy systems, by considering coupling channel effects, the mass of charmonium state $\chi_{c_1}(2P)$ can be lowered to the value of $X(3872)$ [30−36]. In the study of $\psi(4415)$, Cao and Zhao considerd the influence of molecular states $D_{s_1}\bar{D}_s$ and $D_{s_0}\bar{D}^*_s$ in the unitarized picture [37]. Luo *et al.* calculated the mass spectrum of $\Lambda_c(2P,(3/2)^-)$ by considering the coupling channel effects of D^*N, which provided a good explanation for the charmed baryon state $\Lambda_c(2940)^+$ reported by BaBar Collaboration [38]. Furthermore, the potential of placing $X(3915)$, which is produced through the two-photon fusion process, as a charmonium family member $\chi_{c0}(2P)$ is closely related to the coupling channel effects.

These studies prompt us to continue delving into and developing the unquenched quark model. Recently, this has become a crucial topic in hadron physics, driven by the discovery of numerous new hadronic states and the accumulation of relevant experimental data. Generally, the transition operator, which mix the quark-antiquark and four-quark components, is obtained from the $^3P_0$ model in these theoretical calculations. Some of the previous work found that the virtual quark pair creation in hadronic system leads to a very large mass shifts [39, 40]. The large mass shift will challenge the validity of the valence quark model in describing the ground state hadrons and convergence of UQM. Furthermore, the convergence problem was noted by Ferretti and Santopinto, and it can be addressed by considering only the contribution from the closest set of meson-meson intermediate states and taking the contribution from other states as some type of global constant [41]. In our previous study [40, 42], we attempted to solve this problem by modifying the transition operator, i.e. introducing energy and separation damping factors. With the improved transition operator, the mass shifts of the low-lying light mesons [40] and charmonium [42] are significantly reduced. The proportion of the two-quark component increases to approximately 90%. This in turn suppresses the influence of the four-quark components. This ensures the validity of the constituent valence quark model in describing the low-lying hadron states.

Given the incorporation of the four-quark components, the model parameters used in the valence quark model should be adjusted. In this study, with the improved transition operator, the meson spectrum is computed by solving the eigenequation of the unquenched quark model Hamiltonian. Then, by fitting the experimental data of the low-lying mesons, the model parameters are determined. The involved low-lying mesons in the fitting include π, ρ, ω, η, $\eta_c(1S)$, $\eta_c(2S)$, $J/\psi(1S)$, $J/\psi(2S)$, $\chi_{cJ}(1P)(J=0,1,2)$, and $h_c(1P)$, a total of 12 mesons. Using the obtained new set of model parameters, we calculated the high-lying excited-state energy spectrum of charmonium $\chi_{cJ}(2P)(J=0,1,2)$ and $1D$ $c\bar{c}$ mesons. For certain exotic states, such as $X(3872)$, can be effectively described in the unquenched quark model.

The paper is organized as follows. In Sec. II, the chiral quark model and GEM are presented. In Sec. III, we introduce the modified transition operator. The discussion of the results is provided in Sec. IV. The last section is devoted to the summary of the current study.

II. CHIRAL QUARK MODEL

In the nonrelativistic quark model, we obtained the meson spectrum by solving the Schrödinger equation:

$$H\Psi^{IJ}_{M_IM_J}(1,2)=E^{IJ}\Psi^{IJ}_{M_IM_J}(1,2), \tag{3}$$

where 1, 2 denote the quark and antiquark, respectively. $\Psi^{IJ}_{M_IM_J}(1,2)$ denotes the wave function of a meson comprised of a quark and antiquark with quantum numbers IJ^P and can be expressed as:

$$\Psi^{IJ}_{M_IM_J}(1,2)=\sum_\alpha C_\alpha\left[\psi_l(\mathbf{r})\chi_s(1,2)\right]^J_{M_J}\omega^c(1,2)\phi^I_{M_I}(1,2), \tag{4}$$

where $\psi_l(\mathbf{r})$, $\chi_s(1,2)$, $\omega^c(1,2)$, and $\phi^I(1,2)$ denote orbit, spin, color, and flavor wave functions, respectively. Furthermore, α denotes the intermediate quantum numbers, l,s and potential flavor indices. In our calculations, the orbital wave functions can be expanded using a set of Gaussians as follows:

$$\psi_{lm}(\mathbf{r})=\sum_{n=1}^{n_{\max}}c_n\psi^G_{nlm}(\mathbf{r}), \tag{5a}$$

$$\psi^G_{nlm}(\mathbf{r}) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}), \tag{5b}$$

where the Gaussian size parameters are selected according to the following geometric progression.

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{\max}}}{r_1}\right)^{\frac{1}{n_{\max}-1}}. \tag{6}$$

This procedure enables optimization of the ranges using only a small number of Gaussians.

Hence, the wave function in Eq. (4) can be expressed as follows:

$$\Psi^{IJ}_{M_I M_J}(1,2) = \sum_{n\alpha} C_\alpha c_n \left[\psi^G_{nl}(\mathbf{r})\chi_s(1,2)\right]^J_{M_J} \omega^c(1,2)\phi^I_{M_I}(1,2). \tag{7}$$

We employ Rayleigh-Ritz variational principle for solving the Schrödinger equation, which leads to a generalized eigenvalue problem due to the non-orthogonality of Gaussians

$$\sum_{n',\alpha'}(H^{IJ}_{n\alpha,n'\alpha'} - E^{IJ} N^{IJ}_{n\alpha,n'\alpha'})C^{IJ}_{n'\alpha'} = 0, \tag{8a}$$

$$H^{IJ}_{n\alpha,n'\alpha'} = \langle \Phi^{IJ}_{M_I M_J,n\alpha} | H | \Phi^{IJ}_{M_I M_J,n'\alpha'} \rangle, \tag{8b}$$

$$N^{IJ}_{n\alpha,n'\alpha'} = \langle \Phi^{IJ}_{M_I M_J,n\alpha} | 1 | \Phi^{IJ}_{M_I M_J,n'\alpha'} \rangle, \tag{8c}$$

with $\Phi^{IJ}_{M_I M_J,n\alpha} = [\psi^G_{nl}(\mathbf{r})\chi_s(1,2)]^J_{M_J}\omega^c(1,2)\phi^I_{M_I}(1,2)$, $C^{IJ}_{n\alpha} = C_\alpha c_n$.

Furthermore, we obtain the mass of the four-quark system by solving the Schrödinger equation:

$$H\Psi^{IJ}_{M_I M_J}(4q) = E^{IJ}\Psi^{IJ}_{M_I M_J}(4q), \tag{9}$$

where $\Psi^{IJ}_{M_I M_J}(4q)$ denotes the wave function of the four-quark system, which can be constructed as follows. In our calculations, we only consider the color singlet-singlet meson-meson picture for the four quark system. First, we express the wave functions of two meson clusters,

$$\Psi^{I_1 J_1}_{M_{I_1} M_{J_1}}(1,2) = \sum_{\alpha_1 n_1} C^{\alpha_1}_{n_1} \times \left[\psi^G_{n_1 l_1}(\mathbf{r}_{12})\chi_{s_1}(1,2)\right]^{J_1}_{M_{J_1}} \omega^{c_1}(1,2)\phi^{I_1}_{M_{I_1}}(1,2), \tag{10a}$$

$$\Psi^{I_2 J_2}_{M_{I_2} M_{J_2}}(3,4) = \sum_{\alpha_2 n_2} C^{\alpha_2}_{n_2} \times \left[\psi^G_{n_2 l_2}(\mathbf{r}_{34})\chi_{s_2}(3,4)\right]^{J_2}_{M_{J_2}} \omega^{c_2}(3,4)\phi^{I_2}_{M_{I_2}}(3,4), \tag{10b}$$

Then, the total wave function of the four-quark state can be expressed as:

$$\begin{aligned}\Psi^{IJ}_{M_I M_J}(4q) &= \mathcal{A}\sum_{L_r}\left[\Psi^{I_1 J_1}(1,2)\Psi^{I_2 J_2}(3,4)\psi_{L_r}(\mathbf{r}_{1234})\right]^{IJ}_{M_I M_J} \\ &= \sum_{\alpha_1\alpha_2 n_1 n_2 L_r} C^{\alpha_1}_{n_1} C^{\alpha_2}_{n_2}\left[\left[\psi^G_{n_1 l_1}(\mathbf{r}_{12})\chi_{s_1}(1,2)\right]^{J_1}\right. \\ &\quad \times \left.\left[\psi^G_{n_2 l_2}(\mathbf{r}_{34})\chi_{s_2}(3,4)\right]^{J_2}\psi_{L_r}(\mathbf{r}_{1234})\right]^J_{M_J} \\ &\quad \times [\omega^{c_1}(1,2)\omega^{c_2}(3,4)]^{[1]}\left[\phi^{I_1}(1,2)\phi^{I_2}(3,4)\right]^I_{M_I},\end{aligned} \tag{11}$$

Here, $\mathcal{A}$ denotes the antisymmetrization operator, if all quarks (antiquarks) are considered as identical particles, then

$$\mathcal{A} = \frac{1}{2}(1 - P_{13} - P_{24} + P_{13}P_{24}). \tag{12}$$

$\psi_{L_r}(\mathbf{r}_{1234})$ denotes the relative wave function between two clusters, which is also expanded in a set of Gaussians. L_r denotes the relative orbital angular momentum.

The Hamiltonian of the chiral quark model for the four-quark system comprises three parts: quark rest mass, kinetic energy, and potential energy (four-quark system is taken as an example):

$$\begin{aligned}H &= \sum_{i=1}^4 m_i + \frac{p_{12}^2}{2\mu_{12}} + \frac{p_{34}^2}{2\mu_{34}} + \frac{p_r^2}{2\mu_r} \\ &\quad + \sum_{i<j=1}^4\left(V^C_{\rm CON}(\boldsymbol{r}_{ij}) + V^C_{\rm OGE}(\boldsymbol{r}_{ij})\right. \\ &\quad \left. + V^{\rm SO}_{\rm CON}(\boldsymbol{r}_{ij}) + V^{\rm SO}_{\rm OGE}(\boldsymbol{r}_{ij}) + \sum_{\chi=\pi,K,\eta} V^\chi_{ij} + V^\sigma_{ij}\right).\end{aligned} \tag{13}$$

Where m_i denotes the constituent mass of ith quark (antiquark). $\frac{\mathbf{p}^2_{\mathbf{ij}}}{2\mu_{ij}}$ $(ij = 12;34)$ and $\frac{\mathbf{p}^2_{\mathbf{r}}}{2\mu_r}$ denote the inner kinetic of two clusters and relative motion kinetic between two clusters, respectively, where

$$\mathbf{p}_{12} = \frac{m_2\mathbf{p}_1 - m_1\mathbf{p}_2}{m_1 + m_2}, \tag{14a}$$

$$\mathbf{p}_{34} = \frac{m_4\mathbf{p}_3 - m_3\mathbf{p}_4}{m_3 + m_4}, \tag{14b}$$

$$\mathbf{p}_r = \frac{(m_3+m_4)\mathbf{p}_{12}-(m_1+m_2)\mathbf{p}_{34}}{m_1+m_2+m_3+m_4}, \tag{14c}$$

$$\mu_{ij} = \frac{m_i m_j}{m_i+m_j}, \tag{14d}$$

$$\mu_r = \frac{(m_1+m_2)(m_3+m_4)}{m_1+m_2+m_3+m_4}. \tag{14e}$$

$V^C_{\rm CON}$ and $V^C_{\rm OGE}$ denote the central parts of the confinement and one-gluon-exchange. $V^{\rm SO}_{\rm CON}$ and $V^{\rm SO}_{\rm OGE}$ denote the spin-orbit interaction potential energy. In our calculations, a quadratic confining potential is adopted. For the mesons, the distance between q and $\bar{q}$ is relatively small. Hence, the difference between the linear potential and quadratic potential is very small by adjusting the confinement strengths. Both of them can conform to the linear Regge trajectories for $q\bar{q}$ mesons. $V^{\chi=\pi,K,\eta}_{ij}$, and σ exchange represents the one Goldstone boson exchange. Chiral symmetry suggests dividing quarks into two different sectors: light quarks (u, d and s), where the chiral symmetry is spontaneously broken, and heavy quarks (c and b), where the symmetry is explicitly broken. The origin of the constituent quark mass can be traced back to the spontaneous breaking of chiral symmetry and consequently constituent quarks should interact through the exchange of Goldstone bosons. The detailed derivation process has been determined in several theoretical papers [43, 44]. Here, we only show the expressions of these potentials to save space.

The detailed expressions of the potentials are [45]:

$$V^C_{\rm CON}(\boldsymbol{r}_{ij}) = (-a_c r^2_{ij}-\Delta)\boldsymbol{\lambda}^c_i\cdot\boldsymbol{\lambda}^c_j, \tag{15a}$$

$$V^{\rm SO}_{\rm CON}(\boldsymbol{r}_{ij}) = \boldsymbol{\lambda}^c_i\cdot\boldsymbol{\lambda}^c_j\cdot\frac{-a_c}{2m_i^2m_j^2}\left\{\Big((m_i^2+m_j^2)(1-2a_s) + 4m_im_j(1-a_s)\Big)(\boldsymbol{S}_+\cdot\boldsymbol{L})+(m_j^2-m_i^2)(1-2a_s)(\boldsymbol{S}_-\cdot\boldsymbol{L})\right\}, \tag{15b}$$

$$V^C_{\rm OGE}(\boldsymbol{r}_{ij}) = \frac{\alpha_s}{4}\boldsymbol{\lambda}^c_i\cdot\boldsymbol{\lambda}^c_j\left[\frac{1}{r_{ij}}-\frac{2\pi}{3m_im_j}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\delta(\boldsymbol{r}_{ij})\right], \tag{15c}$$

$$V^{\rm SO}_{\rm OGE}(\boldsymbol{r}_{ij}) = -\frac{1}{16}\cdot\frac{\alpha_s}{m_i^2m_j^2}\boldsymbol{\lambda}^c_i\cdot\boldsymbol{\lambda}^c_j\left\{\frac{1}{r^3_{ij}}-\frac{{\rm e}^{-r_{ij}/r_g(\mu)}}{r^3_{ij}}\cdot\left(1+\frac{r_{ij}}{r_g(\mu)}\right)\right\}\left\{\Big((m_i+m_j)^2+2m_im_j\Big)(\boldsymbol{S}_+\cdot\boldsymbol{L})+(m_j^2-m_i^2)(\boldsymbol{S}_-\cdot\boldsymbol{L})\right\}, \tag{15d}$$

$$\delta(\boldsymbol{r}_{ij}) = \frac{{\rm e}^{-r_{ij}/r_0(\mu_{ij})}}{4\pi r_{ij}r_0^2(\mu_{ij})}, \mathbf{S}_\pm = \mathbf{S}_1\pm\mathbf{S}_2, \tag{15e}$$

$$V_\pi(\boldsymbol{r}_{ij}) = \frac{g^2_{ch}}{4\pi}\frac{m^2_\pi}{12m_im_j}\frac{\Lambda^2_\pi}{\Lambda^2_\pi-m^2_\pi}m_\pi v^\pi_{ij}\sum_{a=1}^3\lambda^a_i\lambda^a_j, \tag{15f}$$

$$V_K(\boldsymbol{r}_{ij}) = \frac{g^2_{ch}}{4\pi}\frac{m^2_K}{12m_im_j}\frac{\Lambda^2_K}{\Lambda^2_K-m^2_K}m_K v^K_{ij}\sum_{a=4}^7\lambda^a_i\lambda^a_j, \tag{15g}$$

$$V_\eta(\boldsymbol{r}_{ij}) = \frac{g^2_{ch}}{4\pi}\frac{m^2_\eta}{12m_im_j}\frac{\Lambda^2_\eta}{\Lambda^2_\eta-m^2_\eta}m_\eta v^\eta_{ij}\times\left[\lambda^8_i\lambda^8_j\cos\theta_P-\lambda^0_i\lambda^0_j\sin\theta_P\right], \tag{15h}$$

$$v^\chi_{ij}(\boldsymbol{r}_{ij}) = \left[Y(m_\chi r_{ij})-\frac{\Lambda^3_\chi}{m^3_\chi}Y(\Lambda_\chi r_{ij})\right]\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j, \tag{15i}$$

$$V_\sigma(\boldsymbol{r}_{ij}) = -\frac{g^2_{ch}}{4\pi}\frac{\Lambda^2_\sigma}{\Lambda^2_\sigma-m^2_\sigma}m_\sigma\times\left[Y(m_\sigma r_{ij})-\frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r_{ij})\right], \tag{15j}$$

where $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ denote the spin of the two meson clusters. $Y(x)={\rm e}^{-x}/x$; $r_0(\mu_{ij})=s_0/\mu_{ij}$; $\boldsymbol{\sigma}$ denote $SU(2)$ Pauli matrices; $\boldsymbol{\lambda}$, $\boldsymbol{\lambda}^c$ denote $SU(3)$ flavor, color Gell-Mann matrices, respectively. The form factor parameter Λ_χ $(\chi=\pi,K,\eta,\sigma)$ is introduced to remove the short-range contribution of Goldstone bosons exchanges. Furthermore, $g^2_{ch}/4\pi$ denotes the chiral coupling constant, determined from the π-nucleon coupling. Additionally, α_s denotes an effective scale-dependent running coupling [45],

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln\left[(\mu^2_{ij}+\mu^2_0)/\Lambda^2_0\right]}. \tag{16}$$

In our calculations for the two-quark system, the central and noncentral potential energies are included. However, in the four-quark system calculations, we observe that the influence of the noncentral potential energy on the mass shift of the state is minimal, and thereby, it is omitted.

Finally, we show the model parameters in Table 1. In the table, $\theta_p(^\circ)$ equals -15. The angle θ_p is the mixing angle between η_1 and η_8. $|\eta_1\rangle=\cos(\theta_p)|\eta_1\rangle+\sin(\theta_p)|\eta_8\rangle$, $|\eta'\rangle=\sin(\theta_p)|\eta_1\rangle+\cos(\theta_p)|\eta_8\rangle$, with $|\eta_1\rangle=(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$ and $|\eta_8\rangle=(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$. Furthermore, Λ_0 parameter is an adjustable parameter to parameterize the running coupling constant, and it is not related to $\Lambda_{\rm QCD}$. As stated in Ref. [20], the usual one-loop expression of the running coupling constant diverges when $Q\to\Lambda_{\rm QCD}$. Hence, the effective formula of the scale-dependent strong coupling constant is used in chiral quark model. It should be noted that, as reported in Ref. [20], the confinement item takes the screened form $V_{ij}^C = \left(-a_c(1-\mathrm{e}^{-\mu_c r_{ij}})+\Delta\right)(\lambda_i^c\cdot\lambda_j^c)$, and in our present calculations, the usual quadratic confinement $V_{ij}^C=(-a_c r_{ij}^2-\Delta)\lambda_i^c\cdot\lambda_j^c$ is employed. Hence, some parameters, such as quark mass, a_c, and Δ, differ. In the nonrelativistic valence quark model, using the model parameters, we calculated the masses of certain mesons from light to heavy, and the results are shown in the third column of Table 3. It can be shown that most of the ground-state mesons are consistent with the experiment values. However, for some excited charmonium states, the quark model cannot describe them very well.

III. TRANSITION OPERATOR

The $^3P_0$ quark-pair creation model [47–49] has been widely applied to OZI-rule-allowed two-body strong decays of hadrons [50–55]. If the quark and antiquark in the original meson are labeled by 1, 2, and the quark and antiquark ($u\bar{u}$, $d\bar{d}$, $s\bar{s}$) generated in the vacuum are numbered as 3, 4, then the transition operator of $^3P_0$ model can be expresssed:

$$T_0 = -3\gamma\sum_m\langle 1m1(-m)|00\rangle\int \mathrm{d}\mathbf{p}_3\mathrm{d}\mathbf{p}_4\delta^3(\mathbf{p}_3+\mathbf{p}_4)\times\mathcal{Y}_1^m\left(\frac{\mathbf{p}_3-\mathbf{p}_4}{2}\right)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34}b_2^\dagger(\mathbf{p}_3)d_3^\dagger(\mathbf{p}_4), \tag{17}$$

where, $\chi_{1-m}^{34},\phi_0^{34},\omega_0^{34}$ denote spin, flavor, and color wave functions of the created quark pair, respectively. Furthermore, $\mathcal{Y}_1^m(\frac{\mathbf{p}_3-\mathbf{p}_4}{2}) = pY_1^m(\hat{\mathbf{p}})$ is the solid spherical harmonics. Additionally, γ describes the probability for creating a quark-antiquark pair with momenta $\mathbf{p}_3$ and $\mathbf{p}_4$ from the vacuum. It is normally determined by fitting the strong decay widths of hadrons. This yields $\gamma=6.95$ for $u\bar{u}$ and $d\bar{d}$ pair creation, and $\gamma=6.95/\sqrt{3}$ for $s\bar{s}$ pair creation [56].

To reduce the mass shift due to the coupled-channel effects, the transition operator in Eq. (17) should be modified. In Ref. [40], two suppression factors are introduced, namely energy damping factor and distance damping factor. The first factor is $\exp[-r^2/(4f^2)]$ ($\exp[-f^2p^2]$ in momentum space. Specifically, $\mathbf{r}=\mathbf{r_3}-\mathbf{r_4}$ is the distance between the quark and antiquark created in the vacuum, considering the effect of quark-antiquark energy created in the vacuum and it suppresses the contribution from meson-meson states with high energy. Furthermore, when the distance between the bare meson and a pair of charmed mesons becomes smaller, the energy of tetraquark will increase, and the momentum of the created quark (antiquark) will be high. At this point, the energy damping factor $\exp[-f^2p^2]$ comes into play. Hence, the mass shift of the charmed mesons is still suppressed and the convergence is guaranteed. The second factor is $\exp[-R_{AV}^2/R_0^2]$, which considers the effect that the created quark-antiquark pair should not be far away from the source meson. Here, R_{AV} represents the distance between the created quark-antiquark pair and source meson. It can be expressed as:

$$\mathbf{R_{AV}} = \mathbf{R_A}-\mathbf{R_V}; \tag{18a}$$

$$\mathbf{R_A} = \frac{m_1\mathbf{r_1}+m_2\mathbf{r_2}}{m_1+m_2}; \tag{18b}$$

$$\mathbf{R_V} = \frac{m_3\mathbf{r_3}+m_4\mathbf{r_4}}{m_3+m_4} = \frac{\mathbf{r_3}+\mathbf{r_4}}{2}\quad (m_3=m_4). \tag{18c}$$

Hence, the modified transition operator can be expressed as:

$$T_1 = -3\gamma\sum_m\langle 1m1(-m)|00\rangle\int \mathrm{d}\mathbf{r_3}\mathrm{d}\mathbf{r_4}\left(\frac{1}{2\pi}\right)^{\frac{3}{2}}\mathrm{i}r2^{-\frac{5}{2}}f^{-5}\, Y_{1m}(\hat{\mathbf{r}})\mathrm{e}^{-\frac{r^2}{4f^2}}\mathrm{e}^{-\frac{R_{AV}^2}{R_0^2}}\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34}b_3^\dagger(\mathbf{r_3})d_4^\dagger(\mathbf{r_4}), \tag{19}$$

By fitting the decay width of $\rho\to\pi\pi$ and with the requirement that the mass shift is approximately 10% of the bare mass, parameters f, R_0, and γ were fixed,

$$\gamma=32.2,\qquad f=0.5\,\mathrm{fm},\qquad R_0=1\,\mathrm{fm}. \tag{20}$$

IV. NUMERICAL RESULTS

In UQM, we obtain the eigenvalues of systems (quark-antiquark plus four-quark components) by solving the Schrödinger equation as follows:

$$H\Psi=E\Psi, \tag{21}$$

where Ψ and H denote the wave function and the Hamiltonian of the system, respectively. It can be expressed as:

$$\Psi=c_1\Psi_{2q}+c_2\Psi_{4q}\,, \tag{22}$$

$$H=H_{2q}+H_{4q}+T\,. \tag{23}$$

Specifically, H_{2q} only acts on the wave function of two-quark system, Ψ_{2q}, and H_{4q} only acts on the wave function of four-quark system, Ψ_{4q}. The transition operator T is responsible for mixing the quark-antiquark and four-quark components.

In this manner, we can obtain the matrix elements of the Hamiltonian as follows:

$$\langle\Psi|H|\Psi\rangle = \langle c_1\Psi_{2q}+c_2\Psi_{4q}|H|c_1\Psi_{2q}+c_2\Psi_{4q}\rangle = c_1^2\langle\Psi_{2q}|H_{2q}|\Psi_{2q}\rangle + c_2^2\langle\Psi_{4q}|H_{4q}|\Psi_{4q}\rangle + c_1c_2^*\langle\Psi_{4q}|T|\Psi_{2q}\rangle + c_1^*c_2\langle\Psi_{2q}|T^\dagger|\Psi_{4q}\rangle, \quad (24)$$

Furthermore, block-matrix structure for the Hamiltonian and overlap can be expressed as:

$$(H) = \begin{bmatrix} (H_{2q}) & (H_{24}) \\ (H_{42}) & (H_{4q}) \end{bmatrix}, (N) = \begin{bmatrix} (N_{2q}) & (0) \\ (0) & (N_{4q}) \end{bmatrix}, \quad (25)$$

with

$$(H_{2q}) = \langle\Psi_{2q}|H_{2q}|\Psi_{2q}\rangle, \quad (26)$$

$$(H_{24}) = \langle\Psi_{4q}|T|\Psi_{2q}\rangle, \quad (27)$$

$$(H_{4q}) = \langle\Psi_{4q}|H_{4q}|\Psi_{4q}\rangle, \quad (28)$$

$$(N_{2q}) = \langle\Psi_{2q}|1|\Psi_{2q}\rangle, \quad (29)$$

$$(N_{4q}) = \langle\Psi_{4q}|1|\Psi_{4q}\rangle. \quad (30)$$

Where (H_{2q}) and (H_{4q}) denote the matrix for the pure two-quark system and pure four-quark system, respectively, and (N_{2q}) and (N_{4q}) are their respective overlap matrices. Furthermore, (H_{24}) denotes the coupling matrix of two-quark system and four-quark system.

Finally, the eigenvalues (E_n) and eigenvectors (C_n) of the system are obtained by solving the generalized eigenproblem as follows:

$$\left[\ (H)-E_n(N)\ \right]\left[\ C_n\ \right]=0. \quad (31)$$

By employing the model parameters in Table 1 and considering the original transition operator T_0 in Eq. (17), we calculated the mass shifts for the light ground-state mesons (π,ρ,ω,η) [40] and some charmonium $c\bar{c}$ states [42] in our previous study. The results show that for the light ground-state mesons, the coupled-channel effects generate alarmingly large negative mass shifts, and the average value is approximately 2000 MeV. For $c\bar{c}$, the mass shifts in [42] vary among states, and the average is approximately 500 MeV. This type of large mass shift will challenge the validity of the valence quark model as a good zeroth order approximation in describing the low-lying hadron spectrum.

Therefore, we introduced modifications to the transition operator to develop a more realistic unquenching procedure. By adopting the modified transition operator in Eq. (19), we also demonstrated our new mass shifts for the light ground-state mesons (π,ρ,ω,η) [40] and for some charmonium $c\bar{c}$ states [42]. For the light ground-state mesons, the mass shifts have been reduced to be approximately 10%−25% of a given meson's bare mass, and for $c\bar{c}$, the unquenching correction is only 1%−4% of a given meson's bare mass. Hence, the effects of including energy damping factor and distance damping factor on the mass shifts are relatively stable.

Table 1. Model parameters, determined by fitting the meson spectrum, leaving room for unquenching contributions in the case of light-quark systems.

Quark masses/MeV	$m_u=m_d$	313
	m_s	536
	m_c	1728
	m_b	5112
Goldstone bosons ($\text{fm}^{-1}\sim 200\,\text{MeV}$)	m_π	0.70
	m_σ	3.42
	m_η	2.77
	m_K	2.51
	$\Lambda_\pi=\Lambda_\sigma$	4.2
	$\Lambda_\eta=\Lambda_K$	5.2
	$g_{ch}^2/(4\pi)$	0.54
	$\theta_p/(^\circ)$	−15
Confinement	$a_c/(\text{MeV fm}^{-2})$	101
	Δ/MeV	−78.3
OGE	α_0	3.67
	Λ_0/fm^{-1}	0.033
	μ_0/MeV	36.98
	s_0/MeV	28.17

In our previous study [40, 42], we made minor adjustments to few parameters, which increased the bare masses of mesons. After considering the effects of coupled channels, the mass shifts resulted in a reduction of the state masses, which were then compared with experimental data. However, in the current study, we adjusted almost all the model parameters (except the parameters related to Goldstone boson exchange) for recalculating the masses of the light quark states and charmonium states in the unquenched quark model and comparing them with experimental results. Hence, the aim was to perform a realistic calculation of meson spectra.

Then, by fitting the experimental data of the low-lying mesons (π, ρ, ω, η, $\eta_c(1S)$, $\eta_c(2S)$, $J/\psi(1S)$, $J/\psi(2S)$, $\chi_{cJ}(1P)\,(J=0,1,2)$, $h_c(1P)$, totally twelve mesons), the model parameters are determined and provided in Table 2. The parameters related to the confinement, one-gluon-

Table 2. Adjusted quark model parameters.

Quark masses/MeV	$m_u = m_d$	361
	m_s	477
	m_c	1700
	m_b	5112
Goldstone bosons ($\text{fm}^{-1} \sim 200$ MeV)	m_π	0.70
	m_σ	3.42
	m_η	2.77
	m_K	2.51
	$\Lambda_\pi = \Lambda_\sigma$	4.2
	$\Lambda_\eta = \Lambda_K$	5.2
	$g_{ch}^2/(4\pi)$	0.54
	$\theta_p/(^\circ)$	−15
Confinement	a_c/(MeV fm^{-2})	120
	Δ/MeV	−53
OGE	α_{uu}	0.72
	α_{us}	0.75
	α_{cc}	0.39
	α_{cu}	0.44
	α_{cs}	0.38
	s_0/MeV	34

exchange potentials, and the masses of quarks are all re-adjusted. However, the model parameters related to the Goldstone boson exchange potentials are unchanged. In the nonrelativistic valence quark model, using the new adjusted parameters in Table 2, we re-calculated the masses of some mesons from light to heavy and listed them in the fourth column (M_2) of Table 3. By comparing the values in third (M_1) and fourth columns(M_2), we observed that with the new quark model parameters, the masses of mesons become larger and new theoretical threshold information is provided.

With the new quark model parameters, as well as the modified transition operator T_1 in Eq. (19), we calculated the mass shifts of the light ground-state mesons and some charmonia, and the results are provided in Table 4 and Table 5. In the tables, for the selection of tetraquark channels, some factors, such as parity conservation, conservation of angular momentum, isospin conservation, and exchange symmetry, should be considered. For the exchange symmetry, it only applies to the identical quarks.

From Table 6, we can determine that with the new set of the quark model parameters, the bare masses of the light ground-state mesons are increased. However, the mass shifts are not very sensitive to these model parameters when compared with our previous study [40]. Eventually, the unquenched masses of mesons are effectively consistent with the experimental values.

Table 3. Mass spectrum in the chiral quark model in comparison with the experimental data [46]. M_1 represents the mass spectrum with the model parameters in Table 1, and M_2 represents the mass spectrum with the new adjusted quark model parameters in Table 2 (unit: MeV).

Name	$J^{P(C)}$	M_1	M_2	PDG [46]
π	0^-	134.9	182.6	135.0
K	0^-	489.4	242.6	493.7
ρ	1^{--}	772.3	922.6	775.3
K^*	1^-	913.6	980.9	892.0
ω	1^{--}	701.6	852.4	782.7
η	0^{-+}	669.2	738.7	547.9
$\phi(1020)$	1^{--}	1015.9	1117.9	1019.5
D^0	0^-	1861.9	2065.2	1864.8
D^{*0}	1^-	1980.6	2162.5	2006.9
D_s^+	0^-	1950.1	2147.8	1968.4
D_s^{*+}	1^-	2079.9	2231.6	2112.2
B^-	0^-	5280.7	5462.9	5279.3
B^*	1^-	5319.6	5501.6	5324.7
B_s^0	0^-	5367.4	5503.4	5366.9
B_s^*	1^-	5410.2	5543.9	5415.4
$\eta_c(1S)$	0^{-+}	2964.4	3063.4	2983.9
$\eta_c(2S)$	0^{-+}	3507.8	3651.2	3637.5
J/ψ	1^{--}	3096.4	3187.7	3096.0
$\psi(2S)$	1^{--}	3605.0	3744.4	3686.1
$\chi_{c_0}(1P)$	0^{++}	3362.8	3471.3	3414.7
$\chi_{c_0}(2P)$	0^{++}	3814.7	3966.7	$\chi_{c_0}(3915)$?
$\chi_{c_1}(1P)$	1^{++}	3393.9	3509.3	3510.7
$\chi_{c_1}(2P)$	1^{++}	3851.9	4011.3	$\chi_{c_1}(3872)$?
$\chi_{c_2}(1P)$	2^{++}	3435.8	3559.2	3556.2
$\chi_{c_2}(2P)$	2^{++}	3901.1	4068.9	$\chi_{c_2}(3930)$?
$h_c(1P)$	1^{+-}	3416.1	3535.2	3525.4
$h_c(2P)$	1^{+-}	3877.4	4040.4	$Z_c(3900)$?

An open channel exists in our calculations. The mass of ρ meson is larger than the sum of masses of two pions and it can decay to $\pi\pi$. For the open channel, which implies that final state energy is lower than the bare mass of the meson, the mass shift of the state will change with the Gaussian distribution of the relative motion between two mesons. Specifically, we determine that the mass shift will periodically change with an increase in spatial volume. In our calculations, we selected the biggest mass shift as the contribution of this open channel. For $\pi\pi$ state, it is a scattering one with discrete energy levels that vary with the Gaussian distribution in the theoretical calculations due to the limitation of finite volume. When

Table 4. Mass shifts (unit: MeV) computed for non-strange mesons with quantum numbers $IJ^{-}(I=0,1;J=0,1)$ with new quark model parameters and modified transition operator T_1 in Eq. (19): $f=0.5$ fm, $\gamma=32.2$, $R_0=1$ fm. (η is an isospin 0 partner to the pion.)

(IJ^P)	$\pi(10^-)$	$\rho(11^-)$	$\omega(01^-)$	$\eta(00^-)$
bare mass (Theo.)	182.6	922.6	852.4	738.7
$\pi\pi$	...	−32.9	...	...
$\pi\rho$	−13.9	...	−37.2	...
$\pi\omega$	...	−15.1	...	...
$\eta\rho$	...	−9.7	...	...
$\rho\rho$	...	−31.4	...	−32.8
$\rho\omega$	−11.9	...	...	...
$\eta\omega$	...	...	−9.2	...
$\omega\omega$	...	...	...	−11.5
$K\bar{K}$	...	−6.0	−3.6	...
$K\bar{K}^{\star}(\bar{K}K^{\star})$	−4.1	−8.4	−7.4	−9.2
$K^{\star}\bar{K}^{\star}$	−7.3	−21.5	−19.9	−14.5
Total mass shift	−37.2	−125.0	−77.3	−68.0
Unquenched mass	145.4	797.6	775.1	670.7
Exp	139	772	782	547

considering the coupling of $\pi\pi$ and ρ, the strength of coupling will be increased as one of the energy of $\pi\pi$ state is close to that of ρ, and the induced mass shift will increase. We consider the biggest one as the mass shift of state ρ to $\pi\pi$ state. Furthermore, if we expand the space further with higher r_n values, the same biggest mass shift will be repeated. Based on the table, we can also determine that for the open channel $\pi\pi$, the mass shift is larger than the other close channels. In Table 3, we determine that M_2 of the $c\bar{c}$ states are below 4100 MeV, and the sums of the masses of D and $D^{(*)}$ are larger than 4100 MeV. Hence, for charmonium mesons, no open channels exist with the new adjusted quark model parameters. In the case of these processes (close channel), the mass shifts will not vary with an increase in space.

The results for the mass spectrum of the $c\bar{c}$ charmonium mesons with the new quark model parameters are shown in Table 5. For the ground-state $\eta_c(1S)$ and $J/\psi(1S)$, the experimental masses are fitted well. For the excited $\eta_c(2S)$ and $J/\psi(2S)$, the theoretical unquenched masses are close to the experiment values. The fitted masses for $\chi_{c_J}(1P)(J=0,1,2)$ and $h_c(1P)$ are slightly bigger, approximately 90 MeV. This improves for $\chi_{c_J}(2P)$ $(J=0,1,2)$ and $h_c(2P)$. Specifically, for $\chi_{c_1}(2P)$, the unquenched mass is 3876.6 MeV, which is good agreement with experimental value of the exotic state $X(3872)$. Furthermore, in our study, $\chi_{c_2}(2P)$ has a theoretical mass of 3919.5 MeV, and the mass is very close to the exotic state $\chi_{c_2}(3930)$. For higher charmonium $1D$ states, for example, the mass of 1^1D_2 is approximately 3790.4 MeV.

Table 5. Mass shifts computed for $c\bar{c}$ charmonium mesons with the new quark model parameters and the modified transition operator T_1 in Eq. (19): $f=0.5$ fm, $\gamma=32.2$, $R_0=1$ fm. (Units of MeV)

Bare $c\bar{c}$ state			Mass shifts by channels									$c\bar{c}+qq\bar{q}\bar{q}$
State$(n^{2S+1}L_J)$	Bare mass	Exp	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$	$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$	Total	Unquenched mass
$\eta_c(1S)(1^1S_0)$	3063.4	2983.9	...	−11.8	−11.8	−22.7	...	−2.9	−2.9	−5.7	−57.8	3005.6
$\eta_c(2S)(2^1S_0)$	3651.2	3637.5	...	−22.1	−22.1	−40.4	...	−4.0	−4.0	−7.7	−100.3	3550.9
$J/\psi(1S)(1^3S_1)$	3187.7	3096.0	−4.7	−9.1	−9.1	−30.8	−1.1	−2.2	−2.2	−7.6	−66.8	3120.9
$\psi(2S)(2^3S_1)$	3744.4	3686.1	−9.1	−16.5	−16.5	−52.4	−1.5	−2.9	−2.9	−9.8	−111.6	3632.8
$\chi_{c_0}(1P)(1^3P_0)(S+D)$	3471.3	3414.7	−13.6	...	...	−56.8	−2.7	...	...	−12.5	−85.6	3385.7
$\chi_{c_1}(1P)(1^3P_1)(S+D)$	3509.3	3510.7	...	−16.5	−16.5	−41.9	...	−3.4	−3.4	−9.2	−90.9	3418.4
$\chi_{c_2}(1P)(1^3P_2)(S+D)$	3559.2	3556.2	−10.0	−14.1	−14.1	−42.7	−2.1	−3.0	−3.0	−9.0	−98.0	3461.2
$h_c(1P)(1^1P_1)(S+D)$	3535.2	3525.4	...	−20.0	−20.0	−37.3	...	−4.1	−4.1	−8.2	−93.7	3441.5
$\chi_{c_0}(2P)(2^3P_0)(S+D)$	3966.7	$\chi_{c0}(3915)$?	−29.0	...	...	−78.9	−3.7	...	...	−13.8	−125.4	3841.3
$\chi_{c_1}(2P)(2^3P_1)(S+D)$	4011.3	$\chi_{c_1}(3872)$?	...	−29.0	−29.0	−58.2	...	−4.2	−4.2	−10.1	−134.7	3876.6
$\chi_{c_2}(2P)(2^3P_2)(S+D)$	4068.9	$\chi_{c_2}(3930)$?	−18.2	−22.4	−22.4	−66.4	−2.5	−3.5	−3.5	−10.5	−149.4	3919.5
$h_c(2P)(2^1P_1)(S+D)$	4040.4	$Z_c(3930)$?	...	−33.1	−33.1	−56.1	...	−4.9	−4.9	−9.3	−141.4	3899.0
$\eta_{c_2}(1D)(1^1D_2)$	3824.9	?	...	−7.9	−7.9	−13.6	...	−1.3	−1.3	−2.5	−34.5	3790.4
$\psi(1D)(1^3D_1)$	3799.8	$\psi(3770)$?	−14.5	−6.2	−6.2	−4.3	−2.3	−1.1	−1.1	−0.8	−36.5	3763.3
$\psi_2(1D)(1^3D_2)$	3817.1	$\psi_2(3823)$?	...	−11.6	−11.6	−6.9	...	−2.0	−2.0	−1.2	−35.3	3781.8
$\psi_3(1D)(1^3D_3)$	3839.8	$\psi_3(3842)$?	...	...	...	−28.4	...	...	...	−5.0	−33.4	3806.4

Table 6. Mass shifts computed for $\chi_{c_1}(2P)$ state when the relative angular momentum L_r is in S-wave and D-wave. (Units of MeV)。

Bare $c\bar{c}$ state			Mass shifts by channels									$c\bar{c}+qq\bar{q}\bar{q}$
State$(n^{2S+1}L_J)$	Bare mass	Exp	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$	$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$	Total	Unquenched mass
$\chi_{c_1}(2P)(2^3P_1)(S)$	4011.3	$\chi_{c_1}(3872)$?	...	−17.5	−17.5	...	...	−2.4	−2.4	...	−39.8	
$\chi_{c_1}(2P)(2^3P_1)(D)$	4011.3	$\chi_{c_1}(3872)$?	...	−11.5	−11.5	-58.2	...	−1.8	−1.8	-10.1	−94.9	3876.6

In future, we look forward to having a more experimental data about this state. Furthermore, $\psi(3770)$ is also described very well in the unquenched quark model, and it can be a good candidate of 1^3D_1 state, with the unquenched mass as 3763.3 MeV. Furthermore, the charmonium states $\psi_2(3823)$ and $\psi_3(3842)$ are very likely candidates of 1^3D_2 and 1^3D_3, with the theoretical unquenched masses corresponding to 3781.8 and 3806.4 MeV, respectively.

It should be noted that in our calculations, the angular momentum for the two mesons, l_1 and l_2 in Eq. (10), equals zero. With respect to the charmonium $1S$, $2S$, and $1D$ states, the relative angular momentum L_r between two mesons in Eq. (11) may equal 1 (P wave) or 3 (F wave) by considering the parity conservation. Here, we only consider L_r as corresponding to 1 for a simplification. For $1P$ and $2P$ states, we not only consider $L_r=0$, but also $L_r=2$. In Table 6, we demonstrated the mass shifts of $\chi_{c_1}(2P)$ state when the relative angular momentum L_r is in S-wave and D-wave, respectively. Based on the table, it can be observed that the mass shifts from D-wave $D\bar{D}^*$ and $D_s\bar{D}_s^*$ are smaller than that from corresponding S-wave states. For the contributions from D-wave, $D^*\bar{D}^*$ and $D_s^*\bar{D}_s^*$ are larger than the corresponding S-wave states. Other $1P$ and $2P$ states also follow similar patterns. Generally, D-wave channels exhibit larger energies than those of S-wave channels, and they should have less contribution to the mass of $c\bar{c}$ state. The inversion of the contribution from S- and D-wave channels may lead to the problem of convergency. Hence, future research on this issue is expected. Additionally our conclusions are consistent with that of Ref. [36].

Further, we analyze the fractions of the two-quark ($q\bar{q}$) system and the four-quark (meson-meson) system for light ground-states in Table 7 and for the charmonium states in Table 8. From Table 7, we can see that the probability fractions of $q\bar{q}$ components are all over 90%. But for ρ meson, the $q\bar{q}$ component accounts for 39.2%, because it can decay to the open channel $\pi\pi$. This open channel makes the largest mass shift contribution and the fraction of four-quark components $\pi\pi$ are rather large, 48.4%. By the way, the picture of rho meson in UQM is that a quark-antiquark core surrounded by meson cloud. The decay constant and electromagnetic form factor mainly depend on the part of the wave function in the core, so we expect that these quantities will not be changed in UQM.

Table 7. Fractions (%) of two- and four-quark components for the light ground-state mesons in the unquenched quark model.

	π	ρ	ω	η
Bare $q\bar{q}$	98.3	39.2	90	95.8
$\pi\pi$	...	48.4	...	...
$\pi\rho$	0.7	...	4.6	...
$\pi\omega$	...	2.8	...	...
$\eta\rho$	...	0.7	...	...
$\eta\omega$	...	...	0.7	...
$\rho\rho$	...	2.1	...	1.9
$\rho\omega$	0.5	...	...	...
$\omega\omega$	...	...	...	0.7
KK	...	4.4	2.7	...
KK^*	0.2	1	0.8	0.8
K^*K^*	0.3	1.4	1.2	0.8

In Table 8, we can observe that the dominant components of charmonium states are $c\bar{c}$, which correspond to 70%−97% in the unquenched quark model. For $\chi_{cJ}(1P)$ $(J=0,1,2)$ and $h_c(1P)$ state, the dominant component $c\bar{c}$ accounts for more than 90%. Furthermore, the main four-quark components are all $D^*\bar{D}^*$, ~3%. For $\chi_{c_1}(2P)$ state, the unquenched mass is in good agreement with the experimental value of $X(3872)$. The picture of the exotic state $X(3872)$ is $c\bar{c}$ state mixed with four-quark components. The dominant component of $X(3872)$ is still $c\bar{c}$, approximately 78.4%, and the fraction of $D\bar{D}^*+D^*\bar{D}$ is approximately 12.4%, $D^*\bar{D}^*$, 7.1%, $D_s\bar{D}_s^*+D_s^*\bar{D}_s$, 1% and $D_s^*\bar{D}_s^*$, and 1.1%. Our results of $X(3872)$ are qualitatively consistent with some previous studies [36, 57−59]. However, in other studies, the dominant components of $X(3872)$ are meson-meson ones, and the fraction of $c\bar{c}$ is small, for example, 7%−32% in [60], 7.5%−11.2% in [61], 14.7% in [41]. Additionally, Kalashnikova exhibits a slightly larger fraction of $c\bar{c}$, approximately 54.3% [30]. Furthermore, in our study, we conducted calculations on the four-quark system $\bar{D}^*D$ and did not observe any bound state although there is an attraction between $\bar{D}^*$ and D. Therefore, in our present framework of the unquenched quark, we consider that $X(3872)$ can be classified as a member of the charmonium family $\chi_{c_1}(2P)$.

For $\chi_{c_2}(3930)$, the candidate of $\chi_{c_2}(2P)$ state in the un-

Table 8. Fractions (%) of two- and four-quark components for the charmonium mesons in the unquenched quark model.

	Bare $q\bar{q}$	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$	$D_s^*\bar{D}_s$	$D_s^*\bar{D}_s^*$
$\eta_c(1S)(1^1S_0)$	97	...	0.7	0.7	1.1	...	0.1	0.1	0.3
$\eta_c(2S)(2^1S_0)$	90.6	...	2.3	2.3	3.6	...	0.3	0.3	0.6
$J/\psi(1S)(1^3S_1)$	95.8	0.3	0.6	0.6	1.9	0.1	0.1	0.1	0.5
$\psi(2S)(2^3S_1)$	87.8	1.3	2.0	2.0	5.2	0.2	0.3	0.3	0.9
$\chi_{c0}(1P)(1^3P_0)(S+D)$	93.9	1.3	...	...	3.8	0.2	...	...	0.8
$\chi_{c_1}(1P)(1^3P_1)(S+D)$	93.2	...	1.4	1.4	3.0	...	0.2	0.2	0.6
$\chi_{c_2}(1P)(1^3P_2)(S+D)$	91.8	0.9	1.2	1.2	3.6	0.2	0.2	0.2	0.7
$h_c(1P)(1^1P_1)(S+D)$	92.6	...	1.7	1.7	2.9	...	0.3	0.3	0.5
$\chi_{c0}(2P)(2^3P_0)(S+D)$	81	7.9	...	...	9.1	0.6	...	...	1.4
$\chi_{c_1}(2P)(2^3P_1)(S+D)$	78.4	...	6.2	6.2	7.1	...	0.5	0.5	1.1
$\chi_{c_2}(2P)(2^3P_2)(S+D)$	72.2	5.2	4.3	4.3	11.3	0.4	0.5	0.5	1.3
$h_c(2P)(2^1P_1)(S+D)$	75.3	...	6.9	6.9	8.4	...	0.7	0.7	1.1
$\eta_{c2}(1D)(1^1D_2)$	95.3	...	1.3	1.3	1.7	...	0.1	0.1	0.2
$\psi(1D)(1^3D_1)$	94.2	2.8	0.9	0.9	0.5	0.3	0.1	0.1	0.2
$\psi_2(1D)(1^3D_2)$	95	...	1.8	1.8	0.9	...	0.2	0.2	0.1
$\psi_3(1D)(1^3D_3)$	95.9	...	...	...	3.6	...	...	...	0.5

quenched quark model, the dominant constituent is also $c\bar{c}$, approximately 72.2%. The largest four-quark component is $D^*\bar{D}^*$, 11.3%, and the $D_s^{(*)}\bar{D}_s^{(*)}$ occupied a very small percentage. For higher excited states $1D$, the fraction of $c\bar{c}$ is much higher than that of χ_{cJ} $(2P)(J=0,1,2)$ and $h_c(2P)$ states, which correspond to 95%. For 1^1D_2 and 1^3D_2, the main four-quark component is $D\bar{D}^*+D^*\bar{D}$, 2.6% and 3.6%, respectively. For 1^3D_1, the main four-quark component is $D\bar{D}$, 2.8%. For 1^3D_3, $D^*\bar{D}^*$ accounts for 3.6%.

V. SUMMARY

To provide a unified description of the ordinary meson and exotic states in experiments, a new quark model – the unquenched quark model is developed. As a preliminary study, we calculated the unquenched masses of the ordinary light ground-state mesons (π, ρ, ω, η), as well as some charmnonium $c\bar{c}$ states in UQM.

In UQM, the coupling of the two quark component and high Fock four-quark component is considered. For the four-quark component, there are different configurations, including meson-meson structure, hidden-color structure, and diquark-antidiquark structure. In our present study, only four-quark components in meson-meson structure are considered. Additionally, in future studies, the effects and convergence of the higher configuration of the four-quark components will be investigated.

The modified transition operator, which relates the valence part to the high Fock components, is also applied. Two simple, physically motivated improvements are introduced. One improvement involves suppressing the contribution from the intermediate dressing states with large momentum. Furthermore, another state involves favoring the quark-antiquark creation near the source hadron. With these improvements, alarmingly mass shifts are reduced and the success of valence quark model in describing low-lying spectrum of meson is maintained.

Furthermore, by fitting the experimental values of π, ρ, ω, η, $\eta_c(1S)$, $\eta_c(2S)$, $J/\psi(1S)$, $J/\psi(2S)$, $\chi_{cJ}(1P)$ $(J=0,1,2)$, and $h_c(1P)$, a total of twelve mesons, we obtained a set new quark model parameters. We calculated the high excited $c\bar{c}$ states with the new model parameters in UQM to explain certain exotic states observed in experiments. Furthermore, certain well-known exotic states can be described very well. Simultaneously, the masses of light ground-state mesons and low-lying charmnia, η_c and J/ψ, are reproduced well. For example, our calculation shows that the unquenched mass of $\chi_{c_1}(2P)$ is very close to the experimental value of $X(3872)$, and the dominant component of $X(3872)$ is $c\bar{c}$, approximately 78.4%. $\chi_{c_2}(2P)$ is a good candidate of $\chi_{c_2}(3930)$, and the dominant constituent is also $c\bar{c}$, approximately 72.2%. $\psi(3770)$ is highly likely to be the charmonium 1^3D_1 state. $\psi_2(3823)$ and $\psi_3(3842)$ may be the candidate of 1^3D_2 and 1^3D_3 states, respectively. All of the $1D$ states are $c\bar{c}$ dominant states, with the fractions of approximately 95%.

Hence, the unquenched quark model is a promising phenomenological method for unifying the description of ordinary mesons and exotic mesons. However, there are still some problems with the convergency of the Fock expansion. Further improvement of the transition operator are expected. With the accumulation of experiment data, it will further aid in verifying the reasonability of the improvements in the quark model.

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