Studying the localized CP violation and the branching fraction of the $\bar{B}^0 \to K^-\pi^+\pi^- \pi^+$ decay

Jing-Juan Qi, Zhen-Yang Wang, Jing Xu, Xin-Heng Guo

Abstract: In this work, we study the localized CP violation and the branching fraction of the four-body decay $\bar{B}^0 \to K^-\pi^+\pi^- \pi^+$ by employing a quasi-two-body QCD factorization approach. Considering the interference of $\bar{B}^0 \to K_0^*(700)\rho^0(770) \to K^-\pi^+\pi^- \pi^+$ and $\bar{B}^0 \to K^+(892)f_0(500) \to K^-\pi^+\pi^- \pi^+$ channels, we predict $\mathcal{A}_{CP}(\bar{B}^0 \to K^-\pi^+\pi^- \pi^+) \in [0.15, 0.28]$ and $\mathcal{B}(\bar{B}^0 \to K^-\pi^+\pi^- \pi^+) \in [1.73, 5.10]\times 10^{-7}$, respectively, which shows that this two channels' interference mechanism can induce the localized CP violation to this four-body decay. Meanwhile, within the two quark model framework for the scalar mesons $f_0(500)$ and $K_0^*(700)$, we calculate the direct CP violations and branching fractions of the $\bar{B}^0 \to K_0^*(700)\rho^0(770)$ and $\bar{B}^0 \to K^+(892)f_0(500)$ decays, respectively. The corresponding results are $\mathcal{A}_{CP}(\bar{B}^0 \to K_0^*(700)\rho^0(770)) \in [0.20, 0.36]$, $\mathcal{A}_{CP}(\bar{B}^0 \to K^+(892)f_0(500)) \in [0.08, 0.12]$, $\mathcal{B}(\bar{B}^0 \to K_0^*(700)\rho^0(770)) \in [6.76, 18.93]\times 10^{-8}$ and $\mathcal{B}(\bar{B}^0 \to K^+(892)f_0(500)) \in [2.66, 4.80]\times 10^{-6}$, respectively, indicating the CP violations of these two two-body decays are both positive and the branching fractions are quite different. These studies provide a new way to investigate the aforementioned four-body decay and could be helpful in clarifying the configuration of the structure of light scalar meson.

Keywords: decays of bottom mesons, CP violation, perturbative calculations

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1 Introduction

Charge-Parity (CP) violation is one of the most fundamental and important properties of the weak interaction. Nonleptonic decays of hadrons containing a heavy quark play an important role in testing the Standard Model (SM) picture for the CP violation mechanism in flavor physics, improving our understanding of nonperturbative and perturbative QCD and exploring new physics beyond the SM. CP violation is related to the weak complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the mixing of different generations of quarks [1, 2]. Besides the weak phase, a large strong phase is usually also needed for a large CP violation. Generally, this strong phase is provided by QCD loop corrections and some phenomenological models.

Recently, more attentions have been focused on the studies of the two- or three-body heavy meson decays both theoretically and experimentally [3-12], while for the four-body nonleptonic decays of these heavy mesons there are limited studies [13-15]. Because of the complicated phase spaces and relatively smaller branching fractions, four-body decays of heavy mesons are hard to be investigated. However, in the aspect of studying the intermediate resonances, four-body decays of heavy mesons can provide rich information, especially for the unclear compositions of scalar mesons like $f_0(500)$ ($\sigma$), $K^*(700)$ ($\kappa$), $a_0(980)$ and $f_0(980)$. Up to now, the descriptions of...
the inner structures for the light scalar states are still unclear and even controversial, which could be, for example, $q\bar{q}$, $q\bar{q}gg$, meson-meson bound states or even those supplemented with a scalar glueball [16-19]. Studying four-body decays of heavy mesons in addition to two- or three-body decays can provide useful information for clarifying configurations of light scalar mesons. In fact, with the great development of the large hadron collider beauty (LHCb) and Belle-II experiments, more and more four-body decay modes involving one or two scalar states in the $B$ and $D$ meson decays are expected to be measured with good precision in the future.

As mentioned above, four-body meson decays are generally dominated by intermediate resonances, which means that they proceed through quasi-two-body or quasi-three-body decays. In our work, we will adopt the quasi-two-body decay mechanism to study the four-body decay $B^0 \to K^{-}\pi^+\pi^-\pi^+$, i.e. $B^0 \to K^0(700)\rho^0(770) \to K^-\pi^+\pi^-\pi^+$ and $B^0 \to K^+(892)f_0(500) \to K^-\pi^+\pi^+$, where the light scalars $f_0(500)$ and $K^*(700)$ will be considered as lowest-lying and first excited $q\bar{q}$ states [20], respectively. We can then explore whether the localized CP violation of the four-body decay $B^0 \to K^-\pi^+\pi^-\pi^+$ can be induced by these two channels' interference.

Theoretically, to calculate the hadronic matrix elements of $B$ or $D$ weak decays, some approaches, such as QCD factorization (QCDF) [6, 21], the perturbative QCD(pQCD) [22] and the soft-collinear effective theory(SCET) [23], have been fully developed and extensively employed in recent years. Unfortunately, in the collinear factorization approximation, the calculation of annihilation corrections always suffers from the end-point divergence. In the QCDF approach, such divergence is usually parameterized in a model-independent manner [6, 21] and will be explicitly expressed in Sect. II.

The remainder of this paper is organized as follows. In Sec. 2, we present our theoretical framework. The numerical results are given in Sec. 3 and we summarize our work in Sec. 4. Appendix A recapitulates explicit expressions of hard spectator-scattering and weak annihilation amplitudes. The factorizable amplitudes of two-body decays are summarized in Appendix B. Related theoretical parameters are listed in Appendix C.

2 Theoretical framework

2.1 Kinematics of the four-body decay

The kinematics of the process $B^0 \to K^-\rho^0(p_2)\pi^-\pi^+(p_3)p^+(p_4)$ is described in terms of the five variables displayed in Fig. 1 [24, 25] in which

(i) the invariant mass squared of the $K\pi$ system $s_{K\pi} = (p_1 + p_2)^2 = m_{K\pi}^2$;

(ii) the invariant mass squared of the $\pi\pi$ system $s_{\pi\pi} = (p_3 + p_4)^2 = m_{\pi\pi}^2$;

(iii) $\theta_e$ is the angle of the $\pi^+$ in the $\pi^-\pi^+\pi^+$ center-of-mass frame $\Sigma_{\pi\pi}$ with respect to the $\pi^-$ line of flight in the $B^0$ rest frame $\Sigma_{B^0}$;

(iv) $\theta_K$ is the angle of the $K^-$ in the $K\pi$ center-of-mass system $\Sigma_{K\pi}$ with respect to the $K^-$ line of flight in $\Sigma_{B^0}$;

(v) $\phi$ is the angle between the $K\pi$ and $\pi\pi$ planes.

The physical ranges are

$$4m_{\pi\pi}^2 \leq s_{\pi\pi} \leq (m_{B^0} - m_{K\pi})^2,$$

$$m_{K\pi}^2 \leq s_{K\pi} \leq (m_{B^0} - m_{\pi\pi})^2,$$

$$0 \leq \theta_e, \theta_K \leq \pi, \quad 0 \leq \phi \leq 2\pi. \quad (1)$$

We consider the localize CP violation of the $B^0 \to K^-\rho^0(p_2)\pi^-\pi^+(p_3)p^+(p_4)$ decay when the invariant mass of $\pi\pi$ is near the masses of $f_0(500)$ (including $\rho^0(770)$), and the invariant mass of $K\pi$ is near the masses of $K^0(700)$ (including $K^*(892)$), respectively. We adopt

$$\left( m_{f_0(500)} - \frac{\Gamma_{f_0(500)}}{2} \right)^2 \leq s_{\pi\pi} \leq \left( m_{f_0(500)} + \frac{\Gamma_{f_0(500)}}{2} \right)^2,$$

$$\left( m_{K^0(700)} - \frac{\Gamma_{K^0(700)}}{2} \right)^2 \leq s_{K\pi} \leq \left( m_{K^0(700)} + \frac{\Gamma_{K^0(700)}}{2} \right)^2. \quad (2)$$

In Eqs. (2), $m_{f_0(500)}$ and $m_{K^0(700)}$ are the masses of $f_0(500)$ and $K^0(700)$ mesons, respectively. $\Gamma_{f_0(500)}$ and $\Gamma_{K^0(700)}$ are the widths of the corresponding mesons, respectively.

Instead of the individual momenta $p_1$, $p_2$, $p_3$, $p_4$, it is more convenient to use the following kinematic variables

$$P = p_1 + p_2, \quad Q = p_1 - p_2,$$

$$L = p_3 + p_4, \quad N = p_3 - p_4. \quad (3)$$

It follows that

$$P^2 = s_{K\pi}, \quad Q^2 = 2m_K^2 + p_2^2 - s_{K\pi}, \quad L^2 = s_{\pi\pi},$$

$$P \cdot L = \frac{1}{2}(m_B^2 - s_{K\pi} - s_{\pi\pi}), \quad P \cdot N = X \cos \theta_1. \quad (4)$$

where the function $X$ is defined as

$$X(s_{K\pi}, s_{\pi\pi}) = \left( P \cdot L \right)^2 - s_{K\pi} s_{\pi\pi} \leq \frac{1}{2} \left| s_{K\pi}^2 \right| \left( m_B^2, s_{K\pi}, s_{\pi\pi} \right),$$

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz. \quad (5)$$
2.2 $B$ decay in QCD factorization

The effective weak Hamiltonian for nonleptonic $B$ weak decays is [6]

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i,p,k} \sum_{d,d',s} \lambda_i^{(D)}(c_{i1}O_{i1}^p + c_2O_{i2}^p + c_{i3}O_{i3}),
$$

where $G_F$ represents the Fermi constant, $\lambda_i^{(D)} = V_{i1}V_{i2}^*$, and $V_{i1}$ and $V_{i2}$ are the CKM matrix elements, $\lambda_1(i = 1 - 10, 7y, 8g)$ are Wilson coefficients, $O_{i,1}^p$ are the tree level operators, $O_{7-8}$ are the QCD penguin operators, $O_{7-8}$ arise from electroweak penguin diagrams, and $O_{7g}$ and $O_{8g}$ are the electromagnetic and chromomagnetic dipole operators, respectively.

With the effective Hamiltonian in Eq. (6), the QCDF method has been fully developed and extensively employed to calculate the hadronic two-body $B$ decays. The spectator scattering and annihilation amplitudes are expressed with the convolution of scattering functions and the light-cone wave functions of the participating mesons [6]. The explicit expressions for the basic building blocks of the spectator scattering and annihilation amplitudes have been given in Ref. [6], which are also listed in Appendix A for convenience. The annihilation contributions $A_n^j (n = 1, 2, 3)$ can be simplified to [26]:

$$
A_1^j (VS) = 6\pi\alpha_s \left[ 3\mu_s \left( B_1(3X_1 + 4 - \pi^2) + B_1(10X_1) \right) - \frac{23}{18} \frac{10}{\pi^2} \right] - \frac{956}{9} - \frac{100}{3} \pi^2 \right] - r_{kx}^i X(A-2),
$$

$$
A_2^j (VS) = 6\pi\alpha_s \left[ 3\mu_s \left( B_1(3X_1 + 29 - 3\pi^2) + B_1(X_1) \right) + 2956 \frac{10}{3} \pi^2 \right] - r_{kx}^i X(A-2),
$$

$$
A_3^j (VS) = 6\pi\alpha_s \left[ - r_{kx}^i \mu_s \left( 9B_1(X_1 + 4 - \pi^2) + 10B_1(3X_1 - 19 + 6) \pi^2 \right) + \frac{2956}{9} \frac{10}{3} \pi^2 \right] X(A-2),
$$

$$
A_4^j (VS) = 6\pi\alpha_s \left[ - r_{kx}^i \mu_s \left( 9B_1(X_1 + 4 - \pi^2) + 10B_1(3X_1 - 19 + 6) \pi^2 \right) + \frac{2956}{9} \frac{10}{3} \pi^2 \right] X(A-2),
$$

for $M_1M_2 = VS$, and

$$
A_1^i (SV) = -A_2^i (SV), \quad A_2^i (SV) = -A_1^i (SV),
$$

$$
A_1^i (SV) = A_3^i (SV), \quad A_2^i (SV) = -A_2^i (SV),
$$

for $M_1M_2 = SV$, where the superscripts $i$ and $f$ refer to gluon emission from the initial and final state quarks, respectively. The model-dependent parameter $X_A$ is used to estimate the end point contributions, and expressed as

$$
X_A = (1 + \rho_A \phi_A) \ln \frac{m_s}{\Lambda},
$$

with $\Lambda$ being a typical scale of order 500 MeV, $\rho_A$ an unknown real parameter and $\phi_A$ the free strong phase in the range $[0,2\pi]$. For the spectator scattering contributions, the calculation of twist-3 distribution amplitudes also suffers from the end point divergence, which is usually dealt with in the same manner as in Eq. (9) and labeled by $X_H$. In our work, when dealing with the end-point divergences from the hard spectator scattering and weak annihilation contributions, we will follow the assumption $X_H = X_A$ for the $B$ two-body decays [20].

2.3 Four-body decay amplitudes and localized CP violation

For the $B^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decay, we consider the contributions from $B^0 \rightarrow K_0(700)^0(892) \rightarrow K^- \pi^+ \pi^- \pi^+$ and $B^0 \rightarrow K^- (892) \rightarrow K^- \pi^+ \pi^- \pi^+$ channels. For convenience, $f_0(500), f_0(770), K_0(700)$ and $K^- (892)$ mesons will be denoted as $\sigma, \rho, \kappa$ and $K^+$, respectively. The amplitudes of these two channels are

$$
M(B^0 \rightarrow \kappa \rho \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^-) = \langle \kappa | H_{\text{eff}} | B^0 \rangle \langle K^- | H_{\text{eff}} | \kappa \rangle \langle \pi^+ \pi^- \pi^+ \pi^- | \sigma \rangle, \quad S_\kappa S_\rho,
$$

$$
M(B^0 \rightarrow \kappa^+ \sigma \rightarrow K^- \pi^+ \pi^- \pi^+) = \langle \kappa^+ | H_{\text{eff}} | B^0 \rangle \langle K^- | H_{\text{eff}} | \kappa^+ \rangle \langle \pi^- \pi^+ \pi^- | \sigma \rangle, \quad S_\kappa S_\sigma,
$$

where $H_{\text{eff}}$, $H_{\text{eff}}$, $H_{\text{eff}}$, $H_{\text{eff}}$, $H_{\text{eff}}$, $H_{\text{eff}}$, $H_{\text{eff}}$, $H_{\text{eff}}$ are the strong Hamiltonians for $\rho \rightarrow \pi^+ \pi^-$, $\sigma \rightarrow \pi^+ \pi^-$, $\kappa \rightarrow K^- \pi^+$ and $K^+ \rightarrow K^- \pi^+$ decays, respectively. $S_\kappa$, $S_\rho$, $S_\kappa$ and $S_\sigma$ are the reciprocal of the dynamical functions of the corresponding mesons. Since the width of $\sigma$ is larger than the other three mesons, we shall adopt the Breit-Wigner function and the Bugg model [27, 28] to deal with the distributions of the first three mesons ($\kappa$, $\rho$ and $K^+$) and $\sigma$ meson, respectively.

In the Breit-Wigner model, $S_\kappa$ takes the form $S - m_k^2 + im_k\Gamma_k$, $k = 1, 2, 3$ corresponding to $\kappa$, $\rho$ and $K^+$ mesons, respectively. $S = s_{\pi\pi}$ or $S = s_{\kappa\kappa}$ when dealing with $\pi\pi$ or $\kappa\kappa$ systems.

The Bugg model is used to parameterize the distribution of $\sigma$ [27, 28].

$$
S_{\sigma}(s) = M_2^2 - s - g_2^2(s) \frac{\gamma_{\text{had}}}{M_2^2} \Gamma(s) - iM_{\text{tot}}(s)|/M_{\Gamma}(s),
$$

where $\gamma(s) = j_1(s) - R^2(s)$ with $j_1(s) = 1/2 + \rho_1 \ln(1/s)$.
\[ \Gamma_{\text{tot}}(s) = \sum_{i=1}^{4} \Gamma_i(s) \text{ and} \]
\[ M\Gamma_1(s) = s - s_0 \rho_1(s), \]
\[ M\Gamma_2(s) = 0.6g_1^2(s)(s/M^2)^2 \exp(-\alpha_2(s - 4m_0^2))\rho_2(s), \]
\[ M\Gamma_3(s) = 0.2g_1^2(s)(s/M^2)^2 \exp(-\alpha_3(s - 4m_0^2))\rho_3(s), \]
\[ M\Gamma_4(s) = M\sum_{s < 4s} \rho_{4s}(s), \]
where we abbreviate \( s_{2\pi} \) as \( s \), related parameters are fixed to be \( M = 0.953 \text{ GeV}, s_A = 0.14m_0^2, c_1 = 1.302 \text{ GeV}^2, c_2 = 0.340, \) \( A = 2.426 \text{ GeV}^2 \) and \( g_{s_{2\pi}} = 0.011 \text{ GeV} \), as given in the fourth column of Table I in Ref. [27]. The parameters \( \rho_{2,3} \) are the phase-space factors of the decay channels \( \pi \pi, KK \) and \( \eta \eta \), respectively, which are defined as [27]
\[ \rho_1(s) = \sqrt{1 - 4m_0^2/s}, \quad (14) \]

\[ M(B_0^0 \to \bar{k} \rho \to K^-\pi^+\pi^-\pi^+) = \frac{iG_Fg_{K\bar{K}}g_{\rho\pi\pi}}{6\lambda} \sum_{\rho K} \lambda_p^{(s)} \left( \begin{array}{c} f_{K}^2 \frac{m_0}{m_K} - \frac{1}{2} \alpha_p^{(s)}(K^+\pi^-) \\ \beta_p \frac{m_0}{m_K} - \frac{1}{2} \alpha_p^{(s)}(K^0\pi^-) \\ \lambda_p \frac{m_0}{m_K} - \frac{1}{2} \alpha_p^{(s)}(K^0\pi^-) \end{array} \right), \quad (17) \]

and
\[ M(B_0^0 \to \bar{k} \rho \to K^+\pi^-\pi^-\pi^-) = -\frac{iG_Fg_{K\bar{K}}g_{\rho\pi\pi}}{6\lambda} \sum_{\rho K} \lambda_p^{(s)} \left( \begin{array}{c} f_{K}^2 \frac{m_0}{m_K} - \frac{1}{2} \alpha_p^{(s)}(K^+\pi^-) \\ \beta_p \frac{m_0}{m_K} - \frac{1}{2} \alpha_p^{(s)}(K^0\pi^-) \\ \lambda_p \frac{m_0}{m_K} - \frac{1}{2} \alpha_p^{(s)}(K^0\pi^-) \end{array} \right), \quad (18) \]

respectively, where \( g_{K\pi}, g_{\rho\pi\pi}, g_{K\bar{K}K}, g_{\rho\sigma\pi} \) are the strong coupling constants of the corresponding decays, which are listed in Eq. (C4), \( F_1^{\bar{k}\rho}(m_0^2), A_0^{\rho\pi}(m_0^2), A_0^{\bar{k}K}(m_0^2) \) and \( F_1^{\rho\sigma}(m_0^2) \) are form factors for \( B_0^0 \) to \( \bar{k}, \rho, K^+ \) and \( \sigma \) transitions, respectively, \( f_{\bar{k}}, f_{\rho}, f_{K^+} \) and \( f_{\sigma} \) are decay constants of \( \bar{k}, \rho, K^+ \) and \( \sigma \) mesons, respectively, \( \beta_p \) and \( \lambda_p \) are decay constants of \( \sigma \) coming from the up and strange quark components, respectively.

There could be a relative strong phase \( \delta \) between the two interference amplitudes, which value depends on experimental data and theoretical models. Since little information about \( \delta \) can be provided by experiments, we choose to adopt the same method as that in Refs. [7, 33, 34], i.e. setting \( \delta = 0 \). The total decay amplitude of \( B_0^0 \to K^-\pi^+\pi^-\pi^- \) including both \( B_0^0 \to \bar{k} \rho \to K^-\pi^+\pi^-\pi^- \)

with \( m_1 = m_{\bar{k}}, m_2 = m_K \) and \( m_3 = m_{\pi} \).

When dealing with the final states interactions, unitarized chiral perturbation theory is an effective method which have been studied in Refs. [29-32]. Now we will adopt the method in Refs. [7, 28]
\[ \langle M_1 M_2 | H_2 | V \rangle = g_{2V M_1, M_2} | (p_M - p_M) \rangle. \quad (15) \]

and
\[ \langle M_1 M_2 | H_2 | S \rangle = g_{SM M_1, M_2}, \quad (16) \]

respectively, where \( g_{2V M_1, M_2} \) and \( g_{SM M_1, M_2} \) are the strong coupling constants of the corresponding vector and scalar mesons decays, respectively. Generally, these coupling constants can be derived from experiments which have been listed in Eq. (C4).

Within the QCDF framework in Ref. [6], we can get the decay amplitudes of \( B_0^0 \to \bar{k} \rho, K^+ \sigma \) which have been listed in Appendix B. Combining Eqs. (35), (15) and (10), (36), (16)and (11), respectively, the amplitudes of \( B_0^0 \to \bar{k} \rho \to K^-\pi^+\pi^-\pi^- \) and \( B_0^0 \to K^+\pi^-\pi^-\pi^- \), channels can be written as
and \( \bar{B}^0 \to \bar{K}^- \sigma \to K^- \pi^+ \pi^- \pi^- \) channels can be written as
\[
M = M(\bar{B}^0 \to \bar{\kappa} \rho \to K^- \pi^+ \pi^- \pi^-) + M(\bar{B}^0 \to \bar{K}^- \sigma \to K^- \pi^+ \pi^- \pi^-). \tag{19}
\]
The differential CP asymmetry parameter can be defined as
\[
A_{\text{CP}} = \frac{|M|^2 - |M'|^2}{|M|^2 + |M'|^2}. \tag{20}
\]

The localized integration \( \text{CP} \) asymmetry can be measured by experiments and takes the following form:
\[
A_{\text{CP}} = \frac{\int d\Omega(|M|^2 - |M'|^2)}{\int d\Omega(|M|^2 + |M'|^2)}, \tag{21}
\]
where \( \Omega \) represents the phase space given in Eq. (2) with \( d\Omega = \Delta s_{\pi \pi} \Delta s_{KK} \cos \theta_{\pi \pi} \cos \theta_{KK} d\phi \).

As for the decay rate, one has \([13]\)
\[
d^2\Gamma = \frac{1}{4(4\pi)^6m_{\rho}^2} \sigma(s_{\pi \pi})X(s_{\pi \pi}, s_{KK}) \sum_{\text{spins}} |M|^2 d\Omega, \tag{22}
\]
with
\[
\sigma(s_{\pi \pi}) = \sqrt{1 - 4m_{\rho}^2/s_{\pi \pi}}. \tag{23}
\]

This leads to the branching fraction
\[
\mathcal{B} = \frac{1}{\Gamma_{\rho}} \int d^2\Gamma, \tag{24}
\]
where \( \Gamma_{\rho} \) is the decay width of the \( \bar{B}^0 \) meson.

3 Numerical results

Within the QCDF approach, we get the amplitudes of the two-body decays \( \bar{B}^0 \to \bar{\kappa} \rho \) and \( \bar{B}^0 \to \bar{K}^- \sigma \), where the light scalar \( \sigma \) and \( \kappa \) mesons are considered as the lowest-lying and first excited \( q\bar{q} \) states \([20]\). As for the parameters for the end-point divergences, we take \( \rho_{H(A)} = 0.5 \) and arbitrary strong phases \( \phi_{\rho}(H) \). All the form factors are evaluated at \( q^2 = 0 \) due to the smallness of \( m_{\rho}^2 \), \( m_{\kappa}^2 \), \( m_{\sigma}^2 \) and \( m_{\bar{K}}^2 \), compared with \( m_{\pi}^2 \). We also simply set \( F_{\rho}(s_0) = 0.3 \) and assign its uncertainty to be \( \pm 0.1 \). With the given parameters, we obtain the \( CP \) violations and branching fractions of the \( \bar{B}^0 \to \bar{\kappa} \rho \) and \( \bar{B}^0 \to \bar{K}^- \sigma \) decays substituting Eqs. (35), (36) into (20), respectively. The results are \( A_{\text{CP}}(\bar{B}^0 \to \bar{\kappa} \rho) \in [0.20,0.36] \), \( A_{\text{CP}}(\bar{B}^0 \to \bar{K}^- \sigma) \in [0.08,0.12] \), \( B(\bar{B}^0 \to \bar{\kappa} \rho) \in [6.76,18.93] \times 10^{-8} \), and \( B(\bar{B}^0 \to \bar{K}^- \sigma) \in [2.66,4.80] \times 10^{-6}, \) respectively. Obviously, the \( CP \) violations of these two-body decays are both positive, with the \( CP \) violation in \( \bar{B}^0 \to \bar{K}^- \sigma \) decay being smaller than that in \( \bar{B}^0 \to \bar{\kappa} \rho \). The magnitudes of the branching fractions in these two-body decays are different with the former being about two orders smaller than the latter. When dealing with the four-body decay \( \bar{B}^0 \to K^- \pi^+ \pi^- \pi^- \), we adopt \( B(\rho \to \pi^+ \pi^-) \approx 1 \), \( B(\sigma \to \pi^+ \pi^-) \approx \frac{1}{2} \), \( B(\bar{K}^- \to K^\pi) \approx 1 \), \( B(\bar{K}^- \to K^- \pi^+) \approx \frac{1}{2} \). Then, substituting Eq. (19) into (21) and (24), respectively, we get the localized \( CP \) violation and branching fraction of the four-body decay \( \bar{B}^0 \to K^- \pi^+ \pi^- \pi^- \), with the results \( A_{\text{CP}}(\bar{B}^0 \to K^- \pi^+ \pi^- \pi^-) \in [0.15,0.28] \) and \( B(\bar{B}^0 \to K^- \pi^+ \pi^- \pi^-) \in [1.73,5.10] \times 10^{-7} \). Compared with the uncertainties from the Gegenbauer moments, we find those from the divergence parameters are much larger. It is obvious that the sign of the localized \( CP \) violation of \( \bar{B}^0 \to K^- \pi^+ \pi^- \pi^- \) is positive when the invariant masses of \( \pi \pi \) and \( \kappa \pi \) are near the masses of \( \rho(\sigma) \) and \( \kappa(K^-) \), respectively. This indicates that the interference of \( \bar{B}^0 \to \bar{\kappa} \rho \to K^- \pi^+ \pi^- \pi^- \) and \( \bar{B}^0 \to \bar{K}^- \sigma \to K^- \pi^+ \pi^- \pi^- \) channels can induce the localized \( CP \) violation to the four-body decay \( \bar{B}^0 \to K^- \pi^+ \pi^- \pi^- \). Our theoretical results shown here are predictions for ongoing experiments at LHCb and Belle-II. If our predictions are confirmed by experiments in the future, the viewpoint that \( K^*_0(700) \) and \( f_0(500) \) have the \( q\bar{q} \) composition should be well supported. However, to exclude other possible structures, more investigations will be needed due to uncertainties from both theory and experiments.

4 Summary

By studying the quasi-two-body decays within the QCDF approach, we predicted the localized \( CP \) violation and branching fraction of the four-body decay \( \bar{B}^0 \to K^- \pi^+ \pi^- \pi^- \) due to the interference of the two channels \( \bar{B}^0 \to K^*_0(700)\rho(770)(\to \bar{\kappa} \rho) \to K^- \pi^+ \pi^- \pi^- \) and \( \bar{B}^0 \to K^*_+(892)f_0(500)(\to \bar{K}^- \sigma) \to K^- \pi^+ \pi^- \pi^- \), with the results \( A_{\text{CP}}(\bar{B}^0 \to K^- \pi^+ \pi^- \pi^-) \in [0.15,0.28] \) and \( B(\bar{B}^0 \to K^- \pi^+ \pi^- \pi^-) \in [1.73,5.10] \times 10^{-7} \). It is obvious that the sign of the localized \( CP \) violation of \( \bar{B}^0 \to K^- \pi^+ \pi^- \pi^- \) is positive. In the two quark model for the scalar mesons, we also obtained the \( CP \) violations and branching fractions of the two-body decays \( \bar{B}^0 \to K^*_0(700)\rho(770) \) and \( \bar{B}^0 \to K^*_+(892)f_0(500) \) as \( A_{\text{CP}}(\bar{B}^0 \to K^*_0(700)\rho(770)) \in [0.20,0.36], A_{\text{CP}}(\bar{B}^0 \to K^*_+(892)f_0(500)) \in [0.08,0.12], B(\bar{B}^0 \to K^*_0(700)\rho(770)) \in [6.76,18.93] \times 10^{-8} \) and \( B(\bar{B}^0 \to K^*_+(892)f_0(500)) \in [2.66,4.80] \times 10^{-6} \), respectively. Obviously, the \( CP \) violations of these two-body decays are both positive, and the \( CP \) violation in \( \bar{B}^0 \to K^*_+(892)f_0(500) \) is smaller than that in \( \bar{B}^0 \to K^*_0(700)\rho(770) \). Furthermore, the branching fractions in these two body decays are quite different, with the former being two orders smaller than the latter. Our results will be tested by the precise data from future LHCb and Belle-II experiments. In the present work, we assumed that \( f_0(500) \) and \( K^*_0(700) \) are dominated by the \( q\bar{q} \) configuration. Possible other structures of \( f_0(500) \) and \( K^*_0(700) \) could affect the results in our interference model.
which will need further investigation. If predictions of CP violation and branch ratio in our work are both confirmed by future experiment, we agree and support the view that scalar mesons have the $q\bar{q}$ composition. But there is still a long way to study the structure of scalar mesons.

Appendix A: explicit expressions of hard spectator-scattering and weak annihilation amplitudes

For the hard spectator terms, we obtain \[ (A6) \]

$$H_i(M_1M_2) = \int_0^1 \int_0^1 \frac{d\xi}{\xi} \frac{d\eta}{\eta} \Phi(\xi) \Phi(\eta) \frac{m^2}{B_0} \left( \Phi_{M_1}(\xi) + i r_{M_1} \frac{\xi}{\Phi_{M_0}(\eta)} \right),$$

for $i = 1 - 4, 9, 10$, where the upper sign is for $M_1 = V$ and the lower sign for $M_1 = S$.

$$H_i(M_1M_2) = \int_0^1 \int_0^1 \frac{d\xi}{\xi} \frac{d\eta}{\eta} \Phi(\xi) \Phi(\eta) \frac{m^2}{B_0} \left( \Phi_{M_1}(\xi) + i r_{M_1} \frac{\xi}{\Phi_{M_0}(\eta)} \right),$$

and the twist-3 ones are respectively

$$H_i(M_1M_2) = \int_0^1 \int_0^1 \frac{d\xi}{\xi} \frac{d\eta}{\eta} \Phi(\xi) \Phi(\eta) \frac{m^2}{B_0} \left( \Phi_{M_1}(\xi) + i r_{M_1} \frac{\xi}{\Phi_{M_0}(\eta)} \right).$$

The twist-2 light-cone distribution amplitudes (LCDA) for the pseudoscalar and vector mesons are respectively \[ (A7) \]

$$D(SV) = F_{1B_0}(0)m_{SV}^2, \quad D(VS) = A_{0B_0}(0)m_{VS}^2,$$

and $c_i^{MC_i}$ (i = 1, 2) are "chirally-enhanced" terms defined as

$$c_i^{MC_i} = \frac{2m_i}{m_0} f_i^{MC_i} - f_v q_i^{MC_i}$$

for $i = 5, 7$ and $H_i = 0$.

The twist-2 light-cone distribution amplitudes (LCDA) for the pseudoscalar and vector mesons are respectively \[ (A8) \]

$$H_i = \pi\alpha s \int_0^1 dy dy \left[ \Phi_{M_1}(y) \Phi_{M_2}(y) \left( \frac{1 + \frac{m_0}{m_0} + \frac{m_0}{m_0} \Phi_{M_1}(y) \Phi_{M_2}(y) \frac{2}{2} \right) \right] \quad \text{for } M_1M_2 = VS,$$

$$A_i = \pi\alpha s \int_0^1 dy dy \left[ \Phi_{M_1}(y) \Phi_{M_2}(y) \left( \frac{1 + \frac{m_0}{m_0} + \frac{m_0}{m_0} \Phi_{M_1}(y) \Phi_{M_2}(y) \frac{2}{2} \right) \right] \quad \text{for } M_1M_2 = SV,$$

$$A_i = \pi\alpha s \int_0^1 dy dy \left[ \Phi_{M_1}(y) \Phi_{M_2}(y) \left( \frac{1 + \frac{m_0}{m_0} + \frac{m_0}{m_0} \Phi_{M_1}(y) \Phi_{M_2}(y) \frac{2}{2} \right) \right] \quad \text{for } M_1M_2 = VS,$$

$$A_i = \pi\alpha s \int_0^1 dy dy \left[ \Phi_{M_1}(y) \Phi_{M_2}(y) \left( \frac{1 + \frac{m_0}{m_0} + \frac{m_0}{m_0} \Phi_{M_1}(y) \Phi_{M_2}(y) \frac{2}{2} \right) \right] \quad \text{for } M_1M_2 = SV,$$

$$A_i = \pi\alpha s \int_0^1 dy dy \left[ \Phi_{M_1}(y) \Phi_{M_2}(y) \left( \frac{1 + \frac{m_0}{m_0} + \frac{m_0}{m_0} \Phi_{M_1}(y) \Phi_{M_2}(y) \frac{2}{2} \right) \right] \quad \text{for } M_1M_2 = SV.$$
\[ M(B^0 \rightarrow \bar{K} \rho) = \frac{ieG_F}{2} \sum_{p, p', \rho', \rho} \left( \frac{2 \mu_p \bar{F}^{\rho'}(m_{\rho'}^2)\bar{m}_{\rho'}\rho_p}{\sqrt{2} \delta_{p'23}} \left( \bar{s}_{\rho}(\bar{K}') - \bar{s}_{\rho}(\bar{K}' \rho) \right) + f_{\rho} f_{\rho'} f_{\bar{K}} f_{\bar{K}' \rho} \left( \bar{s}_{\rho}(\bar{K}') - \bar{s}_{\rho}(\bar{K}' \rho) \right) \right) \]

\[ M(B^0 \rightarrow \bar{K}^* \rho) = \frac{ieG_F}{2} \sum_{p, p', \rho', \rho} \left( \frac{2 \mu_p \bar{F}^{\rho'}(m_{\rho'}^2)\bar{m}_{\rho'}\rho_p}{\sqrt{2} \delta_{p'23}} \left( \bar{s}_{\rho}(\bar{K}' \rho) - \bar{s}_{\rho}(\bar{K}' \rho) \right) + f_{\rho} f_{\rho'} f_{\bar{K}} f_{\bar{K}' \rho} \left( \bar{s}_{\rho}(\bar{K}' \rho) - \bar{s}_{\rho}(\bar{K}' \rho) \right) \right) \]

**Appendix C: theoretical input parameters**

The predictions obtained in the QCDF approach depend on many input parameters. The values of the Wolfenstein parameters are taken from Ref. [36]: \( \bar{\rho} = 0.117 \pm 0.021, \bar{\eta} = 0.353 \pm 0.013 \).

The effective Wilson coefficients used in our calculations are taken from Ref. [28]:

\[ C_1 = -0.3125, \quad C_2 = -1.1502, \quad C_3 = 2.120 \times 10^{-3}, \quad C_4 = -4.869 \times 10^{-2}, \quad C_5 = -1.552 \times 10^{-2}, \]

For the masses used in \( B^0 \) decays, we use the following values (in GeV) [36]:

\[ m_s = 0.0035, \quad m_c = 0.119, \quad m_b = 4.2, \quad m_{B^0} = 0.14, \]

\[ m_{B^{(s)}} = 0.494, \quad m_{B^{(s)}(0)} = 0.50, \quad m_{B^{(s)}(770)} = 0.824, \quad m_{B^{(s)}(892)} = 0.775, \]

\[ m_{B^{(s)}(770)} = 0.095, \quad m_{B^{(s)}(892)} = 0.028, \]

and widths are (in GeV) [36]:

\[ \Gamma_{B^{(s)}(770)} = 0.149, \quad \Gamma_{B^{(s)}(0)} = 0.047, \quad \Gamma_{B^{(s)}(892)} = 0.047. \]

The strong coupling constants are determined from the measured partial widths through the relations [7, 37]:

\[ G^{SM}_{M, M'} = \sqrt{\frac{8 \pi m^2}{p_{V, S}(S)}} \Gamma_{V \rightarrow M, M'} , \quad G^{SV}_{M, M'} = \sqrt{\frac{6 \pi m^2}{p_{V}(V)}} \Gamma_{V \rightarrow M, M'} , \]

where \( p_{V, S} \) are the magnitudes of the three momenta of the final state mesons in the rest frame of \( S \) and \( V \) mesons, respectively.

The following related decay constants (in GeV) are used [20, 35]:

\[ f_{\rho} = 0.131, \quad f_{\rho'} = 0.21 \pm 0.02, \quad f_{\rho'} = 0.156 \pm 0.007, \quad f'_{\rho}(500) = 0.21 \pm 0.003, \quad f'_{\rho}(770) = 0.4829 \pm 0.076, \]

\[ f_{\rho}(500) = 0.34 \pm 0.02, \quad f_{\rho}(770) = 0.216 \pm 0.003, \quad f_{\rho}(892) = 0.22 \pm 0.005, \]

\[ f_{\rho}(892) = 0.185 \pm 0.010. \]

As for the form factors, we use [35, 20]:

\[ F_{0, f}(0) = 0.35 \pm 0.04, \quad F_{0, f}(500) = 0.45 \pm 0.15. \]

The values of Gegenbauer moments at \( \mu = 1 \)GeV are taken from [35, 20]:

\[ a_0 = 0, \quad a_2 = 0.15 \pm 0.07, \quad a_4 = 0.14 \pm 0.06, \quad a_2^{K(892)} = 0.03 \pm 0.02, \quad a_4^{K(892)} = 0.04 \pm 0.03, \quad a_2^{K(892)} = 0.11 \pm 0.09. \]

**References**