Analysis of the strong decays of SU(3) partners of the Ω(2012) baryon

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Abstract: We estimate the coupling constants and decay widths of the $SU(3)$ partners of the Ω(2012) hyperon, as discovered by the BELLE Collaboration, using the distribution amplitudes of the octet baryons within light cone sum rules method. Our study includes a comparison of the obtained results for the relevant decay widths with those derived within the framework of the flavor $SU(3)$ analysis. We observe a good agreement between the predictions of both approaches. Moreover, our results on the decay width of the $\Omega \to \Xi K$ is compatible with the existing experimental result within the uncertainties of the model predictions. These results can provide helpful insights for determining the nature of the $SU(3)$ partners of the Ω(2012) baryon.

Keywords: coupling constants, Light cone sum rule, hyperon

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I. INTRODUCTION

In 2018, the BELLE Collaboration made an exciting announcement regarding the discovery of the Ω(2012) hyperon. This discovery was based on the $\Omega^{*-} \to \Xi^0 K^-$ and $\Omega^{*-} \to \Xi^- K_s^0$ decay channels, with a measured mass of $m = 2012.4 \pm 0.7\,(\text{stat}) \pm 0.6\,(\text{sys})\,\text{MeV}$, and decay width of $\Gamma_{tot} = 6.4^{+2.5}_{-2.0}\,(\text{stat}) \pm 1.6\,(\text{sys})\,\text{MeV}$ [1]. However, knowing only the mass of the state is not sufficient enough to determine the quantum numbers of a state. For instance, within the QCD sum rule method, the mass of the Ω(2012) baryon is estimated, assuming it to be either 1P or 2S excitation state [2]. Both assumptions yield the same mass value, although the estimated residues differ. Thus, additional physical quantities, such as the decay width, are necessary to identify the quantum numbers of newly discovered particles.

In a previous study [3], the $\Omega(2012) \to \Xi^0 K^-$ transition was investigated, and its corresponding decay width was estimated by considering two possible scenarios for Ω(2012): either a $1P$ or $2S$ state. A comparison of the total decay widths obtained in this work led to the conclusion that the Ω(2012) is itself a $J^P = \frac{3}{2}^-$ state. Moreover, predictions from various theoretical models also converge on the likely quantum numbers $J^P = \frac{3}{2}^-$ for the observed state [4−15].

In this study, considering Ω(2012) as $J^P = \frac{3}{2}^-$ state, strong couplings of SU(3) partners of this state are investigated within the framework of light cone sum rules (LCSR) by using the octet baryon's distribution amplitudes (DAs). It should be noted that this problem was also studied in [16] using the flavor SU(3) symmetry approach.

The structure of this paper is as follows: Section II introduces the LCSR for the strong couplings of the transitions $\frac{3}{2}^- \to \frac{1}{2}^+ + \text{pseudoscalar mesons}$. Section III provides a numerical analysis of the LCSR, focusing on the relevant strong couplings. Within this section, we also present the computed values of the decay widths based on the obtained coupling constants. Additionally, we compare our results with those obtained from the flavor SU(3) symmetry method. Finally, our conclusions are summarized in the last section.

II. LCSR FOR THE STRONG COUPLINGS OF $SU(3)$ PARTNERS OF Ω(2012)

To calculate the strong couplings of $SU(3)$ partners, denoted as $\frac{3}{2}^-$ states in the following discussions, we introduce the vacuum-to-octet baryon correlation function:

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{iqx} \left\langle 0|T\left\{\eta_\mu(0) J_\nu(x)\right\}|O(p)\right\rangle , \quad (1)$$

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where η_μ represents the interpolating current of the decuplet baryons, $J_\nu = \bar{q}_1\gamma_\nu\gamma_5 q_2$ is the interpolating current of the pseudoscalar mesons, and $|O(p)\rangle$ represents the octet baryon state.

The interpolating current of the decuplet baryons can be written as:

$$\eta_\mu = \varepsilon^{abc} A \Big\{ \left(q_1^{aT} C\gamma_\mu q_2^b\right) q_3^c + \left(q_2^{aT} C\gamma_\mu q_3^b\right) q_1^c + \left(q_3^{aT} C\gamma_\mu q_1^b\right) q_2^c \Big\}\,, \tag{2}$$

where a,b,c are the color indices, C is the charge conjugation operator, and A is the normalization factor. The quark content of the decuplet baryons and the normalization factor A are presented in Table 1.

To derive the Light Cone Sum Rules for the strong coupling constants, the approach involves computing the correlation function in two ways: in terms of hadrons and in terms of quark-gluon fields within the deep Euclidean domain. By applying the quark-hadron duality ansatz, the relevant sum rules can be derived.

The strong coupling constants appear in double dispersion relation for the correlation function given in Eq. (1). Hence, to calculate these constants, double dispersion relation for the correlation function needs to be calculated. The double dispersion relation is obtained by analytical continuation of the imaginary part of the corresponding invariant amplitudes with respect to the variables p'^2 and q^2 in spin-3/2 and pseudoscalar meson channels, respectively.

Before delving into the details of the calculations, it is important to highlight the following aspect: the interpolating current for the decuplet baryons interacts not only with the ground positive parity states $J^P = \frac{3}{2}^+$ but also with the negative parity states $J^P = \frac{3}{2}^-$ and even with states of $J^P = \frac{1}{2}^-$.

To eliminate the contributions from unwanted states, $J^P = \frac{3}{2}^+$ and $J^P = \frac{1}{2}^-$, a technique involving linear contributions of different Lorentz structures is employed (for more details about this approach refer to [17]).

Table 1. The quark content of the decuplet baryons and the normalization factor A.

	A	q_1	q_2	q_3
Δ^+	$\sqrt{\frac{1}{3}}$	u	u	d
$\Sigma^+(3/2)$	$\sqrt{\frac{1}{3}}$	u	u	s
$\Sigma^0(3/2)$	$\sqrt{\frac{2}{3}}$	u	d	s
$\Sigma^-(3/2)$	$\sqrt{\frac{1}{3}}$	d	d	s
$\Xi^0(3/2)$	$\sqrt{\frac{1}{3}}$	s	s	u
$\Xi^-(3/2)$	$\sqrt{\frac{1}{3}}$	s	s	d
Ω^-	1	s	s	s

Following the standard procedure, we insert the total set of baryons with $J^P = \frac{3}{2}$ into the correlation function as well as the corresponding pseudoscalar mesons. Then, we get,

$$\Pi_{\mu\nu}(p,q) = \sum_{i=\pm} \frac{\left\langle 0|\eta_\mu|\frac{3}{2}^i(p')\right\rangle}{m_i^2 - p'^2}\,\frac{\left\langle \frac{3}{2}^i(p')\mathcal{P}(q)|O(p)\right\rangle}{m_{\mathcal{P}}^2 - q^2}\,\langle 0|J_\nu(x)|\mathcal{P}(q)\rangle\,, \tag{3}$$

where summation is over positive and negative states, and $m_{\mathcal{P}}$ is the mass of the corresponding pseudoscalar meson $\mathcal{P}$ with momentum q. The matrix elements in the above equation are defined as,

$$\begin{aligned}
&\left\langle 0\middle|t\eta_\mu\middle|t\frac{3}{2}^+(p')\right\rangle = \lambda_+ u_\mu(p')\,,\\
&\left\langle 0\middle|t\eta_\mu\middle|t\frac{3}{2}^-(p')\right\rangle = \lambda_-\gamma_5 u_\mu(p')\,,\\
&\left\langle \frac{3}{2}^+(p')\mathcal{P}(q)\middle|tO(p)\right\rangle = g_+\bar{u}_\alpha(p')u(p)q^\alpha\,,\\
&\left\langle \frac{3}{2}^-(p')\mathcal{P}(q)\middle|tO(p)\right\rangle = g_-\bar{u}_\alpha(p')\gamma_5 u(p)q^\alpha\,,\\
&\langle 0|J_\nu|\mathcal{P}(q)\rangle = i f_{\mathcal{P}} q_\nu\,,
\end{aligned} \tag{4}$$

where $\lambda_\pm$ are the residues of the related $\frac{3}{2}^\pm$ baryons, $g_\pm$ stands for the coupling constants of the $J^P = \frac{3}{2}^\pm$ baryons with the octet baryons and the pseudoscalar mesons, $f_{\mathcal{P}}$ is the decay constant of the pseudoscalar meson and q denotes its 4-momentum, and $u_\mu(p')$ and $u(p)$ are the Rarita-Schwinger and Dirac spinors respectively. Performing summation over the spins of the Rarita-Schwinger spinors with the help of the following formula,

$$\sum_{s'} u_\mu(p',s')\bar{u}_\alpha(p',s') = -(\not{p}'+m)\left[g_{\mu\alpha} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2p'_\mu p'_\alpha}{3m^2} + \frac{p'_\mu\gamma_\alpha - p'_\alpha\gamma_\mu}{3m}\right], \tag{5}$$

and using Eqs. (3) and (4) one can obtain the expression of the correlation function from the hadronic part. It should be reminded here that the interpolating current interacts not only with spin $\frac{3}{2}$ states, but also with spin $\frac{1}{2}$ states.

Using the condition $\gamma^\mu\eta_\mu = 0$, it can easily be shown that

$$\left\langle 0\middle|t\eta_\mu\middle|t\frac{1}{2}(p')\right\rangle \sim \left[\alpha\gamma_\mu - \beta p'_\mu\right]u(p')\,. \tag{6}$$

It follows from this equation that any structure containing γ_μ or p'_μ is "contaminated" by the contributions of spin $\frac{1}{2}$-states. Hence, to remove the contributions of spin $\frac{1}{2}$-states, these structures are all discarded.

Another problem is all Dirac structures not being independent of each other. To overcome this issue, Dirac structures need to be arranged in a specific order. In the present work we choose the ordering $\gamma_\mu\ \not{p}'\ \not{q}\gamma_\nu$.

Keeping these notes in mind, and using Eqs. (3), (4) and (5), we obtain the correlation function from the phenomenological part as follows:

$$\Pi_{\mu\nu} = \frac{\lambda_+ g_+(-\not{q} + m_+ + m_O) q_\mu q_\nu f_{\mathcal{P}}}{(m_+^2 - p'^2)(m_{\mathcal{P}}^2 - q^2)} u(p) + \frac{\lambda_- g_-(\not{q} + m_- - m_O) q_\mu q_\nu f_{\mathcal{P}}}{(m_-^2 - p'^2)(m_{\mathcal{P}}^2 - q^2)} u(p) + \ldots, \qquad (7)$$

where m_O is the mass of the relevant octet baryon, $m_+(m_-)$ is the mass of the spin-$\frac{3}{2}$ positive (negative) parity baryon. Here, ... indicates the contributions of the excited states and continuum.

As a last step, we need to eliminate the contributions of $J^P = \frac{3}{2}^+$ states. For this purpose, the linear combinations of the invariant functions corresponding to different Lorentz structures are considered.

We now turn our attention to the calculation of the correlation function by using the operator product expansion (OPE) in the deep Euclidean region for the variables $p'^2 = (p-q)^2$, and $q^2 \ll 0$. To calculate the OPE, the explicit forms of the interpolating current are placed in the correlator and possible contractions are performed between quark fields by using Wick's theorem. As an example, for the correlation function of $\Sigma^0(3/2) \to NK$, we get,

$$\begin{aligned}\Pi_{\mu\nu} = \sqrt{\frac{2}{3}} \int d^4x e^{iqx} \epsilon_{abc} (C\gamma_\mu)_{\alpha\beta} (\gamma_\nu\gamma_5)_{\rho\sigma} & \Big\{ \langle 0|u_\alpha^a(0) u_\sigma^b(x) d_\beta^c(0)|N\rangle S_{\gamma\rho}(-x) \\ & + \langle 0|u_\gamma^c(0) u_\sigma^b(x) d_\alpha^c(0)|N\rangle S_{\beta\rho}(-x) \\ & + \langle 0|u_\beta^a(0) u_\sigma^b(x) d_\gamma^c(-x)|N\rangle S_{\alpha\rho}(-x) \Big\} \end{aligned} \qquad (8)$$

where $S(-x)$ is the strange quark propagator. From this expression it follows that the OPE results is obtained by convolution of the quark propagator to the sum of the nucleon distribution amplitudes, obtained from $\epsilon^{abc}\langle 0|u_\alpha^a(0) u_\beta^b(x) d_\gamma^c(0)|N\rangle$ matrix element. Diagrammatic description of the Eq.(8) is given in Fig. 1. Once we use the explicit expressions of the quark propagators and definition of DAs of octet baryons, the following master integral appears in the coefficients of different Lorentz structures

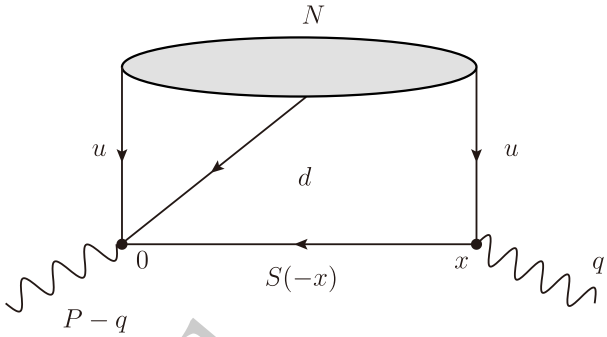

Fig. 1. Diagrammatic representation of the correlation function is depicted. The wavy lines describes the external currents, solid lined corresponds to the quark fields and shaded regions corresponds to the DAs of the nucleon.

$$I_{n,k} = \int du \frac{u^k}{\left[m^2 - (pu-q)^2\right]^n}\ ;\ n = 1,2,3.$$

For the calculation of the double spectral densities it is enough to find the double spectral representations of the master integrals. The details of the calculations for the spectral density for $n = 1$ case are presented in Appendix A. The cases $n = 2$ and $n = 3$ are calculated in the similar manner.

The invariant amplitudes are related to the spectral densities via double dispersion relation as follows:

$$\Pi[(p-q)^2, q^2] = \int ds_1 \int ds_2 \frac{\rho(s_1,s_2)}{[s_1 - (p-q)^2](s_2 - q^2)} + \cdots \qquad (9)$$

The spectral density can be obtained from $\Pi[(p-q)^2, q^2]$ by applying two subsequent double Borel transformations (for more details of the calculation see Appendix A.)

Matching the OPE results with the double dispersion relations for the relevant Lorentz structures for the hadrons, applying the quark-hadron duality ansatz, and performing double Borel transformation with respect to the variables $-(p-q)^2$ and $-q^2$, we obtain the LCSR for the relevant coupling constants whose explicit form can be written as,

$$\begin{aligned} g_- = & \frac{e^{m_-^2/M_1^2} e^{m_{\mathcal{P}}^2/M_2^2}}{f_{\mathcal{P}} \lambda_-(m_+ + m_-)} \frac{1}{\pi^2} \int_0^{s_0} ds_1 \\ & \times \int_{t_1(s_1)}^{min(s'_0, t_2(s_1))} ds_2\, e^{-s_1/M_1^2} e^{-s_2/M_2^2}\, \mathrm{Im}_{s_1} \mathrm{Im}_{s_2} \\ & \left\{ \Pi_1(m_+ - m_O) + \Pi_2 \right\}, \end{aligned} \qquad (10)$$

where Π_1 and Π_2 are the invariant functions of the Lorentz structures $\not{q} q_\mu q_\nu$ and $q_\mu q_\nu$, respectively, and

$$t_{1,2} = s_1 + m_O^2 \mp 2m_O\sqrt{s_1 - m^2}$$

where m is the corresponding mass of light quarks. Here, s_0' is the continuum threshold in pseudoscalar meson channel. The continuum threshold s_0' is chosen as a mass square of the first radial excitation of the corresponding pseudoscalar meson. Finally, note that in the $m_O \to 0$ limit, the applied method needs to modified (for more details, see [18,19]).

III. NUMERICAL ANALYSIS

The present section is devoted to the numerical analysis of the coupling constants derived in the previous section within LCSR. The main nonperturbative input of the considered LCSR is the distribution amplitudes of the octet baryons, namely N, Σ and Ξ. The explicit expressions of the relevant DAs are obtained in [20–23]. The DAs contain the normalization constants f, λ_1 and λ_2 which are determined from the analysis of mass sum rules as well as lattice QCD [24,25]. The normalization constant of the leading twist f, (for N, Σ, and Ξ baryons) is defined via the matrix element of the local current (all quark fields are at the same point).

$$\epsilon^{abc}\langle 0|(q_1^a(0)C\not{n}q_2^b(0))\gamma_5\not{n}q_3^c(0)|O(p)\rangle = f(pn)\not{n}u(p)\,. \tag{11}$$

Besides, the DAs of higher twist contributions involve two additional normalization constants, λ_1 and λ_2 and they are defined as the matrix elements of local three quark twist-four operators,

$$\epsilon^{abc}\langle 0|(q_1^a(0)C\gamma_\mu q_2^b(0))\gamma_5\gamma^\mu q_3^c(0)|O(p)\rangle = \lambda_1 m_O u(p)\,,$$
$$\epsilon^{abc}\langle 0|(q_1^a(0)C\sigma_{\mu\nu} q_2^b(0))\gamma_5\sigma^{\mu\nu} q_3^c(0)|O(p)\rangle = \lambda_2 m_O u(p)\,, \tag{12}$$

where n is the light-like vector and $u(p)$ is the Dirac bispinor.

The normalization constants f, λ_1, and λ_2 for Λ baryon can be obtained from Eqs. (11) and (12) with the help of following replacements:

$$C\not{n} \to C\gamma_5\not{n} \quad \gamma_5\not{n} \to \not{n} \qquad \text{for f} \tag{13}$$

$$C\gamma_\mu \to C\gamma_5\gamma_\mu \quad \gamma_5\gamma_\mu \to \gamma_\mu \qquad \text{for } \lambda_1 \tag{14}$$

$$C\sigma_{\mu\nu} \to C\gamma_5 \quad \gamma_5\sigma_{\mu\nu} \to 1 \qquad \text{for } \lambda_2 \tag{15}$$

In our analysis, we used the values of these parameters obtained from lattice QCD that are presented in Table 2 for completeness. The masses of the $SU(3)$ partners of $\Omega(2012)$ are obtained in [16] and presented below.

$$m_- = \begin{cases} 1700\pm 90\ \text{MeV} & \text{for } \Delta, \\ 1805\pm 100\ \text{MeV} & \text{for } \Sigma(3/2), \\ 1910\pm 110\ \text{MeV} & \text{for } \Xi(3/2), \\ 2012.4\pm 0.9\ \text{MeV} & \text{for } \Omega\ [26]. \end{cases}$$

These mass values are used in our numerical analysis. For the masses of the ground state baryons, we adapted values from PDG [26]. In addition, the value of the quark condensate is taken as $\langle\bar{q}q\rangle = -(246^{+28}_{-19}\ \text{MeV})^3$ [17] and $\langle\bar{s}s\rangle = 0.8\langle\bar{q}q\rangle$ [27].

The residues of the negative parity $J^P = \frac{3}{2}^-$ baryons are related to the residues of the radial excitations of the decuplet baryons as follows,

$$\lambda_- = \lambda_{rad}\sqrt{\frac{m_- - m_+}{m_- + m_+}}\,.$$

The residues of the radial excitations of the decuplet baryons are calculated in [2]. Using these results one can easily determine the residues of the $J^P = \frac{3}{2}^-$ baryons.

The working regions of the Borel mass parameters

Table 2. Numerical values of the parameters f, λ_1 and λ_2 given in units of 10^{-3} GeV^2.

	f	λ_1	λ_2
N	3.54	−44.9	93.4
Σ	5.31	−46.1	85.2
Ξ	6.11	−49.8	99.5
Λ	4.87	−42.2	98.9

Table 3. Working regions of the Borel mass parameters and continuum threshold s_0.

	Borel mass parameters M_1^2(GeV^2)	M_2^2(GeV^2)	Continuum threshold s_0(GeV^2)	Continuum threshold s_0'(GeV^2)
$\Delta \to N\pi$	$3\div 4$	$0.25\div 0.35$	5.0 ± 0.2	1.7
$\Sigma(3/2) \to NK$	$3\div 4$	$0.25\div 0.35$	5.5 ± 0.2	2.0
$\Sigma(3/2) \to \Lambda\pi$	$3\div 4$	$0.42\div 0.44$	5.5 ± 0.2	1.7
$\Sigma(3/2) \to \Sigma\pi$	$3\div 4$	$0.42\div 0.44$	5.5 ± 0.2	1.7
$\Xi(3/2) \to \Lambda K$	$3\div 4$	$0.45\div 0.47$	6.0 ± 0.2	2.0
$\Xi(3/2) \to \Sigma K$	$3\div 4$	$0.60\div 0.65$	6.0 ± 0.2	2.0
$\Xi(3/2) \to \Xi\pi$	$3\div 4$	$0.50\div 0.60$	6.0 ± 0.2	1.7
$\Omega \to \Xi K$	$3\div 4$	$0.55\div 0.65$	6.5 ± 0.2	2.0

and continuum thresholds, s_0 and s_0' used in the numerical analysis are presented in Table 3. Determination of working region of Borel parameters is based on the criteria that both power corrections and continuum contributions should be suppressed. Moreover, The continuum threshold s_0 is obtained from the condition that the mass of the considered states reproduce the experimental values about 10% accuracy.

Having the values of all input parameters at hand, we can proceed to perform the numerical analysis of the relevant coupling constants. As an example, in Fig. 2, we present the dependency of the coupling constant on M_1^2 at the fixed values of the continuum thresholds s_0, s_0' and M_2^2 for the $\Omega \to \Xi K$ transition since this transition has already been discovered. From this figure, we observe that there exists good stability of the coupling constant when M_1^2 varies in its working region (see Table 3). The obtained coupling constants are presented in Table 4. The errors in the results for the coupling constants can be attributed to the uncertainties in the input parameters as well as errors to the Borel mass parameters M_1^2, M_2^2, and continuum threshold s_0 and s_0'.

Having determined coupling constants, we can calculate the decay widths of the corresponding transitions. Using the matrix elements for the considered $\frac{3}{2}^- \to \frac{1}{2}^+ +$pseudoscalar meson transitions, the decay width can be written as,

$$\Gamma = \frac{g_-^2}{24\pi m_-^2}\left[(m_- - m_O)^2 - m_{\mathcal{P}}^2\right]|t\vec{p}|t^3\,, \tag{16}$$

where

$$|t\vec{p}|t = \frac{1}{2m_-}\sqrt{m_-^4 + m_O^4 + m_{\mathcal{P}}^4 - 2m_-^2 m_O^2 - 2m_-^2 m_{\mathcal{P}}^2 - 2m_O^2 m_{\mathcal{P}}^2}\,,$$

is the momentum of octet baryon, m_O and $m_{\mathcal{P}}$ are the mass of the octet baryon and pseudoscalar meson, respectively, Using the values of the coupling constants obtained within this work, we estimated the decay widths of the relevant transitions that are summarized in Table 4. For comparison we also present the results of the decay widths obtained from the flavor $SU(3)$ analysis [16]. We would like to make the following remark at this point. From the expression of the decay width, we see that it is quite sensitive to the mass splitting among the $SU(3)$ partners of the Ω(2012) and ground state baryons. Thus, for a fair comparison, we used the same mass values as in [16].

As a final remark, we compare our results with the values obtained within the framework of the flavor $SU(3)$ method [16]. In this analysis, the coupling constant for $\Omega \to \Xi K$ is taken as the input parameter, and all the remaining couplings are expressed in terms of this coupling with the help of $SU(3)$ symmetry relations. Using the experimental value of the decay width $\Omega \to \Xi K$, one can determine the coupling constant of this transition with the help of Eq. (16), and hence all the other coupling constants can be determined. When we compare our results for the coupling constants and decay widths of the considered decays with those obtained within flavor $SU(3)$ analysis, we see they are compatible within the uncertainties of the the model predictions. Small deviations in the results can be attributed to the $SU(3)$ violation effects, uncertainties of the input parameters of the theory. Furthermore, our prediction on decay width for $\Omega \to \Xi K$ is compatible with the ones observed by BELLE Collaboration within the uncertainties [1]. Also note that the coupling constant, hence the decay width of $\Omega \to \Xi K$ within light cone sum rules method was calculated by using the DAs of pseudoscalar mesons in [3]. However, in present work, we recalculated these quantities within the same framework by using the DAs of Ξ baryon. In this method, the calculations of the theoretical part of the sum rules can be achieved by using only one quark propagator, however in [3], two quark propagators are needed and

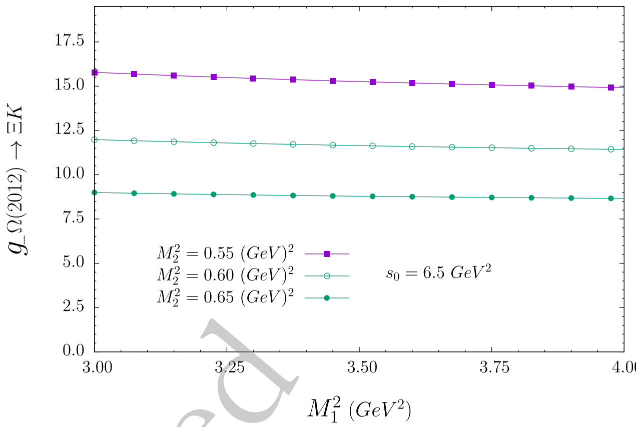

Fig. 2. (color online) The dependency of the coupling constant of the $\Omega(2012) \to \Xi^- K^+$ transition on the Borel mass parameter M_1^2, at several fixed values of the Borel parameter M_2^2, and the continuum threshold $s_0 = 6.5\, GeV^2$.

Table 4. Decay widths of the $J^P = \frac{3}{2}^-$ baryons.

Decay channels	g_- (GeV^{-1})	Γ(MeV) (This work)	Γ(MeV) [16]
$\Delta \to N\pi$	12 ± 3	$71.6 \times (1.0 \pm 0.5)$	39-58
$\Sigma \to NK$	6 ± 2	$11.1 \times (1.0 \pm 0.6)$	7-12
$\Sigma \to \Lambda\pi$	9 ± 3	$23.7 \times (1.0 \pm 0.6)$	11-18
$\Sigma \to \Sigma\pi$	5 ± 1	$4.5 \times (1.0 \pm 0.4)$	4-7
$\Xi \to \Lambda K$	10 ± 2	$15.5 \times (1.0 \pm 0.4)$	5-10
$\Xi \to \Sigma K$	6 ± 2	$2.7 \times (1.0 \pm 0.6)$	2-5
$\Xi \to \Xi\pi$	7 ± 2	$6.9 \times (1.0 \pm 0.5)$	5-9
$\Omega \to \Xi K$	12 ± 3.5	$7.4 \times (1.0 \pm 0.6)$	-

that makes the calculations difficult since each quark propagator contains many terms. Another advantage of the present method appears in dealing with the contributions of baryons with different parities, especially when mass splittings are small. In this method, no pollution will arise due to negative parity baryons. However, with the methods used in [3], the problem of separation of the contributions of positive baryons remains unsolved. Another difference between two methods is that in this study we take into account both Borel mass parameters M_1^2 and M_2^2 but in [3], $M_1^2 = M_2^2$ is considered. On the other hand, the uncertainties of the parameters entering to distribution amplitudes of baryons are larger than compared to meson DAs. Once the errors are minimized in the determination of these parameters, more precise results can be obtained. When we compare our results on coupling constant for $\Omega \to \Xi K$, we find out that our result is consistent with the one in [3] within the uncertainties.

IV. CONCLUSION

In conclusion, we employed the LCSR method to compute the strong coupling constants and decay widths for the $SU(3)$ partners of the $\Omega(2012)$ baryon in $\frac{3}{2}^- \to \frac{1}{2}^+ +$ pseudoscalar meson transitions. The "contamination" caused by the $J^P = \frac{3}{2}^+$ baryons are eliminated by considering the linear combinations of the sum rules obtained from different Lorentz structures. By comparing our decay width results with the findings of [16], we ascertain the compatibility of our decay width predictions with the outcomes of the flavor $SU(3)$ symmetry analysis. Small discrepancy between the two methods' predictions may be attributed to the $SU(3)$ violation effects. Moreover, our estimated decay width for $\Omega \to \Xi K$ transition is also compatible with the measurement of BELLE collaboration within the uncertainty limits. In addition our result on coupling constant for $\Omega \to \Xi K$ calculated using DAs of Ξ are also consistent with the prediction [3] where the DAs of pseudoscalar mesons are used.

Our results on the branching ratios can give useful hints about the nature of the $SU(3)$ partners of $\Omega(2012)$ baryon.

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APPENDIX A: DERIVATION OF THE SPECTRAL DENSITY

Here we demonstrate the details of the derivation of the spectral density (See also [28]).

After applying the double Borel transformation over the variables variables $-p'^2$ and $-q^2$ to Eq. (9), we get,

$$\Pi^{\mathcal{B}_1}(M_1^2, M_2^2) = \int ds_1 \int ds_2 e^{-s_1/M_1^2 - s_2/M_2^2} \rho(s_1, s_2) . \quad \text{(A1)}$$

Before implementing the second double Borel transformation, we introduce new variables $\sigma_1 = \frac{1}{M_i^2}$. The second Borel transformation can be performed over the new Borel parameter τ_i by using the relation,

$$\mathcal{B}_\tau e^{-s\sigma} = \delta\left(\frac{1}{\tau} - s\right) . \quad \text{(A2)}$$

As a result, we have

$$\mathcal{B}_{\tau_1}\mathcal{B}_{\tau_2}\Pi^{\mathcal{B}_1}(M_1^2, M_2^2) = \rho\left(\frac{1}{\tau_1}, \frac{1}{\tau_2}\right) . \quad \text{(A3)}$$

Hence, double spectral density can be obtained as follows,

$$\rho(s_1, s_2) = \mathcal{B}_{\frac{1}{s_1}}(\sigma_1)\mathcal{B}_{\frac{1}{s_2}}(\sigma_2)\Pi^{\mathcal{B}}\left(\frac{1}{\sigma_1}, \frac{1}{\sigma_2}\right) .$$

Let us now pay our attention to the double spectral density for the $n = 1$ case. Using

$$-(pu - q)^2 = -u(p - q)^2 - \bar{u}q^2 + u\bar{u}m_O^2 ,$$

where $\bar{u} = 1 - u$. $I_{1,k}$ can be written as,

$$\begin{aligned} I_{1,k} &= \int du \frac{u^k}{\left[m^2 - u(p-q)^2 - \bar{u}q^2 + \bar{u}um_O^2\right]} \\ &= \int du \frac{u^k}{\mathcal{D}} , \end{aligned}$$

where m is the corresponding quark mass. Using the Schwinger representation for the denominator and carrying out the first double Borel transformation over the variables $-(p-q)^2$ and $-q^2$, we get

$$\begin{aligned} I_{1,k} &= \frac{\sigma_2^k}{(\sigma_1+\sigma_2)^{k+1}} exp\left[-m_O^2 \frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2} - m^2(\sigma_1+\sigma_2)\right] , \\ &= \frac{\sigma_2^k}{(\sigma_1+\sigma_2)^{k+1}} exp\left[m_O^2 \frac{\sigma_1^2+\sigma_2^2}{2(\sigma_1+\sigma_2)} - \left(m^2 + \frac{m_O^2}{2}\right) \right. \\ &\quad \left. \times (\sigma_1+\sigma_2)\right] , \end{aligned}$$

where $\sigma_i = \dfrac{1}{M_i^2}$. In order to perform the second double Borel transformation we use the relation,

$$\sqrt{\frac{\sigma_1+\sigma_2}{2\pi}}\int_{-\infty}^{+\infty} dx_i\, exp\left[-\frac{\sigma_1+\sigma_2}{2}x_i^2-\sigma_i m_O x_i\right] = exp\left[\frac{m_O^2\sigma_i^2}{2(\sigma_1+\sigma_2)}\right] .$$

Then we get,

$$\begin{aligned}
I_{1,k}^{\mathcal{B}} &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} dx_1\int_{-\infty}^{+\infty} dx_2\,\frac{\sigma_2^k}{(\sigma_1+\sigma_2)^k}\,exp\left[-\sigma_1\left(m^2+\frac{(m_O+x_1)^2+x_2^2}{2}\right)-\sigma_2\left(m^2+\frac{(m_O+x_2)^2+x_1^2}{2}\right)\right]\\
&= \frac{1}{2\pi}\frac{1}{\Gamma(k)}\int_{-\infty}^{+\infty} dx_1\int_{-\infty}^{+\infty} dx_2\int_0^{\infty} dt\, t^{k-1}\sigma_2^k\, exp\left[-\sigma_1\left(m^2+\frac{(m_O+x_1)^2+x_2^2}{2}+t\right)-\sigma_2\left(m^2+\frac{(m_O+x_2)^2+x_1^2}{2}+t\right)\right]\\
&= \frac{1}{2\pi}\frac{1}{\Gamma(k)}\int_{-\infty}^{+\infty} dx_1\int_{-\infty}^{+\infty} dx_2\int_0^{\infty} dt\, t^{k-1}\, exp\left[-\sigma_1\left(m^2+\frac{(m_O+x_1)^2+x_2^2}{2}+t\right)\right]\\
&\times\left(-\frac{\partial}{\partial t}\right)^k exp\left[-\sigma_2\left(m^2+\frac{(m_O+x_2)^2+x_1^2}{2}+t\right)\right] .
\end{aligned}$$

After performing the second Borel transformation, we obtain the the spectral density corresponding to $I_{1,k}$ as is given below,

$$\begin{aligned}
\rho_{1,k}(s_1,s_2) &= \frac{1}{2\pi}\frac{1}{\Gamma(k)}\left(-\frac{\partial}{\partial s_2}\right)^k\int_{-\infty}^{+\infty} dx_1\int_{-\infty}^{+\infty} dx_2\int_0^{\infty} dt\, t^{k-1}\delta\left[s_1-\left(m^2+\frac{(m_O+x_1)^2+x_2^2}{2}+t\right)\right]\\
&\times\delta\left[s_2-\left(m^2+\frac{(m_O+x_2)^2+x_1^2}{2}+t\right)\right]\\
&= \frac{1}{2\pi}\frac{1}{\Gamma(k)}\left(-\frac{\partial}{\partial s_2}\right)^k\int_{-\infty}^{+\infty} dx_1\int_{-\infty}^{+\infty} dx_2\int_0^{\infty} dt\, t^{k-1}\delta\left[s_1-\left(m^2+\frac{(m_O+x_1)^2+x_2^2}{2}+t\right)\right]\\
&\times\delta\left[s_2-\left(m^2+\frac{(m_O+x_2)^2+x_1^2}{2}+t\right)\right].
\end{aligned}$$

Using two Dirac delta functions, one can easily perform integrals over t and x_2 whose result is given below,

$$\rho_{1,k}(s_1,s_2) = \frac{1}{2\pi\Gamma(k)m_O}\left(-\frac{\partial}{\partial s_2}\right)^k\int_{-\infty}^{+\infty} dx_1\left[s_1-\left(m^2+\frac{(m_O+x_1)^2+x_2^2}{2}\right)\right]^{k-1}\times\Theta\left[s_1-\left(m^2+\frac{(m_O+x_1)^2+x_2^2}{2}\right)\right] ,$$

where

$$x_2 = \frac{s_2-s_1}{m_O}+x_1 ,$$

and $\Theta(x)$ is the Heaviside step function which restricts the integral over x_1 between the limits $y_\pm(s_1,s_2)$ where

$$y_\pm(s_1,s_2) = \frac{-m_O^2+s_1-s_2\pm\sqrt{\Delta}}{2m_O} ,$$

and

$$\Delta = -m_O^4-(s_1-s_2)^2+2m_O^2(-2m^2+s_1+s_2) .$$

Thus as a result of above summarized calculations, the spectral density can takes the following form,

$$\begin{aligned}
\rho_{1,k}(s_1,s_2) &= \frac{1}{2\pi}\frac{1}{\Gamma(k)}\frac{1}{m_O}\left(-\frac{\partial}{\partial s_2}\right)^k\\
&\times\int_{y_-}^{y_+} dx\left[(y_+-x)(x-y_-)\right]^k\Theta(\Delta) .
\end{aligned}$$

In order to evaluate the x integral, we introduce a new variable through the relation,

$$x = (y_+-y_-)y+y_- ,$$

so that the spectral density can be written as,

$$\rho_{1,k}(s_1,s_2) = \frac{1}{2\pi}\frac{\Gamma(k)}{\Gamma(2k)}\frac{1}{m_O^{2k}}\left(-\frac{\partial}{\partial s_2}\right)^k\left[\Delta^{k-\frac{1}{2}}\Theta(\Delta)\right] . \quad \text{(A4)}$$

Double spectral densities for $I_{2,k}$ and $I_{3,k}$ can be calculated with the help of the following relations,

$$I_{2,k} = \left(-\frac{\partial}{\partial m^2} \right) I_{1,k} \text{ , and,}$$

$$I_{3,k} = \frac{1}{2} \left(-\frac{\partial}{\partial m^2} \right)^2 I_{1,k} \text{ ,}$$

(see also [17] for the calculation of the spectral densities $I_{2,k}$ and $I_{3,k}$).

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