Cluster radioactivity half-lives within deformed Gamow-like model*

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Abstract: In the present work, based on a Gamow-like model, considering the deformation effect of Coulomb potential, where the effective nuclear radius constant is parameterized, we systematically investigate the cluster radioactivity half-lives of 25 trans-lead nuclei. For comparison, a universal decay law (UDL) proposed by Qi *et al*. [Phys. Rev. C **80**, 044326 (2009)], a new semi-empirical formula for exotic cluster decay proposed by Balasubramaniam *et al.* [Phys. Rev. C **70**, 017301 (2004)], and a scaling law proposed by Horoi [J. Phys. G: Nucl. Part. Phys. **30**, 945 (2004)] are also used. The calculated results within the deformed Gamow-like model are in better agreement with the experimental half-lives. The deformation effect is also discussed within both the Gamow-like and deforemed Gamow-like models. Moreover, we extend this model to predict the cluster radioactivity half-lives of 49 nuclei whose decay energies are energetically allowed or observed but not yet quantified in NUBASE2020.

Keywords: cluster radioactivity, half-lives, Gamow-like model, deformation

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I. INTRODUCTION

Cluster radioactivity is a dominant spontaneous decay mode in unstable nuclei [1−7]. It was first predicted by Săndulescu, Poenaru, and Greiner in 1980 and then experimentally confirmed by Rose and Jones in 1984 [8−10]. Cluster radioactivity is generally believed to share the same quantum mechanism as alpha decay, proton radioactivity, two proton radioactivity, and fission [11, 12]. However, its emitted fragments, whose mass is known to be greater than that of an α particle but less than that of fission fragments, makes it a special case [13]. Benefitting from the rapid development of radioactive beam techniques and establishment of modern devices, increasing experimental data have been obtained and various exotic nuclear phenomenon experimentally observed, which make it necessary to examine and advance the existing theoretical model for cluster radioactivity [14, 15].

Determination of the cluster decay half-lives serves as an effective probe for abundant and important information about nuclear structure and nuclear reactions for heavy and superheavy nuclei. There are generally two scenarios to describe cluster radioactivity half-lives [16−19]. One is the α-like model, which considers the emitted cluster as pre-born in the parent nucleus before it penetrates the interacting potential barrier with corresponding preformation probabilities [20−23]. The other is the fission-like model, which assumes that the emitted cluster forms along with constant geometric shape deformations occurring in its nuclear barrier penetration process from the parent nucleus to the scission configuration [24−26]. Based on the Gamow theory, Zdeb *et al.* first established a Gamow-like model to estimate α decay and cluster radioactivity half-lives of spherical nuclei, in-

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cluding an adjustable parameter characterizing nuclear radius that showed satisfactory agreement with the experimental data. Thereafter, the Gamow-like model successfully evaluated proton radioactivity and two proton radioactivity half-lives [27−34]. Regarding heavy and superheavy nuclei, especially for cluster radioactivity far to its obvious asymmetric shape, deformation should be considered carefully when estimating half-lives due to its effect on inner and outer turning points [35]. In the present work, we extend the Gamow-like model by considering the deformation for both the daughter and emitted cluster to calculate cluster radioactivity half-lives, where the effective nuclear radius constant is taken as an adjustable parameter.

The remainder of this article is organized as follows. The detailed theoretical framework for the deformed Gamow-like model and three semi-empirical formulas of the cluster radioactivity half-lives are presented in Sec. II. Numerical results and detailed discussion are given in Sec. III. Finally, Sec. IV presents a brief summary.

II. THEORETICAL FRAMEWORK

The cluster radioactivity half-life of a nucleus can be determined as [15]

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P P_c}, \tag{1}$$

where λ represents the decay constant and can be obtained by the product of the assault frequency ν, barrier penetration probability P, and cluster preformation probability P_c.

In this work, the assault frequency ν can be estimated as the harmonic oscillator frequency in the Nilsson potential [28]:

$$h\nu = \hbar w \cong \frac{41}{A^{\frac{1}{3}}}. \tag{2}$$

Here $\hbar$ and A represent the reduced Planck constant and mass number of parent nucleus, respectively. The penetration probability P can be calculated using the Wentzel-Kramers-Brillouin (WKB) approximation. In the deformed Gamow-like model, it can be obtained through averaging P_θ in all directions under the effect of deformation, which can be expressed as [32]

$$P = \frac{1}{2}\int_0^\pi P_\theta \sin \mathrm{d}\theta. \tag{3}$$

Here, θ is the orientation angle with respect to the daughter nucleus's symmetry axis and radius vector. P_θ is the polar-angle-dependent tunneling penetration probability of cluster emission. P_θ can be expressed as [32]

$$P_\theta = \exp\left\{-\frac{2}{\hbar}\int_{\mathrm{R_{in}}(\theta)}^{\mathrm{R_{out}}(\theta)} \sqrt{2\mu|\mathrm{V(r,\theta)-Q_c}|\mathrm{dr}}\right\}, \tag{4}$$

where $\mu = \dfrac{M_c M_d}{M_c + M_d}$ is the reduced mass with M_c and M_d being the masses of the emitted cluster and daughter nucleus, respectively. The decay energy Q_c is obtained by [10]

$$Q_c = B(A_d,Z_d) + B(A_c,Z_c) - B(A,Z). \tag{5}$$

Here, $B(A_c,Z_c)$, $B(A_d,Z_d)$, and $B(A,Z)$ represent the binding energy of the emitted cluster, daughter, and parent nuclei taken from the latest 2020 atomic mass table AME2020 [36] and NUBASE2020 [37]. A_i $(i=c,d)$ and Z_i $(i=c,d)$ represent the mass and charge number of the emitted cluster and daughter nucleus, respectively. $R_{\mathrm{in}}(\theta) = R_c(\theta) + R_d(\theta)$, where $R_c(\theta)$ and $R_d(\theta)$ are the radii of the emitted cluster and daughter nucleus, respectively, such that $R_{\mathrm{out}}(\theta)$ satisfies the condition $V(R_{\mathrm{out}}(\theta)) = Q_c$ are the classical turning points. The radii of the emitted cluster and daughter nucleus can be expressed as [32]

$$R_i(\theta) = r_0 A_i^{\frac{1}{3}}[1+\sum_\lambda \beta_\lambda Y_{\lambda 0}(\theta)](i=c,d), \tag{6}$$

where r_0 is the effective nuclear radius constant. β_λ denotes the collection of deformation parameters of the emitted cluster and daughter nucleus with respect to the quadrupole, hexadecapole, and hexacontatetrapole deformations. $Y_{\lambda 0}(\theta)$ is the spherical harmonic function.

In this work, the total interacting potential between the emitted cluster and daughter nucleus is written as [27]

$$V(r,\theta) = \begin{cases} -V_0, & 0 \le r \le R_{\mathrm{in}}(\theta), \\ V_C(r,\theta) + V_l(r) & r \ge R_{\mathrm{in}}(\theta). \end{cases} \tag{7}$$

$V_0 = 25A_c$ denotes the depth of the nuclear potential. The Coulomb interaction is given by [38]

$$V_C(r) = \frac{\mathrm{e}^2 Z_c Z_d}{r} + 3\mathrm{e}^2 Z_c Z_d \sum_\lambda \frac{1}{\lambda+1} \times \frac{R_d^\lambda(\theta)}{r^{\lambda+1}} Y_{\lambda 0}(\theta)\left[\beta_\lambda + \frac{4}{7}\beta_\lambda^2 Y_{\lambda 0}(\theta)\right]. \tag{8}$$

V_l is the centrifugal potential. In this work, we choose it as the Langer modified form because $l(l+1) \to (l+\frac{1}{2})^2$ is a necessary correction for one-dimensional problems. This can be written as [29]

$$V_l(r) = \frac{\hbar^2(l+\frac{1}{2})^2}{2\mu r^2}, \tag{9}$$

where l is the angular momentum carried by the emitted cluster. It can be obtained by

$$l = \begin{cases} \Delta_j, & \text{for even } \Delta_{\mathrm{j}} \text{ and } \pi_{\mathrm{p}} = \pi_{\mathrm{d}}, \\ \Delta_j + 1, & \text{for even } \Delta_{\mathrm{j}} \text{ and } \pi_{\mathrm{p}} \neq \pi_{\mathrm{d}}, \\ \Delta_j, & \text{for odd } \Delta_{\mathrm{j}} \text{ and } \pi_{\mathrm{p}} \neq \pi_{\mathrm{d}}, \\ \Delta_j + 1, & \text{for odd } \Delta_{\mathrm{j}} \text{ and } \pi_{\mathrm{p}} = \pi_{\mathrm{d}}, \end{cases} \tag{10}$$

where $\Delta_j = |j_p - j_d - j_c|$. $j_c, \pi_c, j_p, \pi_p, j_d, \pi_d$ represent the isopin and parity values of the emitted cluster, parent, and daughter nuclei, respectively.

In this work, the preformation probability P_c is calculated through an empirical formula proposed by Ren *et al.*, expressed as [39]

$$\log_{10} P_c = \begin{cases} -(0.011674 Z_c Z_d - 2.035466), \\ \text{for even-even nuclei}, \\ -(0.011674 Z_c Z_d - 2.035466) - 1.175, \\ \text{for odd-A nuclei}. \end{cases} \tag{11}$$

A. Semi-empirical formula

1. *Universal decay law*

In 2009, employing the charged-particle emission microscopic mechanism within α-like R-matrix theory, Qi *et al.* proposed the universal decay law (UDL) [40], which can be written as

$$\log_{10}(T_{1/2})(s) = a Z_c Z_d \sqrt{\frac{\mathcal{A}}{Q_c}} + b\sqrt{\mathcal{A} Z_c Z_d (A_c^{1/3} + A_d^{1/3})} + c, \tag{12}$$

where $\mathcal{A} = \dfrac{A_c A_d}{A_c + A_d}$ is the reduced mass of the emitted cluster-daughter nucleus system measured in units of nucleon mass. $a = 0.4314$, $b = -0.3921$, and $c = -32.7044$ are the adjustable parameters.

2. *Balasubramaniam's semi-empirical formula for cluster radioactivity*

In 2004, Balasubramaniam *et al.* generalized the Geiger-Nuttall law taking into account the mass and charge asymmetries for cluster radioactivity half-lives [41], which can be expressed as

$$\log_{10}(T_{1/2}) = \frac{a A_c \eta + b Z_c \eta_z}{\sqrt{Q_c}} + c, \tag{13}$$

where $\eta = \dfrac{A_d - A_c}{A}$ and $\eta_z = \dfrac{Z_d - Z_c}{Z}$ are the mass and charge asymmetries, respectively. $a = 10.603$, $b = 78.027$, and $c = -80.669$ are the adjustable parameters.

3. *Scaling law*

In 2004, based on the Geiger-Nuttall law, Horoi proposed a scaling law of α decay and cluster radioactivity half-lives, which can be expressed as [42]

$$\log_{10}(T_{1/2}) = (a_1 \mu^x + b_1)\left[\frac{(Z_e Z_d)^y}{\sqrt{Q}} - 7\right] + (a_2 \mu^x + b_2), \tag{14}$$

where Z_e and Z_d are the charge number of emitted α particle and/or cluster and daughter nucleus, respectively. Q is the decay energy of α decay and/or cluster radioactivity. $a_1 = 9.1$, $b_1 = -10.2$, $a_2 = 7.39$, $b_2 = -23.2$, $x = 0.416$, and $y = 0.613$ are the parameters adjusted by fitting the experimental data of 119 α decay and 11 cluster radioactivities emitted from even-even parent nucleus.

III. RESULTS AND DISCUSSION

The aim of the present work is to systematically study the half-lives of 25 trans-lead nuclei based on the Gamow-like model by considering the deformation effect of Coulomb potential for both cluster and daughter nucleus. Under this model, in the present work, the only adjustable parameter, the effective nuclear radius constant, is obtained as 1.19 by fitting the experimental half-lives of those nuclei, which align with the empirical region 1.1 ~ 1.2 and is physically reasonable according to Refs. [9, 43, 44]. The detailed calculated results are well reflected in Table 1. In this table, the first to fourth columns represent the decay process, cluster radioactivity decay energy Q_c, experimental cluster radioactivity half-lives taken from Refs. [45] and [46] in logarithmic form, and angular momentum l carried by the emitted cluster, labeled as Decay, Q_c (MeV), $\log T_{1/2}^{\mathrm{exp}}$, and l, respectively. The quadrupole, hexadecapole, and hexacontatetrapole deformation parameters taken from Ref. [47] of the daughter and emitted cluster nucleus, denoted as β_{di} ($i = 2,4,6$) and β_{ci} ($i = 2,4,6$), respectively, are listed in the fifth to tenth columns. The eleventh to fourteenth columns are the calculated results of UDL, Balasubramaniam's semi-empirical formula, scaling law, and our work in logarithmic form, represented as $\log T_{1/2}^{\mathrm{UDL}}$, $\log T_{1/2}^{\mathrm{Bala}}$, $\log T_{1/2}^{\mathrm{SL}}$ and $\log T_{1/2}^{\mathrm{Cal}}$, respectively. From this table, it is clear that the calculated half-lives within the deformed Gamow-like model are basically consistent with experimental half-lives, as well as the calculated results

Table 1. Comparisons between the experimental cluster radioactivity half-lives (in seconds) in logarithm form and the calculated ones using different formulas.

Decay	Q_c/MeV	$\log T_{1/2}^{\rm exp}$	l	β_{d2}	β_{d4}	β_{d6}	β_{c2}	β_{c4}	β_{c6}	$\log T_{1/2}^{\rm UDL}$	$\log T_{1/2}^{\rm Bala}$	$\log T_{1/2}^{\rm SL}$	$\log T_{1/2}^{\rm Cal}$
$^{221}\text{Fr}\to^{207}\text{Tl}+^{14}\text{C}$	31.29	14.56	3	0	0	0	−0.361	0	0	12.70	14.65	13.54	15.20
$^{221}\text{Ra}\to^{207}\text{Pb}+^{14}\text{C}$	32.40	13.39	3	0	0	0	−0.361	0	0	11.46	13.14	12.27	14.16
$^{222}\text{Ra}\to^{208}\text{Pb}+^{14}\text{C}$	33.05	11.22	0	0	0	0	−0.361	0	0	10.07	12.23	11.00	11.69
$^{223}\text{Ra}\to^{209}\text{Pb}+^{14}\text{C}$	31.83	15.05	4	−0.011	0	0	−0.361	0	0	12.57	14.01	13.42	15.12
$^{224}\text{Ra}\to^{210}\text{Pb}+^{14}\text{C}$	30.53	15.87	0	0	0	0	−0.361	0	0	15.38	16.01	16.14	16.22
$^{226}\text{Ra}\to^{212}\text{Pb}+^{14}\text{C}$	28.20	21.20	0	0	0	0	−0.361	0	0	20.95	19.97	21.53	21.07
$^{223}\text{Ac}\to^{209}\text{Bi}+^{14}\text{C}$	33.06	12.60	2	−0.011	0	0	−0.361	0	0	11.08	12.35	11.93	13.73
$^{225}\text{Ac}\to^{211}\text{Bi}+^{14}\text{C}$	30.48	17.16	4	−0.01	0	0	−0.361	0	0	16.61	16.24	17.26	18.56
$^{228}\text{Th}\to^{208}\text{Pb}+^{20}\text{O}$	44.72	20.73	0	0	0	0	0.01	−0.024	0.02	21.97	22.22	21.2	21.92
$^{230}\text{Th}\to^{206}\text{Hg}+^{24}\text{Ne}$	57.76	24.63	0	0	0	0	−0.063	0.013	−0.03	25.39	25.68	23.92	24.69
$^{231}\text{Pa}\to^{207}\text{Tl}+^{24}\text{Ne}$	60.41	22.89	1	0	0	0	−0.063	0.013	−0.03	22.27	23.59	21.32	23.41
$^{232}\text{U}\to^{208}\text{Pb}+^{24}\text{Ne}$	62.31	20.39	0	0	0	0	−0.063	0.013	−0.03	20.59	22.26	19.94	20.93
$^{233}\text{U}\to^{209}\text{Pb}+^{24}\text{Ne}$	60.49	24.84	2	−0.011	0	0	−0.063	0.013	−0.03	23.63	23.83	22.55	24.51
$^{234}\text{U}\to^{210}\text{Pb}+^{24}\text{Ne}$	58.82	25.93	0	0	0	0	−0.063	0.013	−0.03	26.52	25.32	25.03	25.72
$^{235}\text{U}\to^{211}\text{Pb}+^{24}\text{Ne}$	57.36	27.42	1	0	0	0	−0.063	0.013	−0.03	29.16	26.69	27.31	29.07
$^{233}\text{U}\to^{208}\text{Pb}+^{25}\text{Ne}$	60.70	24.84	2	0	0	0	0.053	0.002	−0.03	24.00	24.43	23.05	24.74
$^{234}\text{U}\to^{208}\text{Pb}+^{26}\text{Ne}$	59.41	25.93	0	0	0	0	0.121	−0.052	−0.035	27.01	26.37	25.84	25.35
$^{234}\text{U}\to^{206}\text{Hg}+^{28}\text{Mg}$	74.1	25.53	0	0	0	0	0.277	−0.073	0.008	25.77	25.95	24.76	24.02
$^{236}\text{U}\to^{208}\text{Hg}+^{28}\text{Mg}$	70.73	27.58	0	0	0	0	0.277	−0.073	0.008	31.25	28.54	29.28	28.32
$^{236}\text{Pu}\to^{208}\text{Pb}+^{28}\text{Mg}$	79.67	21.52	0	0	0	0	0.277	−0.073	0.008	20.64	22.82	20.83	20.24
$^{238}\text{Pu}\to^{210}\text{Pb}+^{28}\text{Mg}$	75.91	25.70	0	0	0	0	0.277	−0.073	0.008	26.26	25.42	25.42	24.54
$^{236}\text{U}\to^{206}\text{Hg}+^{30}\text{Mg}$	72.27	27.58	0	0	0	0	0.119	−0.005	−0.031	29.94	28.64	28.69	27.93
$^{238}\text{Pu}\to^{208}\text{Pb}+^{30}\text{Mg}$	76.80	25.70	0	0	0	0	0.119	−0.005	−0.031	26.06	26.05	25.71	25.04
$^{238}\text{Pu}\to^{206}\text{Hg}+^{32}\text{Si}$	91.19	25.28	0	0	0	0	−0.124	−0.03	−0.033	25.48	25.63	25.7	25.63
$^{242}\text{Cm}\to^{208}\text{Pb}+^{34}\text{Si}$	96.51	23.15	0	0	0	0	0	0	−0.039	22.35	24.46	24.21	23.60

obtained using UDL, Balasubramaniam's semi-empirical formula, and scaling law.

For evaluating the robustness of deformed Gamow-like model to the application for cluster radioactivity, the standard deviation σ between the experimental cluster radioactivity half-lives and calculated ones is also introduced in the following. In this work, it is defined as follows

$$\sigma = \sqrt{\sum(\log_{10}T_{1/2}^{\rm exp} - \log_{10}T_{1/2}^{\rm cal})^2/n}, \qquad (15)$$

where $\log_{10}T_{1/2}^{\rm exp}$ and $\log_{10}T_{1/2}^{\rm cal}$ are the experimental cluster radioactivity half-lives and the calculated ones in logarithm form, respectively. n is the number of nuclei involved for each case. The calculated results are listed in Table 2. It is evident from the data that the σ values for all 25 nuclei within the deformed Gamow-like model are smaller than those obtained using UDL, Balasubramaniam's semi-empirical formula, and scaling law. Comparing the results by UDL, the σ value from our work shows a decrease of 41.26%. This shows that deformed Gamow-like model can reproduce the experimental data well.

Table 2. Standard deviation σ between the experimental data and calculated ones by using different formulas for cluster radioactivity.

σ	UDL	Bala	SL	Cal
	1.3828	0.8980	0.9976	0.8122

Furthermore, for the sake of intuitively comparing these results, the differences between the experimental cluster radioactivity half-lives and calculated ones using UDL, Balasubramaniam's semi-empirical formula, scaling law, and deformed Gamow-like model in logarithmic form are plotted in Fig. 1. It is clear that the differences for the calculated results from the deformed Gamow-like

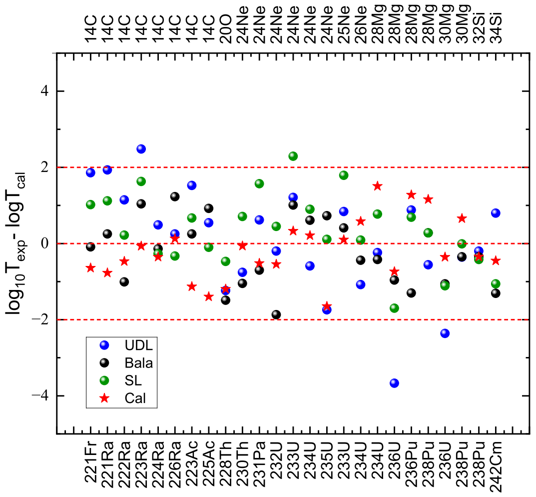

Fig. 1. (color online) Discrepancy between the experimental cluster radioactivity half-lives and calculated ones obtained using different semi-empirical formulas as well as the deformed Gamow-like model in logarithm form.

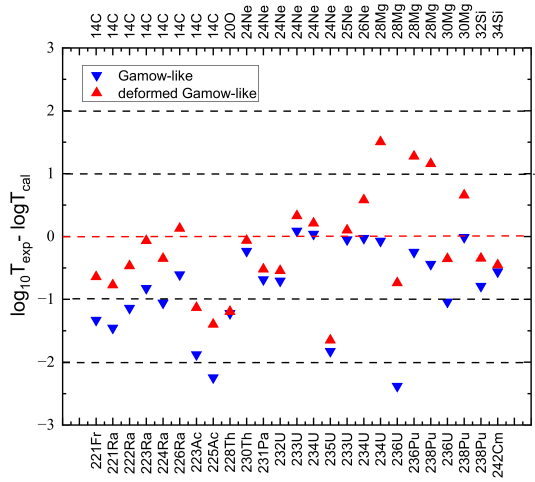

Fig. 2. (color online) Discrepancy between the experimental cluster radioactivity half-lives and calculated ones obtained by Gamow-like and deformed Gamow-like models in logarithm form.

model as well as Balasubramaniam's formula in logarithmic form are overall within a range of ±2. The deviations are closely distributed around the zero line, which indicates that our results as well as the calculated ones using Balasubramaniam's formula can reproduce the experimental data well. By contrast, in the case of $^{236}U \rightarrow ^{208}Hg + ^{28}Mg$, there is a discrepancy of 3.668 from UDL. Also, there are 3 cases where the 25 nuclei are ±2 away from the UDL results. In the case of $^{233}U \rightarrow ^{209}Pb + ^{24}Ne$, the discrepancy is –2.29 compared with scaling law. Additionally, the distribution for the discrepancy of UDL and scaling law exhibits a degree of dispersion. The results in Fig. 1 further demonstrate that the deformed Gamow-like model is in good agreement with the experimental cluster radioactivity half-lives.

To evaluate the strength of deformation in cluster radioactivity, the deformation effect has also been discussed by comparing the half-lives calculated by the Gamow-like and deformed Gamow-like models. The calculated results are presented in Table 3. In this table, the labels are the same as those in Table 1, except for the last two columns denoted as deGa and Ga, which represent the calculated results obtained by the deformed Gamow-like and Gamow-like models in logarithmic form, respectively. From this table, it can be clearly seen that the calculated results considering the deformation can better reproduce the experimental cluster radioactivity half-lives. Furthermore, we have also acquired the σ value within the non-deformed Gamow-like model using Eq. (15), which is 1.0912, an increase of 34.35% compared to the deformed one. To visually analyze the effect of deformation on the Gamow-like model, the differences between the experimental cluster radioactivity half-lives and calculated ones obtained using the Gamow-like and deformed Gamow-like models are plotted in Fig. 2. From this figure, one can see that the calculated results considering deformation are much more in concentratedly distributed along the zero line, which indicates that deformation should be carefully considered when estimating cluster radioactivity half-lives.

Encouraged by the good agreement between the experimental cluster radioactivity half-lives and calculated ones within the deformed Gamow-like model, in the following, we extend this model to predict the half-lives for 49 nuclei that may be appropriate for unobserved ones according to Ref. [45] whose cluster radioactivity is energetically allowed or observed but yet not quantified in NUBASE2020. The detailed calculation results are presented in Table 4. The notations of this table are the same as those of Table 1. From this table, it is clear that the predicted results in our work closely align with the predicted ones obtained using UDL, Balasubramaniam's semi-empirical formula, and scaling law. This illustrates that our predictions are reliable. To set realistic expectations for future experimental confirmation, the uncertainties of Q_c and theoretically predicted half-lives within the deformed Gamow-like model have also been provided. Because Q_c cannot be directly measured through experiments, uncertainties in binding energies will lead to uncertainties in Q_c compared to the theoretically predicted half-lives. The uncertainties of Q_c are obtained through the error propagation formula based on our previous work [48]. We hope this work can provide a theoretical guide for future experiments.

IV. SUMMARY

In summary, based on the deformed Gamow-like model while considering the deformation effect of Cou-

Table 3. Comparisons between experimental cluster radioactivity half-lives (in seconds) using deformed Gamow-like and non-deformed Gamow-like models in logarithm form.

Decay	Q_c/MeV	$\log T_{1/2}^{\rm exp}$	l	β_{d2}	β_{d4}	β_{d6}	β_{c2}	β_{c4}	β_{c6}	$\log T_{1/2}^{\rm deGa}$	$\log T_{1/2}^{\rm Ga}$
$^{221}{\rm Fr}\to^{207}{\rm Tl}+^{14}{\rm C}$	31.29	14.56	3	0	0	0	−0.361	0	0	15.20	15.89
$^{221}{\rm Ra}\to^{207}{\rm Pb}+^{14}{\rm C}$	32.40	13.39	3	0	0	0	−0.361	0	0	14.16	14.84
$^{222}{\rm Ra}\to^{208}{\rm Pb}+^{14}{\rm C}$	33.05	11.22	0	0	0	0	−0.361	0	0	11.69	12.36
$^{223}{\rm Ra}\to^{209}{\rm Pb}+^{14}{\rm C}$	31.83	15.05	4	−0.011	0	0	−0.361	0	0	15.12	15.88
$^{224}{\rm Ra}\to^{210}{\rm Pb}+^{14}{\rm C}$	30.53	15.87	0	0	0	0	−0.361	0	0	16.22	16.92
$^{226}{\rm Ra}\to^{212}{\rm Pb}+^{14}{\rm C}$	28.20	21.20	0	0	0	0	−0.361	0	0	21.07	21.80
$^{223}{\rm Ac}\to^{209}{\rm Bi}+^{14}{\rm C}$	33.06	12.60	2	−0.011	0	0	−0.361	0	0	13.73	14.48
$^{225}{\rm Ac}\to^{211}{\rm Bi}+^{14}{\rm C}$	30.48	17.16	4	−0.01	0	0	−0.361	0	0	18.56	19.41
$^{228}{\rm Th}\to^{208}{\rm Pb}+^{20}{\rm O}$	44.72	20.73	0	0	0	0	0.01	−0.024	0.02	21.92	21.95
$^{230}{\rm Th}\to^{206}{\rm Hg}+^{24}{\rm Ne}$	57.76	24.63	0	0	0	0	−0.063	0.013	−0.03	24.69	24.86
$^{231}{\rm Pa}\to^{207}{\rm Tl}+^{24}{\rm Ne}$	60.41	22.89	1	0	0	0	−0.063	0.013	−0.03	23.41	23.57
$^{232}{\rm U}\to^{208}{\rm Pb}+^{24}{\rm Ne}$	62.31	20.39	0	0	0	0	−0.063	0.013	−0.03	20.93	21.10
$^{233}{\rm U}\to^{209}{\rm Pb}+^{24}{\rm Ne}$	60.49	24.84	2	−0.011	0	0	−0.063	0.013	−0.03	24.51	24.75
$^{234}{\rm U}\to^{210}{\rm Pb}+^{24}{\rm Ne}$	58.82	25.93	0	0	0	0	−0.063	0.013	−0.03	25.72	25.89
$^{235}{\rm U}\to^{211}{\rm Pb}+^{24}{\rm Ne}$	57.36	27.42	1	0	0	0	−0.063	0.013	−0.03	29.07	29.25
$^{233}{\rm U}\to^{208}{\rm Pb}+^{25}{\rm Ne}$	60.70	24.84	2	0	0	0	0.053	0.002	−0.03	24.74	24.89
$^{234}{\rm U}\to^{208}{\rm Pb}+^{26}{\rm Ne}$	59.41	25.93	0	0	0	0	0.121	−0.052	−0.035	25.35	25.96
$^{234}{\rm U}\to^{206}{\rm Hg}+^{28}{\rm Mg}$	74.1	25.53	0	0	0	0	0.277	−0.073	0.008	24.02	25.60
$^{236}{\rm U}\to^{208}{\rm Hg}+^{28}{\rm Mg}$	70.73	27.58	0	0	0	0	0.277	−0.073	0.008	28.32	29.96
$^{236}{\rm Pu}\to^{208}{\rm Pb}+^{28}{\rm Mg}$	79.67	21.52	0	0	0	0	0.277	−0.073	0.008	20.24	21.77
$^{238}{\rm Pu}\to^{210}{\rm Pb}+^{28}{\rm Mg}$	75.91	25.70	0	0	0	0	0.277	−0.073	0.008	24.54	26.14
$^{236}{\rm U}\to^{206}{\rm Hg}+^{30}{\rm Mg}$	72.27	27.58	0	0	0	0	0.119	−0.005	−0.031	27.93	28.62
$^{238}{\rm Pu}\to^{208}{\rm Pb}+^{30}{\rm Mg}$	76.80	25.70	0	0	0	0	0.119	−0.005	−0.031	25.04	25.71
$^{238}{\rm Pu}\to^{206}{\rm Hg}+^{32}{\rm Si}$	91.19	25.28	0	0	0	0	−0.124	−0.03	−0.033	25.63	26.07
$^{242}{\rm Cm}\to^{208}{\rm Pb}+^{34}{\rm Si}$	96.51	23.15	0	0	0	0	0	0	−0.039	23.60	23.71

Table 4. Predicted half-lives for possible cluster radioactive nuclei.

Decay	Q_c/MeV	$\log T_{1/2}^{\rm exp}$	l	β_{d2}	β_{d4}	β_{d6}	β_{c2}	β_{c4}	β_{c6}	$\log T_{1/2}^{\rm UDL}$	$\log T_{1/2}^{\rm Bala}$	$\log T_{1/2}^{\rm SL}$	$\log T_{1/2}^{\rm Cal}$
$^{219}{\rm Rn}\to^{205}{\rm Hg}+^{14}{\rm C}$	$28.10^{+0.004254}_{-0.004254}$	−	3	0	0	0	−0.361	0	0	19.09	19.75	19.65	$20.73^{+0.009402}_{-0.009404}$
$^{220}{\rm Rn}\to^{206}{\rm Hg}+^{14}{\rm C}$	$28.54^{+0.020045}_{-0.020045}$	−	0	0	0	0	−0.361	0	0	17.95	18.99	18.61	$18.43^{+0.043124}_{-0.043071}$
$^{223}{\rm Ra}\to^{205}{\rm Hg}+^{18}{\rm O}$	$40.30^{+0.004254}_{-0.004254}$	−	1	0	0	0	0.01	0.048	−0.019	26.44	24.99	24.74	$27.28^{+0.008052}_{-0.008050}$
$^{225}{\rm Ra}\to^{211}{\rm Pb}+^{14}{\rm C}$	$29.47^{+0.002809}_{-0.002809}$	−	4	0	0	0	−0.361	0	0	17.84	17.76	18.53	$19.73^{+0.005891}_{-0.005894}$
$^{225}{\rm Ra}\to^{205}{\rm Hg}+^{20}{\rm O}$	$40.48^{+0.004406}_{-0.004406}$	−	1	0	0	0	0.01	−0.024	0.02	28.27	27	26.86	$28.41^{+0.008663}_{-0.008661}$
$^{226}{\rm Ra}\to^{206}{\rm Hg}+^{20}{\rm O}$	$40.82^{+0.020068}_{-0.020068}$	−	0	0	0	0	0.01	−0.024	0.02	27.46	26.59	26.18	$26.52^{+0.038897}_{-0.038856}$
$^{227}{\rm Ac}\to^{207}{\rm Tl}+^{20}{\rm O}$	$43.09^{+0.005264}_{-0.005264}$	−	1	0	0	0	0.01	−0.024	0.02	23.95	23.94	22.99	$24.75^{+0.009408}_{-0.009406}$
$^{229}{\rm Ac}\to^{206}{\rm Hg}+^{23}{\rm F}$	$48.35^{+0.036222}_{-0.036222}$	−	2	0	0	0	0.01	−0.024	0.02	28.93	27.93	27.3	$27.83^{+0.063431}_{-0.063344}$
$^{226}{\rm Th}\to^{208}{\rm Pb}+^{18}{\rm O}$	$45.73^{+0.002283}_{-0.002283}$	>16.76	0	0	0	0	0.01	0.048	−0.019	18.14	18.96	17.38	$19.10^{+0.003553}_{-0.003553}$
$^{226}{\rm Th}\to^{212}{\rm Po}+^{14}{\rm C}$	$30.55^{+0.002332}_{-0.002332}$	>15.36	0	0	0	0	−0.361	0	0	17.55	16.27	18.1	$18.14^{+0.004727}_{-0.004730}$
$^{227}{\rm Th}\to^{209}{\rm Pb}+^{18}{\rm O}$	$44.20^{-0.002234}_{-0.002234}$	−	4	−0.011	0	0	0.01	0.048	−0.019	21.00	20.68	19.96	$22.83^{+0.003706}_{-0.003699}$
$^{228}{\rm Th}\to^{206}{\rm Hg}+^{22}{\rm Ne}$	$55.74^{+0.020045}_{-0.020045}$	−	0	0	0	0	0.384	0.096	−0.007	27.48	25.83	25.14	$23.41^{+0.029822}_{-0.029809}$

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Table 4-continued from previous page

Decay	Q_c/MeV	$\log T_{1/2}^{\rm exp}$	l	β_{d2}	β_{d4}	β_{d6}	β_{c2}	β_{c4}	β_{c6}	$\log T_{1/2}^{\rm UDL}$	$\log T_{1/2}^{\rm Bala}$	$\log T_{1/2}^{\rm SL}$	$\log T_{1/2}^{\rm Cal}$
^{229}Th $\to^{209}$ Pb $+^{20}$O	$43.40^{+0.002470}_{-0.002470}$	–	2	−0.011	0	0	0.01	−0.024	0.02	24.64	23.8	23.64	$25.39^{+0.004423}_{-0.004422}$
^{229}Th $\to^{205}$ Hg $+^{24}$Ne	$57.83^{+0.004319}_{-0.004319}$	–	3	0	0	0	−0.063	0.013	−0.03	25.34	25.59	23.8	$25.90^{+0.006419}_{-0.006416}$
^{231}Th $\to^{207}$ Hg $+^{24}$Ne	$56.25^{+0.030024}_{-0.030024}$	–	2	−0.011	0	0	−0.063	0.013	−0.03	28.12	27.12	26.29	$28.06^{+0.046940}_{-0.046898}$
^{231}Th $\to^{206}$ Hg $+^{25}$Ne	$56.80^{+0.035245}_{-0.035245}$	–	2	0	0	0	0.053	0.002	−0.03	27.92	27.42	26.31	$27.82^{+0.055103}_{-0.055045}$
^{232}Th $\to^{208}$ Hg $+^{24}$Ne	$54.67^{+0.030027}_{-0.030027}$	>29.2	0	0	0	0	−0.063	0.013	−0.03	31.12	28.89	28.89	$29.41^{+0.049526}_{-0.049470}$
^{232}Th $\to^{206}$ Hg $+^{26}$Ne	$55.91^{+0.026933}_{-0.026933}$	>29.2	0	0	0	0	0.121	−0.052	−0.035	30.37	29.1	28.66	$27.99^{+0.043870}_{-0.043826}$
^{227}Pa $\to^{209}$ Bi $+^{18}$O	$45.87^{+0.002993}_{-0.002993}$	–	2	−0.011	0	0	0.01	0.048	−0.019	19.16	19	18.27	$21.22^{+0.004714}_{-0.004707}$
^{229}Pa $\to^{207}$ Tl $+^{22}$Ne	$58.96^{+0.005292}_{-0.005292}$	–	2	0	0	0	0.384	0.096	−0.007	23.31	23.16	21.67	$21.34^{+0.007188}_{-0.007190}$
^{230}U $\to^{208}$ Pb $+^{22}$Ne	$61.39^{+0.001977}_{-0.001977}$	>18.2	0	0	0	0	0.384	0.096	−0.007	20.73	21.35	19.55	$18.15^{+0.002523}_{-0.002530}$
^{230}U $\to^{206}$ Pb $+^{24}$Ne	$61.35^{+0.002040}_{-0.002040}$	>18.2	0	0	0	0	−0.063	0.013	−0.03	22.34	23	21.27	$22.35^{+0.002805}_{-0.002803}$
^{232}U $\to^{204}$ Hg $+^{28}$Mg	$74.32^{+0.001455}_{-0.001455}$	>22.26	0	−0.052	−0.022	−0.008	0.277	−0.073	0.008	25.59	25.73	24.47	$24.35^{+0.001826}_{-0.001825}$
^{233}U $\to^{205}$ Hg $+^{28}$Mg	$74.23^{+0.004286}_{-0.004286}$	>27.59	3	0	0	0	0.277	−0.073	0.008	25.66	25.84	24.6	$25.17^{+0.005358}_{-0.005359}$
^{235}U $\to^{211}$ Pb $+^{24}$Ne	$57.36^{+0.002577}_{-0.002577}$	>27.65	1	0	0	0	−0.063	0.013	−0.03	29.16	26.69	27.31	$29.07^{+0.004013}_{-0.004019}$
^{235}U $\to^{210}$ Pb $+^{25}$Ne	$57.68^{+0.029053}_{-0.029053}$	>27.653	3	0	0	0	0.053	0.002	−0.03	29.41	27.21	27.72	$29.17^{+0.045623}_{-0.045580}$
^{235}U $\to^{207}$ Hg $+^{28}$Mg	$72.43^{+0.030019}_{-0.030019}$	>28.451	1	−0.011	0	0	0.277	−0.073	0.008	28.45	27.22	26.98	$27.46^{+0.039446}_{-0.039414}$
^{235}U $\to^{206}$ Hg $+^{29}$Mg	$72.48^{+0.020030}_{-0.020030}$	>28.45	3	0	0	0	0.23	−0.057	0.016	29.03	27.83	27.66	$27.94^{+0.026669}_{-0.026646}$
^{236}U $\to^{212}$ Pb $+^{24}$Ne	$55.95^{+0.002142}_{-0.002142}$	>26.27	0	0	0	0	−0.063	0.013	−0.03	31.83	28.07	29.6	$30.08^{+0.003493}_{-0.003499}$
^{236}U $\to^{210}$ Pb $+^{26}$Ne	$56.69^{+0.018085}_{-0.018085}$	>26.27	0	0	0	0	0.121	−0.052	−0.035	32.10	28.98	30.27	$29.49^{+0.029670}_{-0.029646}$
^{236}U $\to^{208}$ Hg $+^{28}$Mg	$70.73^{+0.007070}_{-0.030019}$	>26.27	0	0	0	0	0.277	−0.073	0.008	31.25	28.54	29.28	$28.32^{+0.041234}_{-0.041203}$
^{236}U $\to^{206}$ Hg $+^{30}$Mg	$72.27^{+0.020070}_{-0.020070}$	>26.27	0	0	0	0	0.119	−0.005	−0.031	29.94	28.64	28.69	$27.93^{+0.027372}_{-0.027345}$
^{238}U $\to^{208}$ Hg $+^{30}$Mg	$69.46^{+0.030053}_{-0.030053}$	–	0	0	0	0	0.119	−0.005	−0.031	34.79	30.91	32.72	$31.78^{+0.044251}_{-0.044211}$
^{231}Np $\to^{209}$ Bi $+^{22}$Ne	$61.90^{-0.007208}_{-0.007208}$	–	3	−0.011	0	0	0.384	0.096	−0.007	21.37	21.18	20.1	$20.13^{+0.009243}_{-0.009234}$
^{233}Np $\to^{209}$ Bi $+^{24}$Ne	$62.16^{+0.007226}_{-0.007226}$	–	3	−0.011	0	0	−0.063	0.013	−0.03	22.37	22.64	21.46	$23.59^{+0.009840}_{-0.009844}$
^{235}Np $\to^{207}$ Tl $+^{28}$Mg	$77.10^{+-0.005145}_{-0.005145}$	–	2	0	0	0	0.277	−0.073	0.008	22.82	24.2	22.48	$23.02^{+0.006063}_{-0.006071}$
^{237}Np $\to^{207}$ Tl $+^{30}$Mg	$74.79^{+0.005272}_{-0.005272}$	>27.57	2	0	0	0	0.119	−0.005	−0.031	27.54	27.13	26.82	$27.32^{+0.006839}_{-0.006842}$
^{237}Pu $\to^{209}$ Pb $+^{28}$Mg	$77.73^{+0.002158}_{-0.002158}$	–	1	−0.011	0	0	0.277	−0.073	0.008	23.49	24.14	23.16	$23.76^{+0.002553}_{-0.002555}$
^{237}Pu $\to^{208}$ Pb $+^{29}$Mg	$77.45^{+0.001732}_{-0.001732}$	–	3	0	0	0	0.23	−0.057	0.016	24.51	24.95	24.18	$24.57^{+0.002093}_{-0.002094}$
^{237}Pu $\to^{205}$ Hg $+^{32}$Si	$91.46^{+0.004218}_{-0.004218}$	–	4	0	0	0	−0.124	−0.03	−0.033	25.17	25.43	25.39	$26.65^{+0.004649}_{-0.004652}$
^{239}Pu $\to^{209}$ Pb $+^{30}$Mg	$75.08^{+0.002383}_{-0.002383}$	–	4	−0.011	0	0	0.119	−0.005	−0.031	28.78	27.29	27.95	$28.57^{+0.003125}_{-0.003136}$
^{239}Pu $\to^{205}$ Hg $+^{34}$Si	$90.87^{+0.004212}_{-0.004212}$	–	1	0	0	0	0	0	−0.039	26.83	26.85	27.29	$27.94^{+0.004828}_{-0.004826}$
^{237}Am $\to^{209}$ Bi $+^{28}$Mg	$79.85^{+0.007876}_{-0.007876}$	–	2	−0.011	0	0	0.277	−0.073	0.008	22.06	23.02	22.02	$22.76^{+0.008983}_{-0.008983}$
^{239}Am $\to^{207}$ Tl $+^{32}$Si	$94.50^{+0.005205}_{-0.005205}$	–	3	0	0	0	−0.124	−0.03	−0.033	22.65	24.14	23.66	$24.82^{+0.005460}_{-0.005460}$
^{241}Am $\to^{207}$ Tl $+^{34}$Si	$93.96^{+0.005171}_{-0.005171}$	>24.41	3	0	0	0	0	0	−0.039	24.13	25.51	25.44	$26.04^{+0.005637}_{-0.005639}$
^{240}Cm $\to^{208}$ Pb $+^{32}$Si	$97.55^{+0.001789}_{-0.001789}$	–	0	0	0	0	−0.124	−0.03	−0.033	20.31	22.87	22.02	$21.97^{+0.001793}_{-0.001785}$
^{241}Cm $\to^{209}$ Pb $+^{32}$Si	$95.39^{+0.002140}_{-0.002140}$	–	4	−0.011	0	0	−0.124	−0.03	−0.033	23.19	24.07	24.26	$25.25^{+0.002238}_{-0.002245}$
^{243}Cm $\to^{209}$ Pb $+^{34}$Si	$94.79^{+0.002243}_{-0.002243}$	–	2	−0.011	0	0	0	0	−0.039	24.77	25.47	26.12	$26.60^{+0.002445}_{-0.002447}$
^{244}Cm $\to^{210}$ Pb $+^{34}$Si	$93.17^{+0.001924}_{-0.001924}$	–	0	0	0	0	0	0	−0.039	27.06	26.43	27.92	$27.12^{+0.002173}_{-0.002172}$

lomb potential, we systematically studied the cluster radioactivity half-lives of 25 nuclei. The calculated cluster radioactivity half-lives obtained using the deformed Gamow-like model were found to be in better agreement with experimental data compared with the results obtained by UDL, Balasubramaniam's formula, and scaling law with the corresponding root-mean-square (rms) deviation: 1.3828, 0.8980, and 0.9976, respectively. Furthermore, the deformation effect was discussed. The calculated results obtained by the deformed Gamow-like model can better reproduce the experimental half-lives compared to the Gamow-like model, which indicates that the deformation effect should be carefully considered. Finally, we extended this model to predict the cluster radioactivity half-lives of 49 nuclei whose decay energies are energetically allowed or observed but not yet quantified, while the decay energy Q_c was calculated from the latest 2020 atomic mass table. The predictions of this model are very consistent with those obtained by UDL, Balasubramaniam's formula, and scaling law. The results of this work may be helpful for future experiments.

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