

Synthesis of the superheavy elements beyond Og: extrapolating from ^{48}Ca to ^{50}Ti and $^{54}\text{Cr}^*$

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Abstract: Theoretical predictions on the optimal reaction energies are essential for producing superheavy elements (SHEs) beyond Og. Owing to the limitation of targets, synthesizing elements 119 and 120 will require beams of ^{50}Ti and/or ^{54}Cr ions. However, is it reliable to theoretically extrapolate from the well-investigated ^{48}Ca induced reactions to those with heavier projectiles? In this work, we answer this question from two perspectives: radial and mass asymmetry degrees of freedom. The Smoluchowski diffusion equation is employed in the mass asymmetry degree of freedom for the first time, in which, by fitting the calculations to experimental evaporation residue cross sections (ERCS) for the reactions of ^{48}Ca as projectiles with the actinide targets, we find a strong linear correlation between the contact distance (D_{cont}) and center-of-mass energy excess above the Coulomb barrier ($E_{\text{c.m.}} - B_0$) and introduce a parametrization formula. The calculations based on the fitted formula reproduce the available experimental data of the ERCS satisfactorily. Furthermore, using recent experimental data, we extrapolate the calculation in the reactions $^{50}\text{Ti} + ^{242}\text{Pu}$, $^{50}\text{Ti} + ^{244}\text{Pu}$, and $^{54}\text{Cr} + ^{238}\text{U}$. The calculations reproduce the experimental data rather well within the experimental errors in both perspectives. Our results demonstrate that theoretically extrapolating the projectile from ^{48}Ca to ^{50}Ti and ^{54}Cr for synthesizing SHEs beyond Og is relatively reliable.

Keywords: superheavy nuclei, fusion reaction, Smoluchowski diffusion equation, evaporation residue cross section

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I. INTRODUCTION

The synthesis of superheavy elements (SHEs) is a frontier of research in nuclear physics [1–5]. With regard to hot fusion reactions, remarkable progress has been achieved in the synthesis of SHEs using double magic projectile ^{48}Ca and actinide targets [6–8]. In recent years, to open the eighth period, worldwide efforts have been made to produce SHEs beyond Oganesson (Og). However, no correlated decay chains were observed. A key challenge is determining the optimal incident energy (OIE), which primarily depends on theoretical predictions [5, 9–10].

Note that because of the plentiful experimental data, most of the theories can describe the ^{48}Ca induced fusion-evaporation reactions quite well. However, for SHEs beyond Og to be synthesized, projectiles heavier than ^{48}Ca such as ^{50}Ti , ^{54}Cr , ^{58}Fe , and ^{64}Ni are considered among the most promising candidates [11]. Is it reliable to extrapolate theoretically from the well-investigated ^{48}Ca in-

duced reactions to those with heavier projectiles for predicting the OIEs?

Owing to the complexity of the synthesis mechanisms of SHEs, particularly the presence of delicate ambiguities [12], the fusion process is not fully understood theoretically [13–14]. The fusion probability is particularly important for revealing the mechanism of synthesizing SHEs, and it is often calculated using diffusion models [15–16], master equations [17–27], or empirical formulas [28]. These theories describe the dynamics of the formation of compound nuclei from different degrees of freedom, primarily the radial degree of freedom: distance R between the centers of the nuclei (corresponding to the elongation of a mononucleus) and the mass asymmetry degree of freedom: $\eta = \frac{A_1 - A_2}{A_1 + A_2}$ (A_1 and A_2 are the mass numbers of the two nuclei that constitute the compound nuclei).

Owing to these inherent ambiguities and the diversity of theoretical frameworks, the reliability of predictions

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for SHEs with $Z=119$ and 120 has become an urgent concern. Currently, most theoretical models describe existing experiments fairly well. However, despite extensive theoretical studies on the synthesis of SHEs with $Z=119$ and 120 , predictions across different theoretical models exhibit significant uncertainty and model dependence [5]. In this case, several critical aspects must be addressed to provide reliable theoretical predictions: (i) How can the uncertainty of the theoretical model be evaluated? In Ref. [29], the Bayesian uncertainty quantification method was employed to evaluate the uncertainty of the calculated evaporation residue cross sections (ERCS) in the dinuclear system model. (ii) How can we reduce the model dependence of theoretical predictions? One weak model-dependence method (the OIE law) is proposed based on the strong correlation between OIEs, the Coulomb barrier height of side collisions, and the Q value [9]. (iii) Is extrapolating predictions from ^{48}Ca to ^{54}Cr reliable? In this work, utilizing the latest experimental data of ^{50}Ti and ^{54}Cr projectiles introduced reactions [5, 11], we quantify the reliability of extrapolation from ^{48}Ca to ^{54}Cr within a one-dimensional model by employing the Smoluchowski diffusion equation, which is examined from two distinct perspectives: the radial and mass asymmetry degrees of freedom.

The Smoluchowski diffusion equation provides a concise physical image, in which the fusion probability is described by an analytical formula. It has been effectively applied to investigate SHE synthesis based on the radial degree of freedom for both cold and hot fusion reactions [15, 30–32]. During the fusion stage, the system undergoes diffusion across a one-dimensional parabolic barrier to overcome the fusion barrier and form the compound nucleus. Additionally, as shown in the following, the only adjustable parameter is obtained from systematic behaviors, which will relatively reduce the uncertainty introduced by the theoretical model, making it suitable for investigating the extrapolation properties. In this study, we adopt the Smoluchowski diffusion equation to investigate the theoretical extrapolation ability from ^{48}Ca to heavier projectiles ($^{50}\text{Ti}/^{54}\text{Cr}$) for synthesizing SHEs. To strengthen the reliability, in addition to the radial degree of freedom, we employ the Smoluchowski diffusion equation in the mass asymmetry degree of freedom for the first time.

II. THEORETICAL DESCRIPTIONS

Theoretically, the synthesis of SHEs can be divided into three steps, and the ERCS is calculated as the summation over all partial waves J :

$$\begin{aligned} \sigma_{\text{ER}}(E_{\text{c.m.}}) \\ = \frac{\pi \hbar^2}{2\mu E_{\text{c.m.}}} \sum_J (2J+1) T(E_{\text{c.m.}}, J) P_{\text{fus}}(E_{\text{c.m.}}, J) W_{\text{sur}}(E^*, J), \quad (1) \end{aligned}$$

where $E_{\text{c.m.}}$ denotes the incident energy in the center-of-mass system. The excitation energy $E^* = E_{\text{c.m.}} + Q$. The values of Q calculated by using the Myers mass table [33]. $T(E_{\text{c.m.}}, J)$ is the transmission probability. $P_{\text{fus}}(E_{\text{c.m.}}, J)$ is the fusion probability, and $W_{\text{sur}}(E_{\text{c.m.}}, J)$ denotes the survival probability.

In the capture process, the projectile nucleus overcomes the Coulomb potential barrier to form a dinuclear system. The calculation of the Coulomb barrier relies on the nucleus-nucleus interaction potential. The interaction potential consists of Coulomb potential V_C and nuclear potential V_N . V_C is expressed by the Wong formula:

$$\begin{aligned} V_C(R, \beta_1, \beta_2, \theta_1, \theta_2) \\ = \frac{Z_1 Z_2 e^2}{R} + \sqrt{\frac{9}{20\pi}} \left(\frac{Z_1 Z_2 e^2}{R^3} \right) \sum_{i=1,2} R_i^2 \beta_2^{(i)} P_2(\cos \theta_i) \\ + \left(\frac{3}{7\pi} \right) \left(\frac{Z_1 Z_2 e^2}{R^3} \right) \sum_{i=1,2} R_i^2 [\beta_2^{(i)} P_2(\cos \theta_i)]^2. \quad (2) \end{aligned}$$

V_N is expressed by the double-folding method:

$$\begin{aligned} V_N(\mathbf{R}) = C_0 \left\{ \frac{F_{\text{in}} - F_{\text{ex}}}{\rho_0} \left[\int \rho_1^2(\mathbf{r}) \rho_2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \right. \right. \\ \left. \left. + \int \rho_1(\mathbf{r}) \rho_2^2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \right] \right. \\ \left. + F_{\text{ex}} \int \rho_1(\mathbf{r}) \rho_2(\mathbf{r} - \mathbf{R}) d\mathbf{r} \right\}, \quad (3) \end{aligned}$$

ρ_1 and ρ_2 are the nuclear density distribution functions. Details can be found in Ref. [29]. In this work, the diffuseness parameter was set to $a = 0.54$ fm for both projectile and target.

Because the OIE for the SHE synthesis reactions investigated in this work is dominated by the $3n$ evaporation channel at high excitation energies, the influence of barrier distributions is weak for determining OIE. Therefore, to avoid introducing empirical parameters and minimize model uncertainty, we consider only simple single-barrier penetration without empirical barrier distribution in the capture process. The penetration probability is given by the well-known Hill-Wheeler formula [34]:

$$\begin{aligned} T(E_{\text{c.m.}}, J) \\ = \frac{1}{1 + \exp \left\{ -\frac{2\pi}{\hbar \omega(J)} \left[E_{\text{c.m.}} - B - \frac{\hbar^2}{2\mu R_B^2(J)} J(J+1) \right] \right\}}, \quad (4) \end{aligned}$$

where the interaction potential at the maximum is defined as the barrier height B , and the corresponding position is the barrier radius $R_B(J)$ for the J th partial wave. Taking the interaction potential of reaction $^{48}\text{Ca} + ^{243}\text{Am}$ as an ex-

ample, we depict B and R_B in Fig. 1. The potential around the Coulomb barrier can be approximated by an inverted parabola with curvature $\hbar\omega(J)$. The barrier curvature $\hbar\omega(J)$ can be calculated using the formula $\omega(J) =$

$$\sqrt{-\frac{\hbar^2}{\mu} \frac{\partial^2}{\partial R^2} V(R, J)} \Big|_{R=R_B(J)}.$$

P_{fus} is calculated by using the Smoluchowski diffusion equation [35]:

$$G \frac{\partial W}{\partial t} = -(bxW)' + TW''.$$
 (5)

The Brownian particle moves along the stretching degree of freedom x in a viscous fluid, which is characterized by a repulsive parabolic potential $V(x) = -bx^2/2$. The constant G is a friction coefficient. The initial probability distribution W is assumed to be a delta function at the event injection point S_{inj} . Additionally, we assume that W is a Gaussian distribution function that elongates over time. When the interaction time approaches infinity, the probability of the Gaussian distribution being on the right side of the maximum point of the potential barrier is

$$P_{\text{fus}}(E_{\text{c.m.}}, J) = \frac{1}{2} \left(1 - \text{erf} \sqrt{\frac{B_{\text{fus}}(J)}{T}} \right).$$
 (6)

$P_{\text{fus}}(E_{\text{c.m.}}, J)$ results from the angular momentum (J)-dependent potential energy surface of the colliding system. T is the temperature of the fusing system. $B_{\text{fus}}(J)$ is the inner fusion barrier that a binuclear system must overcome to form a compound nucleus during the subsequent fusion process after capture. We study the complex fusion process in terms of the evolution of two degrees of freedom: radial and mass asymmetry degrees of freedom, respectively. In this study, we neglect shell effects during the fusion process. For hot fusion reactions with relatively high excitation energy, these effects are largely suppressed, making this approximation reasonable [36]. In particular, when the excitation energy reaches 30–50 MeV, the temperature dependent shell effects may have a minimal impact on the fusion barrier. Furthermore, as described by W. J. Swiatecki [15], the smooth parabolic parameterization of the potential energy surface adopted in this work facilitates the derivation of analytical expressions in Eq. (3). Finally, this systematic analysis of parameters effectively reduces the errors introduced by neglecting shell effects, ensuring that our calculated results remain relatively reliable. Specific details are discussed later.

The excited compound nucleus can evaporate light particles, such as neutrons, protons, and α particles. We employ the Monte Carlo method to calculate the decay probabilities in each channel. In the i th deexcitation step,

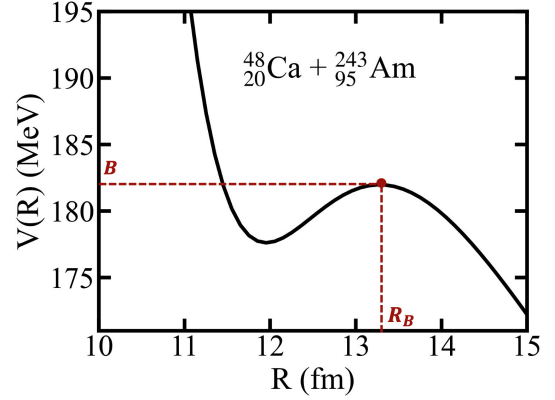


Fig. 1. (color online) Nucleus-nucleus interaction potential of the reaction $^{48}\text{Ca} + ^{243}\text{Am}$.

the probability of evaporating the neutron (n) channel can be expressed as

$$P_n(E_i^*) = \frac{\Gamma_n(E_i^*)}{\Gamma_{\text{tot}}(E_i^*)},$$
 (7)

where $\Gamma_{\text{tot}} = \Gamma_n + \Gamma_p + \Gamma_\alpha + \Gamma_\gamma + \Gamma_f$, which is addressed in detail in Refs. [37–38]. The partial decay widths for the evaporation of neutron can be estimated by the Weisskopf-Ewing theory [39].

$$\begin{aligned} \Gamma_n(E^*, J) &= \frac{(2s_n + 1)m_n}{\pi^2 \hbar^2 \rho(E^*, J)} \\ &\times \int_{I_n} \varepsilon \rho(E^* - B_n - \varepsilon, J) \sigma_{\text{inv}}(\varepsilon) d\varepsilon. \end{aligned}$$
 (8)

Compared with the standard Bohr-Wheeler fission width [40], we assume that the quantal penetration and reflection of the barrier can be represented by the Hill-Wheeler approximation [41]. In contrast to the standard Bohr-Wheeler fission width, we employ the modified version in which the fission barrier is treated as excitation energy dependent [14]. The modified Bohr-Wheeler fission width is

$$\begin{aligned} \Gamma_f(E^*, J) &= \frac{1}{2\pi \rho_f(E^*, J)} \\ &\times \int_{I_f} \frac{\rho_f(E^* - B_f - \varepsilon, J) d\varepsilon}{1 + \exp[-2\pi(E^* - B_f - \varepsilon)/\hbar\omega]}. \end{aligned}$$
 (9)

Here, $\hbar\omega = 1$ MeV. The fission barrier height $B_f(E^*)$ in Γ_f is given as [42]

$$B_f(E^*) = -E_{\text{sh}}^0 e^{-E^*/E_d},$$
 (10)

where E_{sh}^0 is the shell correction energy, which is taken from Ref. [43]. E_d is the damping factor of the shell ef-

fects. In this work, $E_d = 22$ MeV. The level density parameters of neutron evaporation and fission channels are $a_n = A/12$ MeV $^{-1}$ and $a_f = B_f a_n$. Here, $B_f = 1.05$.

III. RESULTS AND DISCUSSION

A. Fusion at the radial degree of freedom

The fusion barrier $B_{\text{fus}}(J)$ is a key physical quantity for calculating the fusion probability. From the perspective of the radial degree of freedom (mostly related to quadrupole moment Q_{20}), the fusion can be considered as the reverse process of the fission with fixed octupole moment Q_{30} . $B_{\text{fus}}(J)$ in Eq. (3) can be calculated as

$$B_{\text{fus}}^R(J) = E_{\text{sd}} - E_{\text{inj}} + E_{\text{sd}}^{\text{rot}}(J) - E_{\text{inj}}^{\text{rot}}(J). \quad (11)$$

In this work, the deformation energies including the saddle point E_{sd} and injection point E_{inj} are calculated using the finite range liquid drop model (FRLDM) [44–45]. The injection point S_{inj} denotes the closest touching configuration (R_{min}) in capture process and an initial condition undergoing the shape evolution toward the compound nucleus by overcoming an inner barrier, expressed as $S_{\text{inj}} = R_{\text{min}} - (R_P + R_T)$. Details of rotational energy $E_{\text{sd}}^{\text{rot}}(J)$ and $E_{\text{inj}}^{\text{rot}}(J)$ are given in Ref. [46].

As shown in Fig. 2(a), the fusion process involves overcoming the inner fusion barrier with the evolution of the configurations from injection point to the saddle point. The position of injection point plays an important role in the fusion probability. The expression for S_{inj} can be derived from experimental data by fitting the calculations to experimentally measured maximum values of the ERCS. The fitting results of the S_{inj} values as a function of the excess of the center-of-mass energy $E_{\text{c.m.}}$ over the Coulomb barrier B_0 are shown in Fig. 2(b), where B_0 represents the nucleus-nucleus interaction potential in the entrance channel, which prevent the system from capture. The strong linear correlation between S_{inj} and $E_{\text{c.m.}} - B_0$ is shown, and the systematics can be fitted as

$$S_{\text{inj}} = 2.253 \text{ fm} - 0.0165 \times (E_{\text{c.m.}} - B_0) \text{ fm/MeV}. \quad (12)$$

We observe from Fig. 2(b) that the injection distance S_{inj} increases as the value of $E_{\text{c.m.}} - B_0$ decreases. This is because, for a low incident energy, the system has sufficient time for nucleon rearrangement, thereby facilitating neck formation at the relatively large distance. In contrast, for a high incident energy, a more compressed configuration for neck formation is required. The same behavior is also shown in Ref. [65]. The strong correlation observed between the S_{inj} values and the corresponding energies $E_{\text{c.m.}} - B_0$ provides support for the fission barriers

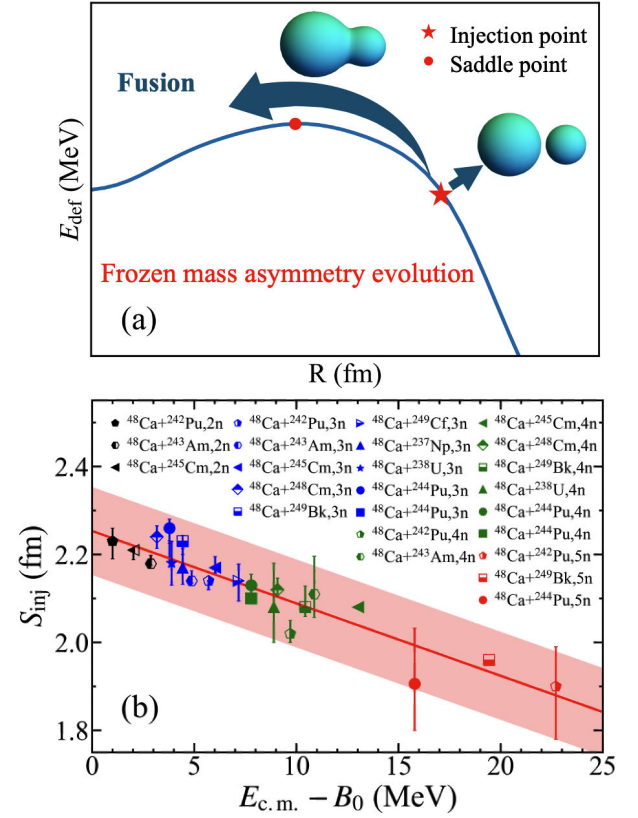


Fig. 2. (color online) (a) Schematic depiction of the fusion process in the radial degree of freedom. Red pentagram and red circle represent injection and saddle points, respectively. (b) Systematics of the S_{inj} for the hot fusion reactions with ^{48}Ca projectiles as a function of $E_{\text{c.m.}} - B_0$, deduced from analysis of experimental data [7–8, 47–64]. The red shaded area in the figure represents the overall error of all S_{inj} , which is obtained through weighted least squares fitting. The error bars for each S_{inj} are derived from the upper and lower limits of the error bars of the experimental values.

proposed by Kowal et al. [66].

Eq. (12) determines the systematics of the injection point. Fig. 3 shows a comparison of the calculated ERCS using Eq. (12) with experimental data. The calculated ERCS (solid lines) can reproduce the experimental data within the error bar rather well. Therefore, the systematical behavior for S_{inj} , as well as the thorough description of the experimental data, shows that the Smoluchowski diffusion equation at the radial degree of freedom is a reasonable approach for investigating the extrapolation behavior to heavier projectiles.

B. Fusion at the mass asymmetry degree of freedom

Fission is a slow process dominated by neck evolution dynamics. Similarly, the fusion process has sufficient time for shape relaxation. To strengthen the reliability of theoretical results, the fusion process is worth studying from different perspectives: the mass asym-

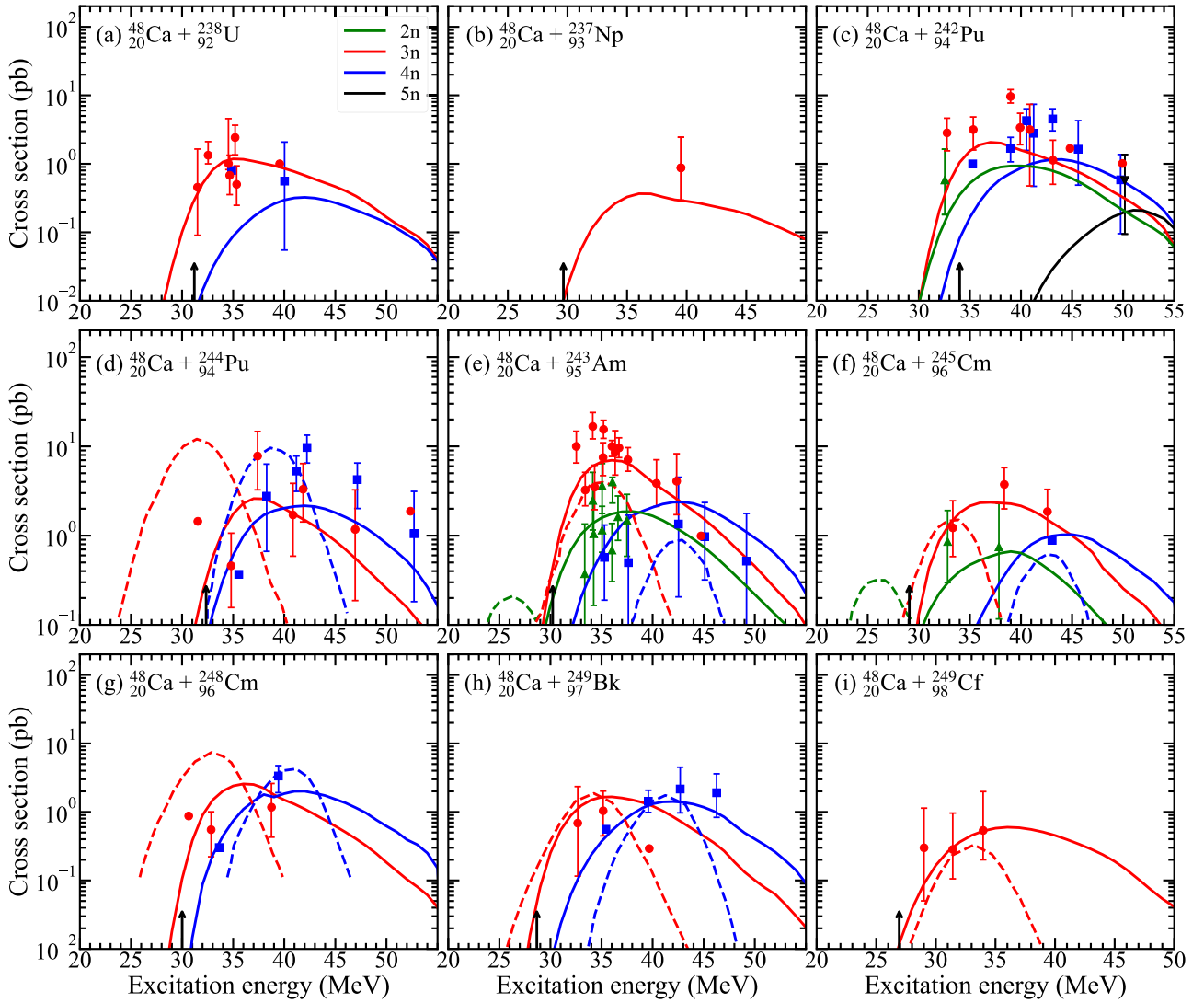


Fig. 3. (color online) In terms of radial degree of freedom (solid lines), the calculated ERCS are compared with the available experimental data for the reactions $^{48}\text{Ca} + ^{238}\text{U}$ [47] (a), $^{48}\text{Ca} + ^{237}\text{Np}$ [8, 48] (b), $^{48}\text{Ca} + ^{242}\text{Pu}$ [47, 49–50] (c), $^{48}\text{Ca} + ^{244}\text{Pu}$ [7, 51–53] (d), $^{48}\text{Ca} + ^{243}\text{Am}$ [48, 54–55] (e), $^{48}\text{Ca} + ^{245}\text{Cm}$ [8, 51, 56] (f), $^{48}\text{Ca} + ^{248}\text{Cm}$ [47, 57–59] (g), $^{48}\text{Ca} + ^{249}\text{Bk}$ [8, 48, 60–64] (h), and $^{48}\text{Ca} + ^{249}\text{Cf}$ [56, 62] (i). The calculated ERCS in the channels 2n, 3n, 4n, and 5n are denoted by the green, red, blue, and black lines, respectively. Here, dash lines denote the results from previous theoretical work [67]. Vertical arrows denote the excitation energies corresponding to collisions at $E_{\text{c.m.}} = B_0$ for each reaction.

metry degree of freedom (mostly related to Q_{30}). In low-energy heavy-ion nuclear reactions, nucleon transfer and mass rearrangement between colliding partners play a crucial role.

Based on these physical features, the mass asymmetry is often selected as a key macroscopic variable to describe fusion process. We also obtain an approximately inverted parabolic potential energy surface and investigate the fusion using the Smoluchowski diffusion equation. In the nuclear fusion process under the mass-asymmetry degree of freedom, the evolution in radial is frozen and the nucleon transfer occurs at the contact position which can be described by the distance between surfaces of the two colliding nuclei (D_{cont}). The reaction of

the system towards fusion generally refers to the transfer of nucleons from the lighter nucleus (either the projectile or the target) to the heavier nucleus, evolving in the direction of increasing the mass asymmetry η . The evolution of this process can be described by the diffusion of the mass asymmetry degree of freedom η (see Fig. 4(a)) [68].

Following the capture of the projectile by the target nucleus after overcoming the Coulomb barrier, forming a composite system, fusion occurs and the compound nucleus is formed when the dinuclear system overcomes the inner fusion barrier B_{fus} . The more asymmetry configurations than those on the B.G. point are considered as the occurrence of fusion [37]. B_{fus} is calculated to be equal to the difference between the driving potential at the B.G.

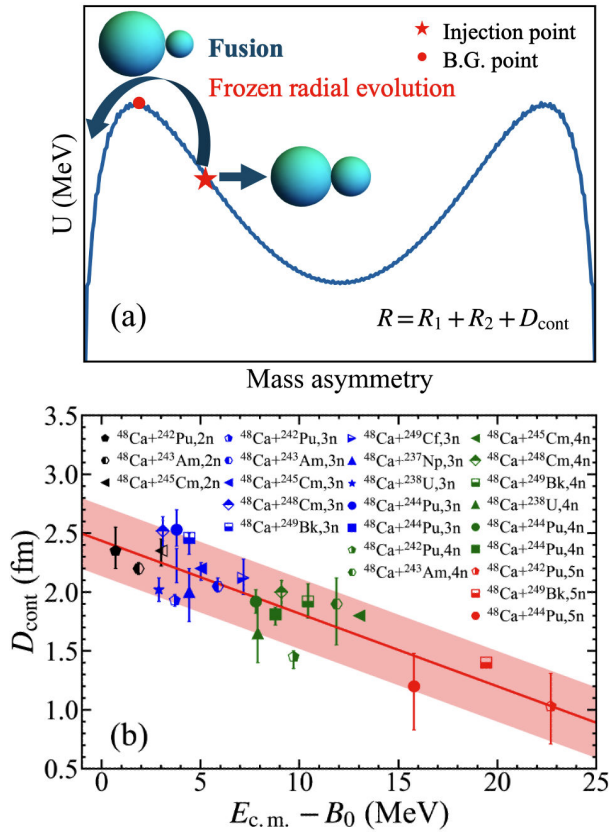


Fig. 4. (color online) (a) Schematic depiction of the fusion process in mass asymmetry degree of freedom. Red pentagram and red circle represent injection and B.G. points, respectively. (b) Same as in Fig. 2(b), dependence of the D_{cont} on $E_{\text{c.m.}} - B_0$. The error bars are calculated in the same manner as those in Fig. 2.

point and the driving potential at the injection point (η of the projectile-target combination), as shown in Fig. 4(a). B_{fus} is primarily determined by the details of the potential energy surface (PES), which is the potential energy of the dinuclear system along the η direction and can be expressed as

$$U(Z_1, N_1, R = R_1 + R_2 + D_{\text{cont}}) = \Delta(Z_1, N_1) + \Delta(Z_2, N_2) + V(Z_1, N_1, R = R_1 + R_2 + D_{\text{cont}}), \quad (13)$$

where R_1 and R_2 are the radii of the two nuclei. $\Delta(Z_i, N_i) (i=1,2)$ is the mass excess of the i th fragment [69]. $V(Z_1, N_1, R = R_1 + R_2 + D_{\text{cont}})$ is the nucleus-nucleus interaction potential. Here, D_{cont} refers to the surface distance between two colliding nuclei for the nucleon transfer process [37].

$B_{\text{fus}}^\eta(J)$ can be calculated as

$$B_{\text{fus}}^\eta(J) = U(\eta_{\text{B.G.}}) - U(\eta_i). \quad (14)$$

Similarly, the relationship between the contact distance D_{cont} and $E_{\text{c.m.}} - B_0$ is shown in Fig. 4(b); within the error bar, a linear relationship between D_{cont} and $E_{\text{c.m.}} - B_0$ is obtained. This interesting behavior represents that the consideration of the Smoluchowski diffusion equation at the mass asymmetry degree of freedom is reasonable. The systematical behavior can be expressed as

$$D_{\text{cont}} = 2.435 \text{ fm} - 0.0618 \times (E_{\text{c.m.}} - B_0) \text{ fm/MeV}. \quad (15)$$

Unexpectedly, we observe that the inner fusion barrier exhibits a systematic similarity in the mass asymmetry and radial degrees of freedom. This suggests that, to some extent, the descriptions of the fusion process of both degrees of freedom share similarities and are comparable. Both perspectives are important reaction degrees of freedom in the fusion process. Studying the fusion reactions from multiple perspectives is crucial for verifying the reliability of the theoretical extrapolation from ^{48}Ca to ^{54}Cr .

To verify the method, by using Eq. (15), we compare the calculated ERCS in ^{48}Ca induced reactions with the experimental data, as shown in Fig. 5. The calculated results (dashed lines) agree well with both the experimental ERCS and the optimal energies. These results give us confidence based on the Smoluchowski diffusion equation at mass asymmetry degree of freedom to investigate the ERCS of fusion reactions leading to new elements. In Fig. 5, we also compare our results with those from the previous work Ref. [67]. We find that the 3n evaporation channel is dominant, which is consistent with the results in Ref. [67]. We observe that for the reactions $^{48}\text{Ca} + ^{243}\text{Am}$ and $^{48}\text{Ca} + ^{245}\text{Cm}$, our theoretical predictions agree better with experimental results than those reported in Ref. [67]. This is probably because in Ref. [67], the capture transmission coefficients T are calculated using a simple sharp cutoff approximation. This approach is known to underestimate the capture cross-section below the Coulomb barrier.

C. Extrapolation of projectiles with ^{50}Ti and ^{54}Cr

In the above calculations, we notice that the Smoluchowski diffusion equation based on the radial and mass asymmetry degrees of freedom describes the experimental data in ^{48}Ca induced reactions quite well. As we have mentioned above, we must clarify whether it is reasonable to extrapolate the model including the systematical behaviors of S_{inj} and D_{cont} in ^{50}Ti and ^{54}Cr induced reactions. Based on the values of S_{inj} and D_{cont} determined by Eqs. (12) and (15). The calculated results in the reactions $^{50}\text{Ti} + ^{242}\text{Pu}$, $^{50}\text{Ti} + ^{244}\text{Pu}$, and $^{54}\text{Cr} + ^{238}\text{U}$ are shown in Fig. 6. The calculated results for all reactions agree well with the experimental data for both perspectives of radial and mass asymmetry degrees of freedom. The radi-

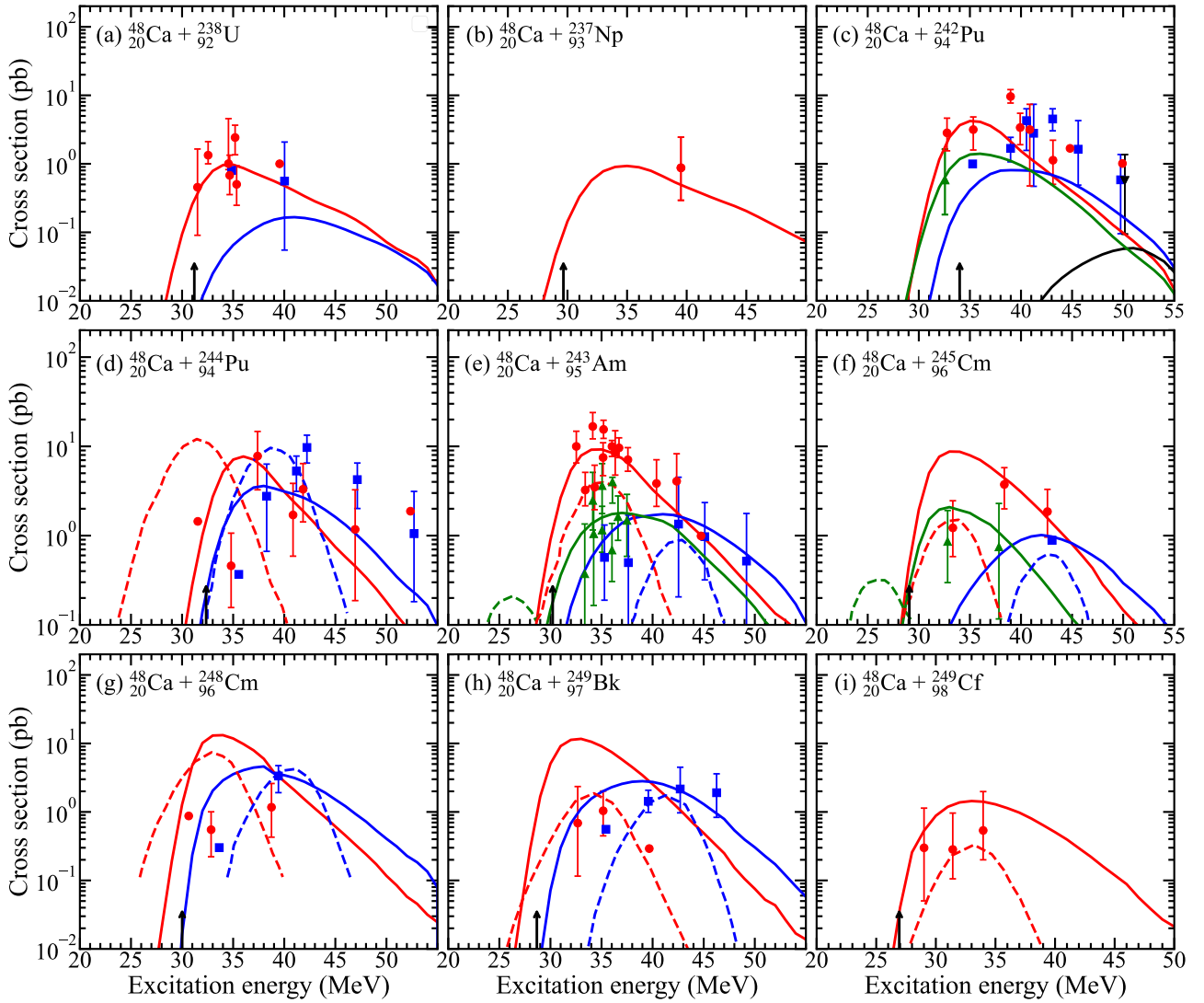


Fig. 5. (color online) In terms of mass asymmetry degree of freedom (solid lines), the calculated ERCS are compared with the available experimental data for the reactions $^{48}\text{Ca} + ^{238}\text{U}$ [47] (a), $^{48}\text{Ca} + ^{237}\text{Np}$ [8, 48] (b), $^{48}\text{Ca} + ^{242}\text{Pu}$ [47, 49–50] (c), $^{48}\text{Ca} + ^{244}\text{Pu}$ [7, 51–53] (d), $^{48}\text{Ca} + ^{243}\text{Am}$ [48, 54–55] (e), $^{48}\text{Ca} + ^{245}\text{Cm}$ [8, 51, 56] (f), $^{48}\text{Ca} + ^{248}\text{Cm}$ [47, 57–59] (g), $^{48}\text{Ca} + ^{249}\text{Bk}$ [8, 48, 60–64] (h), and $^{48}\text{Ca} + ^{249}\text{Cf}$ [56, 62] (i). The calculated ERCS in the channels $2n$, $3n$, $4n$, and $5n$ are denoted by the green, red, blue, and black lines, respectively. Here, dash lines denote the results from previous theoretical work [67]. Vertical arrows denote the excitation energies corresponding to collisions at $E_{\text{c.m.}} = B_0$ for each reaction.

al and mass asymmetric degrees of freedom are key degrees of freedom in fusion processes. These two degrees of freedom are typically considered in some theoretical models separately. For example, in the dinuclear system (DNS) model, the radial degree of freedom is frozen and the fusion process takes place in the mass asymmetry degree of freedom. In the fusion by diffusion (FBD) model, the fusion probability is calculated using the barrier, and penetrating the barrier occurs in the radial degree of freedom. In this work, we independently study the fusion process from two perspectives: radial and mass asymmetric degrees of freedom. We find that the parameter S_{inj} under radial degrees of freedom and D_{cont} under mass asymmetric degrees of freedom have a strong linear cor-

relation with $E_{\text{c.m.}} - B_0$. From a quantitative perspective, we also give the error range of S_{inj} and D_{cont} . The final calculated ERCS is in good agreement with the experimental data. In Fig. 6, the extrapolation calculation results are in good agreement with the experimental data within the error range. We observe that the reliability of this extrapolation is relatively high. Our investigation shows that studying fusion via either the radial degree of freedom or the mass asymmetry degree of freedom leads to remarkably similar and comparable outcomes. Consequently, both can be reliably employed to investigate complex fusion mechanisms. We would like to state that the transition from ^{48}Ca to heavier projectiles, such as ^{50}Ti and ^{54}Cr is reasonable and reliable according to cur-

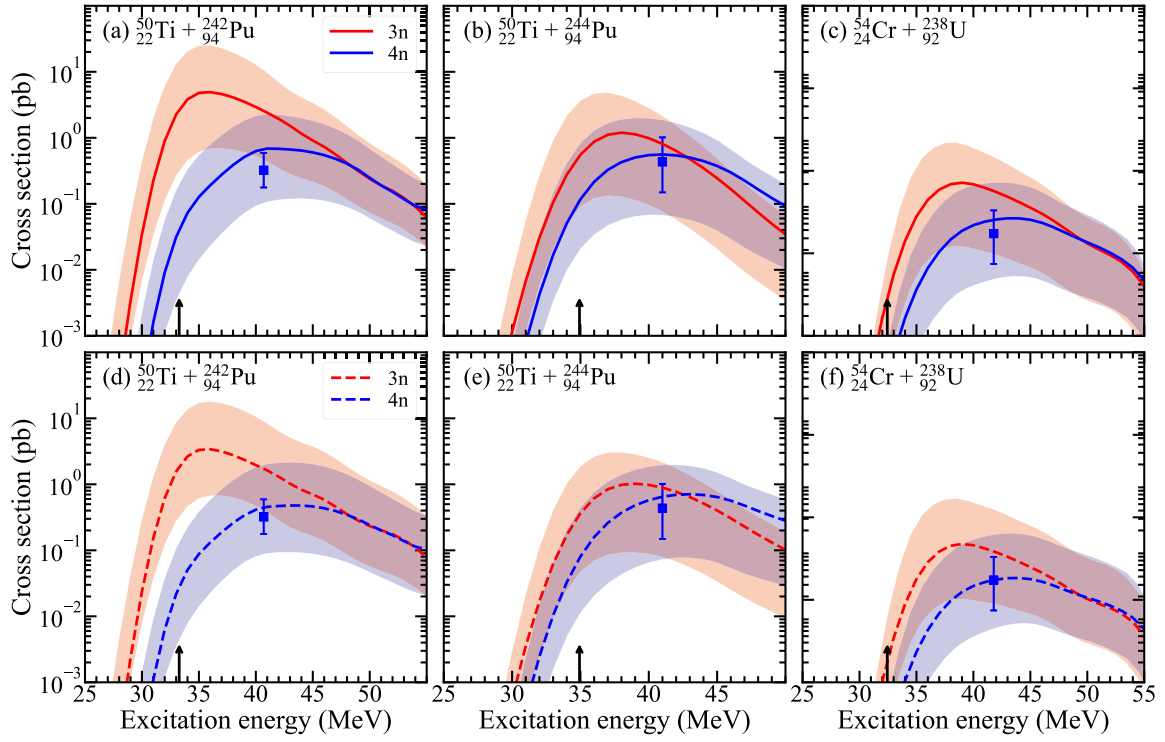


Fig. 6. (color online) Comparison of the ERCS for reactions $^{50}\text{Ti} + ^{242}\text{Pu}$, $^{50}\text{Ti} + ^{244}\text{Pu}$, and $^{54}\text{Cr} + ^{238}\text{U}$ in the radial degree of freedom (solid lines) and mass asymmetry degree of freedom (dashed lines) together with corresponding error corridors. Blue full squares represent experimental data for the 4n reaction channel of the reactions $^{50}\text{Ti} + ^{242}\text{Pu}$ [11], $^{50}\text{Ti} + ^{244}\text{Pu}$ [5], and $^{54}\text{Cr} + ^{238}\text{U}$ [11].

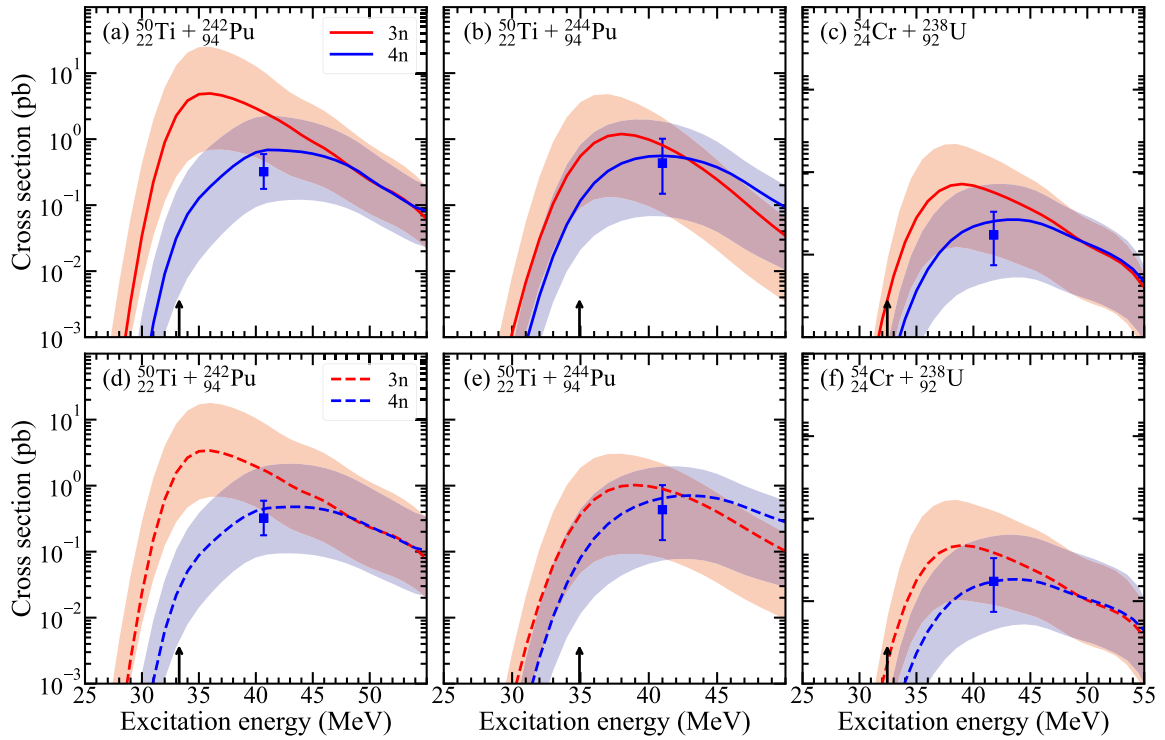


Fig. 7. (color online) ERCS production of new elements 119 through the reactions $^{50}\text{Ti} + ^{249}\text{Bk}$, $^{51}\text{V} + ^{248}\text{Cm}$, and $^{54}\text{Cr} + ^{243}\text{Am}$ predicted in radial degree of freedom (solid lines) and mass asymmetry degree of freedom (dashed lines) together with corresponding error corridors. The gray solid circles represent the maximum value of the ERCS for the 3n channels, corresponding to the optimal excitation energies.

rent theories.

Calculations show that the ERCS for the heavier projectiles ^{50}Ti and ^{54}Cr in the synthesis of SHEs with $Z = 116$ are nearly one order of magnitude lower than that for the reaction induced by the ^{48}Ca projectile. This reduction is primarily due to the lower mass asymmetry in the entrance channel of the reactions involving ^{50}Ti and ^{54}Cr as projectiles, coupled with a higher fusion barrier. Consequently, the system has a lower probability of fusion through diffusion.

To further investigate ERCS of synthesizing SHEs with $Z = 119$. In Fig. 7, we conducted a study on the possibility of synthesizing the superheavy nucleus $Z = 119$ using ^{50}Ti , ^{51}V , and ^{54}Cr as the projectiles. In the radial degree of freedom, the OIEs for producing the element with $Z = 119$ via the reactions $^{50}\text{Ti} + ^{249}\text{Bk}$, $^{51}\text{V} + ^{248}\text{Cm}$, and $^{54}\text{Cr} + ^{243}\text{Am}$ are estimated to be 224.5, 231.3, and 242.3 MeV, respectively. Correspondingly, under the mass asymmetric degree of freedom, the OIEs are predicted to be 224.5, 231.3, and 240.3 MeV, respectively. The predicted OIEs are close to the results from the OIE law proposed in Ref. [9], particularly for the reactions $^{50}\text{Ti} + ^{249}\text{Bk}$ and $^{54}\text{Cr} + ^{243}\text{Am}$.

IV. SUMMARY

The ERCS for synthesizing SHEs based on the different perspectives (radial and mass asymmetry degrees of

freedom) within the concept of Smoluchowski diffusion equation are investigated. By calibrating the injection point distance S_{inj} and contact distance D_{cont} as adjustable parameters using the experimental ERCS data from ^{48}Ca induced fusion reactions, we observe the strong linear systematic behaviors of S_{inj} and D_{cont} on $E_{\text{c.m.}} - B_0$. The inner fusion barrier shows a systematic similarity in both the radial and mass asymmetry degrees of freedom, which suggests that, to some certain extent, radial and mass asymmetry degrees of freedom are similar and comparable in their description of the fusion process. The parameterization of S_{inj} and D_{cont} are then used for extrapolation to calculate the ERCS in the reactions $^{50}\text{Ti} + ^{242}\text{Pu}$, $^{50}\text{Ti} + ^{244}\text{Pu}$, and $^{54}\text{Cr} + ^{238}\text{U}$. The calculated results agree well with the recent experimental data from LBNL [5] and Dubna [11], which indicate that the theoretical calculations are relatively reliable in extrapolating the projectiles from ^{48}Ca to ^{50}Ti and ^{54}Cr for synthesizing SHEs beyond Og. Finally, we present predictions of ERCS and OIEs for synthesizing superheavy nuclei with $Z = 119$ from both radial and mass asymmetry degrees of freedom. The predicted OIEs in radial and mass asymmetry degrees of freedom are consistent with each other, as well as the results of Ref. [9].

This work also inspires us to utilize microscopic theories (such as density functional theories [32, 70–71] for calculating a multidimensional PES and address it within the framework of the Smoluchowski diffusion equation.

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