U-spin sum rules in charmed baryon decays beyond the *U*-spin limit*

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Abstract: *U*-spin symmetry is an important flavor symmetry for studying the weak decays of charmed hadrons. However, sizable *U*-spin breaking limits its predictive power. In this work, we derive dozens of *U*-spin sum rules for decay rates and decay parameters in charmed baryon decays beyond the exact *U*-spin symmetry limit. These sum rules provide tests of flavor symmetry up to first-order *U*-spin-breaking effects in the charmed baryon sector.

Keywords: U-spin, Charmed baryon, Two-body decay

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I. INTRODUCTION

Charmed baryon decays provide laboratories for studying strong and weak interactions in heavy-to-light baryonic transitions. From a theoretical perspective, quantum chromodynamics (QCD)-inspired approaches are less effective for charmed hadron decays because the expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ are larger than those in *b*-hadron decays. In addition to model calculations, it is useful to study charmed baryon decays based on flavor symmetry. In the literature, the two-body nonleptonic decays of charmed baryons have been widely studied using flavor $SU(3)$ symmetry [1−32] and its $SU(2)$ subgroups, namely isospin [33] and *U*-spin [34−38] symmetries. Recent studies have attempted to include flavor-symmetry-breaking effects [39−42]. *U*-spin symmetry reflects the symmetry between the *d* and *s* quarks, providing broader connections than isospin and *V*-spin symmetries. *U*-spin symmetry can connect the Cabibbo-favored (CF) and doubly Cabibbo-suppressed (DCS) modes, as well as the $c\to s\bar{s}u$ and $c\to d\bar{d}u$ transitions in the singly Cabibbo-suppressed (SCS) modes. By contrast, isospin and *V*-spin symmetries can connect only a single class of charmed hadron decays, because the *d* and *s* quarks belong to different isospin and *V*-spin multiplets. The *d* and *s* quarks in loop diagrams allow *U*-spin to relate the direct *CP* asymmetries of charmed hadron decays.

U-spin breaking is naively estimated to be $\epsilon_U\sim(m_s-m_d)/\Lambda_{\rm had}\sim(20\sim30)\%$, where $\Lambda_{\rm had}$ is the typical strong-interaction scale, with $(m_s-m_d)\approx90$ MeV and $\Lambda_{\rm had}\approx(300\sim500)$ MeV. Theoretical predictions based on *U*-spin symmetry deviate from the exact values. To obtain more accurate theoretical predictions, it is necessary to study *U*-spin sum rules beyond the *U*-spin limit, i.e., exact *U*-spin symmetry, as in *D* meson decays [43−45]. In Refs. [46−52], a perturbative method for analyzing *U*-spin breaking was proposed. In this method, *U*-spin-breaking corrections to arbitrary order are obtained by introducing an *s*-*d* spurion mass operator $M_{\rm Ubrk}$ into the effective Hamiltonian or into the initial and final states. $M_{\rm Ubrk}$ is the $U_3=0$ component of a *U*-spin triplet. Using $M_{\rm Ubrk}$, one can derive *U*-spin sum rules beyond the *U*-spin limit. In this work, we apply this approach to charmed baryon decays. We derive dozens of *U*-spin sum rules for rates and decay parameters in charmed baryon decays beyond the *U*-spin limit, providing precise tests of flavor symmetry.

The rest of this paper is organized as follows. In Sec. II, we use a *U*-spin system as an example to derive rate and decay-parameter sum rules beyond the *U*-spin limit. Additional sum rules for singly and doubly charmed baryon decays are presented in Sec. III. Sec. IV provides a brief summary.

II. THEORETICAL FRAMEWORK

In *U*-spin symmetry, the *d* and *s* quarks form a $U=1/2$ doublet. The mass term for the (d,s) quarks can be written as

$$\mathcal{L}_m^{(d,s)}=-\frac{m_s+m_d}{2}(\bar{d}d+\bar{s}s)-\frac{m_s-m_d}{2}(\bar{s}s-\bar{d}d). \quad (1)$$

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The first term is a U-spin scalar, whereas the second transforms as the $U_3 = 0$ component of a U-spin triplet and is represented by the spurion operator $M_{\rm Ubrk}$ [51]. At the hadronic level, U-spin breaking is encoded not only in the reduced weak matrix elements but also in hadron-mass splittings, phase-space factors, partial-wave momentum factors, decay constants, form factors, and final-state rescattering effects. Consequently, the effective U-spin-breaking parameters are generally complex and depend on the decay system and partial wave. The dimensionless U-spin-breaking expansion parameter ϵ_U can be naively estimated as

$$\epsilon_U \sim \frac{m_s - m_d}{\Lambda_{\rm had}} \approx \frac{90\,{\rm MeV}}{(300 \sim 500)\,{\rm MeV}} \approx (20 \sim 30)\%. \quad (2)$$

A simple example of U-spin breaking is the difference between the kaon and pion decay constants, i.e., $\epsilon_U \sim f_K/f_\pi - 1 \approx 0.2$.

Considering the U-spin system

$$\Xi_c^0 \rightarrow \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \begin{pmatrix} \Sigma^- \\ \Xi^- \end{pmatrix} \quad (3)$$

As an example, we illustrate the derivation of U-spin amplitudes beyond the U-spin limit. There are four decay channels in this U-spin system: $\Xi_c^0 \to \Xi^-\pi^+$, $\Xi_c^0 \to \Sigma^- K^+$, $\Xi_c^0 \to \Xi^- K^+$, and $\Xi_c^0 \to \Sigma^-\pi^+$. The decays $\Xi_c^0 \to \Xi^-\pi^+$ and $\Xi_c^0 \to \Sigma^- K^+$, as well as $\Xi_c^0 \to \Xi^- K^+$ and $\Xi_c^0 \to \Sigma^-\pi^+$, form two U-spin conjugate pairs; that is, the two decay modes in each pair are related by the interchanges $s \leftrightarrow d$ and $\bar{s} \leftrightarrow -\bar{d}$ in all initial and final states. If two decay channels form a U-spin conjugate pair, the third U-spin components of the corresponding initial and final states are equal in magnitude but opposite in sign. For example, the U-spins and corresponding third components of the Ξ_c^0, K^+, π^+, Σ^-, and Ξ^- are $(0,0)$, $(1/2,1/2)$, $(1/2,-1/2)$, $(1/2,1/2)$, and $(1/2,-1/2)$, respectively. The three components of the Hamiltonian responsible for the CF, SCS, and DCS modes, transforming as $\bar{s}d$, $(s\bar{s}-d\bar{d})$, and $\bar{d}s$, respectively, form a U-spin triplet [51].

$$\mathcal{H}_{\rm CF} = -\cos^2\theta_C\,(1,-1), \qquad \mathcal{H}_{\rm SCS} = \sqrt{2}\cos\theta_C\sin\theta_C\,(1,0),$$
$$\mathcal{H}_{\rm DCS} = -\sin^2\theta_C\,(1,1), \quad (4)$$

where θ_C denotes the Cabibbo angle. In the U-spin limit, the U-spin amplitudes for these four decay modes are given by

$$\begin{aligned}
&\mathcal{A}(\Xi_c^0 \to \Xi^-\pi^+) = \langle \Xi^-\pi^+|\mathcal{H}_{\rm CF}|\Xi_c^0\rangle = -\cos^2\theta_C\langle\tfrac{1}{2},-\tfrac{1}{2};\tfrac{1}{2},-\tfrac{1}{2}|1,-1;0,0\rangle = -\cos^2\theta_C\mathcal{A}_1,\\
&\mathcal{A}(\Xi_c^0 \to \Sigma^- K^+) = \langle \Sigma^- K^+|\mathcal{H}_{\rm DCS}|\Xi_c^0\rangle = -\sin^2\theta_C\langle\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{2},\tfrac{1}{2}|1,1;0,0\rangle = -\sin^2\theta_C\mathcal{A}_1,\\
&\mathcal{A}(\Xi_c^0 \to \Xi^- K^+) = \langle \Xi^- K^+|\mathcal{H}_{\rm SCS}|\Xi_c^0\rangle = \sqrt{2}\cos\theta_C\sin\theta_C\langle\tfrac{1}{2},-\tfrac{1}{2};\tfrac{1}{2},\tfrac{1}{2}|1,0;0,0\rangle = \cos\theta_C\sin\theta_C\mathcal{A}_1,\\
&\mathcal{A}(\Xi_c^0 \to \Sigma^-\pi^+) = \langle \Sigma^-\pi^+|\mathcal{H}_{\rm SCS}|\Xi_c^0\rangle = \sqrt{2}\cos\theta_C\sin\theta_C\langle\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{2},-\tfrac{1}{2}|1,0;0,0\rangle = \cos\theta_C\sin\theta_C\mathcal{A}_1. \quad (5)
\end{aligned}$$

We then obtain the following amplitude relations in the U-spin limit:

$$\begin{aligned}
-\mathcal{A}(\Xi_c^0 \to \Xi^-\pi^+) &= -\frac{\mathcal{A}(\Xi_c^0 \to \Sigma^- K^+)}{\tan^2\theta_C}\\
&= \frac{\mathcal{A}(\Xi_c^0 \to \Xi^- K^+)}{\tan\theta_C} = \frac{\mathcal{A}(\Xi_c^0 \to \Sigma^-\pi^+)}{\tan\theta_C} = \mathcal{A}_1. \quad (6)
\end{aligned}$$

At first order in U-spin breaking, an $s-d$ spurion mass operator, $M_{\rm Ubrk}$, with $M_{\rm Ubrk} \propto (s\bar{s}-d\bar{d}) = \sqrt{2}(1,0)$, couples to the initial and final states and to the effective Hamiltonian. According to the angular momentum coupling rules, simultaneous reversal of all magnetic quantum numbers changes the coupling coefficients only by an overall phase [53],

$$\begin{aligned}
&\langle j_1,-m_1;j_2,-m_2|j_3,-m_3;j_4,-m_4\rangle\\
&= (-1)^{j_1+j_2-j_3-j_4}\langle j_1,m_1;j_2,m_2|j_3,m_3;j_4,m_4\rangle. \quad (7)
\end{aligned}$$

Here, j_i denotes the magnitude of the corresponding angular momentum, while m_i denotes its projection along the z axis. A detailed derivation of Eq. (7) is presented in Appendix A. From Eq. (7), the first-order U-spin-breaking corrections for the four decay modes satisfy

$$\begin{aligned}
\mathcal{A}_{\rm Ubrk}(\Xi_c^0 \to \Xi^-\pi^+) &= \frac{(-1)^{|\Delta U|}\mathcal{A}_{\rm Ubrk}(\Xi_c^0 \to \Sigma^- K^+)}{\tan^2\theta_C},\\
\mathcal{A}_{\rm Ubrk}(\Xi_c^0 \to \Sigma^-\pi^+) &= (-1)^{|\Delta U|}\mathcal{A}_{\rm Ubrk}(\Xi_c^0 \to \Xi^- K^+), \quad (8)
\end{aligned}$$

where $\Delta U = U_{\mathcal{B}} + U_M - U_{\mathcal{B}_c} - U_{\mathcal{H}_{\rm eff}} \pm U_{\rm Ubrk}$, and $U_{\mathcal{B}}$, U_M, $U_{\mathcal{B}_c}$, $U_{\mathcal{H}_{\rm eff}}$, and $U_{\rm Ubrk}$ denote the U-spin quantum num-

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bers of the final-state baryon $\mathcal{B}$, final-state meson M, initial-state baryon $\mathcal{B}_c$, effective Hamiltonian $\mathcal{H}_{\rm eff}$, and $s-d$ spurion mass operator $M_{\rm Ubrk}$, respectively. The sign preceding $U_{\rm Ubrk}$ is "+" if $M_{\rm Ubrk}$ couples to one of the final states, and "−" if $M_{\rm Ubrk}$ couples to the initial state or the effective Hamiltonian. Substituting the U-spin quantum numbers of the initial and final states, the effective Hamiltonian, and $M_{\rm Ubrk}$, we obtain $|\Delta U|=1$. Thus, we have

$$\begin{aligned}&\mathcal{A}_{\rm Ubrk}(\Xi_c^0\to\Xi^-\pi^+) = -\frac{\mathcal{A}_{\rm Ubrk}(\Xi_c^0\to\Sigma^-K^+)}{\tan^2\theta_C} = \mathcal{A}_1\times\epsilon_1,\\ &\frac{\mathcal{A}_{\rm Ubrk}(\Xi_c^0\to\Sigma^-\pi^+)}{\tan\theta_C} = -\frac{\mathcal{A}_{\rm Ubrk}(\Xi_c^0\to\Xi^-K^+)}{\tan\theta_C} = \mathcal{A}_1\times\epsilon_2,\end{aligned}\quad (9)$$

where ϵ_1 and ϵ_2 are complex U-spin-breaking parameters.

More generally, nth-order U-spin breaking can be obtained by introducing, into the Hamiltonian or the initial/final state, a total of n powers of the $s-d$ spurion mass operator $M_{\rm Ubrk}$. The nth-order U-spin-breaking corrections for the four decay modes satisfy

$$\begin{aligned}&\mathcal{A}^n_{\rm Ubrk}(\Xi_c^0\to\Xi^-\pi^+) = \frac{(-1)^{|\Delta U^n|}\mathcal{A}_{\rm Ubrk}(\Xi_c^0\to\Sigma^-K^+)}{\tan^2\theta_C},\\ &\mathcal{A}^n_{\rm Ubrk}(\Xi_c^0\to\Sigma^-\pi^+) = (-1)^{|\Delta U^n|}\mathcal{A}^n_{\rm Ubrk}(\Xi_c^0\to\Xi^-K^+),\end{aligned}\quad (10)$$

where $\Delta U^n = U_{\mathcal{B}}+U_M-U_{\mathcal{B}_b}-U_{\mathcal{H}_{\rm eff}}+\sum_1^n(\pm U_{\rm Ubrk})$. According to the angular momentum coupling rules, ΔU^n is odd for odd n and even for even n. Consequently, the U-spin amplitudes for the $\Xi_c^0\to\Xi^-\pi^+$, $\Xi_c^0\to\Sigma^-K^+$, $\Xi_c^0\to\Xi^-K^+$, and $\Xi_c^0\to\Sigma^-\pi^+$ modes at arbitrary order are

$$\begin{aligned}&\mathcal{A}(\Xi_c^0\to\Xi^-\pi^+) = -\cos^2\theta_C\mathcal{A}_1(1-\epsilon_1+a_1\epsilon_1^2-a_1'\epsilon_1^3+...),\\ &\mathcal{A}(\Xi_c^0\to\Sigma^-K^+) = -\sin^2\theta_C\mathcal{A}_1(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...),\\ &\mathcal{A}(\Xi_c^0\to\Xi^-K^+) = \cos\theta_C\sin\theta_C\mathcal{A}_1(1-\epsilon_2+a_2\epsilon_2^2-a_2'\epsilon_2^3+...),\\ &\mathcal{A}(\Xi_c^0\to\Sigma^-\pi^+) = \cos\theta_C\sin\theta_C\mathcal{A}_1(1+\epsilon_2+a_2\epsilon_2^2+a_2'\epsilon_2^3+...).\end{aligned}\quad (11)$$

The nonperturbative coefficients $a_{1,2}$, $a'_{1,2}$,..., can be determined from nonperturbative QCD calculations or extracted from experimental data.

The decay width Γ is

$$\Gamma = \frac{p_c}{8\pi}\frac{(m_{\mathcal{B}_c}+m_{\mathcal{B}})^2-m_M^2}{m_{\mathcal{B}_c}^2}\left(|S|^2+|P|^2\right),\quad (12)$$

where S and P are the S-wave and P-wave amplitudes, respectively, and p_c is the center-of-momentum (CM) momentum in the rest frame of the initial baryon. The U-spin amplitudes given in Eq. (11) apply to all partial-wave amplitudes. From Eq. (11), we derive a rate sum rule that is protected against first-order U-spin breaking:

$$\begin{aligned}&|\mathcal{A}(\Xi_c^0\to\Xi^-\pi^+)|^2+\frac{|\mathcal{A}(\Xi_c^0\to\Sigma^-K^+)|^2}{\tan^4\theta_C}\\ &=\frac{|\mathcal{A}(\Xi_c^0\to\Xi^-K^+)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_c^0\to\Sigma^-\pi^+)|^2}{\tan^2\theta_C}+O(\epsilon^2).\end{aligned}\quad (13)$$

The squared modulus of the amplitude, $|\mathcal{A}(\mathcal{B}_c\to\mathcal{B}M)|^2$, is calculated from the S-wave and P-wave amplitudes as $|S(\mathcal{B}_c\to\mathcal{B}M)|^2+|P(\mathcal{B}_c\to\mathcal{B}M)|^2$. According to Eq. (12), $|\mathcal{A}(\mathcal{B}_c\to\mathcal{B}M)|^2$ can be extracted from the branching fraction of the $\mathcal{B}_c\to\mathcal{B}M$ decay mode.

$$|\mathcal{A}(\mathcal{B}_c\to\mathcal{B}M)|^2 = \frac{8\pi}{p_c\tau_{\mathcal{B}_c}}\frac{m_{\mathcal{B}_c}^2\mathcal{B}r(\mathcal{B}_c\to\mathcal{B}M)}{(m_{\mathcal{B}_c}+m_{\mathcal{B}})^2-m_M^2}.\quad (14)$$

The left- and right-hand sides of Eq. (13) can also be replaced by the product of the moduli of two decay amplitudes.

$$\frac{|\mathcal{A}(\Xi_c^0\to\Xi^-\pi^+)||\mathcal{A}(\Xi_c^0\to\Sigma^-K^+)|}{\tan^2\theta_C}\quad (15)$$

and

$$\frac{|\mathcal{A}(\Xi_c^0\to\Xi^-K^+)||\mathcal{A}(\Xi_c^0\to\Sigma^-\pi^+)|}{\tan^2\theta_C}\quad (16)$$

respectively. The amplitude modulus $|\mathcal{A}(\mathcal{B}_c\to\mathcal{B}M)|$ is obtained from the S-wave and P-wave amplitudes as $\sqrt{|S(\mathcal{B}_c\to\mathcal{B}M)|^2+|P(\mathcal{B}_c\to\mathcal{B}M)|^2}$.

The Lee-Yang parameters α, β, and γ are defined as

$$\alpha = \frac{2\mathcal{R}e(S^*P)}{|S|^2+|P|^2},\qquad \beta = \frac{2\mathcal{I}m(S^*P)}{|S|^2+|P|^2},\qquad \gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}.\quad (17)$$

The amplitude decompositions given in Eq. (11) apply to both the S- and P-wave amplitudes. The decay parameter α for the $\Xi_c^0\to\Xi^-\pi^+$ decay is given by

$$\alpha(\Xi_c^0\to\Xi^-\pi^+) = \frac{2\mathcal{R}e(S^*P)}{|S|^2+|P|^2} = \frac{2\mathcal{R}e[(\mathcal{A}^S_{1/2}(1-\epsilon_1^S...))^*\times(\mathcal{A}^P_{1/2}(1-\epsilon_1^P...))]}{|\mathcal{A}^S_{1/2}(1-\epsilon_1^S...)|^2+|\mathcal{A}^P_{1/2}(1-\epsilon_1^P...)|^2}$$

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$$\simeq \frac{1}{|\mathcal{A}^S_{1/2}|^2+|\mathcal{A}^P_{1/2}|^2}\times\left[2\mathcal{R}e(\mathcal{A}^{S*}_{1/2}\mathcal{A}^P_{1/2})-2\mathcal{R}e\left((\mathcal{A}^S_{1/2}\epsilon^S_1)^*\mathcal{A}^P_{1/2}+\mathcal{A}^{S*}_{1/2}\mathcal{A}^P_{1/2}\epsilon^P_1\right)+2|\mathcal{A}^S_{1/2}|^2\mathcal{R}e(\epsilon^S_1)+2|\mathcal{A}^P_{1/2}|^2\mathcal{R}e(\epsilon^P_1)\right]+O(\epsilon_1^2). \quad (18)$$

The corresponding decay parameter for $\Xi_c^0\to\Sigma^-K^+$ is

$$\begin{aligned}\alpha(\Xi_c^0\to\Sigma^-K^+)\simeq&\frac{1}{|\mathcal{A}^S_{1/2}|^2+|\mathcal{A}^P_{1/2}|^2}\times\left[2\mathcal{R}e(\mathcal{A}^{S*}_{1/2}\mathcal{A}^P_{1/2})+2\mathcal{R}e((\mathcal{A}^S_{1/2}\epsilon^S_1)^*\mathcal{A}^P_{1/2}+\mathcal{A}^{S*}_{1/2}\mathcal{A}^P_{1/2}\epsilon^P_1)\right.\\&\left.-2|\mathcal{A}^S_{1/2}|^2\mathcal{R}e(\epsilon^S_1)-2|\mathcal{A}^P_{1/2}|^2\mathcal{R}e(\epsilon^P_1)\right]+O(\epsilon_1^2).\end{aligned} \quad (19)$$

By summing $\alpha(\Xi_c^0\to\Xi^-\pi^+)$ and $\alpha(\Xi_c^0\to\Sigma^-K^+)$, we obtain

$$\alpha(\Xi_c^0\to\Xi^-\pi^+)+\alpha(\Xi_c^0\to\Sigma^-K^+)\simeq\frac{4\mathcal{R}e(\mathcal{A}^{S*}_{1/2}\mathcal{A}^P_{1/2})}{|\mathcal{A}^S_{1/2}|^2+|\mathcal{A}^P_{1/2}|^2}+O(\epsilon_1^2). \quad (20)$$

Similarly, we have

$$\alpha(\Xi_c^0\to\Xi^-K^+)+\alpha(\Xi_c^0\to\Sigma^-\pi^+)\simeq\frac{4\mathcal{R}e(\mathcal{A}^{S*}_{1/2}\mathcal{A}^P_{1/2})}{|\mathcal{A}^S_{1/2}|^2+|\mathcal{A}^P_{1/2}|^2}+O(\epsilon_2^2). \quad (21)$$

We therefore derive the following sum rule for the decay parameters, which is protected against first-order U-spin-breaking corrections:

$$\begin{aligned}&\alpha(\Xi_c^0\to\Xi^-\pi^+)+\alpha(\Xi_c^0\to\Sigma^-K^+)\\&=\alpha(\Xi_c^0\to\Xi^-K^+)+\alpha(\Xi_c^0\to\Sigma^-\pi^+)+O(\epsilon^2),\end{aligned} \quad (22)$$

Similarly, the following U-spin relations can be derived for the decay parameters β and γ:

$$\begin{aligned}&\beta(\Xi_c^0\to\Xi^-\pi^+)+\beta(\Xi_c^0\to\Sigma^-K^+)\\&=\beta(\Xi_c^0\to\Xi^-K^+)+\beta(\Xi_c^0\to\Sigma^-\pi^+)+O(\epsilon^2),\end{aligned} \quad (23)$$

$$\begin{aligned}&\gamma(\Xi_c^0\to\Xi^-\pi^+)+\gamma(\Xi_c^0\to\Sigma^-K^+)\\&=\gamma(\Xi_c^0\to\Xi^-K^+)+\gamma(\Xi_c^0\to\Sigma^-\pi^+)+O(\epsilon^2).\end{aligned} \quad (24)$$

Owing to the limited experimental data currently available, the complex U-spin breaking parameters ϵ_1^S, ϵ_1^P, ϵ_2^S, and ϵ_2^P cannot yet be determined. Their combination can be estimated approximately from the ratio of the branching fractions $\mathcal{B}r(\Xi_c^0\to\Xi^-K^+)$ and $\mathcal{B}r(\Xi_c^0\to\Xi^-\pi^+)$:

$$\begin{aligned}&\frac{|S|^2\left(\mathcal{R}e(\epsilon^S_1)-\mathcal{R}e(\epsilon^S_2)\right)+|P|^2\left(\mathcal{R}e(\epsilon^P_1)-\mathcal{R}e(\epsilon^P_2)\right)}{|S|^2+|P|^2}\\&\simeq\left[\frac{|\mathcal{A}(\Xi_c^0\to\Xi^-K^+)|^2}{\tan^2\theta_C\,|\mathcal{A}(\Xi_c^0\to\Xi^-\pi^+)|^2}-1\right]/2+O(\epsilon^2)\\&=\left[\frac{\mathcal{B}r(\Xi_c^0\to\Xi^-K^+)}{\tan^2\theta_C\,\mathcal{B}r(\Xi_c^0\to\Xi^-\pi^+)}\frac{\kappa(\Xi_c^0\to\Xi^-\pi^+)}{\kappa(\Xi_c^0\to\Xi^-K^+)}-1\right]/2+O(\epsilon^2),\end{aligned} \quad (25)$$

where $\kappa=p_c[(m_{\mathcal{B}_c}+m_{\mathcal{B}})^2-m_M^2]$. Using the ratio $\mathcal{B}r(\Xi_c^0\to\Xi^-K^+)/\mathcal{B}r(\Xi_c^0\to\Xi^-\pi^+)=0.0275\pm0.0057$ reported by the Belle Collaboration [55], the parameter combination is estimated to be

$$\frac{|S|^2\left(\mathcal{R}e(\epsilon^S_1)-\mathcal{R}e(\epsilon^S_2)\right)+|P|^2\left(\mathcal{R}e(\epsilon^P_1)-\mathcal{R}e(\epsilon^P_2)\right)}{|S|^2+|P|^2}\approx-0.21\pm0.06 \quad (26)$$

which is consistent with the naïve prediction.

III. OTHER DECAY MODES

The U-spin rate sum rules beyond the U-spin symmetry limit can be obtained systematically through a general procedure. If a U-spin system receives contributions from only one U-spin amplitude in the U-spin limit and contains two U-spin conjugate pairs, then a rate U-spin sum rule protected against first-order U-spin breaking must exist in this system. For example, the U-spin system (3) is a ①→②+② system. The number of independent U-spin amplitudes is one. The rate U-spin sum rule (13) is derived by summing the squared moduli of the amplitudes of the two U-spin conjugate pairs and equating the two sums. The number of independent U-spin amplitudes for various U-spin systems is summarized in Table 1. Table 1 shows that such a rate sum rule exists in most U-spin systems of charmed baryon decays, except

Table 1. The number of independent U-spin amplitudes is given for different systems, where ①, ②, ③, and ④ denote the U-spin singlet, doublet, triplet, and quartet, respectively.

System	①→②+②	①→③+③	①→①+③	①→②+④	②→②+③	②→①+②	②→①+④
N	1	1	1	1	2	1	1

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for the ②→ ②+③ system.

Following the above procedure, we find rate sum rules for singly and doubly charmed baryon decays that hold up to first-order *U*-spin breaking as follows:

$$|\mathcal{A}(\Xi_c^0\to\Sigma^+K^-)|^2+\frac{|\mathcal{A}(\Xi_c^0\to p\pi^-)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_c^0\to pK^-)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_c^0\to\Sigma^+\pi^-)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{27}$$

$$\begin{aligned}&3|\mathcal{A}(\Xi_c^0\to\Xi^{*-}\pi^+)|^2+\frac{3|\mathcal{A}(\Xi_c^0\to\Sigma^{*-}K^+)|^2}{\tan^4\theta_C}=|\mathcal{A}(\Xi_c^0\to\Omega^-K^+)|^2+\frac{|\mathcal{A}(\Xi_c^0\to\Delta^-\pi^+)|^2}{\tan^4\theta_C}\\&=\frac{3}{4}\frac{|\mathcal{A}(\Xi_c^0\to\Sigma^{*-}\pi^+)|^2}{\tan^2\theta_C}+\frac{3}{4}\frac{|\mathcal{A}(\Xi_c^0\to\Xi^{*-}K^+)|^2}{\tan^2\theta_C}+O(\epsilon^2),\end{aligned} \tag{28}$$

$$|\mathcal{A}(\Xi_c^0\to\Sigma^{*+}K^-)|^2+\frac{|\mathcal{A}(\Xi_c^0\to\Delta^+\pi^-)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_c^0\to\Delta^+K^-)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_c^0\to\Sigma^{*+}\pi^-)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{29}$$

$$|\mathcal{A}(\Lambda_c^+\to\Delta^{++}K^-)|^2+\frac{|\mathcal{A}(\Xi_c^+\to\Delta^{++}\pi^-)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Lambda_c^+\to\Delta^{++}\pi^-)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_c^+\to\Delta^{++}K^-)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{30}$$

$$|\mathcal{A}(\Xi_{cc}^{++}\to\Xi_c^+\pi^+)|^2+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Lambda_c^+K^+)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Xi_c^+K^+)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Lambda_c^+\pi^+)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{31}$$

$$|\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^0\pi^+)|^2+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^0K^+)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{+}\to\Xi_c^0K^+)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to\Xi_c^0\pi^+)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{32}$$

$$|\mathcal{A}(\Xi_{cc}^{++}\to\Xi_c^{*+}\pi^+)|^2+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^+K^+)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Xi_c^{*+}K^+)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Sigma_c^+\pi^+)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{33}$$

$$|\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{++}K^-)|^2+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{++}\pi^-)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{+}\to\Sigma_c^{++}\pi^-)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to\Sigma_c^{++}K^-)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{34}$$

$$|\mathcal{A}(\Xi_{cc}^{++}\to\Sigma^+D^+)|^2+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to pD_s^+)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{++}\to pD^+)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Sigma^+D_s^+)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{35}$$

$$|\mathcal{A}(\Xi_{cc}^{+}\to\Sigma^+D^0)|^2+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to pD^0)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{+}\to pD^0)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to\Sigma^+D^0)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{36}$$

$$|\mathcal{A}(\Xi_{cc}^{++}\to\Sigma^{*+}D^+)|^2+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Delta^+D_s^+)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Delta^+D^+)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Xi_{cc}^{++}\to\Sigma^{*+}D_s^+)|^2}{\tan^2\theta_C}+O(\epsilon^2), \tag{37}$$

$$|\mathcal{A}(\Xi_{cc}^{+}\to\Sigma^{*+}D^0)|^2+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to\Delta^+D^0)|^2}{\tan^4\theta_C}=\frac{|\mathcal{A}(\Xi_{cc}^{+}\to\Delta^+D^0)|^2}{\tan^2\theta_C}+\frac{|\mathcal{A}(\Omega_{cc}^{+}\to\Sigma^{*+}D^0)|^2}{\tan^2\theta_C}+O(\epsilon^2). \tag{38}$$

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The U-spin amplitude decompositions for the decay channels discussed above are presented in Appendix B. Note that the nonperturbative parameters ϵ_1, ϵ_2,..., and $a_{1,2}$, $a'_{1,2}$,... are distinct for different U-spin systems. The left- and right-hand sides of Eqs. (27) ~ (38) can be replaced by products of the moduli of the corresponding decay amplitudes, as in Eqs. (15) and (16). Similarly, the decay parameters α, β, and γ of these modes satisfy sum rules analogous to those in Eqs. (22)~(24). The derivation is similar to that presented in the previous section. Note that the Lee-Yang parameters α, β, and γ for the decuplet-baryon final states are defined as

$$\alpha = \frac{2\mathcal{R}e(PD^*)}{|P|^2+|D|^2}, \qquad \beta = \frac{2\mathcal{I}m(PD^*)}{|P|^2+|D|^2}, \qquad \gamma = \frac{-|P|^2+|D|^2}{|P|^2+|D|^2}. \tag{39}$$

Due to limited experimental data, we can neither test the validity of the U-spin relations derived in this work nor use them to predict unmeasured decay channels. These sum rules are protected against first-order U-spin-breaking corrections and therefore provide relatively precise tests of U-spin symmetry in the charmed baryon sector.

Compared with bottom-baryon decays, charmed baryon decays exhibit more relations that are protected against first-order U-spin-breaking corrections. In bottom-baryon decays, U-spin relations protected against first-order U-spin breaking exist only in the $b \to c\bar{u}d/s$ and $b \to u\bar{c}d/s$ decays. Decay modes dominated by the $b \to u\bar{u}d/s$ ($b \to c\bar{c}d/s$) transitions also receive contributions from the $b \to u\bar{u}d/s$ ($b \to c\bar{c}d/s$) transitions through c- (u-) and t-quark loop diagrams. The products of CKM matrix elements associated with the c- (u-) and t-quark loop diagrams differ from those associated with the tree diagrams in $b \to u\bar{u}d/s$ ($b \to c\bar{c}d/s$) decays and cannot be eliminated through flavor-symmetry cancellations. Interference among contributions with different CKM structures prevents one from factoring out a common product of CKM matrix elements. In singly Cabibbo-suppressed charmed baryon decays, the products of CKM matrix elements for the $c \to u\bar{d}d$ and $c \to u\bar{s}s$ transitions are $V^*_{cd}V_{ud}$ and $V^*_{cs}V_{us}$, respectively. $V^*_{cd}V_{ud}$ and $V^*_{cs}V_{us}$ can be approximated as $V^*_{cd}V_{ud} \simeq -\cos\theta_C \sin\theta_C$ and $V^*_{cs}V_{us} \simeq \cos\theta_C \sin\theta_C$. Contributions from b-quark loop diagrams are negligible because the CKM factor $V^*_{cb}V_{ub}$ is much smaller than $\cos\theta_C \sin\theta_C$. Consequently, common CKM factors can be factored out in the Cabibbo-favored, singly Cabibbo-suppressed, and doubly Cabibbo-suppressed decays, allowing all these weak transitions to be used in constructing U-spin relations beyond the U-spin limit. Higher-order U-spin relations in bottom-baryon decays connect only two distinct weak transitions, either $b \to c\bar{u}d$ and $b \to c\bar{u}s$ or $b \to u\bar{c}d$ and $b \to u\bar{c}s$. Thus, only a limited number of U-spin relations can be found [54]. In contrast, higher-order U-spin relations in charmed baryon decays connect four distinct weak transitions: $c \to u\bar{d}s$, $c \to u\bar{d}d$, $c \to u\bar{s}s$, and $c \to u\bar{s}d$. Therefore, more higher-order U-spin relations can be found in charmed baryon decays.

At present, experimental measurements of the branching fractions of doubly charmed baryon decays remain highly challenging. The U-spin relations presented in this work are derived within the narrow-width approximation. For decay modes involving decuplet baryons in the final state, the final-state baryons generally have relatively large decay widths, especially the δ baryons. Finite-width effects, resonance line shapes, and interference effects in the corresponding multi-body final states may introduce additional uncertainties into these relations. Compared with decays into decuplet baryons, charmed baryon decays into octet baryons provide cleaner tests of U-spin symmetry. The main difficulty in testing the U-spin sum rules proposed in this work lies in measuring the branching fractions of the doubly Cabibbo-suppressed modes. The doubly Cabibbo-suppressed mode in Eq. (13), $\Xi_c^0 \to \Sigma^- K^+$, is difficult to observe experimentally because the Σ^- must be reconstructed through $\Sigma^- \to n\pi^-$. As a result, Eq. (27) provides one of the most promising relations for testing U-spin symmetry, since both final-state particles of the doubly Cabibbo-suppressed decay mode $\Xi_c^0 \to p\pi^-$ are experimentally accessible. The branching fraction of $\Xi_c^0 \to p\pi^-$ is roughly estimated as $\mathcal{B}r(\Xi_c^0 \to p\pi^-) \simeq \tan^4\theta_C \mathcal{B}r(\Xi_c^0 \to \Sigma^+ K^-) \approx 4.5\times10^{-6}$ [56]. The prospect of measuring $\mathcal{B}r(\Xi_c^0 \to p\pi^-)$ at LHCb can be assessed using the reference channel $\Xi_c^0 \to pK^-K^-\pi^+$. Using the Run 1 data set corresponding to an integrated luminosity of $3\,\text{fb}^{-1}$, LHCb reconstructed $N(\Xi_c^0 \to pK^-K^-\pi^+) = (21.6\pm0.2)\times10^3$ signal candidates [57]. The absolute branching fraction of $\Xi_c^0 \to pK^-K^-\pi^+$ measured by Belle is $\mathcal{B}(\Xi_c^0 \to pK^-K^-\pi^+) = (5.8\pm2.3\pm0.5)\times10^{-3}$ [58]. This gives $\mathcal{B}(\Xi_c^0 \to p\pi^-)/\mathcal{B}(\Xi_c^0 \to pK^-K^-\pi^+) \simeq 7.8\times10^{-4}$. If one assumes identical reconstruction efficiencies for the two decay modes, the expected signal yield in the Run 1 sample would be $N(p\pi^-) \simeq 2.16\times10^4\times7.8\times10^{-4} \approx 17$. Assuming that the signal yield scales linearly with integrated luminosity and neglecting changes in detector performance, trigger efficiencies, and running conditions, Upgrade II of LHCb is expected to collect a data set of $300\,\text{fb}^{-1}$, which could yield around 1700 signal candidates for the $\Xi_c^0 \to p\pi^-$ decay.

IV. SUMMARY

In this work, we studied U-spin sum rules for two-body nonleptonic decays of charmed baryons, including first-order U-spin breaking. Several sum rules for decay rates and decay parameters that remain valid beyond exact U-spin symmetry have been proposed. These sum rules are protected against first-order U-spin breaking. Their uncertainties are therefore expected to be smaller

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than the typical size of *U*-spin breaking. Thus, these *U*-spin sum rules could provide relatively precise tests of *U*-spin symmetry in charmed baryon decays.

APPENDIX A: ANGULAR MOMENTUM COUPLING RULE

In this appendix, we derive the angular-momentum coupling relation in Eq. (7). The matrix element in Eq. (7) is assumed to represent a rotationally invariant coupling between two-particle angular-momentum states. The initial- and final-state angular momenta can be coupled separately to the same intermediate total angular momentum J and its third component M. Therefore, the coupling coefficient can be expanded as

$$\begin{aligned} &C_{m_1m_2;m_3m_4} \equiv \langle j_1,m_1;j_2,m_2|j_3,m_3;j_4,m_4\rangle \\ &= \sum_{J,M} \mathcal{A}_J\, C^{JM}_{j_1m_1,j_2m_2} \left(C^{JM}_{j_3m_3,j_4m_4}\right)^*, \end{aligned} \tag{A1}$$

where

$$C^{JM}_{j_1m_1,j_2m_2} \equiv \langle j_1,m_1;j_2,m_2|J,M\rangle \tag{A2}$$

denotes a Clebsch–Gordan coefficient. The reduced coupling coefficient $\mathcal{A}_J$ is independent of the magnetic quantum numbers. In the Condon–Shortley phase convention, the Clebsch–Gordan coefficients satisfy

$$C^{J,-M}_{j_1,-m_1;j_2,-m_2} = (-1)^{j_1+j_2-J} C^{JM}_{j_1m_1,j_2m_2}. \tag{A3}$$

Similarly, for the final-state angular momenta,

$$C^{J,-M}_{j_3,-m_3;j_4,-m_4} = (-1)^{j_3+j_4-J} C^{JM}_{j_3m_3,j_4m_4}. \tag{A4}$$

When all magnetic quantum numbers are reversed, the coupling coefficient becomes

$$C_{-m_1,-m_2;-m_3,-m_4} = \sum_{J,M} \mathcal{A}_J\, C^{J,-M}_{j_1,-m_1;j_2,-m_2} \left(C^{J,-M}_{j_3,-m_3;j_4,-m_4}\right)^*. \tag{A5}$$

From Eqs. (A3) and (A4), one obtains

$$\begin{aligned} &C_{-m_1,-m_2;-m_3,-m_4} \\ &= \sum_{J,M} \mathcal{A}_J\, (-1)^{j_1+j_2-J}(-1)^{j_3+j_4-J} C^{JM}_{j_1m_1,j_2m_2} \left(C^{JM}_{j_3m_3,j_4m_4}\right)^* \\ &= \sum_{J,M} \mathcal{A}_J\, (-1)^{j_1+j_2+j_3+j_4-2J} C^{JM}_{j_1m_1,j_2m_2} \left(C^{JM}_{j_3m_3,j_4m_4}\right)^*. \end{aligned} \tag{A6}$$

For an allowed coupling of the final-state angular momenta, $j_3+j_4-J \in \mathbb{Z}$. It follows that $2(j_3+j_4-J)$ is an even integer. The exponent in Eq. (A6) can therefore be rearranged as

$$j_1+j_2+j_3+j_4-2J = j_1+j_2-j_3-j_4+2(j_3+j_4-J). \tag{A7}$$

Because $2(j_3+j_4-J)$ is an even integer, it follows that

$$(-1)^{j_1+j_2+j_3+j_4-2J} = (-1)^{j_1+j_2-j_3-j_4}. \tag{A8}$$

Substituting Eq. (A8) into Eq. (A6), we obtain

$$\begin{aligned} &\langle j_1,-m_1;j_2,-m_2|j_3,-m_3;j_4,-m_4\rangle \\ &= (-1)^{j_1+j_2-j_3-j_4} \langle j_1,m_1;j_2,m_2|j_3,m_3;j_4,m_4\rangle. \end{aligned} \tag{A9}$$

APPENDIX B: *U*-SPIN AMPLITUDE DECOMPOSITIONS

In this appendix, we present the *U*-spin amplitudes for the *U*-spin systems analyzed in Eqs. (27) ~ (38).

1. The amplitudes for the *U*-spin system

$$\Xi_c^0 \to \begin{pmatrix} \pi^- \\ K^- \end{pmatrix} \begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} \tag{B1}$$

are expressed as

$$\begin{aligned} &\mathcal{A}(\Xi_c^0 \to \Sigma^+K^-) = -\cos^2\theta_C \mathcal{A}_1(1-\epsilon_1+a_1\epsilon_1^2-a_1'\epsilon_1^3+...), \\ &\mathcal{A}(\Xi_c^0 \to p\pi^-) = -\sin^2\theta_C \mathcal{A}_1(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...), \\ &\mathcal{A}(\Xi_c^0 \to pK^-) = \cos\theta_C\sin\theta_C \mathcal{A}_1(1-\epsilon_2+a_2\epsilon_2^2-a_2'\epsilon_2^3+...), \\ &\mathcal{A}(\Xi_c^0 \to \Sigma^+\pi^-) = \cos\theta_C\sin\theta_C \mathcal{A}_1(1+\epsilon_2+a_2\epsilon_2^2+a_2'\epsilon_2^3+...). \end{aligned} \tag{B2}$$

2. The amplitudes for the *U*-spin system

$$\Xi_c^0 \to \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \begin{pmatrix} \Delta^- \\ \Sigma^{*-} \\ \Xi^{*-} \\ \Omega^- \end{pmatrix} \tag{B3}$$

are expressed as

$$\begin{aligned} &\mathcal{A}(\Xi_c^0 \to \Xi^{*-}\pi^+) = -\frac{1}{2}\cos^2\theta_C \mathcal{A}_1(1-\epsilon_1+a_1\epsilon_1^2-a_1'\epsilon_1^3+...), \\ &\mathcal{A}(\Xi_c^0 \to \Sigma^{*-}K^+) = \frac{1}{2}\sin^2\theta_C \mathcal{A}_1(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...), \end{aligned}$$

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$$\mathcal{A}(\Xi_c^0 \to \Omega^- K^+) = \frac{\sqrt{3}}{2}\cos^2\theta_C \mathcal{A}_1(1-\epsilon_2+a_2\epsilon_2^2-a_2'\epsilon_2^3+...),$$
$$\mathcal{A}(\Xi_c^0 \to \Delta^- \pi^+) = -\frac{\sqrt{3}}{2}\sin^2\theta_C \mathcal{A}_1(1+\epsilon_2+a_2\epsilon_2^2+a_2'\epsilon_2^3+...),$$
$$\mathcal{A}(\Xi_c^0 \to \Sigma^{*-} \pi^+) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1-\epsilon_3+a_3\epsilon_3^2-a_3'\epsilon_3^3+...),$$
$$\mathcal{A}(\Xi_c^0 \to \Xi^{*-} K^+) = -\cos\theta_C \sin\theta_C \mathcal{A}_1(1+\epsilon_3+a_3\epsilon_3^2+a_3'\epsilon_3^3+...). \quad \text{(B4)}$$

3. The amplitudes for the U-spin system

$$\Xi_c^0 \to \begin{pmatrix} \pi^- \\ K^- \end{pmatrix} \begin{pmatrix} \Delta^+ \\ \Sigma^{*+} \end{pmatrix} \quad \text{(B5)}$$

are expressed as

$$\mathcal{A}(\Xi_c^0 \to \Sigma^{*+} K^-) = -\cos^2\theta_C \mathcal{A}_1(1-\epsilon_1+a_1\epsilon_1^2-a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_c^0 \to \Delta^+ \pi^-) = -\sin^2\theta_C \mathcal{A}_1(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_c^0 \to \Delta^+ K^-) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1-\epsilon_2+a_2\epsilon_2^2-a_2'\epsilon_2^3+...),$$
$$\mathcal{A}(\Xi_c^0 \to \Sigma^{*+} \pi^-) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1+\epsilon_2+a_2\epsilon_2^2+a_2'\epsilon_2^3+...). \quad \text{(B6)}$$

4. The amplitudes for the U-spin system

$$\begin{pmatrix} \Lambda_c^+ \\ \Xi_c^+ \end{pmatrix} \to \begin{pmatrix} \pi^- \\ K^- \end{pmatrix} \Delta^{++} \quad \text{(B7)}$$

are expressed as

$$\mathcal{A}(\Lambda_c^+ \to \Delta^{++} K^-) = -\sqrt{\frac{2}{3}}\cos^2\theta_C \mathcal{A}_{1/2}(1-\epsilon_1+a_1\epsilon_1^2 - a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_c^+ \to \Delta^{++} \pi^-) = \sqrt{\frac{2}{3}}\sin^2\theta_C \mathcal{A}_{1/2}(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Lambda_c^+ \to \Delta^{++} \pi^-) = \sqrt{\frac{2}{3}}\cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1-\epsilon_2+a_2\epsilon_2^2 - a_2'\epsilon_2^3+...),$$
$$\mathcal{A}(\Xi_c^+ \to \Delta^{++} K^-) = -\sqrt{\frac{2}{3}}\cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1+\epsilon_2+a_2\epsilon_2^2 + a_2'\epsilon_2^3+...). \quad \text{(B8)}$$

5. The amplitudes for the U-spin system

$$\Xi_{cc}^{++} \to \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \begin{pmatrix} \Lambda_c^+ \\ \Xi_c^+ \end{pmatrix} \quad \text{(B9)}$$

are expressed as

$$\mathcal{A}(\Xi_{cc}^{++} \to \Xi_c^+ \pi^+) = -\cos^2\theta_C \mathcal{A}_1(1-\epsilon_1+a_1\epsilon_1^2-a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_{cc}^{++} \to \Lambda_c^+ K^+) = -\sin^2\theta_C \mathcal{A}_1(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_{cc}^{++} \to \Xi_c^+ K^+) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1-\epsilon_2+a_2\epsilon_2^2-a_2'\epsilon_2^3+...),$$
$$\mathcal{A}(\Xi_{cc}^{++} \to \Lambda_c^+ \pi^+) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1+\epsilon_2+a_2\epsilon_2^2+a_2'\epsilon_2^3+...). \quad \text{(B10)}$$

6. The amplitudes for the U-spin system

$$\begin{pmatrix} \Xi_{cc}^+ \\ \Omega_{cc}^+ \end{pmatrix} \to \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \Xi_c^0 \quad \text{(B11)}$$

are expressed as

$$\mathcal{A}(\Xi_{cc}^+ \to \Xi_c^0 \pi^+) = -\sqrt{\frac{2}{3}}\cos^2\theta_C \mathcal{A}_{1/2}(1-\epsilon_1+a_1\epsilon_1^2-a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Omega_{cc}^+ \to \Xi_c^0 K^+) = \sqrt{\frac{2}{3}}\sin^2\theta_C \mathcal{A}_{1/2}(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_{cc}^+ \to \Xi_c^0 K^+) = \sqrt{\frac{2}{3}}\cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1-\epsilon_2+a_2\epsilon_2^2-a_2'\epsilon_2^3+...),$$
$$\mathcal{A}(\Omega_{cc}^+ \to \Xi_c^0 \pi^+) = -\sqrt{\frac{2}{3}}\cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1+\epsilon_2+a_2\epsilon_2^2+a_2'\epsilon_2^3+...). \quad \text{(B12)}$$

7. The amplitudes for the U-spin system

$$\Xi_{cc}^{++} \to \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \begin{pmatrix} \Sigma_c^+ \\ \Xi_c^{*+} \end{pmatrix} \quad \text{(B13)}$$

are expressed as

$$\mathcal{A}(\Xi_{cc}^{++} \to \Xi_c^{*+} \pi^+) = -\cos^2\theta_C \mathcal{A}_1(1-\epsilon_1+a_1\epsilon_1^2-a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_{cc}^{++} \to \Sigma_c^+ K^+) = -\sin^2\theta_C \mathcal{A}_1(1+\epsilon_1+a_1\epsilon_1^2+a_1'\epsilon_1^3+...),$$
$$\mathcal{A}(\Xi_{cc}^{++} \to \Xi_c^{*+} K^+) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1-\epsilon_2+a_2\epsilon_2^2-a_2'\epsilon_2^3+...),$$
$$\mathcal{A}(\Xi_{cc}^{++} \to \Sigma_c^+ \pi^+) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1+\epsilon_2+a_2\epsilon_2^2+a_2'\epsilon_2^3+...). \quad \text{(B14)}$$

8. The amplitudes for the U-spin system

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$$\begin{pmatrix} \Xi_{cc}^{+} \\ \Omega_{cc}^{+} \end{pmatrix} \to \begin{pmatrix} \pi^{-} \\ K^{-} \end{pmatrix} \Sigma_{c}^{++} \tag{B15}$$

are expressed as

$$\begin{aligned}
\mathcal{A}(\Xi_{cc}^{+} \to \Sigma_{c}^{++} K^{-}) =& -\sqrt{\frac{2}{3}} \cos^2\theta_C \mathcal{A}_{1/2}(1-\epsilon_1 \\
&+ a_1\epsilon_1^2 - a_1'\epsilon_1^3 + ...), \\
\mathcal{A}(\Omega_{cc}^{+} \to \Sigma_{c}^{++} \pi^{-}) =& \sqrt{\frac{2}{3}} \sin^2\theta_C \mathcal{A}_{1/2}(1+\epsilon_1 \\
&+ a_1\epsilon_1^2 + a_1'\epsilon_1^3 + ...), \\
\mathcal{A}(\Xi_{cc}^{+} \to \Sigma_{c}^{++} \pi^{-}) =& \sqrt{\frac{2}{3}} \cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1-\epsilon_2 \\
&+ a_2\epsilon_2^2 - a_2'\epsilon_2^3 + ...), \\
\mathcal{A}(\Omega_{cc}^{+} \to \Sigma_{c}^{++} K^{-}) =& -\sqrt{\frac{2}{3}} \cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1+\epsilon_2 \\
&+ a_2\epsilon_2^2 + a_2'\epsilon_2^3 + ...).
\end{aligned} \tag{B16}$$

9. The amplitudes for the *U*-spin system

$$\Xi_{cc}^{++} \to \begin{pmatrix} D_s^{+} \\ D^{+} \end{pmatrix} \begin{pmatrix} p \\ \Sigma^{+} \end{pmatrix} \tag{B17}$$

are expressed as

$$\begin{aligned}
&\mathcal{A}(\Xi_{cc}^{++} \to \Sigma^{+} D^{+}) = -\cos^2\theta_C \mathcal{A}_1(1-\epsilon_1 + a_1\epsilon_1^2 - a_1'\epsilon_1^3 + ...), \\
&\mathcal{A}(\Xi_{cc}^{++} \to p D_s^{+}) = -\sin^2\theta_C \mathcal{A}_1(1+\epsilon_1 + a_1\epsilon_1^2 + a_1'\epsilon_1^3 + ...), \\
&\mathcal{A}(\Xi_{cc}^{++} \to p D^{+}) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1-\epsilon_2 + a_2\epsilon_2^2 - a_2'\epsilon_2^3 + ...), \\
&\mathcal{A}(\Xi_{cc}^{++} \to \Sigma^{+} D_s^{+}) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1+\epsilon_2 + a_2\epsilon_2^2 + a_2'\epsilon_2^3 + ...).
\end{aligned} \tag{B18}$$

10. The amplitudes for the *U*-spin system

$$\begin{pmatrix} \Xi_{cc}^{+} \\ \Omega_{cc}^{+} \end{pmatrix} \to D^{0} \begin{pmatrix} p \\ \Sigma^{+} \end{pmatrix} \tag{B19}$$

are expressed as

$$\begin{aligned}
\mathcal{A}(\Xi_{cc}^{+} \to \Sigma^{+} D^{0}) =& -\sqrt{\frac{2}{3}} \cos^2\theta_C \mathcal{A}_{1/2}(1-\epsilon_1 \\
&+ a_1\epsilon_1^2 - a_1'\epsilon_1^3 + ...), \\
\mathcal{A}(\Omega_{cc}^{+} \to p D^{0}) =& \sqrt{\frac{2}{3}} \sin^2\theta_C \mathcal{A}_{1/2}(1+\epsilon_1 \\
&+ a_1\epsilon_1^2 + a_1'\epsilon_1^3 + ...), \\
\mathcal{A}(\Xi_{cc}^{+} \to p D^{0}) =& \sqrt{\frac{2}{3}} \cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1-\epsilon_2 \\
&+ a_2\epsilon_2^2 - a_2'\epsilon_2^3 + ...), \\
\mathcal{A}(\Omega_{cc}^{+} \to \Sigma^{+} D^{0}) =& -\sqrt{\frac{2}{3}} \cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1+\epsilon_2 \\
&+ a_2\epsilon_2^2 + a_2'\epsilon_2^3 + ...).
\end{aligned} \tag{B20}$$

11. The amplitudes for the *U*-spin system

$$\Xi_{cc}^{++} \to \begin{pmatrix} D_s^{+} \\ D^{+} \end{pmatrix} \begin{pmatrix} \Delta^{+} \\ \Sigma^{*+} \end{pmatrix} \tag{B21}$$

are expressed as

$$\begin{aligned}
&\mathcal{A}(\Xi_{cc}^{++} \to \Sigma^{*+} D^{+}) = -\cos^2\theta_C \mathcal{A}_1(1-\epsilon_1 + a_1\epsilon_1^2 - a_1'\epsilon_1^3 + ...), \\
&\mathcal{A}(\Xi_{cc}^{++} \to \Delta^{+} D_s^{+}) = -\sin^2\theta_C \mathcal{A}_1(1+\epsilon_1 + a_1\epsilon_1^2 + a_1'\epsilon_1^3 + ...), \\
&\mathcal{A}(\Xi_{cc}^{++} \to \Delta^{+} D^{+}) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1-\epsilon_2 + a_2\epsilon_2^2 - a_2'\epsilon_2^3 + ...), \\
&\mathcal{A}(\Xi_{cc}^{++} \to \Sigma^{*+} D_s^{+}) = \cos\theta_C \sin\theta_C \mathcal{A}_1(1+\epsilon_2 + a_2\epsilon_2^2 + a_2'\epsilon_2^3 + ...).
\end{aligned} \tag{B22}$$

12. The amplitudes for the *U*-spin system

$$\begin{pmatrix} \Xi_{cc}^{+} \\ \Omega_{cc}^{+} \end{pmatrix} \to D^{0} \begin{pmatrix} \Delta^{+} \\ \Sigma^{*+} \end{pmatrix} \tag{B23}$$

are expressed as

$$\begin{aligned}
\mathcal{A}(\Xi_{cc}^{+} \to \Sigma^{*+} D^{0}) =& -\sqrt{\frac{2}{3}} \cos^2\theta_C \mathcal{A}_{1/2}(1-\epsilon_1 \\
&+ a_1\epsilon_1^2 - a_1'\epsilon_1^3 + ...), \\
\mathcal{A}(\Omega_{cc}^{+} \to \Delta^{+} D^{0}) =& \sqrt{\frac{2}{3}} \sin^2\theta_C \mathcal{A}_{1/2}(1+\epsilon_1 \\
&+ a_1\epsilon_1^2 + a_1'\epsilon_1^3 + ...), \\
\mathcal{A}(\Xi_{cc}^{+} \to \Delta^{+} D^{0}) =& \sqrt{\frac{2}{3}} \cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1-\epsilon_2 \\
&+ a_2\epsilon_2^2 - a_2'\epsilon_2^3 + ...), \\
\mathcal{A}(\Omega_{cc}^{+} \to \Sigma^{*+} D^{0}) =& -\sqrt{\frac{2}{3}} \cos\theta_C \sin\theta_C \mathcal{A}_{1/2}(1+\epsilon_2 \\
&+ a_2\epsilon_2^2 + a_2'\epsilon_2^3 + ...).
\end{aligned} \tag{B24}$$

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